AP® Calculus AB
2014 Scoring Guidelines

Question 1

Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30,$ the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

(a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30.$ Indicate units of measure.

(b) Find the value of $A'(15).$ Using correct units, interpret the meaning of the value in the context of the problem.

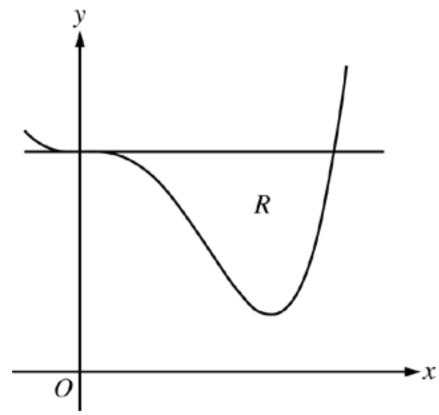
(c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30.$

(d) For $t > 30,$ $L(t),$ the linear approximation to A at $t = 30,$ is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

(a) $\dfrac{A(30) - A(0)}{30 - 0} = -0.197$ (or -0.196) lbs/day	1 : answer with units
(b) $A'(15) = -0.164$ (or -0.163) The amount of grass clippings in the bin is decreasing at a rate of 0.164 (or 0.163) lbs/day at time $t = 15$ days.	$2 : \begin{cases} 1 : A'(15) \\ 1 : \text{interpretation} \end{cases}$
(c) $A(t) = \dfrac{1}{30}\int_0^{30} A(t)\,dt \Rightarrow t = 12.415$ (or 12.414)	$2 : \begin{cases} 1 : \dfrac{1}{30}\int_0^{30} A(t)\,dt \\ 1 : \text{answer} \end{cases}$
(d) $L(t) = A(30) + A'(30)\cdot(t - 30)$ $A'(30) = -0.055976$ $A(30) = 0.782928$ $L(t) = 0.5 \Rightarrow t = 35.054$	$4 : \begin{cases} 2 : \text{expression for } L(t) \\ 1 : L(t) = 0.5 \\ 1 : \text{answer} \end{cases}$

Question 2

Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4,$ as shown in the figure above.



(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2.$

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in $R.$ Find the volume of the solid.

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value $k.$

(a) $f(x) = 4 \Rightarrow x = 0, 2.3$

$$\text{Volume} = \pi\int_0^{2.3}\left[(4+2)^2 - (f(x)+2)^2\right]dx$$
$$= 98.868 \text{ (or } 98.867)$$

$4 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(b) $$\text{Volume} = \int_0^{2.3}\frac{1}{2}(4 - f(x))^2\,dx$$
$$= 3.574 \text{ (or } 3.573)$$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

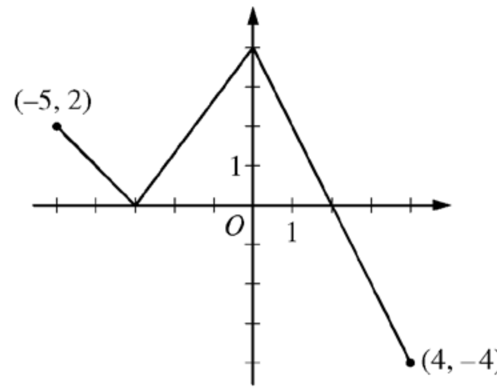
(c) $$\int_0^k (4 - f(x))\,dx = \int_k^{2.3}(4 - f(x))\,dx$$

$2 : \begin{cases} 1 : \text{area of one region} \\ 1 : \text{equation} \end{cases}$

Question 3

The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above.

Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.



Graph of f

(a) Find $g(3)$.

(b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

(c) The function h is defined by $h(x) = \dfrac{g(x)}{5x}$. Find $h'(3)$.

(d) The function p is defined by $p(x) = f\left(x^2 - x\right)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

(a) $g(3) = \int_{-3}^{3} f(t)\,dt = 6 + 4 - 1 = 9$

1 : answer

(b) $g'(x) = f(x)$

The graph of g is increasing and concave down on the intervals $-5 < x < -3$ and $0 < x < 2$ because $g' = f$ is positive and decreasing on these intervals.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

(c) $h'(x) = \dfrac{5xg'(x) - g(x)5}{(5x)^2} = \dfrac{5xg'(x) - 5g(x)}{25x^2}$

$$h'(3) = \frac{(5)(3)g'(3) - 5g(3)}{25 \cdot 3^2}$$

$$= \frac{15(-2) - 5(9)}{225} = \frac{-75}{225} = -\frac{1}{3}$$

$3 : \begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$

(d) $p'(x) = f'\left(x^2 - x\right)(2x - 1)$

$p'(-1) = f'(2)(-3) = (-2)(-3) = 6$

$3 : \begin{cases} 2 : p'(x) \\ 1 : \text{answer} \end{cases}$

Question 4

Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	–120	–150

(a) Find the average acceleration of train A over the interval $2 \le t \le 8$.

(b) Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

(c) At time $t = 2$, train A's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A, in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.

(d) A second train, train B, travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

(a) average accel $= \dfrac{v_A(8) - v_A(2)}{8 - 2} = \dfrac{-120 - 100}{6} = -\dfrac{110}{3}$ m/min^2

1 : average acceleration

(b) v_A is differentiable $\Rightarrow$ v_A is continuous

$v_A(8) = -120 < -100 < 40 = v_A(5)$

Therefore, by the Intermediate Value Theorem, there is a time t, $5 < t < 8$, such that $v_A(t) = -100$.

$2 : \begin{cases} 1 : v_A(8) < -100 < v_A(5) \\ 1 : \text{conclusion, using IVT} \end{cases}$

(c) $s_A(12) = s_A(2) + \int_2^{12} v_A(t)\,dt = 300 + \int_2^{12} v_A(t)\,dt$

$$\int_2^{12} v_A(t)\,dt \approx 3 \cdot \frac{100 + 40}{2} + 3 \cdot \frac{40 - 120}{2} + 4 \cdot \frac{-120 - 150}{2} = -450$$

$s_A(12) \approx 300 - 450 = -150$

The position of Train A at time $t = 12$ minutes is approximately 150 meters west of Origin Station.

$3 : \begin{cases} 1 : \text{position expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{position at time } t = 12 \end{cases}$

(d) Let x be train A's position, y train B's position, and z the distance between train A and train B.

$z^2 = x^2 + y^2 \Rightarrow 2z\dfrac{dz}{dt} = 2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt}$

$x = 300,\ y = 400 \Rightarrow z = 500$

$v_B(2) = -20 + 120 + 25 = 125$

$500\dfrac{dz}{dt} = (300)(100) + (400)(125)$

$\dfrac{dz}{dt} = \dfrac{80000}{500} = 160$ meters per minute

$3 : \begin{cases} 2 : \text{implicit differentiation of distance relationship} \\ 1 : \text{answer} \end{cases}$

Question 5

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

(a) Find the x-coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.

(b) Explain why there must be a value c, for $-1 < c < 1$, such that $f''(c) = 0$.

(c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

(d) Evaluate $\int_{-2}^{3} f'(g(x))g'(x)\,dx$.

(a) $x = 1$ is the only critical point at which f' changes sign from negative to positive. Therefore, f has a relative minimum at $x = 1$.

1 : answer with justification

(b) f' is differentiable $\Rightarrow$ f' is continuous on the interval $-1 \le x \le 1$

$$\frac{f'(1) - f'(-1)}{1 - (-1)} = \frac{0 - 0}{2} = 0$$

Therefore, by the Mean Value Theorem, there is at least one value c, $-1 < c < 1$, such that $f''(c) = 0$.

$2 : \begin{cases} 1 : f'(1) - f'(-1) = 0 \\ 1 : \text{explanation, using Mean Value Theorem} \end{cases}$

(c) $h'(x) = \frac{1}{f(x)} \cdot f'(x)$

$$h'(3) = \frac{1}{f(3)} \cdot f'(3) = \frac{1}{7} \cdot \frac{1}{2} = \frac{1}{14}$$

$3 : \begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$

(d)
$$\begin{aligned} \int_{-2}^{3} f'(g(x))g'(x)\,dx &= \left[f(g(x)) \right]_{x=-2}^{x=3} \\ &= f(g(3)) - f(g(-2)) \\ &= f(1) - f(-1) \\ &= 2 - 8 = -6 \end{aligned}$$

$3 : \begin{cases} 2 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$

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Question 6

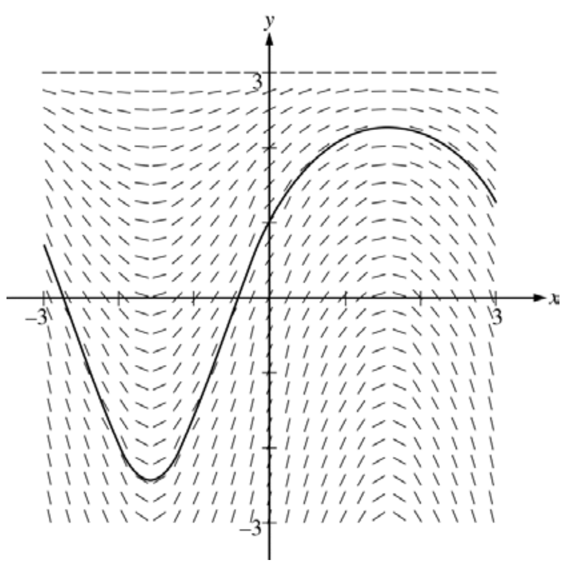
Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.

(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

(a)



1 : solution curve

(b) $\left.\frac{dy}{dx}\right|_{(x,\, y)=(0,\, 1)} = 2\cos 0 = 2$

An equation for the tangent line is $y = 2x + 1$.

$f(0.2) \approx 2(0.2) + 1 = 1.4$

$2 : \begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(c) $\frac{dy}{dx} = (3 - y)\cos x$

$\int \frac{dy}{3 - y} = \int \cos x \, dx$

$-\ln|3 - y| = \sin x + C$

$-\ln 2 = \sin 0 + C \Rightarrow C = -\ln 2$

$-\ln|3 - y| = \sin x - \ln 2$

Because $y(0) = 1$, $y < 3$, so $|3 - y| = 3 - y$

$3 - y = 2e^{-\sin x}$

$y = 3 - 2e^{-\sin x}$

Note: this solution is valid for all real numbers.

$6 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables