C12 - 5.0 - Int Particle Motion Notes

$$F(b) = F(a) + \int_a^b f(x)\,dx$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

FUNDAMENTAL THEOREM OF CALCULUS

A particle moves along a straight line East.. $a(t) = 4cost$ $v(0) = 1$ $s(0) = -2$ $[0,7]$

$a(3) = ?$

$a(t) = 4cost$
$a(3) = 4cos3$

$a(3) = -3.96\frac{m}{s^2}$

$a(5) = ?$

$a(t) = 4cost$
$a(5) = 4cos5$

$a(5) = 1.13\frac{m}{s^2}$

$v(t) = ?$

$v(t) = v(0) + \int_0^t a(t)\,dt$

$v(t) = 1 + \int_0^t 4cost\,dt$

$v(t) = 1 + (4\sin(t) - 2\sin(0))$

$v(t) = 4sint + 1$

$v(3) = ?$

$v(t) = 4sint + 1$
$v(3) = 4sin3 + 1$

$v(3) = 1.56\frac{m}{s}$

Moving Right

$v(3) = v(0) + \int_0^3 a(t)\,dt$

$v(3) = 1 + \int_0^3 4cost\,dt$

$v(3) = 1 + (4\sin(3) - 2\sin(0))$

$v(3) = 1.56\frac{m}{s}$

$v(5) = ?$

$v(t) = 4sint + 1$
$v(5) = 4sin5 + 1$

$v(5) = -2.84\frac{m}{s}$

Moving Left

$s(t) = ?$

$s(t) = s(0) + \int_0^t v(t)\,dt$

$s(t) = s(0) + \int_0^t (4sint + 1)\,dt$

$s(t) = -2 + (-4cost + 1t)\Big|_0^t$

$s(t) = -2 + ((-4cost + 1t) - (-4cos0 + 1(0)))$

$s(t) = -4cost + 1t + 2$

$s(t) = -4cost + 1t + 2$
$s(2) = -4cos2 + 1(2) + 2$

$s(2) = 5.66\ m$

$s(2) = ?$

$s(t) = s(0) + \int_0^t v(t)\,dt$

$s(2) = s(0) + \int_0^2 v(t)\,dt$

$s(2) = -2 + \int_0^2 (4sint + 1)\,dt$

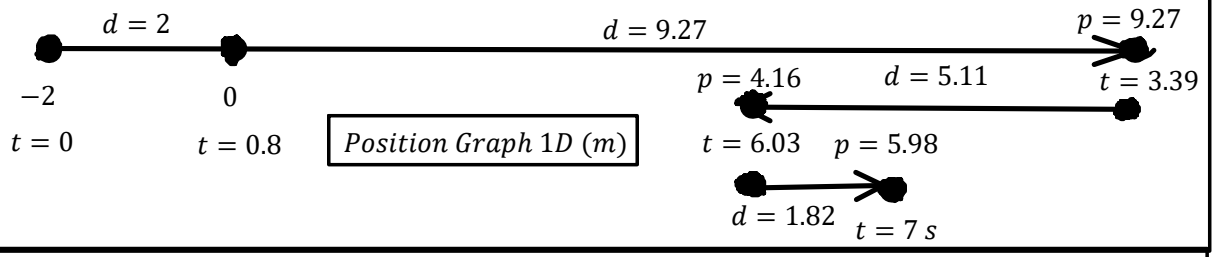
$s(2) = -2 + \big((-4\cos(t) + 1t)\big)\Big|_0^2$

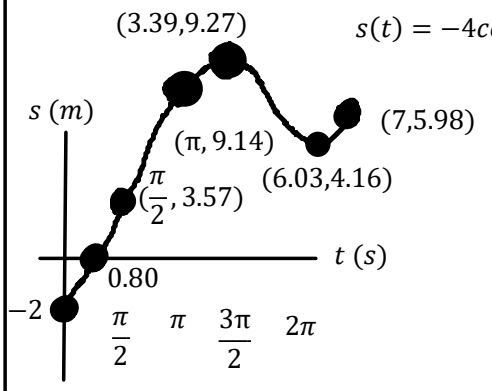
$s(2) = -2 + ((-4\cos(2) + 1(2)) - (-4\cos(0) + 1(0)))$

$s(2) = 4 - 4cos2$

$s(2) = 5.66\ m$

C12 - 5.0 - Int Particle Motion Notes

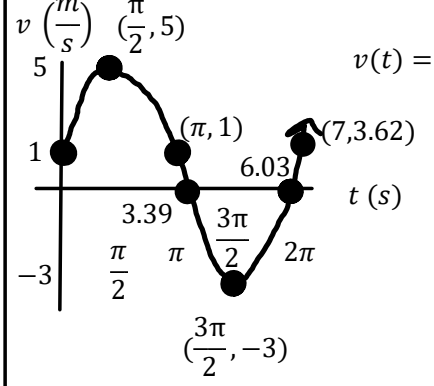




$s(t) = -4cost + 1t + 2$

$s(t) = -4cost + 1t + 2$
$s'(t) = 4sint + 1$
$v(t) = 4sint + 1$
$0 = 4sint + 1$
$sint = -\frac{1}{4}$ *QIII, IV*
$t_r = \sin^{-1}(+\frac{1}{4})$
$t_r = 0.25$
$t = 3.39\ s\ ,\ 6.03\ s$

t	$s(t)$
0	-2
0.8	0
1	0.84
2	5.66
3	8.96
4	8.61
5	5.87
6	4.15
7	5.98

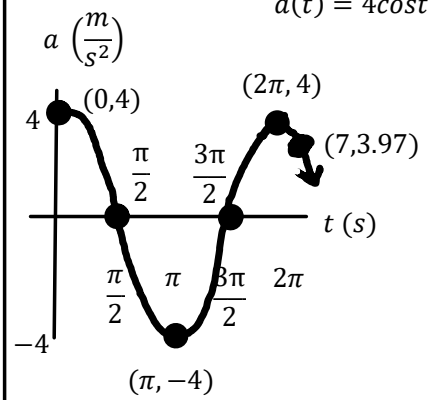


$v(t) = 4sint + 1$

$v(t) = 4sint + 1$
$v'(t) = 4cost$
$a(t) = 4cost$
$0 = 4cost$
$cost = 0$

$t = \frac{\pi}{2}\ s\ ,\ \frac{3\pi}{2}\ s$

t	$v(t)$
0	1
1	4.37
2	4.63
3	1.56
3.39	0
4	-2.03
5	-2.84
6	-0.12
6.03	0
7	*3.62*



$a(t) = 4cost$

t	$a(t)$
0	4
1	2.16
$\pi/2$	0
2	-1.67
3	-3.96
4	-2.62
$3\pi/2$	0
5	1.13
6	3.99
7	3.97

C12 - 5.0 - Int Particle Motion Notes

Stopped v = 0

When is the particle at rest?

$v = 0$

$t = 3.39\ s\ , 6.03\ s$

When is the particle farthest from the origin?

$v = 0$ & s(t) Abs Max or Min

$v(t) = 0$ $t = 3.39\ s$

$p(3.39) = 9.27\ m\ [W]$

Check Endpoints

~~$s(0) = -2$~~

~~$s(6.03) = 4.16$~~

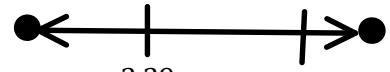
$s(7) = 3.62)$

When is the particle at the origin

$s(t) = 0$

$t = 0.80\ s$

When does the particle change direction?



$s' = v$ $+$ $-$ $+$

$v = 0$ & $v + \rightarrow -$ or $- \rightarrow +$

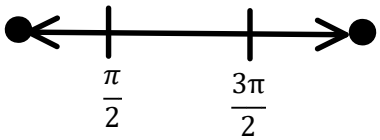
$t = 3.39\ s\ , 6.03\ s$

$(3.39, 9.27)$ Abs Max

$(7, 5.98)$ Loc Max

$(0, -2)$ Loc/Abs Min*

$(6.03, 4.16)$ Loc Min



$v' = a$ $+$ $-$ $+$

$t = \frac{\pi}{2}\ s\ , \frac{3\pi}{2}\ s$

$(\frac{\pi}{2}, 5)$ Abs Max

$(7, 3.62)$ Loc Max

$(0, 1)$ Loc Min

$(\frac{3\pi}{2}, -1)$ Abs Min

When is the particles acceleration greatest?

$a = max$

$a = 4$ $t = 0\ s, 2\pi\ s$

When is the particles acceleration least?

$a = min$

$a = -4$ $t = \pi\ s$

When is the particle moving in the positive direction?

$v > 0$

$(0, 3.39)$ $(6.03, 7)$

When is the particle moving in the negative direction?

$v < 0$

$(3.39, 6.03)$

When is the particle speeding up?

$v, a\ same\ sign$

Speeding up in the negative direction → $v = +\ , a = +$ $v = -\ a = -$

$\left(0, \frac{\pi}{2}\right)$ $\left(3.39, \frac{3\pi}{2}\right)$

When is the particle slowing down?

$v, a\ different\ sign$

$v = +\ , a = -$ Going forward Slowing Down

$v = -\ a = +$ Going backward Slowing Down

$\left(\frac{\pi}{2}, 3.39\right)$ $\left(\frac{3\pi}{2}, 7\right)$

Acceleration in the negative direction

Acceleration in the positive direction

C12 - 5.0 - Int Particle Motion Disp/Dist Notes

What is the displacement : $\vec{d}$ (0,3)
travelled by the particle? (0,7)

$$\vec{d} = \int_a^b (v(t))dt$$
$$\vec{d} = \int_0^3 (4sint + 1)dt$$
$$\vec{d} = \int_0^3 (4sint + 1)dt$$
$$\vec{d} = -4cost + t - (-4cost + t)$$
$$\vec{d} = -4cos3 + 3 - (-4cos0 + 0)$$

$\vec{d} = 10.96\ m\ [E]$

$$\vec{d} = \int_a^b (v(t))dt$$
$$\vec{d} = \int_0^7 (4sint + 1)dt$$
$$\vec{d} = \int_0^7 (4sint + 1)dt$$
$$\vec{d} = -4cost + t - (-4cost + t)$$
$$\vec{d} = -4cos7 + 7 - (-4cos0 + 0)$$

$\vec{d} = 7.98\ [E]$ → $5.98 + 2 = 7.98$

What is the distance : d (0,3)
travelled by the particle? (0,7)

$CP : t = 3.39$
$t = 6.03$

$$d = \int_a^b |v(t)|dt$$
$$d = \int_0^3 |(v(t)|dt$$
$$d = \int_0^3 |4sint + 1|dt$$

Calc : Math 9

$d = 10.96\ m$

$s(0) = -2\ or\ 2[W]$

$s(0.8) = 0$

$$s(3) = -4cost + 1t + 2$$
$$s(3) = -4cos3 + 1(3) + 2$$

$s(3) = 8.96\ m\ [E]$ → $8.96 + 2 = 10.96$

$s(3.39) = 9.27\ [E]$

$$s(7) = -4cost + 1t + 2$$
$$s(7) = -4cos7 + 1(7) + 2$$

$s(7) = 5.98\ m\ [E]$

$$d = \int_a^b |v(t)|dt$$
$$d = \int_0^7 |4sint + 1|dt$$
$$d = \int_0^{3.39} (4sint + 1)dt + \int_{3.39}^7 |4sint + t|dt$$
$$d = \int_0^{3.39} (4sint + 1)dt - \int_{3.39}^{6.03} (4sint + 1)dt + \int_{6.03}^7 (4sint + 1)dt$$
$$d = \big((-4cost + t) - (-4cost + t)\big) - \big((-4cost + t) - (-4cost + t)\big) + \big((-4cost + t) - (-4cost + t)\big)$$
$$d = \big((-4cos3.39 + 3.39) - (-4cos0 + 0)\big) - \big((-4cos6.03 + 6.03) - (-4cos3.39 + 3.39)\big) + \big((-4cos7 + 7) - (-4cos6.03 + 6.03)\big)$$
$$d = (11.27) - (-5.11) + (1.82)$$

Negative Area so
Minus (3.39,6.03)

$d = 18.2\ m$ → $d = +2 + 9.27 + 5.11 + 1.82 = 18.2$