Calculus 12 Notes

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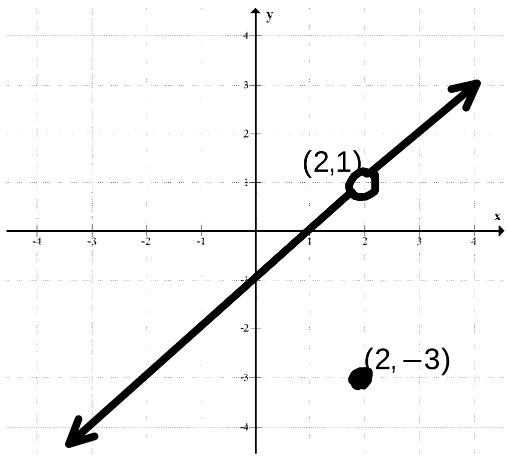
C12 - 1.1 - Limits Removable Discontinuities Graph Notes

Limit: What y is approaching.

What is y approaching as x approaches 2? $\lim_{x \to 2} f(x) = ?$



$f(x) = \begin{cases} x - 1 & ; x \neq 2 \\ -3 & ; x = 2 \end{cases}$

$f(x) = \dfrac{(x-1)(x-2)}{(x-2)}$

$f(2) = DNE$

x	y
1.9	.9
1.999	.999
2	DNE
2.001	1.001
2.1	1.1

Domain: $(-\infty, 2) U (2, \infty)$

Hole: $x - 2 = 0$
$x = 2$

$f(2) = -1$ (2,1)

$\lim_{x \to 2} f(x) = 1$

The Limit of f(x), as x approaches 2, equals 1.

y approaches 1 as x approaches 2.

$\lim_{x \to 2^-} f(x) = 1$ *and* $\lim_{x \to 2^+} f(x) = 1$

Left Hand Limit = Right Hand Limit

One Sided Limits

$\lim_{x \to c^+} f(x) = L$ *The Limit of f(x), as x approaches c, from the positive side (right), equals L.*

$\lim_{x \to c^-} f(x) = L$ *The Limit of f(x), as x approaches c, from the negative side (left), equals L*

Limit Exists if and only if: *Left hand Limit = Right Hand Limit*

$\lim_{x \to c} f(x) = L$ $\lim_{x \to c^-} f(x) = L$ *and* $\lim_{x \to c^+} f(x) = L$

or

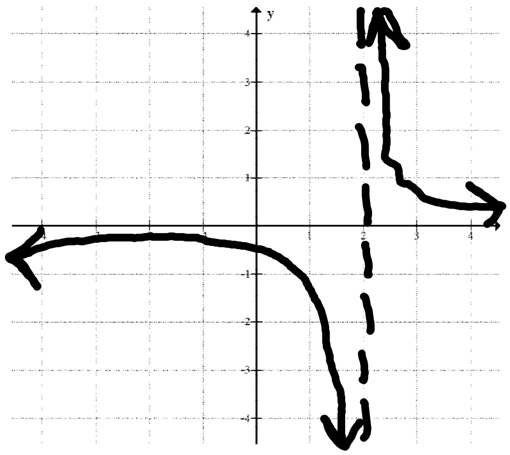
Limit Does Not Exist

$\lim_{x \to c} f(x) = DNE$

C12 - 1.1 - Limits Infinite/Jump Discontinuity Graph Notes

What is y approaching as x approaches 2?

$$\lim_{x \to 2} f(x) = ?$$



$$f(x) = \frac{1}{x-2}$$

$$\frac{1}{2.001-2}$$
$$\frac{1}{.001}$$
$$\frac{1}{\left(\frac{1}{1000}\right)}$$
$$1 \times \frac{1000}{1}$$
$$1000$$

x	y
1.9	-10
1.999	-1000
2	DNE
2.001	1000
2.1	10

$$\lim_{x \to 2} f(x) = DNE$$

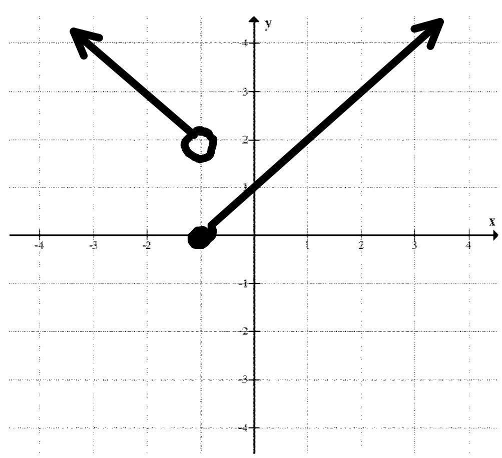
The Limit of $f(x)$, *as x approaches* 2, *Does Not Exist*

$$\lim_{x \to 2^-} f(x) = -\infty \quad \text{and} \quad \lim_{x \to 2^+} f(x) = +\infty$$

Left Hand Limit ≠ *Right Hand Limit*

What is y approaching as x approaches −1?

$$\lim_{x \to -1} f(x) = ?$$



$$F(x) = \begin{cases} -x+1 & ; x < 1 \\ x+1 & ; x \geq -1 \end{cases}$$

x	y
-1.1	2.1
-1.001	2.001
-1	0
-.999	.001
-.9	.1

The Limit of $f(x)$, *as x approaches* −1, *Does Not Exist*

$$\lim_{x \to -1} f(x) = DNE$$

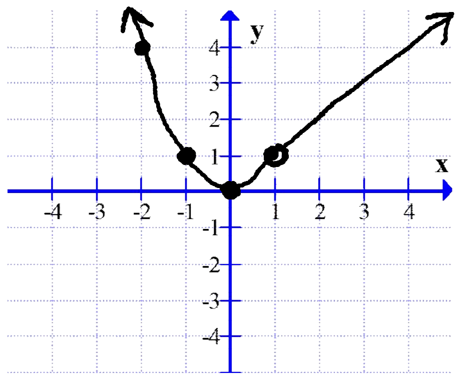
$$\lim_{x \to -1^-} f(x) = 2 \quad \text{and} \quad \lim_{x \to -1^+} f(x) = 0$$

Left Hand Limit ≠ *Right Hand Limit*

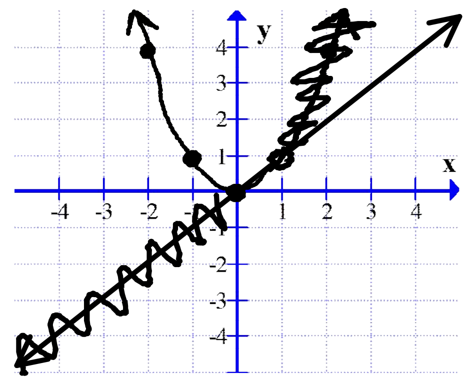
C12 - 1.1 - Limits Continuity Graph Equation Notes

$$f(x) = \begin{cases} x^2, & x < 1 \\ x, & x \geq 1 \end{cases}$$

1^- ; $x < 1$
1^+ ; $x > 1$



$y = x^2$; $x < 1$ $y = x$; $x > 1$



$\lim_{x \to 1^-} f(x) =$ x^2
$\lim_{x \to 1^-}$ $(1^-)^2$
1

$x \to 1^-$
$x \to 1$

$\lim_{x \to 1^+} f(x) =$ x
$\lim_{x \to 1^+}$ $(1^+)^2$
1

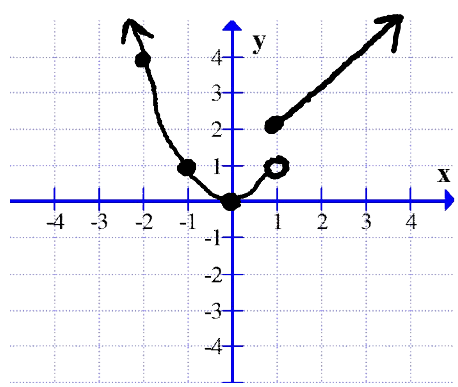
$x \to 1^+$
$x^2 \to 1$

$\lim_{x \to 1} f(x) =$ 1

Continuous

$\lim_{x \to 1^+} = \lim_{x \to 1^+} = \lim_{x \to 1} = 1$

$$f(x) = \begin{cases} x^2, & x < 1 \\ x + 1, & x \geq 1 \end{cases}$$

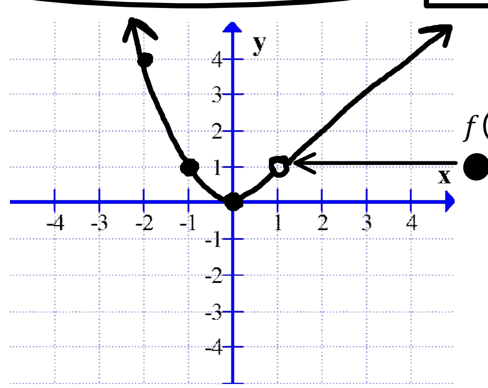


$$f(x) = \begin{cases} x^2, & x < 1 \\ x, & x \geq 1 \end{cases}$$

$f(x) =$ 1, $x = 1$

Discontinuous

Continuous



$f(x) = 1, x = 1$

Removable discontinuity

$\lim_{x \to 1^-} f(x) =$ x^2
$\lim_{x \to 1^-}$ $(1^-)^2$
1

$x \to 1^+$
$x^2 \to 1$

$\lim_{x \to 1^+} f(x) =$ $x + 1$
$\lim_{x \to 1^+}$ $(1^+)^2 + 1$
2

$x \to 1^-$
$x + 1 \to 2$

$\lim_{x \to 1} f(x) =$ DNE

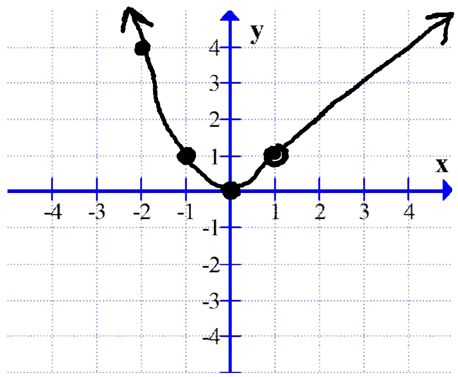
Discontinuous

$\lim_{x \to 1^+} \neq \lim_{x \to 1^+} \to \lim_{x \to 1} = DNE$

C12 - 1.1 - Limits Differentiability Graph Equation Notes

$$f(x) = \begin{cases} x^2, & x < 1 \\ x, & x \geq 1 \end{cases}$$

1^- ; $x < 1$
1^+ ; $x > 1$



$$f'(x) = \lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h}$$

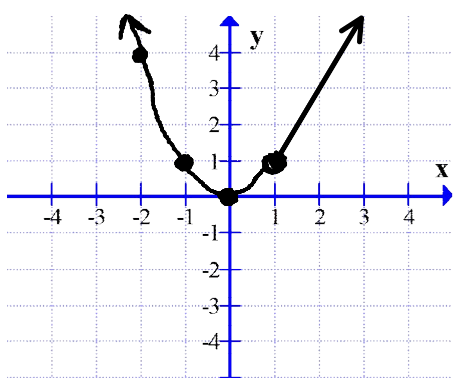
$$f'(x) = \lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h}$$

(2) $\neq$ (1)

$2x = 1$ $2 \neq 1$	Power

Not Differentiable

$$f(x) = \begin{cases} x^2, & x < 1 \\ 2x - 1, & x \geq 1 \end{cases}$$



$$f'(x) = \lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h}$$

(2) = (2)

$2x = 2$ $2 = 2$	Power

Differentiable

C12 - 1.2 - Limits Algebra Conj/LCD Notes

Find the Limits

$$\lim_{x \to 9} \frac{9-x}{3-\sqrt{x}}$$

$$\lim_{x \to 9} \frac{9-x}{3-\sqrt{x}} \times \boxed{\frac{3+\sqrt{x}}{3+\sqrt{x}}}$$ ← Conjugate

$$\lim_{x \to 9} \frac{(9-x)(3+\sqrt{x})}{9-x}$$ Simplify

$(3-\sqrt{x})(3+\sqrt{x}) =$
$9 + 3\sqrt{x} - 3\sqrt{x} - x =$ FL
$9 - x$

$$\lim_{x \to 9} 3+\sqrt{x}$$

$3+\sqrt{9}$
$3+3$ Substitute

6

$$\lim_{x \to 0} \frac{\frac{1}{x+3}-\frac{1}{3}}{x}$$ $LCD = 3(x+3)$

$$\lim_{x \to 0} \frac{\frac{3-(x+3)}{3(x+3)}}{\frac{x}{1}}$$ Add Fractions

$$\lim_{x \to 0} \frac{\frac{-x}{3(x+3)}}{\frac{x}{1}}$$ Simplify

$$\lim_{x \to 0} \frac{-x}{3(x+3)} \times \frac{1}{x}$$ Flip and Multiply

$$\lim_{x \to 0} -\frac{1}{3(x+3)}$$ Simplify

$-\frac{1}{9}$ Substitute

OR

$$\frac{\frac{1}{x+3}-\frac{1}{3}}{\frac{x}{1}}$$

$$\frac{3-(x+3)}{3x(x+3)}$$ Multiply Top and Bottom by LCD

$$\frac{-x}{3x(x+3)}$$ $LCD: 3(x+3)$

$$\frac{-1}{3(x+3)}$$

C12 - 1.2 - Limits Trig Algebra Notes

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{\sin 2x}{2x} =$$

$$1$$

$$\lim_{x \to 0} \frac{\sin 2x}{x}$$

$$\lim_{x \to 0} \frac{\sin 2x}{x} \times \frac{2}{2}$$

$$\lim_{x \to 0} \frac{\sin 2x}{2x} \times 2$$

$$1 \times 2$$

$$2$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

$$\frac{1 - \cos x}{x} \times \frac{1 + \cos x}{1 + \cos x}$$ Conjugate

$$\frac{\sin^2 x}{x(1 + \cos x)}$$ Separate Fractions

$$\frac{\sin x}{x} \times \frac{\sin x}{1 + \cos x}$$

$$\lim_{x \to 0} \frac{\sin x}{x} \times \frac{\sin x}{1 + \cos x}$$

$$1 \times \frac{0}{1 + 1}$$

$$0$$

$$\lim_{x \to 0} \frac{\tan x}{x} = 1$$

$$\frac{\tan x}{x}$$ Proof

$$\frac{\frac{\sin x}{\cos x}}{\frac{x}{1}}$$ Defn

$$\frac{\sin x}{\cos x} \times \frac{1}{x}$$ Flip and Multiply

$$\frac{\sin x}{x} \times \frac{1}{\cos x}$$

$$\lim_{x \to 0} \frac{\sin x}{x} \times \frac{1}{\cos x}$$

$$1 \times \frac{1}{1}$$

$$1$$

$$\lim_{x \to 0} \frac{\tan 4x}{\tan 3x}$$

$$\lim_{x \to 0} \frac{\frac{\tan 4x}{1} \times \frac{4x}{4x}}{\frac{\tan 3x}{1} \times \frac{3x}{3x}}$$

$$\lim_{x \to 0} \frac{\frac{\tan 4x}{4x} \times 4x}{\frac{\tan 3x}{3x} \times 3x}$$

$$\frac{1 \times 4x}{1 \times 3x}$$

$$\frac{4}{3}$$

$$\lim_{x \to 0} \frac{\sin 2x}{4x}$$ Separate Product

$$\lim_{x \to 0} \frac{\sin 2x}{2x} \times \frac{1}{2}$$ $4 = 2 \times 2$

$$1 \times \frac{1}{2}$$

$$\frac{1}{2}$$

$$\lim_{x \to 0} \frac{\sin 2x}{\sin 3x}$$

$$\lim_{x \to 0} \frac{\frac{\sin 2x}{1}}{\frac{\sin 3x}{1}} \times \frac{\frac{2x}{2x}}{\frac{3x}{3x}}$$

$$\lim_{x \to 0} \frac{\frac{\sin 2x}{2x}}{\frac{\sin 3x}{3x}} \times \frac{\frac{2x}{1}}{\frac{3x}{1}}$$

$$\frac{1}{1} \times \frac{2x}{3x}$$

$$\frac{2}{3}$$

C12 - 1.3 - Limits Vertical/Horizontal Asymptotes Notes

How many times does the bottom go into the top?

$0^+ \approx +0.00001$
$0^- \approx -0.00001$

$\frac{1}{0^+} \approx +\infty$ A relatively large number divided by a very small positive number is approximately Infinity $1 \gg 0^+$

$\frac{1}{0^-} \approx -\infty$ A relatively large number divided by a very small negative number is approximately Infinity $1 \gg 0^-$

$\frac{1}{+\infty} \approx 0$ A relatively small number divided by a very large positive number is approximately Zero $1 \ll +\infty$

$\frac{1}{-\infty} \approx 0$ A relatively small number divided by a very large negative number is approximately Zero $1 \ll -\infty$

$\lim\limits_{x \to a^-} = \pm\infty$ OR $\lim\limits_{x \to a^+} = \pm\infty$

$VA: x = a$

$\lim\limits_{x \to 0^+} \frac{1}{x}$ $\frac{1}{0^+}$ $+\infty$

x	y
0^+	∞

$x \to 0^+$, $\frac{1}{x} \to \infty$

$VA: x = 0$

Denominator=0

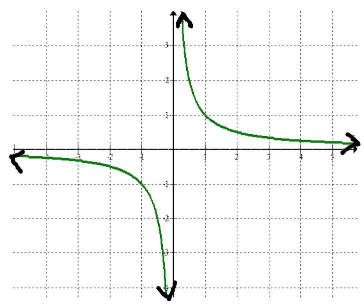
$VA: x = 0$

$\lim\limits_{x \to -\infty} \frac{1}{x}$ $\frac{1}{-\infty}$ 0

$x \to -\infty$, $\frac{1}{x} \to 0^-$

$HA: y = 0$

x	y
-10^{10}	$\approx 0^-$



$y = \frac{1}{x}$

$\lim\limits_{x \to +\infty} \frac{1}{x}$ $\frac{1}{+\infty}$ 0

$x \to \infty$, $\frac{1}{x} \to 0$

$HA: y = 0$

x	y
10^{10}	0^+

$\lim\limits_{x \to +\infty} f(x) = \#$ OR $\lim\limits_{x \to -\infty} f(x) = \#$

$HA: y = \#$

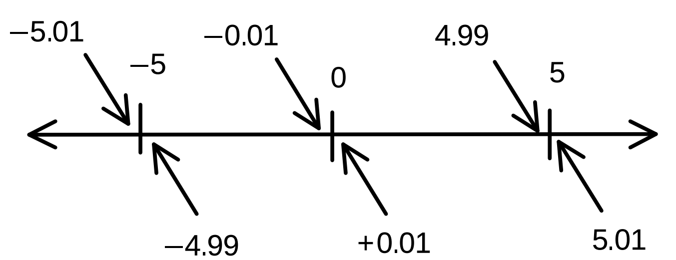
$\lim\limits_{x \to 0^-} \frac{1}{x}$ $\frac{1}{0^-}$ $-\infty$

x	y
0^-	$-\infty$

$x \to 0^-$, $\frac{1}{x} \to -\infty$

$VA: x = 0$

x	y
-10^{10}	$\approx 0^-$
-1	-1
0^-	$-\infty$
0	DNE
0^+	∞
1	1
10^{10}	0^+



$-5^+ \approx -4.999999$
$-5^- \approx -5.000001$

$5^+ \approx 5.000001$
$5^- \approx 4.990999$

$5^+ - 5 = 0^+$
$5 - 5^- = 0^+$

$5^- - 5 = 0^-$
$5 - 5^+ = 0^-$

$-5^+ + 5 = 0^+$
$-5^- - 5 = 0^-$

$\#^\infty = \infty$
$\infty^\infty = \infty$

C12 - 1.3 - Limits Vertical Asymptotes Notes

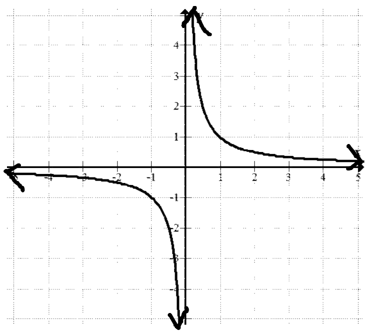
$VA: Denominator = 0$

$\lim_{x \to 0^-} \frac{1}{x}$

$\lim_{x \to 0^-} \frac{1}{0^-}$

$-\infty$

$f(x) = \frac{1}{x}$



VA: $x = 0$

$\lim_{x \to 0^+} \frac{1}{x}$

$\lim_{x \to 0^+} \frac{1}{0^+}$

$+\infty$

DNE

x	y
-0.1	-10
-0.01	-100
-0.001	-1000
0^-	$-\infty$
0	DNE
0^+	$+\infty$
0.001	1000
0.01	100
0.1	10

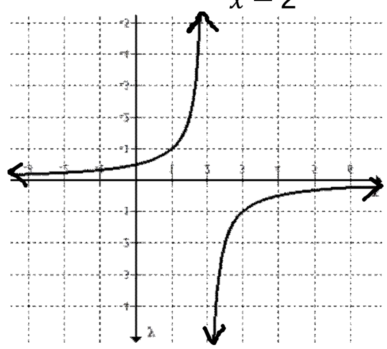
$\lim_{x \to 2^-} -\frac{1}{x-2} =$

$\lim_{x \to 2^-} -\frac{1}{2^- - 2}$

$-\frac{1}{0^-}$

$+\infty$

$f(x) = -\frac{1}{x-2}$



VA: $x - 2 = 0$

$x = 2$

$\lim_{x \to 2^+} -\frac{1}{x-2} =$

$\lim_{x \to 2^+} -\frac{1}{2^+ - 2}$

$-\frac{1}{0^+}$

∞ DNE

x	y
1.9	10
1.99	100
1.999	1000
2^-	∞
2	DNE
2^+	$-\infty$
2.001	-1000
2.01	-100
2.1	-10

$\lim_{x \to 0^-} \frac{1}{x^2}$

$\lim_{x \to 0^-} \frac{1}{(0^-)^2}$

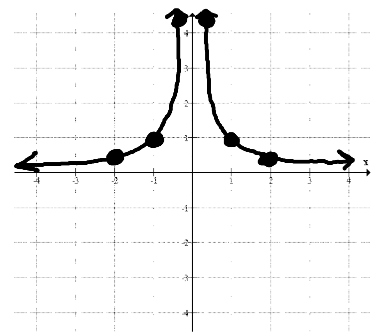
0^+

∞

$\lim_{x \to 0} \frac{1}{x^2} = ?$

$\lim_{x \to 0}$

$f(x) = \frac{1}{x^2}$



VA: $x^2 = 0$

$x = 0$

$\lim_{x \to 0^+} \frac{1}{x^2}$

$\lim_{x \to 0^+} \frac{1}{(0^+)^2}$

0^+

∞ ∞

x	y
-2	$\frac{1}{4}$
-1	1
-0.01	10000
0^-	∞
0	DNE
0^+	∞
0.01	10000
1	1
2	$\frac{1}{4}$

A vertical asymptote by definition is the limit as x approaches the VA from the left-hand side and the right-hand side and equals $+\infty$, $-\infty$ *either or both.*

C12 - 1.4 - Limits Rational HA Notes

A horizontal asymptote by definition is the limit as x approaches ±infinity. Substitute ±infinity for x into a table of values. Or. Divide top and bottom by x to the highest exponent of x in denominator and solve.

Horizontal Asymptote $\lim_{x\to\pm\infty} f(x) = \#\ ; HA\ y = \#$

x	y
$-\infty$	#

x	y
∞	#

Find HA

$$Lim_{x\to\infty} \frac{3x^2+4}{1x^2-1}$$

$$Lim_{x\to\infty} \frac{3+\frac{4}{x^2}}{1-\frac{1}{x^2}}$$

$$Lim_{x\to\infty} \frac{3+\frac{4}{\infty^2}}{1-\frac{1}{\infty^2}}$$

$$\frac{3+0}{1-0}$$

$$3$$

$$HA: y = 3$$

$$\frac{3x^2+4}{1x^2-1}$$

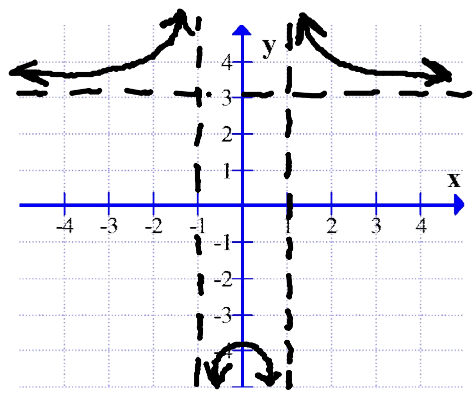
$$\frac{\frac{3x^2}{x^2}+\frac{4}{x^2}}{\frac{x^2}{x^2}-\frac{1}{x^2}}$$

$$\frac{3+\frac{4}{x^2}}{1-\frac{1}{x^2}}$$

$$Lim_{x\to\infty} \frac{3x^2+4}{1x^2-1}$$

$$Lim_{x\to\infty} \frac{3x^2}{1x^2}$$

$$\frac{3}{1}$$



$$Lim_{x\to-\infty} \frac{x^2+5}{3x^3+2x}$$

$$Lim_{x\to-\infty} \frac{\frac{1}{x}+\frac{5}{x^3}}{3+\frac{2}{x^2}}$$

$$Lim_{x\to-\infty} \frac{\frac{1}{-\infty}+\frac{5}{(-\infty)^3}}{3+\frac{2}{(-\infty)^2}}$$

$$\frac{0+0}{3+0}$$

$$0$$

$$HA: y = 0$$

$$\frac{x^2+5}{3x^3+2x}$$

$$\frac{\frac{x^2}{x^3}+\frac{5}{x^3}}{\frac{3x^3}{x^3}+\frac{2x}{x^3}}$$

$$\frac{\frac{1}{x}+\frac{5}{x^3}}{3+\frac{2}{x^2}}$$

$$Lim_{x\to-\infty} \frac{x^2+5}{3x^3+2x}$$

$$Lim_{x\to-\infty} \frac{x^2}{3x^3}$$

$$\frac{1}{3x}$$

$$\frac{1}{3(-\infty)}$$

$$0$$

$$Lim_{x\to\infty} \frac{x^4+3x}{2x+1}$$

$$Lim_{x\to\infty} \frac{x^3+3}{2+\frac{1}{x}}$$

$$Lim_{x\to\infty} \frac{\infty+3}{2+\frac{1}{\infty}}$$

$$Lim_{x\to\infty} \frac{\infty}{2+0}$$

$$\infty$$

$$HA: none$$

$$\frac{x^4+3x}{2x+1}$$

$$\frac{\frac{x^4}{x}+\frac{3x}{x}}{\frac{2x}{x}+\frac{1}{x}}$$

$$\frac{x^3+3}{2+\frac{1}{x}}$$

$$Lim_{x\to\infty} \frac{x^4+3x}{2x+1}$$

$$Lim_{x\to\infty} \frac{x^4}{2x}$$

$$\frac{x^3}{2}$$

$$\frac{(\infty)^3}{2}$$

$$\infty$$

C12 - 1.4 - Limits Exponential HA Notes

Find HA

$\lim_{x\to\infty} 2^x =$

$\lim_{x\to\infty} 2^\infty$

∞

$x \to \infty$
$2^x \to \infty$

$\lim_{x\to -\infty} 2^x =$

$\lim_{x\to -\infty} 2^{-\infty}$

$\lim_{x\to -\infty} \frac{1}{2^\infty}$

0

$x \to -\infty$
$2^{-x} \to 0$

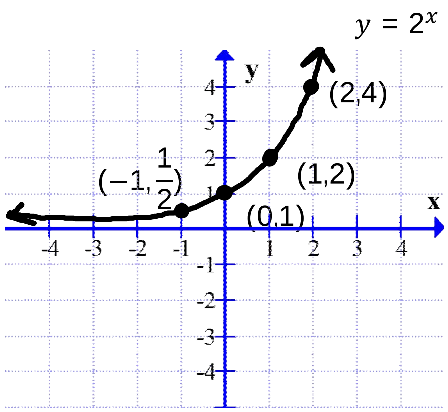
2^{-3}
$\frac{1}{2^3}$

$HA: y = 0$

$\lim_{x\to 0^+} 2^x$
2^{0^-}
1

$\lim_{x\to 0^-} 2^x$
2^{0^+}
1

$\#^0 = 1$



x	y
-1	$\frac{1}{2}$
0	1
1	2
2	4
-10	0^+

$2^{-1} = \frac{1}{2}$
$2^0 = 1$
$2^1 = 2$
$2^2 = 4$

$\lim_{x\to\infty} 2^x - 3 =$

$\lim_{x\to\infty} 2^\infty - 3$

∞

$\lim_{x\to -\infty} 2^x - 3 =$

$\lim_{x\to -\infty} 2^{-\infty} - 3$

$\lim_{x\to -\infty} \frac{1}{2^\infty} - 3$

$0 - 3$

-3

$HA: y = -3$

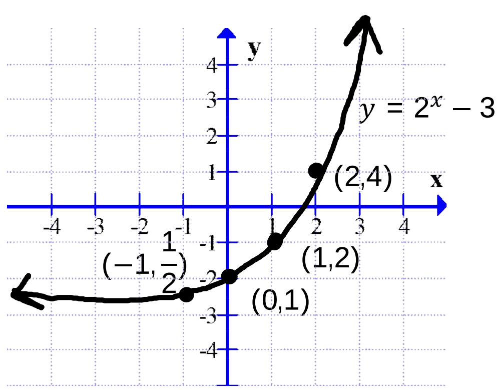
$\lim_{x\to\infty} e^x =$
e^∞
∞

$e = 2.71$

$\lim_{x\to -\infty} e^x =$
$e^{-\infty}$
$\frac{1}{e^\infty}$
$\frac{1}{\infty}$
0

$2^{-3} = \frac{1}{2^3}$

$e^\infty = \infty$



x	y
-1	$-\frac{5}{2}$
0	-2
1	-1
2	1
-10	0^+

$\lim_{x\to\infty} \frac{2^x}{x} =$
$\frac{2^\infty}{\infty}$
∞

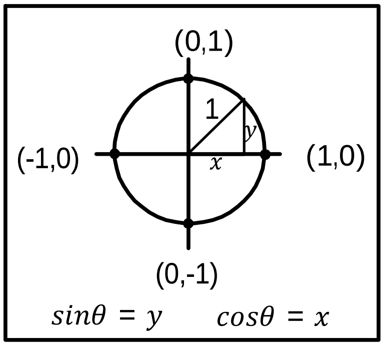
$2^\infty > \infty$

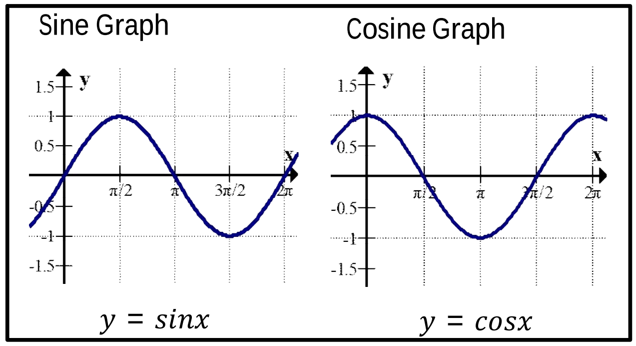
$\lim_{x\to -\infty} 2^{-x}$
$2^{-(-\infty)}$
2^∞
∞

$\lim_{x\to -\infty} \frac{2^x}{x} =$
$\frac{2^{-\infty}}{\infty}$
$\frac{\frac{1}{2^\infty}}{\infty}$
$\frac{\frac{1}{\infty}}{\infty}$
0

$2^{-3} = \frac{1}{2^3}$

C12 - 1.4 - Limits Trig HA Notes





Find HA

$\lim_{x \to \infty} \sin\left(\frac{1}{x}\right) =$

$\lim_{x \to \infty} \sin 0$

$\lim_{x \to \infty}$ 0

$\frac{1}{\infty} = 0$

$\sin\theta = y$

$x \to \infty$
$\frac{1}{x} \to 0$

$\lim_{x \to \infty} \cos\left(\frac{1}{x}\right) =$

$\lim_{x \to \infty} \cos 0$

$\lim_{x \to \infty}$ 1

$\frac{1}{\infty} = 0$

$\cos\theta = y$

$x \to \infty$
$\frac{1}{x} \to 0$

C12 - 1.4 - Limits Absolute Value HA Notes

Find HA

$f(x) = |x|$

Piecewise function: $f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Domain of positive case:

$x \geq 0$

Set what is inside the absolute value greater than or equal to zero.

Domain of negative case:

$x < 0$

Set what is inside the absolute value less than zero.

$\lim_{x \to -\infty} |x| =$

$-x$

∞

$x \to -\infty$
$|x| \to \infty$

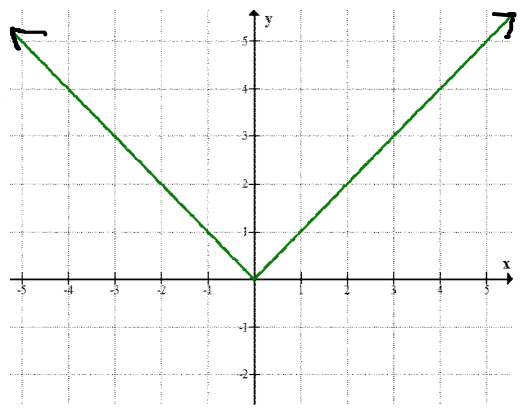
$\lim_{x \to \infty} |x| =$

x

∞

$x \to \infty$
$|x| \to \infty$

$y = -x$



$y = x$

$\lim_{x \to 0^-} |x| =$

$-x$

0

$\lim_{x \to 0^+} |x| =$

x

0

$\lim_{x \to \infty} \frac{|x| + 2}{|x|}$

$\lim_{x \to \infty} \frac{x + 2}{x}$

$\lim_{x \to \infty} \frac{\frac{x}{x} + \frac{2}{x}}{\frac{x}{x}}$

$\lim_{x \to \infty} \frac{1 + \frac{2}{x}}{1}$

$\lim_{x \to \infty} \frac{1 + \frac{2}{\infty}}{1}$

$\frac{1 + 0}{1}$

1

$\lim_{x \to -\infty} \frac{|x| + 2}{|x|}$

$\lim_{x \to -\infty} \frac{-x + 2}{-x}$

$\lim_{x \to -\infty} \frac{\frac{-x}{x} + \frac{2}{x}}{\frac{-x}{x}}$

$\lim_{x \to -\infty} \frac{-1 + \frac{2}{x}}{-1}$

$\lim_{x \to -\infty} \frac{-1 + \frac{2}{\infty}}{-1}$

$\frac{-1 + 0}{-1}$

1

$\sqrt{x^2} = |x| = \pm x$

$\lim_{x \to 2} \frac{2 \cdot x}{|x - 2|}$

&

$\lim_{x \to 3}$

C12 - 1.4 - Limits Square Root HA Notes

$\sqrt{x^2} = |x| = \pm x$

Find HA

$$\lim_{x\to\infty} \frac{5x}{\sqrt{x^2+5}}$$

$$\lim_{x\to\infty} \frac{5}{\sqrt{1+\frac{5}{x^2}}}$$

$$\lim_{x\to\infty} \frac{5}{\sqrt{1+\frac{5}{\infty^2}}}$$

$$\frac{5}{\sqrt{1+0}}$$

5

$$\frac{5x}{\sqrt{x^2+5}}$$

$$\frac{\frac{5x}{\sqrt{x^2}}}{\frac{\sqrt{x^2+5}}{\sqrt{x^2}}}$$

$$\frac{\frac{5x}{|x|}}{\sqrt{\frac{x^2+5}{x^2}}}$$

$\sqrt{x^2} = |x|$

$$\frac{\frac{5x}{x}}{\sqrt{\frac{x^2+5}{x^2}}}$$

$x \to \infty$
$|x| \to x$

$$\frac{5}{\sqrt{1+\frac{5}{x^2}}}$$

$$\frac{x^2+5}{x^2}$$

$$\frac{x^2}{x^2} + \frac{5}{x^2}$$

$$1 + \frac{5}{x^2}$$

Separate Fractions

$$\lim_{x\to-\infty} \frac{5x}{\sqrt{x^2+5}}$$

$$\lim_{x\to-\infty} \frac{-5}{\sqrt{1+\frac{5}{x^2}}}$$

$$\lim_{x\to-\infty} \frac{-5}{\sqrt{1+\frac{5}{\infty^2}}}$$

$$\frac{-5}{\sqrt{1+0}}$$

-5

$$\frac{5x}{\sqrt{x^2+5}}$$

$$\frac{\frac{5x}{\sqrt{x^2}}}{\frac{\sqrt{x^2+5}}{\sqrt{x^2}}}$$

$$\frac{\frac{5x}{|x|}}{\sqrt{\frac{x^2+5}{x^2}}}$$

$$\frac{\frac{5x}{-x}}{\sqrt{\frac{x^2+5}{x^2}}}$$

$x \to -\infty$
$|x| \to -x$

$$\frac{-5}{\sqrt{1+\frac{5}{x^2}}}$$

$$\lim_{x\to\infty} \frac{\sqrt{x^2+3x}}{x}$$

$$\lim_{x\to\infty} \sqrt{1+\frac{3}{x}}$$

$$\lim_{x\to\infty} \sqrt{1+\frac{3}{\infty}}$$

$$\sqrt{1+0}$$

$$\sqrt{1}$$

1

$$\frac{\sqrt{x^2+3x}}{x}$$

$$\frac{\sqrt{x^2+3x}}{\sqrt{x^2}}$$

$$\sqrt{\frac{x^2+3x}{x^2}}$$

$x = \sqrt{x^2}$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$\frac{x^2+3x}{x^2}$$

$$\frac{x^2}{x^2} + \frac{3x}{x^2}$$

$$1 + \frac{3}{x}$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\lim_{x\to\infty} \sqrt{\frac{3x^2+4}{1x^2-1}}$$

$$\lim_{x\to\infty} \sqrt{\frac{3+\frac{4}{x^2}}{1-\frac{1}{x^2}}}$$

$$\lim_{x\to\infty} \sqrt{\frac{3+\frac{4}{\infty^2}}{1-\frac{1}{\infty^2}}}$$

$$\sqrt{\frac{3+0}{1-0}}$$

$\sqrt{3}$

$HA: y = \sqrt{3}$

$$\frac{3x^2+4}{1x^2-1}$$

$$\frac{\frac{3x^2}{x^2} + \frac{4}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}}$$

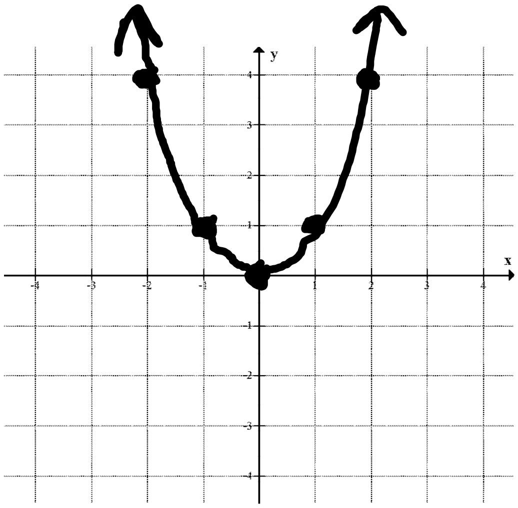
$$\frac{3+\frac{4}{x^2}}{1-\frac{1}{x^2}}$$

C12 - 1.5 - Even Odd One to One Functions Notes

$Even\ and\ Odd\ Functions\ -\ Symmetry$

$Even$: $f(-x) = f(x)$

$f(x) = x^2$



A horizontal flip over the y-axis is same as original

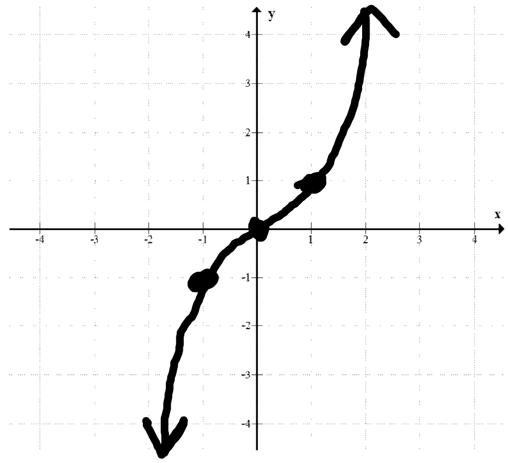
$f(-x) = f(x)$

$(-x)^2 = x^2$
$x^2 = x^2$

✓ If you put -x in for x and it's the same as the original its even

Odd: $f(-x) = -f(x)$

$f(x) = x^3$



A Horizontal Flip Equal to a Vertical Flip!

Symmetric with respect to origin

A horizontal flip over the y-axis is same as a vertical flip over the x-axis.

$f(-x) = -f(x)$

$(-x)^3 = -(x^3)$
$-x^3 = -x^3$

✓ If you put -x in for x and it's the same as distributing a negative into the original its odd

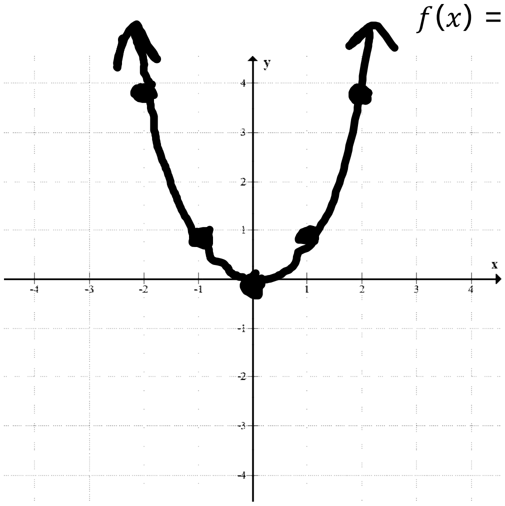
---

$One - to - One\ Function$

$One - to - One$ | Inverse is a Function | Inverse is One to One

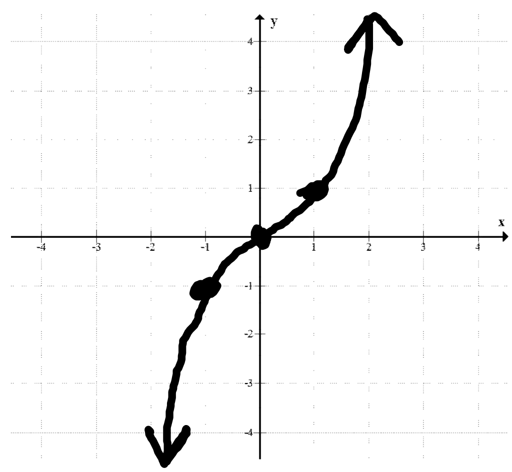
$Only\ one\ x\ value\ for\ every\ y\ value.$
H$orizonal\ and\ Vertical\ line\ test. Run\ your\ pencil\ horizonally\ down\ the\ page$:
$Your\ pencil\ can\ only\ ever\ hit\ the\ graph\ once.$

$f(x) = x^2$



$Not\ One - to - One$

$f(x) = x^3$

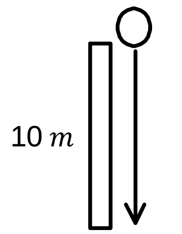


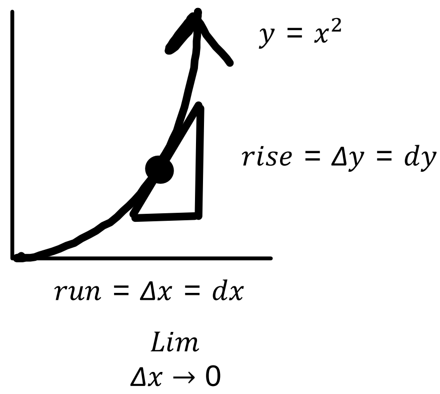
Every input has a different output

$f(a) \neq f(b)$

C12 - 2.1 - Ball Drop hva vs t Notes

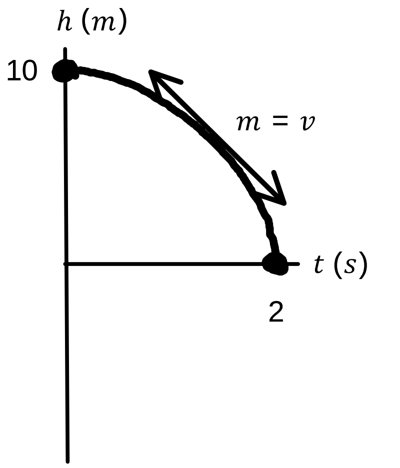
A ball is dropped of a 100 m cliff. Graph height, velocity and acceleration of the ball.



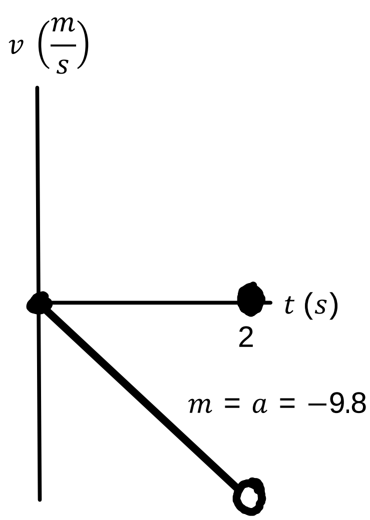


$$m = slope = \frac{rise}{run} = \frac{dy}{dx}$$

$$f'(x) = m = \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = y' = f'(x)$$



t	d
0	100
1	10. ##
2	0.1#

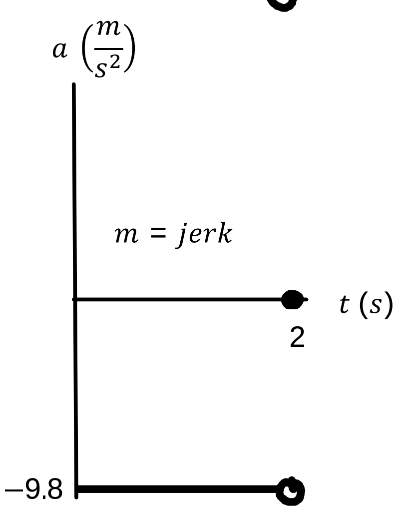


t	v
0	0
1	−9.8
2	−19.6

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

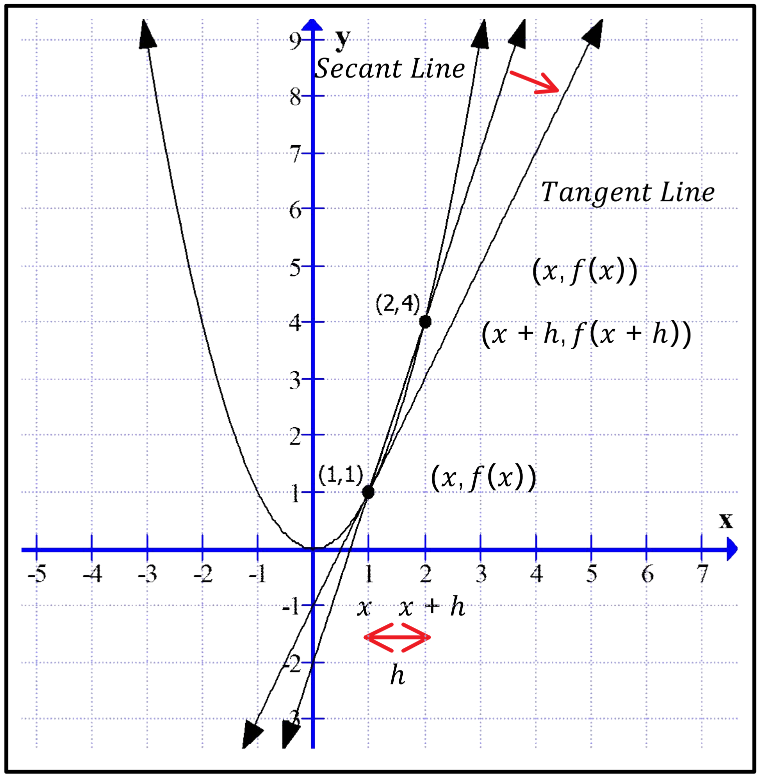
$$m = \frac{-19.6 - (-9.8)}{2 - 1}$$

$$m = -9.8 \frac{m}{s^2}$$



C12 - 2.2 - Definition of Derivative Equation Graph Notes

Find the equation of the tangent line to x^2 *at* $x = 1$.



Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{f(x+h) - f(x)}{x + h - x}$$

$$\boxed{m = \frac{f(x+h) - f(x)}{h}}$$

Definition of the Derivative

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$$\lim_{h \to 0} \frac{(x+h)^2 - (x^2)}{h} \quad \leftarrow \quad f(x) = x^2, \; f(x+h) = (x+h)^2$$

$$\lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$ Foil

$$\lim_{h \to 0} \frac{2xh + h^2}{h}$$ Simplify

$$\lim_{h \to 0} \frac{h(2x + h)}{h}$$ Factor, Simplify

$$\lim_{h \to 0} 2x + h$$ Substitute

$f'(x) = 2x$ *Slope of Tangent*

$f'(1) = 2(1)$ $x = 1$

$f'(1) = 2$ $m = 2$

Power Rule

$y = x^2$
$y' = 2x$

$m = 2(1)$

$m = 2$

$y = x^2$
$y = (1)^2$
$y = 1$ (1,1)

$y - y_1 = m(x - x_1)$

$y - 1 = 2(x - 1)$ (1,1)

$y = 2x - 2 + 1$

$y = 2x - 1$ *Tangent Line*

C12 - 2.2 - Derivative Formula 1,2

Find the equation of the tangent line to x^2 *at* $x = 1$.

$$m = f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$m = f'(1) = \lim_{x \to 1} \frac{x^2 - f(1)}{x - 1}$$

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

$$\lim_{x \to 1} \frac{(x + 1)(x - 1)}{(x - 1)}$$

$$\lim_{x \to 1} x + 1$$

$$1 + 1$$

$$m = f'(1) = 2$$

$$m = f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

$$m = f'(1) = \lim_{h \to 0} \frac{f(1 + h) - f(1)}{h}$$

$$\lim_{h \to 0} \frac{(1 + h)^2 - (1)^2}{h}$$

$$\lim_{h \to 0} \frac{1 + 2h + h^2 - 1}{h}$$

$$\lim_{h \to 0} \frac{2h + h^2}{h}$$

$$\lim_{h \to 0} \frac{h(2 + h)}{h}$$

$$\lim_{h \to 0} 2 + h$$

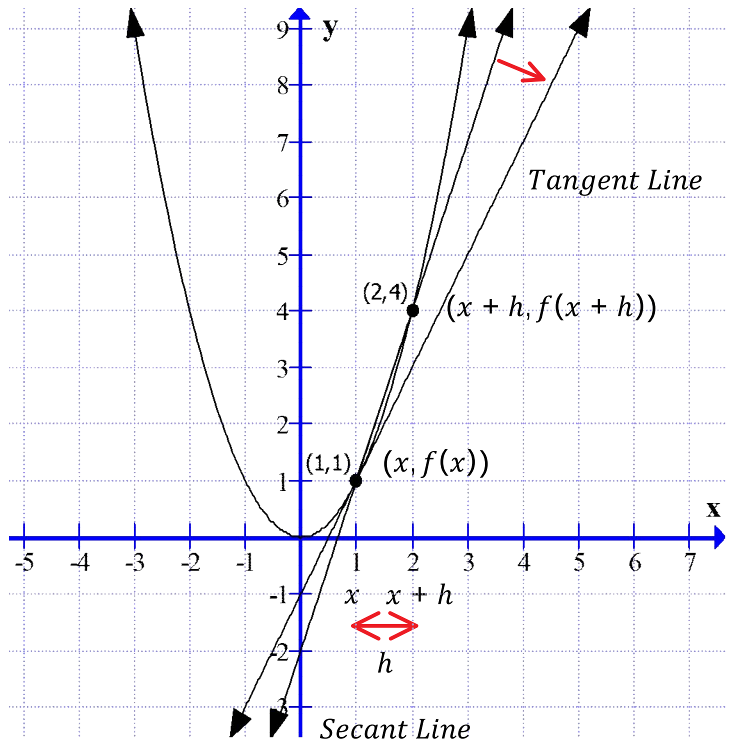
$$2 + 0$$

$$m = f'(1) = 2$$

$f(x) = x^2$ $\quad a = 1 \quad (1,1) \quad f(1) = 1$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

x	y
1	1



$$y - y_1 = m(x - x_1)$$

$$y - 1 = 2(x - 1)$$

$$y = 2x - 1$$

C12 - 2.3 - Poly/Rational Root Derivative Laws Notes

$y' = \frac{dy}{dx} = f'(x)$

Constant/Sum Function Rule

$y = c$
$y' = 0$

$y = cf(x)$
$y' = cf'(x)$

$y = 2$
$y' = 0$

$y = 3^2$
$y' = 0$

$y = 2$
$y = 2x^0$
$y' = 0 \times 2x^{-1}$
$y = 0$

$y = 3x$
$y' = 3$

$y = 3x$
$y = 3x^1$
$y' = 3x^0$
$y' = 3$

$x^0 = 1$

$y = 1x$
$y' = 1$

$y = 3x + 2x$
$y' = 3 + 2$
$y' = 5$

$y = 3x + 2x$
$y = 5x$
$y' = 5$

Power Rule

$y = x^n$
$y' = nx^{n-1}$

$y = x^2$
$y' = 2x^{2-1}$
$y' = 2x$

$y = x^3$
$\frac{dy}{dx} = 3x^{3-1}$
$\frac{dy}{dx} = 3x^2$

$f(x) = 2x^3$
$f'(x) = 3 \times 2x^{3-1}$
$f'(x) = 6x^2$

$y = \frac{1}{x^2}$
$y = x^{-2}$
$y' = -2x^{-3}$
$y' = -\frac{2}{x^3}$

$2^{-3} = \frac{1}{2^3}$

$\sqrt{x} = x^{\frac{1}{2}}$
$\sqrt[3]{x^2} = x^{\frac{2}{3}}$

$y = \sqrt{x}$
$y = x^{\frac{1}{2}}$
$y' = \frac{1}{2}x^{-\frac{1}{2}}$
$y' = \frac{1}{2x^{\frac{1}{2}}}$
$y' = \frac{1}{2\sqrt{x}}$

$y = x^{\frac{3}{2}}$
$y' = \frac{3}{2}x^{\frac{1}{2}}$

$\frac{3}{2} - 1 = \frac{1}{2}$

$y = \sqrt{3x}$
$y = (3x)^{\frac{1}{2}}$
$y' = \frac{1}{2}(3x)^{-\frac{1}{2}} \times 3$ ← Chain Rule
$y' = \frac{3}{2(3x)^{\frac{1}{2}}}$
$y' = \frac{3}{2\sqrt{3x}}$

$y = \sqrt{f(x)}$
$y = (f(x))^{\frac{1}{2}}$
$y' = \frac{1}{2}f'(x)^{-\frac{1}{2}} \times f'(x)$
$y' = \frac{f'(x)}{2\sqrt{f(x)}}$

Product Rule

$y = f(x)g(x)$
$y' = f'(x)g(x) + g'(x)f(x)$

$y = uv$
$y' = u'v + v'u$

$y = (2x + 1)(3x - 2)$
$y' = 2(3x - 2) + 3(2x + 1)$
$y' = 6x - 4 + 6x + 3$
$y' = 12x - 1$

$y = (2x + 1)(3x - 2)$
$y' = 6x^2 - x - 2$
$y' = 12x - 1$

Quotient Rule

$y = \frac{f(x)}{g(x)}$
$y' = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}$

$y = \frac{u}{v}$
$y' = \frac{u'v - v'u}{v^2}$

$y = \frac{x^2}{2x + 1}$
$y' = \frac{2x(2x + 1) - 2(x^2)}{(2x + 1)^2}$
$y' = \frac{4x^2 + 2x - 2x^2}{(2x + 1)^2}$
$y' = \frac{2x^2 + 2x}{(2x + 1)^2}$

$y = \frac{x^2 + 3x^3}{x}$
$y = \frac{x^2}{x} + \frac{3x^3}{x}$
$y = x + 3x^2$
$y' = 1 + 6x$
Separate Fractions

Chain Rule

$y = f(g(x))$
$y' = f'(g(x))(g'(x))$

$y = (3x + 1)^7$
$y' = 7(3x + 1)^{7-1} \times 3$ ← Chain Rule
$y' = 21(3x + 1)^6$

$y = x^7$
$y' = 7x^6 \times 1$ ← Chain Rule

C12 - 2.4 - Exponential Ln Derivatives

$y = e^x$
$y' = e^x lne$
$y' = e^x$

$y = 2^x$
$y' = 2^x ln2$

$y = lnx$
$y' = \frac{1}{xlne}$
$y' = \frac{1}{x}$

$lne = 1$

$y = \log_5 x$
$y' = \frac{1}{xln5}$

$y = e^{2x}$
$y' = e^{2x} \times 2$
$y' = 2e^{2x}$

$y = 5^{3x}$
$y' = 5^{3x} ln5 \times 3$
$y' = 3(5)^{3x} ln5$

$y = ln2x$
$y' = \frac{1}{x} \times 2$
$y' = \frac{2}{x}$

$y = \log_7 x^2$
$y' = \frac{1}{x^2 ln7} \times 2x$
$y' = \frac{2}{xln7}$

$y = xlnx$
$y' = 1(lnx) + \frac{1}{x} \times x$
$y' = lnx + 1$

$y = \ln(lnx)$
$y' = \frac{1}{lnx} \times \frac{1}{x}$
$y' = \frac{1}{xlnx}$

$y = \ln(x^2)$
$y' = \frac{1}{x^2} \times 2x$
$y' = \frac{2}{x}$

$y = (lnx)^2$
$y = 2(lnx) \times \frac{1}{x}$
$y = \frac{2lnx}{x}$

$y = lnx$
$y' = \frac{1}{x} \times x'$
$y' = \frac{x'}{x}$

$y = \ln(f(x))$
$y' = \frac{f'(x)}{f(x)}$

$y = \ln(1 + x^2)$
$y' = \frac{1}{1 + x^2} \times 2x$
$y' = \frac{2x}{1 + x^2}$

$y = \ln(\frac{x+1}{x-1})$
$y = \ln(x + 1) - \ln(x - 1)$
$y' = \frac{1}{x+1} - \frac{1}{x-1}$
$y' = -\frac{2}{(x+1)(x-1)}$

$y = \ln\left(\frac{x+1}{x-1}\right)$
$y' = \frac{1}{\left(\frac{x+1}{x-1}\right)} \times \frac{1(x-1) - 1(x+1)}{(x-1)^2}$
$y' = \frac{x-1}{x+1} \times -\frac{2}{(x-1)^2}$
$y' = -\frac{2}{(x+1)(x-1)}$

This quotient is hard

$\boldsymbol{y = x^x}$
$lny = lnx^x$
$lny = xlnx$
$\frac{1}{y} \times y' = 1(lnx) + \frac{1}{x} \times x$
$\frac{y'}{y} = lnx + 1$
$y' = y[lnx + 1]$

$y' = x^x(lnx + 1)$

Ln both sides

$\frac{d}{dx} lny = \frac{1}{y} \times y'$
$\frac{d}{dx} lny = \frac{y'}{y}$

$y = x^x$

$y = \frac{x+1}{x-1}$ Nobody would ln this
$lny = ln\frac{x+1}{x-1}$ ln *both sides*

$y = \frac{(2x+1)^2}{(x+2)^3}$
$lny = ln\frac{(2x+1)^2}{(x+2)^3}$
$lny = \ln(2x+1)^2 - \ln(x+2)^3$
$lny = 2\ln(2x+1) - 3\ln(x+2)$
$\frac{y'}{y} = 2 \times \frac{1}{2x+1} \times 2 - 3 \times \frac{1}{x+2}$
$y' = y\left(2 \times \frac{1}{2x+1} \times 2 - 3 \times \frac{1}{x+2}\right)$
$y' = \left(\frac{(2x+1)^2}{(x+2)^3}\right)\left(2 \times \frac{1}{2x+1} \times 2 - 3 \times \frac{1}{x+2}\right)$

C12 - 2.5 - Implicit Differentiation Notes

$x^2 + y^2 = 9$
$y = \pm\sqrt{9 - x^2}$

Implicit Functions: Hard or impossible to solve for y

$\frac{d}{dx}x^3$
$3x^2 \times \frac{dx}{dx}$
$3x^2 \times 1$
$3x^2$

Chain Rule
$\frac{dx}{dx} = 1$

$\frac{d}{dx}x = 1$

$\frac{d}{dx}y^3$
$3y^2 \times \frac{dy}{dx}$
$3y^2 \times y'$
$3y^2y'$

Chain Rule
$y' = \frac{dy}{dx}$

$y = f(x)$

$\frac{d}{dx}y = \frac{d}{dx}x^3 = \frac{dy}{dx} = y' = 3x^2$

Power/Chain Rule

$y^2 = x$
$2yy' = 1$
$y' = \frac{1}{2y}$

$x^2 + y^2 = 9$
$2x + 2yy' = 0$
$2yy' = -2x$
$y' = -\frac{x}{y}$

$2x = 3y$
$2 = 3y'$

$xy = 0$
$1y + y'x = 0$
$y'x = -y$
$y' = -\frac{y}{x}$

Product Rule

$y = xy$
$y' = 1(y) + y'(x)$
$y' = y + xy'$
$y' - xy' = y$
$y'(1 - x) = y$
$y' = \frac{y}{1 - x}$

Combine Like Terms
GCF = y'

$xy = 0$
$1y + y'x = 0$
$y'x = -y$
$y' = -\frac{y}{x}$

$2xy$ $2y + y'2x$ $2y + 2y'x$ $2xy$ $2(1(y) + y'(x))$ $2y + 2y'x$	$-xy$ $-1y + y'(-x)$ $-y - y'x$ $-xy$ $-(1y + y'x)$ $-y - y'x$

Power/Product/Chain Rule

$xy^2 = 2$
$1(y^2) + 2yy'(x) = 0$
$y^2 + 2xyy' = 0$
$2xyy' = -y^2$
$y' = \frac{-y^2}{2xy}$

$y^2 = xy$
$2yy' = (1(y) + y'(x))$
$2yy' = y + xy'$
$2yy' - xy' = y$
$y'(2y - x) = y$
$y' = \frac{y}{2y - x}$

$x^2 + xy + y^2 = 9$
$2x + (1(y) + y'(x)) + 2yy' = 0$
$2x + y + xy' + 2yy' = 0$
$xy' + 2yy' = -2x - y$
$y'(x + 2y) = -2x - y$
$y' = \frac{-2x - y}{x + 2y}$
$y' = -\frac{2x + y}{x + 2y}$

Implicit Differentiation. (Don't forget y')

Take Derivative
Combine primes on one side
Everything else on the other side
Factor out y prime
Divide both sides
Sometimes sub y and or y' back in
Possibly sub (x,y) before isolating
Slope/Eq of Tan, don't need to isolate y'

C12 - 2.6 - Trig Derivative Laws Notes

$y = sinx$

$y' = cosx$

$y = cosx$

$y' = -sinx$

$y = secx$

$y' = secxtanx$

$y = cscx$

$y' = -cscxcotx$

$y = tanx$

$y' = \sec^2 x$

$y = cotx$

$y' = -\csc^2 x$

$y = sin2x$
$y' = cos2x \times 2$

$y' = 2cos2x$

$y = \sin^2 x$
$y = (sinx)^2$
$y' = 2sinx \times cosx$
$y' = 2sinxcosx$

$y' = sin2x$

$y = xsinx$

$y' = 1sinx + xcosx$

$y = sin2x$
$y = 2sinxcosx$
$y' = 2\left(cosxcosx + (-sinxsinx)\right)$
$y' = 2(\cos^2 x - \sin^2 x)$

$y' = 2cos2x$

$y = \frac{sinx}{cosx}$

$y' = \frac{cosxcosx - -sinxsinx}{\cos^2 x}$

$y' = \frac{\sin^2 x + \cos^2 x}{\cos^2 x}$

$y' = \frac{1}{\cos^2 x}$

$y' = \sec^2 x$

$y = sinx^2$
$y' = cosx^2 \times 2x$

$y' = 2xcosx^2$

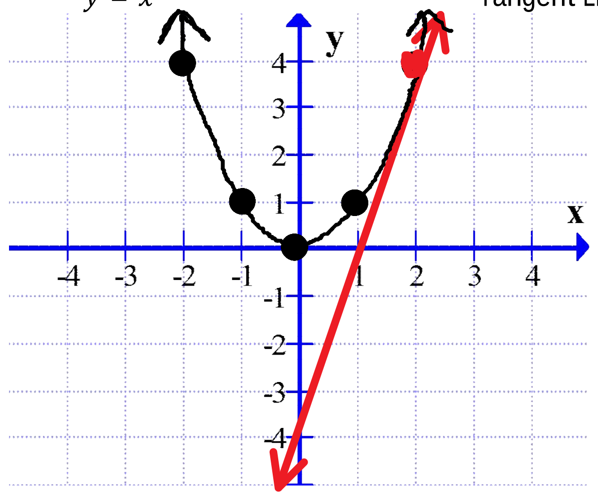
$y = sinxy$

$y' = cosxy(1y + y'x)$

C12 - 2.7 - Eq of Tan Notes

Find the equation of the tangent line at $x = 2$

$y = x^2$ Tangent Line



$y = x^2$
$y = (2)^2$
$y = 4$

(2,4)

Equation of Tangent Line

DERIVATIVE - Take the derivative of the equation
SLOPE - Substitute the X value of the point into the derivative to find the slope value
Y - VALUE - Substitute the X/Y value back into the original equation to figure out the Y/X value
EQUATION - Write down the equation in slope point form or y=mx+b or general form

$y = x^2$
$y' = 2x$

$m = 2(2)$

$m = 4$

$y - y_1 = m(x - x_1)$

$y - 4 = 4(x - 2)$

$y - 4 = 4x - 8$
$y = 4x - 8 + 4$

$y = 4x - 4$

$y = mx + b$
$y = 4x + b$
$4 = 4(2) + b$
$b = -4$

$y = 4x - 4$

Eq of Tan

Derivative
$f'(a) = slope$
$(y - value)$
Tangent Equation

$4x - y - 4 = 0$

Find the equation of the tangent line to $y = x^2$ line parallel to $y = x - 4$

Perpendicular

$m = -\frac{1}{m}$

$y = x^2$
$y' = 2x$

$y = x - 4$

$m = 1$

$y = mx + b$

$2x = 1$
$x = \frac{1}{2}$

$y = x^2$
$y = \left(\frac{1}{2}\right)^2$
$y = \frac{1}{4}$

$y - y_1 = m(x - x_1)$

$y - \frac{1}{4} = 1(x - \frac{1}{2})$

Find the equation of the horizontal tangent line to $y = x^2$

$y = x^2$
$y' = 2x$

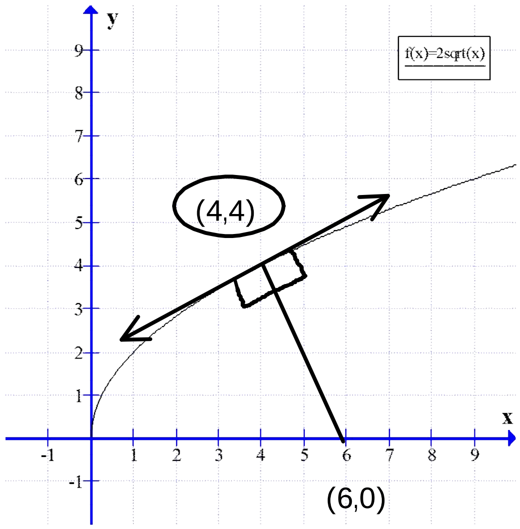
$m = 0$

$2x = 0$
$x = 0$

$y = x^2$
$y = (0)^2$
$y = 0$

$y = 0$

C12 - 2.8 - Perp/Tan Eq to Ext Point Notes



$y = 2\sqrt{x}$

$y = 2\sqrt{x}$

$y' = 2 \times \frac{1}{2} x^{-\left(\frac{1}{2}\right)}$

$y' = \frac{1}{\sqrt{x}}$

$m_\perp = -\sqrt{4}$

$m_\perp = -2$

$m_\perp = -\sqrt{x}$

$m_\perp = -\frac{1}{m}$

Or find the minimum distance.

$Derivative = Slope$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$-\sqrt{x} = \frac{2\sqrt{x} - 0}{x - 6}$

$-x + 6 = \frac{2\sqrt{x}}{\sqrt{x}}$ $\quad x \neq 0$

$-x + 6 = 2$

$x = 4$

$y_2 = 2\sqrt{x}$

(x_1, y_1)
(6,0)

Or Polynomial Factoring

Find the point on the graph closest to the point (6,0) and Equation through both points.

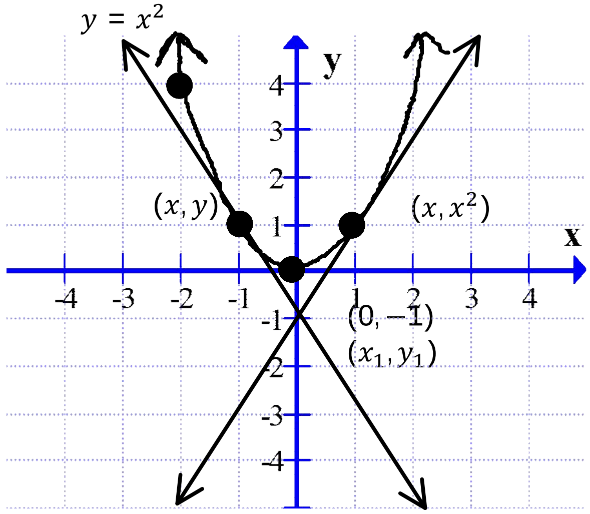
$y - 4 = -2(x - 4)$ $\qquad$ $y - 0 = -2(x - 6)$

$y = -2x + 12$ $\qquad$ $y = -2x + 12$

$2x + y - 12 = 0$

(x_1, y_1)
$(0, -1)$

Find the point(s) on the graph and equation(s) Tangent to the exterior point



$y = x^2$
$y' = 2x$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$y' = m$

$2x = \frac{y - (-1)}{x - 0}$

$2x = \frac{x^2 + 1}{x}$

$2x^2 = x^2 + 1$

$x^2 = 1$

$x = \pm 1$

$y_2 = x^2$

$y = x^2$
$y = (1)^2$
$y = 1$

$y = x^2$
$y = (-1)^2$
$y = 1$

$y = x^2$

$(-1, 1)$ $\qquad$ $(1, 1)$

$y - 1 = 2(x - 1)$ $\qquad$ $y - 1 = -2(x + 1)$

$y = x^2$
$y' = 2x$
$m = 2(x)$
$m = 2(1)$
$m = 2$

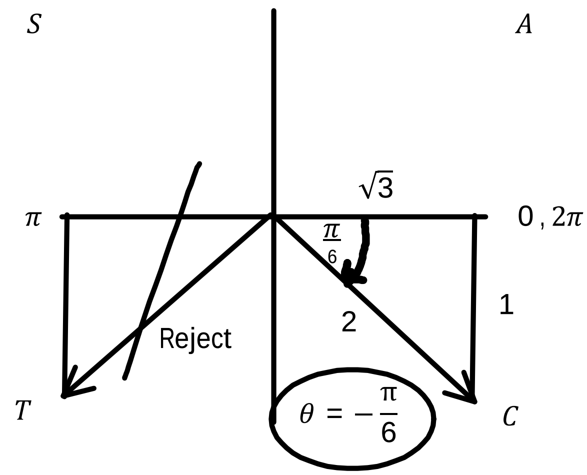
$y = x^2$
$y' = 2x$
$m = 2(x)$
$m = 2(-1)$
$m = -2$

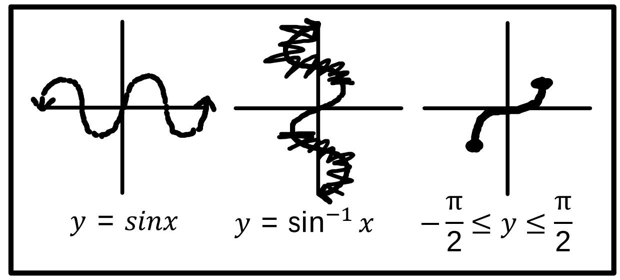
C12 - 2.9 - Inverse Trig Notes

$\sin^{-1}\left(-\frac{1}{2}\right) = ?$

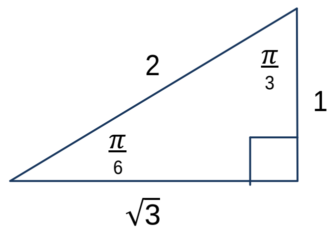
$$\sin\theta = -\frac{1}{2}$$

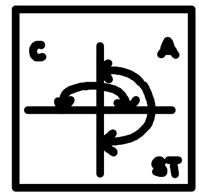
$$\theta = \sin^{-1}\left(\frac{1}{2}\right)$$





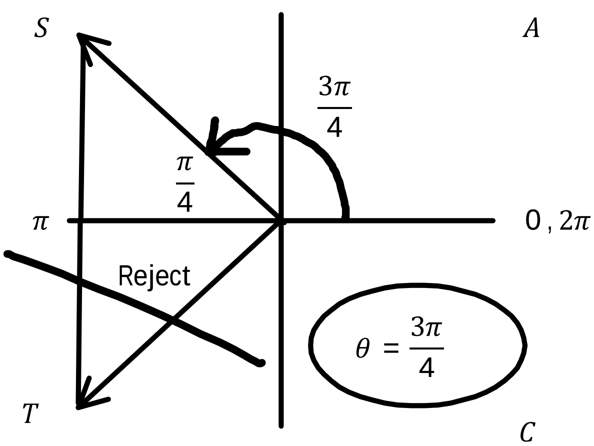
Function Range

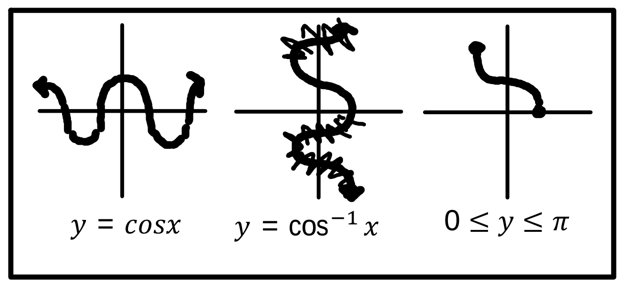




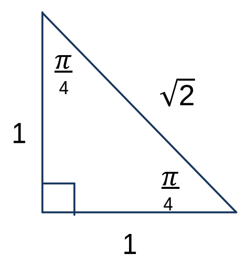
$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = ?$

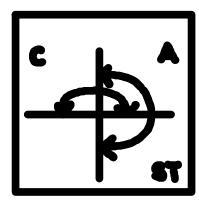
$$\cos\theta = -\frac{1}{\sqrt{2}}$$





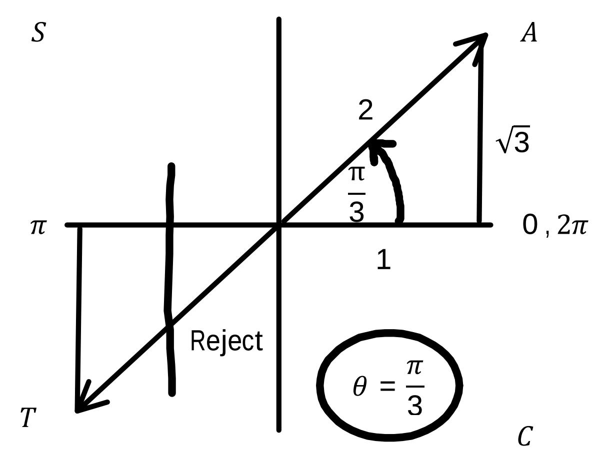
Function Range

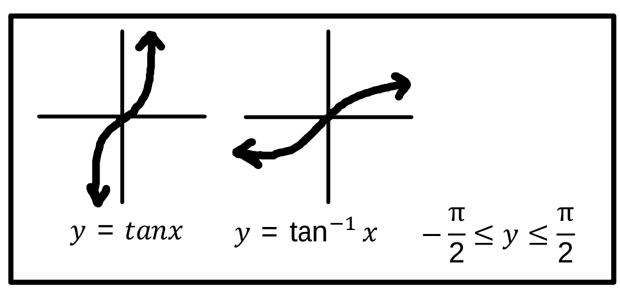




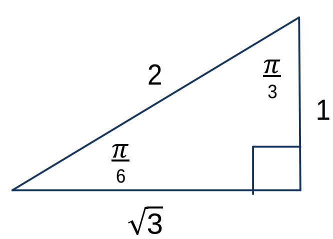
$\tan^{-1}(\sqrt{3}) = ?$

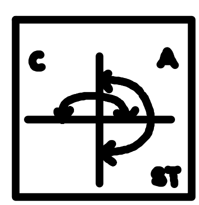
$$\tan\theta = \sqrt{3}$$





Function Range





C12 - 2.10 - Inverse Derivatives Notes

One over the putting the inverse into the derivative.

Find the Derivative of the Inverse at $y = 9$

$f(x) = x^3 + 1$ $\quad (f^{-1})'(9) = ?$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

Inverse Derivative Formula Proof

$$f^{-1}(f(x)) = x$$
$$f'(f^{-1}(x)) \times (f^{-1})'(x) = 1$$
$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

Find the x value when $y = 9$

$$(f^{-1})'(9) = \frac{1}{f'(f^{-1}(9))} = \frac{1}{f'(2)} = \frac{1}{12}$$

$f^{-1}(9) = 2$

$f^{-1}(y) = x$ Inverse Notation

$(2,9)$

$y = x^3 + 1$
$9 = x^3 + 1$
$x^3 = 8$
$x = 2$

$f(2) = 9$

$f(x) = x^3 + 1$
$f'(x) = 3x^2$ Derivative

$f'(x) = 3x^2$
$f'(2) = 3(2)^2$
$f'(2) = 12$

Take Derivative and Substitute 2 in for x

$f(x) = x^3 + 1$
$y = x^3 + 1$
$x = y^3 + 1$
$y = \sqrt[3]{x-1}$
$f^{-1}(x) = \sqrt[3]{x-1}$

$$f^{-1}(x) = \sqrt[3]{x-1}$$
$$(f^{-1})'(x) = \frac{1}{3\sqrt[3]{(x-1)^2}}$$
$$(f^{-1})'(9) = \frac{1}{3\sqrt[3]{(9-1)^2}}$$
$$= \frac{1}{12}$$

Definition of Inverse

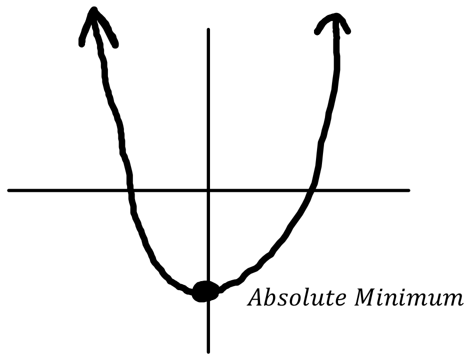
$f(g(x)) = x$
Means
$g(x) = f^{-1}(x)$

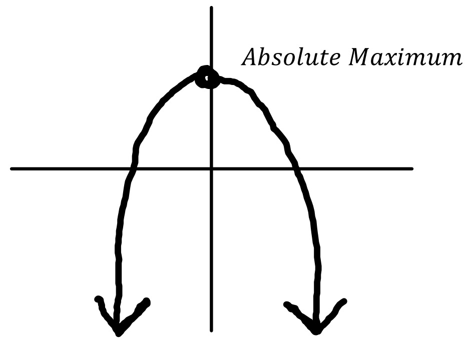
$f(f^{-1}(x)) = x$
$f^{-1}(f(x)) = x$

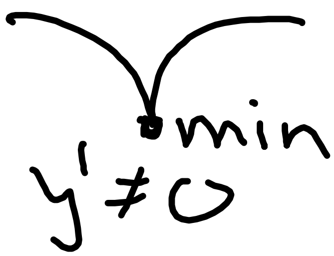
$sin\theta = \frac{o}{h}$
$\theta = \sin^{-1}\left(\frac{o}{h}\right)$

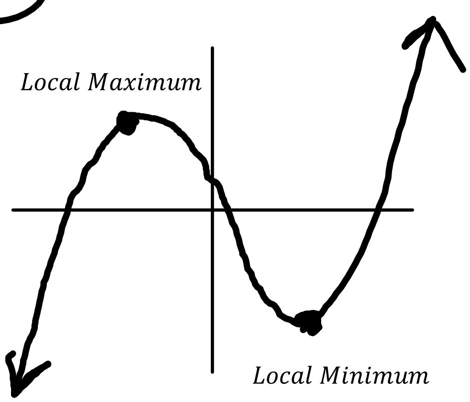
$$\frac{d}{dx}f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$$

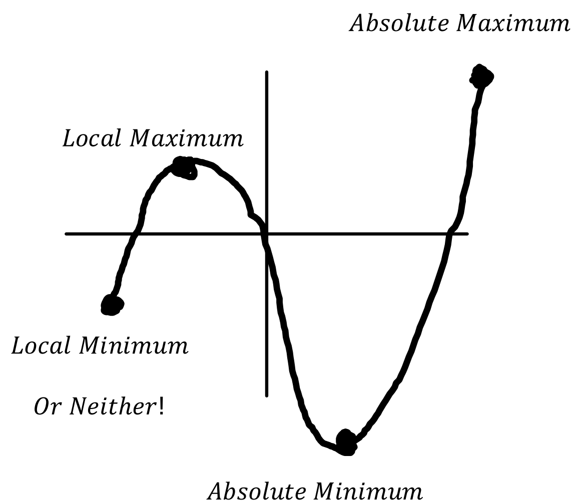
C12 - 3.1 - Absolute/Local Max/Min Notes

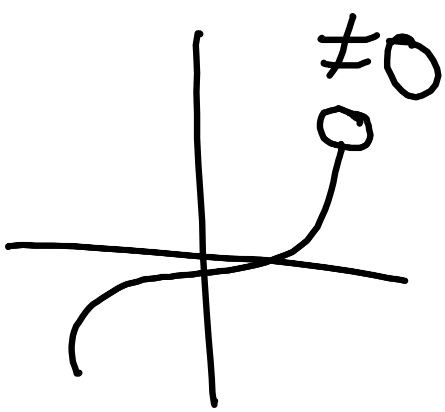


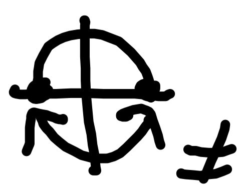


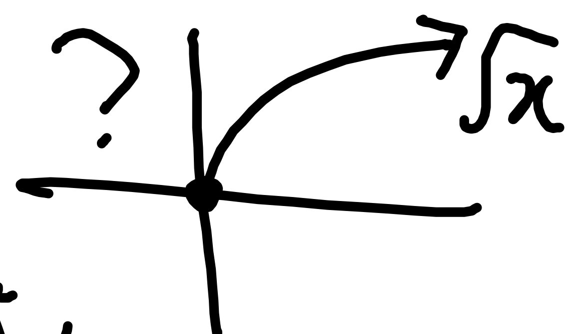












C12 - 3.1 - 1st Derivative Test: Critical Points Notes

$y' = f'(x)$

Find the critical points. Find the 1st derivative and set it equal to zero. Draw a graph and show the location of the horizontal slopes. Identify any maximums or Minimums and Intervals of Increase or Decrease.

$y'\ Test$

$y = x^3 - 12x$

$y' = 3x^2 - 12$ Find the 1st Derivative

$0 = 3x^2 - 12$ Set the Derivative equal to Zero

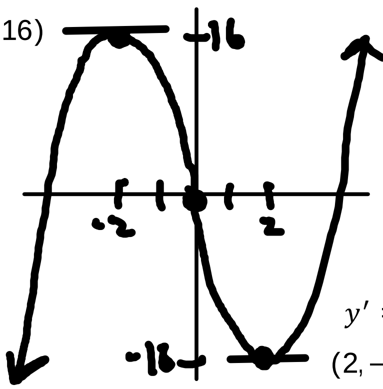
$3x^2 = 12$

$x^2 = 4$

Solve: Critical Values

$x = \pm 2$

$Critical\ Point$ $y' = 0$ $(-2,16)$



$y\ int = (0,0)$

$y = x^3 - 12x$

$y = (0)^3 - 12(0)$

$y = 0$

$y' = 0$

$(2,-16)$ $Critical\ Point$

Prove the 1st derivative is positive to the left of -2. Negative between -2 and 2. And positive to the right of 2.

1st Derivative test

	-5	-2	0	2	5
y'	$+$	0	$-$	0	$+$
y	↗	→	↘	→	↗
y		$(-2,16)$ Max		$(2,-16)$ Min	

$y' = 3x^2 - 12$

$f'(-5) = 3(-5)^2 - 12$

$f'(-5) = +$

$y' = 3x^2 - 12$

$f'(0) = 3(0)^2 - 12$

$f'(0) = -$

$y' = 3x^2 - 12$

$f'(5) = 3(5)^2 - 12$

$f'(5) = +$

$y = x^3 - 12x$

$f(2) = (2)^3 - 12(2)$

$f(2) = -16$

$y = x^3 - 12x$

$f(-2) = (-2)^3 - 12(-2)$

$f(-2) = -16$

$Increasing: (-\infty, -2) U(2, \infty)$ $Decreasing: (-2,2)$

C12 - 3.2 - 2st Derivative Test: Inflection Point Notes

$y'' = f''(x)$

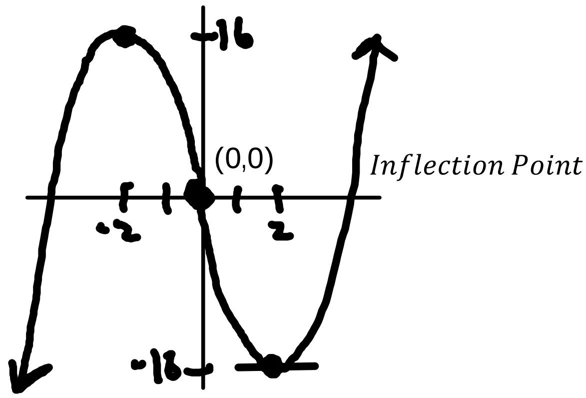
Find the critical points. Find the 2nd derivative and set equal to zero. Draw a graph and show the location of the Inflection Points and the Intervals of Concavity.

y'' Test

$y = x^3 - 12x$
$y' = 3x^2 - 12$
$y'' = 6x$
$0 = 6x$

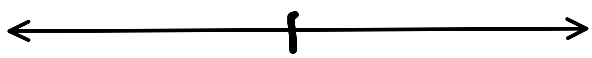
Find the 2nd Derivative
Set the Derivative equal to Zero
Solve: Critical Values

$x = 0$



Prove 2nd derivative is negative to the left of 0 and positive to the right of 0.

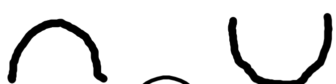
-1 0 1



2nd Derivative test

y'' — 0 +

y



y Max (0,0) IP Min

$Concave\ Down$ $Concave\ Up$

$IP: Inflection\ Point$

$y'' = 6x$
$f''(-1) = 6(-1)$

$f''(-1) = -$

$y'' = 6x$
$f''(-1) = 6(-1)$

$f''(-1) = +$

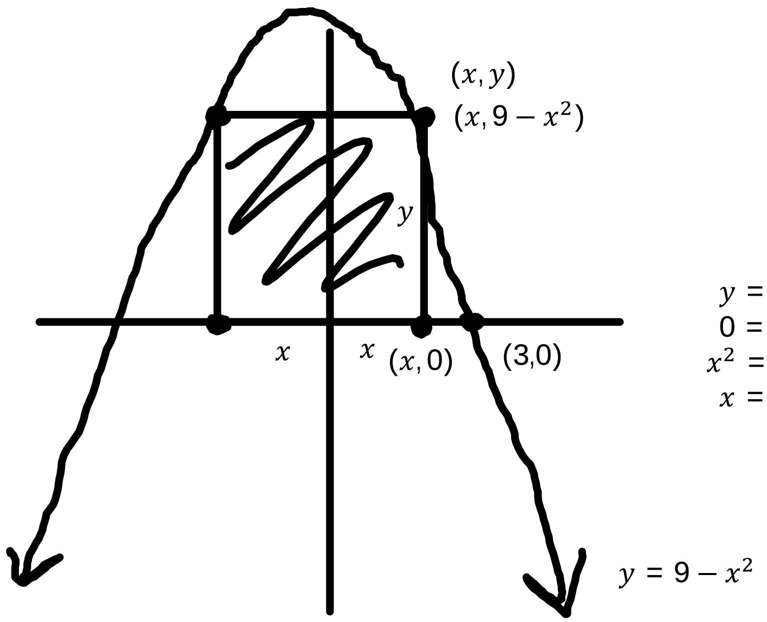
$y = x^3 - 12x$
$y = (0)^3 - 12(0)$

$y = 0$

$Concave\ Down: (-\infty, 0)$

$Concave\ Up: (0, \infty)$

C12 - 3.3 - Max Area Rectangle under Curve Notes



$y = 9 - x^2$
$0 = 9 - x^2$
$x^2 = 9$
$x = \pm 3$

$A = lw$
$A = (2x)(y)$
$A = 2x(9 - x^2)$
$A = 18x - 2x^3$
$A' = 18 - 6x^2$
$0 = 18 - 6x^2$
$6x^2 = 18$
$x^2 = 3$
$x = \pm\sqrt{3}$

	1	$\sqrt{3}$	2
A'	+	0	−
A	↗		↘
A		$(\sqrt{3}, 12\sqrt{3})$ (Max)	

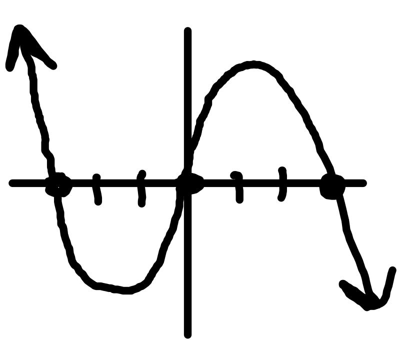
$x \geq 0 \quad x \leq 3$

$0 \geq x \geq 3$

$A' = 18 - 6x^2$
$A' = 18 - 6$
$A' = +$

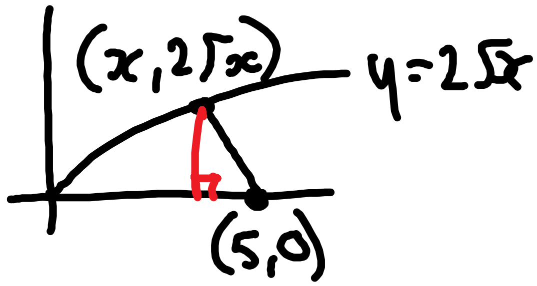
$A' = 18 - 6x^2$
$A' = 18 - 24$
$A' = -$

$A = -2x(x + 3)(x - 3)$



C12 - 3.3 - Min Distance to Curve Notes

Find the point on the graph $2\sqrt{x}$ with the minimum distance to the point (5,0)



$(6,0)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$c = \sqrt{a^2 + b^2}$$

$$d = \sqrt{(x-5)^2 + (2\sqrt{x} - 0)^2}$$

$$d = \sqrt{+25 + 4x \; x^2 - 10x}$$

$$d = \sqrt{x^2 - 6x + 25}$$

$$d = (x^2 - 6x + 25)^{1/2}$$

$$y = \sqrt{f(x)}$$

$$y' = \frac{f'(x)}{2\sqrt{f(x)}}$$

$$d' = \frac{2x - 6}{2\sqrt{x^2 - 6x + 25}}$$

$$0 = \frac{2x - 6}{2\sqrt{x^2 - 6x + 25}}$$

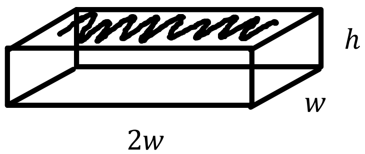
$x = 3$

	1	3	4
y'	$-$		$+$

MIN ✓

$(3, 2\sqrt{3})$

C12 - 3.3 - Min Rect/Cube Area Cost Notes



$V = 8$
$L = 2w$

$Cost_{Base} = \frac{\$4.5}{m^2}$ $\quad Cost_{Sides} = \frac{\$6}{m^2}$ $\quad Cost = Area \times \frac{Cost}{Area}$

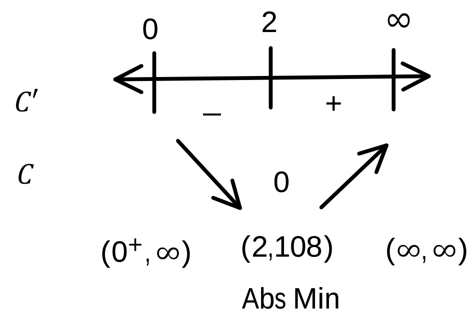
$\boxed{v = Lwh}$

$V = Lwh$
$8 = 2w^2h$
$4 = w^2h$
$h = \frac{4}{w^2}$

$SA^* = 2w^2 + 6wh$

$SA = 2w^2 + 6w\left(\frac{4}{w^2}\right)$
$SA = 2w^2 + 24w^{-1}$
$C = 2w^2 \times 4.5 + 24w^{-1} \times 6$
$C = 9w^2 + 144w^{-1}$
$C' = 18w - 144w^{-2}$
$0 = 18w(1 - 8w^{-3})$

$w = 0$ $\quad 1 - 8w^{-3} = 0$
$1 = \frac{8}{w^3}$
$w^3 = 8$
$w = \sqrt[3]{8}$



Domain
$0 > x > \infty$

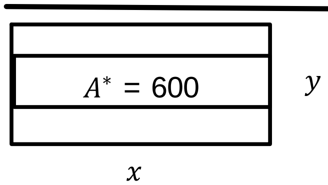
$h = \frac{4}{2^2}$

$h = 1$

$w = 2$

$C = 9w^2 + 144w^{-1}$
$C = 9(2)^2 + 144(2)^{-1}$

$C = 108$



$\boxed{A = lw}$ $\quad \boxed{C = \frac{\$60}{m}}$

$A = lw$
$A = xy$
$600 = xy$
$y = \frac{600}{x}$

$P = 2y + 4x$
$C = 2y \times 60 + 4x \times 60$

Perimeter
Cost

$C = 2\left(\frac{600}{x}\right) \times 60 + 4x \times 60$
$C = \frac{72000}{x} + 240x$
$C = 72000x^{-1} + 240x$
$C' = -72000x^{-2} + 240$
$C' = -\frac{72000}{x^2} + 240$
$0 = -\frac{72000}{x^2} + 240$
$\frac{72000}{x^2} = 240$

Derivative

Derivative=0

$x = \sqrt{300}$

$y = \frac{600}{x}$
$y = \frac{600}{\sqrt{300}}$
$y = \frac{600}{10\sqrt{3}}$
$y = \frac{60}{\sqrt{3}}$

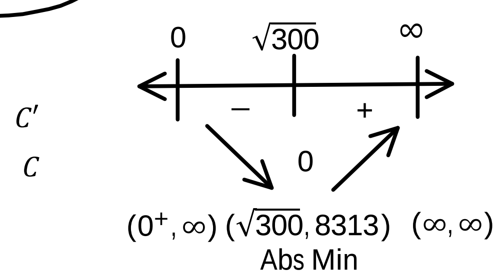
$y = 20\sqrt{3}$

Check Answer

$A = xy$
$A = 20\sqrt{3} \times \sqrt{300}$
$A = 20\sqrt{3} \times 10\sqrt{3}$

$A = 600$

$\boxed{Cost = Length \times \frac{Cost}{length}}$



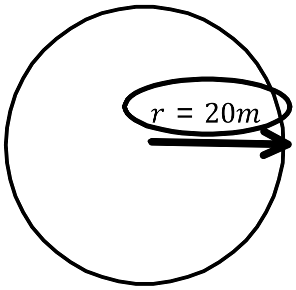
Domain
$0 > x > \infty$

*C12 - 4.1 - Related Rates Circle/Sphere A/V Notes

Find the rate of change.

The radius of a circle is growing at a rate of 4 m/s. What is the rate at which the area within the circle is changing when the radius is 20m?

$\frac{dr}{dt} = 4$ $\qquad$ $\left.\frac{dA}{dt}\right|_{r=20} = ?$

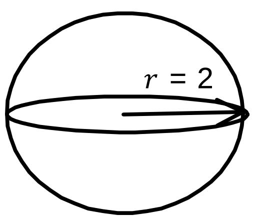


$$A = \pi r^2$$
$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$
$$\frac{dA}{dt} = 2\pi r \cdot (4)$$
$$\frac{dA}{dt} = 8\pi r$$
$$= 8\pi(20)$$
$$= 160\pi$$
$$\frac{dA}{dt} = 160\pi \frac{m^2}{s}$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} \qquad \frac{dA}{dr} = 2\pi r \times \frac{dr}{dr}$$

$$\frac{dr}{dr} = 1$$

The volume of a balloon is increasing at 256 meters cubed per second. How fast is the radius increasing when the radius is two meters?



$\frac{dV}{dt} = 256 \frac{m}{s^3}$ $\qquad$ $\left.\frac{dr}{dt}\right|_{r=2} = ?$

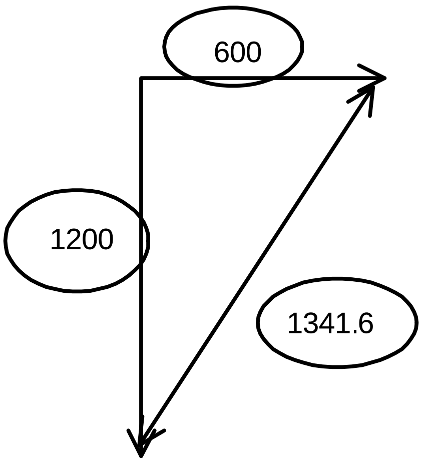
$$V = \frac{4}{3}\pi r^3$$
$$\frac{dV}{dt} = 3 \times \frac{4}{3}\pi r^{3-1} \frac{dr}{dt}$$
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$
$$256 = 4\pi(2)^2 \frac{dr}{dt}$$
$$\frac{dr}{dt} = \frac{16}{\pi}\frac{m}{s}$$

Therefore the radius is changing at $\frac{16}{\pi}\frac{m}{s}$ when the radius is 2 m.

Therefore the area is changing at a rate of $160\pi \frac{m^2}{s}$ when the radius is $20m$.

C12 - 4.2 - Train Pythag/Spotlight Sim Tri Rel Rat Notes

Train 'a' leaves Vancouver heading South at 10 m/s and train 'b' leaves heading East at 5 m/s? How far are they a part after two minutes? What is the speed at which the trains are moving apart at that time?



$\frac{da}{dt} = 10$ $\quad \frac{db}{dt} = 5$ $\quad \frac{dc}{dt}|_{t=2} = ?$

$a^2 + b^2 = c^2$

$1200^2 + 600^2 = c^2$

$c = 1341.6$

$a^2 + b^2 = c^2$

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$

$2(1200)(10) + 2(600)(5) = 2(1341.6)\frac{dc}{dt}$

$30000 = 2683.2\frac{dc}{dt}$

$\frac{dc}{dt} = 11.1\frac{m}{s}$

$2\ minutes = 120\ seconds$

$a = vt$

$a = 10 \times 120$

$a = 1200$

$b = vt$

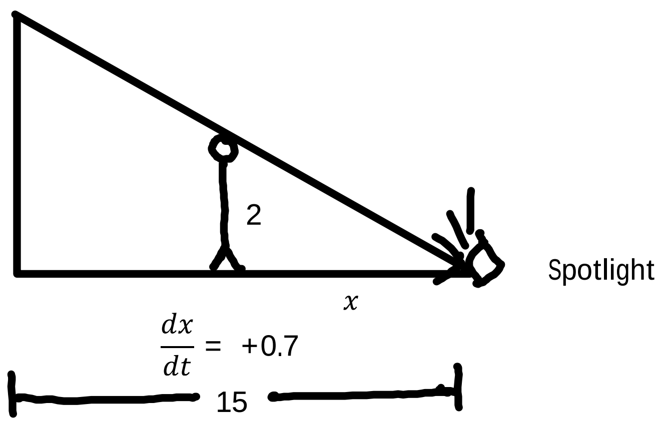
$b = 5 \times 120$

$b = 600$

$d = vt$

A 2 m tall person is walking away from a spotlight, 15 m from a wall, towards the wall at 0.7 m/s. How fast is the shadow on the wall changing when they are 7 m from the spotlight?

$\frac{dy}{dt}|_{x=7} = ?$



$\frac{dx}{dt} = +0.7$

$\frac{y}{15} = \frac{2}{x}$

$xy = 30$

$\frac{dx}{dt}y + \frac{dy}{dt}x = 0$

$0.7(4.29) + \frac{dy}{dt}(7) = 0$

$\frac{dy}{dt} = -\frac{0.7(4.29)}{7}$

$\frac{dy}{dt} = -0.429\frac{m}{s}$

$xy = 30$

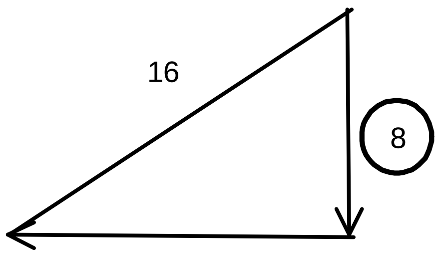
$7y = 30$

$y = \frac{30}{7}$

$y = 4.29$

C12 - 4.2 - Ladder Trig Related Rates Notes

The top of a 16 ft ladder slides down a wall at a rate of 3 ft/s. At what rate is the base of the ladder sliding away from the wall when the latter is at a height of 8 ft on the wall.



$$\frac{dy}{dt} = -3\frac{ft}{s}$$

*Length is shrinking: Derivative is Negative.

$$\frac{dx}{dt}\Big|_{y=8} = ?$$

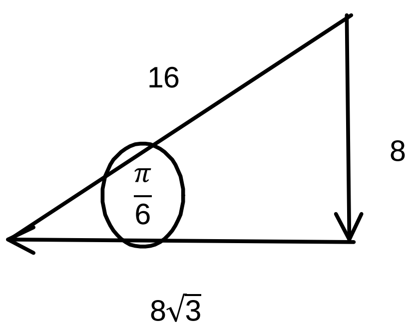
$$x^2 + y^2 = c^2$$
$$x^2 + 8^2 = 16^2$$
$$x = \sqrt{16^2 - 8^2}$$
$$x = \sqrt{192}$$
$$x = 8\sqrt{3}$$

$$x^2 + y^2 = c^2$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2c\frac{dc}{dt}$$

*We can substitute constants into the formula

$$2(8\sqrt{3})\frac{dx}{dt} + 2(8)(-3) = 0$$
$$\frac{dx}{dt} = \frac{3}{\sqrt{3}}$$
$$\frac{dx}{dt} = \sqrt{3}\frac{ft}{s}$$

What is the rate the angle at the bottom of the ladder changing?



$$\sin\theta = \frac{8}{16}$$
$$\theta = \sin^{-1}(\frac{1}{2})$$
$$\theta = \frac{\pi}{6}$$

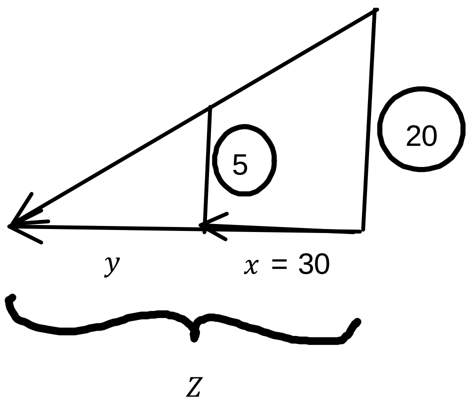
$$\cos\theta = \frac{x}{r}$$
$$\cos\theta = \frac{x}{16}$$
$$-\sin\theta\frac{d\theta}{dt} = \frac{1}{16}\frac{dx}{dt}$$
$$-\frac{8}{16}\frac{d\theta}{dt} = \frac{1}{16}\sqrt{3}$$
$$\frac{d\theta}{dt} = -\frac{\sqrt{3}}{8}\frac{rad}{s}$$

*I used cos because it used the rate I already solved on the top. Using sin and tan is possible but much more difficult based on the information and previously solved. We want our constant on the bottom.

*Real life is in Radians. Degrees are for children.

C12 - 4.2 - Similar Triangles/Cos Law Related Rates Notes

A 5 foot tall woman is walking away from a 20 foot lamp post at 3 m/s. What rate is her shadow increasing when she is 30 feet from the lamp post; and is her shadow getting bigger or smaller. How fast is the tip of her shadow moving?

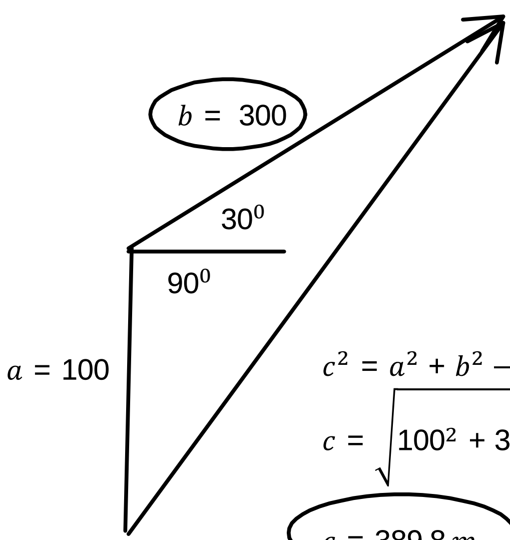


$$\frac{dx}{dt} = 3\frac{m}{s} \qquad \left.\frac{dy}{dt}\right|_{x=30} = ?$$

$$\frac{5}{20} = \frac{y}{x+y}$$
$$5x + 5y = 20y$$
$$5x = 15y$$
$$x = 3y$$
$$\frac{dx}{dt} = 3\frac{dy}{dt}$$
$$3 = 3\frac{dy}{dt}$$
$$\frac{dy}{dt} = 1\frac{ft}{s}$$

$$\frac{dz}{dt} = \frac{dy}{dt} + \frac{dx}{dt}$$
$$\frac{dz}{dt} = 1 + 3$$
$$\frac{dz}{dt} = 4\frac{m}{s}$$

A float plane rising at 30 degrees above the horizontal flies over a boat at an altitude of 100 m at 60 m/s. How fast is the distance between the boat and the plane increasing after five seconds?



$$\frac{db}{dt} = 60 \qquad \left.\frac{dc}{dt}\right|_{t=5} = ?$$

$$v = \frac{d}{t}$$
$$d = vt$$
$$d = 60 \times 5$$
$$d = 300\ m$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c = \sqrt{100^2 + 300^2 - 2(100)(300)\cos\frac{7\pi}{6}}$$
$$c = 389.8\ m$$

*Word Problems in Radians $120^0 = \frac{7\pi}{6}$

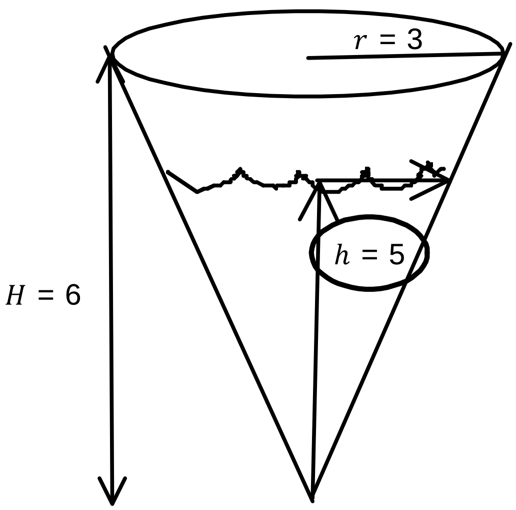
*That would have been a tough product rule if more things were changing

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$2c\frac{dc}{dt} = 0 + 2b\frac{db}{dt} - 2a\cos C\frac{db}{dt}$$
$$2(389.8)\frac{dc}{dt} = 0 + 2(300)(60) - 2(100)\left(-\frac{\sqrt{3}}{2}\right)(60)$$
$$\frac{dc}{dt} = 59.5\frac{m}{s}$$

C12 - 4.3 - Cone V/Similar Triangles Related Rates Notes

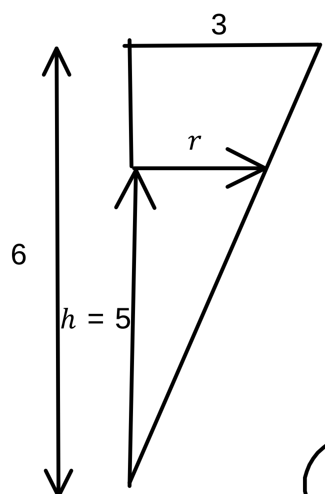
Find the rate of change.

A cone with a radius of 3 cm and height of 6 cm is filling with water where the height of the water level is increasing at a rate of 0.2 cm/s. What is the rate the volume is increasing when the height of the water level is 5 cm.



$$\frac{dh}{dt} = 0.2$$

$$\frac{dV}{dt}\Big|_{h=5} = ?$$



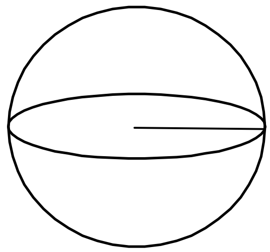
$$\frac{H}{R} = \frac{h}{r}$$
$$\frac{6}{3} = \frac{h}{r}$$
$$2 = \frac{h}{r}$$
$$r = \frac{h}{2}$$

$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$$
$$V = \frac{1}{12}\pi h^3$$
$$\frac{dV}{dt} = 3 \times \frac{1}{12}\pi h^2 \frac{dh}{dt}$$
$$\frac{dV}{dt} = \frac{1}{4}\pi (5)^2 (0.2)$$
$$\frac{dV}{dt} = \frac{5\pi}{4}\frac{cm}{s}$$

*We can't take this product so we must use similar triangles/other info

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(2r\frac{dr}{dt}h + \frac{dh}{dt}r^2\right)$$

C12 - 4.4 - Sphere Tight Rope Notes



$\frac{dV}{dt} = ?$

$\frac{dr}{dt}\Big|_{SA=20} = 2$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi \left(\sqrt{\frac{100}{4\pi}}\right)^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi \times \frac{100}{4\pi} \times 2$

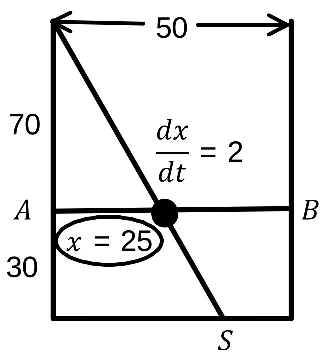
$\frac{dV}{dt} = 200\frac{m^3}{s}$

$SA = 4\pi r^2$

$100 = 4\pi(2)^2$

$r = \sqrt{\frac{100}{4\pi}}$

$r = \frac{10}{2\sqrt{\pi}}\ m$



$\frac{dS}{dt}\Big|_{x=25} = ?$

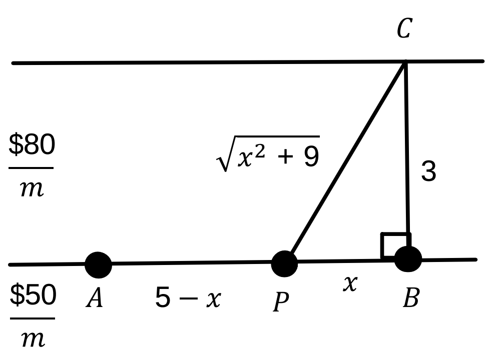
$\frac{x}{70} = \frac{S}{100}$

$x = \frac{7}{10}S$

$\frac{dx}{dt} = \frac{7}{10}\frac{dS}{dt}$

$2 = \frac{7}{10}\frac{dS}{dt}$

$\frac{dS}{dt} = \frac{20}{7}\frac{m}{s}$



$C = 50(5 - x) + 80\sqrt{x^2 + 9}$

$C' =$

$0 =$

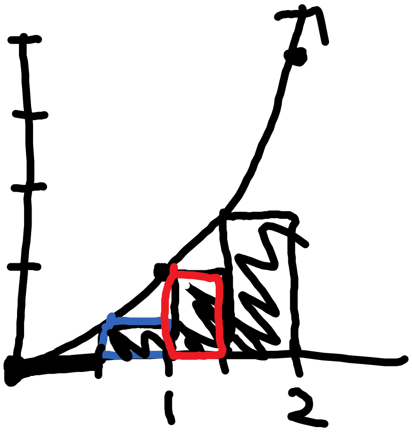
Number Line

$Cost = length \times \frac{cost}{length}$

C12 - 5.1 - Int Reimann's L/R/M RAM & Trap Notes

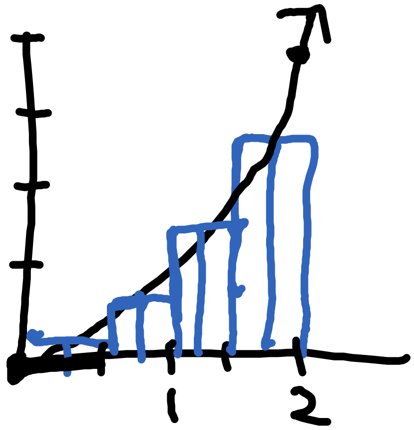
$n = \# \, rectangles$

Find the area under the graph $y = x^2$ from zero to two using four (n=4) rectangles. Using Riemann's LRAM, MRAM & RRAM, and Trapezoidal Rule .



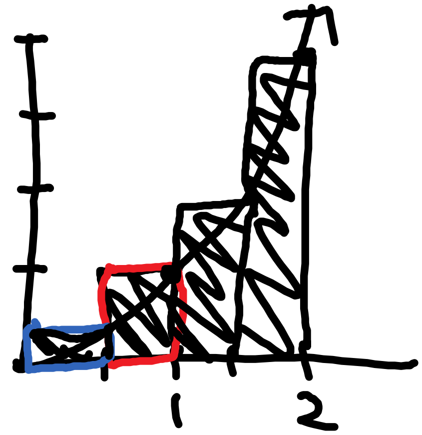
$LEFT \; RAM$

$Height \; is \; LEFT$
$y - value \; of \; section$



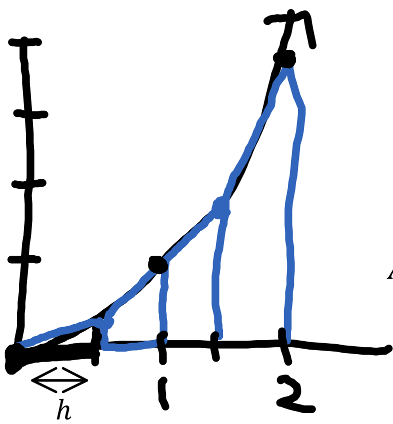
$MIDRAM$

$Height \; is \; MID$
$y - value \; of \; section$



$RIGHT \; RAM$

$Height \; is \; Right$
$y - value \; of \; section$

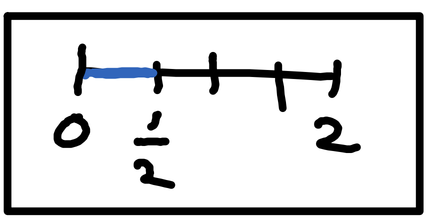


$$A_{TRAP} = \left(\frac{y_0 + y_1}{2}\right) h$$

$$A_{TRAP} = \frac{LRAM + RRAM}{2}$$

$$A_{TRAP} = \left(\frac{y_0 + y_1}{2}\right) h + \left(\frac{y_1 + y_2}{2}\right) h + \left(\frac{y_2 + y_3}{2}\right) h \ldots + \left(\frac{y_{n-1} + y_n}{2}\right) h$$

$$A_{TRAP} = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \cdots 2y_{n-1} + y_n)$$



Factor Out $\frac{h}{2}$
$1\, y_0, 2\, y_1's, 2y_2's, \ldots 2y_{n-1}'s, 1y_n$

$h = horizontal \; width$

x	y

Simpsons

$$A_{SIMP} = \frac{h}{3}(y_0 + 4y_1 + 2y_2 + \cdots 2y_{n-2} + 4y_{n-1} + y_n)$$

Rotates 1-4-2-4-2-4-1

$$h = \frac{b - a}{n} = \frac{2 - 0}{4} = \frac{1}{2}$$

C12 - 5.2 - Int Indefinite Notes

C!

Integral: The Anti-Derivative. Who's Derivative is this? Take the Derivative to Check your Answer.

Constant Rule

$$\int k\,dx = kx + c$$

$(k\text{: a constant})$

Examples

$$\int 5dx = 5\int dx$$

$$= 5x + C$$

Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$(n \neq -1)$

$$\int x^2 dx = \frac{x^{2+1}}{3} + C$$

$$= \frac{x^3}{3} + C$$

$$\int x\,dx = \frac{x^2}{2} + C$$

$$y = \frac{x^2}{2} + C$$
$$y' = \frac{2x}{2}$$
$$y = x$$

$$\int 5x\,dx = 5\int x\,dx$$

$$= \frac{5x^2}{2} + C$$

$$\int \sqrt{x}\,dx = \int x^{\frac{1}{2}} dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$\frac{1}{2} + 1 = \frac{1}{2} + \frac{2}{2} = \frac{3}{2}$$

$$= \frac{2x^{\frac{3}{2}}}{3} + C$$

$$\int 3x^2 dx = \frac{3x^{2+1}}{3} + C$$

$$= x^3 + C$$

$$\int (x^2 + 5)\,dx = \frac{x^3}{3} + 5x + C$$

$$\int (x+2)^2 dx = \int (x^2 + 4x + 4)dx$$

$$= \frac{x^3}{3} + \frac{4x^2}{2} + 4x + C$$

$$= \frac{x^3}{3} + 2x^2 + 4x + C$$

$$\int \frac{x^2 + 2x}{x} dx = \int (x+2)\,dx$$

$$= \frac{x^2}{2} + 2x + c$$

C12 - 5.3 - Int Area Notes

Find the Area under the curve using Integration. Confirm the Area by geometry if possible.

$y = x$ $\quad 0 \le x \le 4$

$$A = \int_a^b f(x)dx$$

$$A = \int_0^4 xdx = \frac{x^2}{2} \Big|_0^4$$

$$= \frac{(4)^2}{2} - \frac{(0)^2}{2}$$

$$= 8$$

FUNDAMENTAL THEOREM OF CALCULUS

$$A = \int_a^b f(x)dx = F(b) - F(a)$$

$F(x)$ *is the antiderivative of* $f(x)$

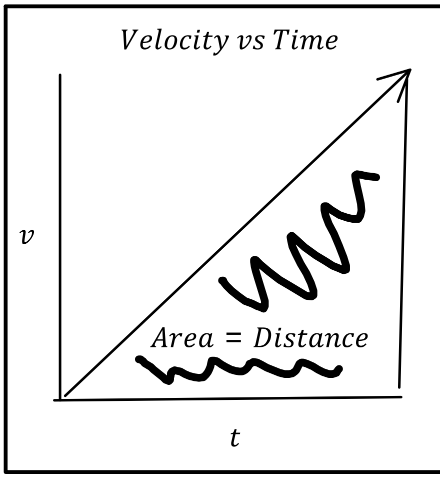
Check with math 9

Check by Geometry

$$A = \frac{bh}{2}$$

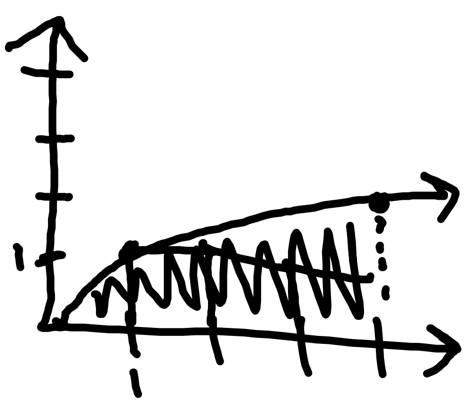
$$A = \frac{4 \times 4}{2}$$

$$A = 8$$



Find the area under the curve using Integration. Confirm the area by geometry if possible.

$y = \sqrt{x} = x^{\frac{1}{2}}$ $\quad 0 \le x \le 4$



$$\int_0^4 x^{\frac{1}{2}}dx = \frac{2x^{\frac{3}{2}}}{3} \Big|_0^4$$

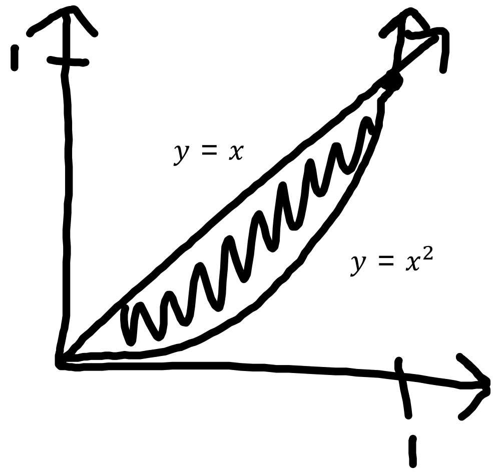
$$= \frac{2(4)^{\frac{3}{2}}}{3} - \frac{(0)^{\frac{3}{2}}}{2}$$

$$= \frac{16}{3}$$

C12 - 5.3 - Int Area Between Notes

Find the area between the curves using Integration.

$y = x$ $\qquad$ $y = x^2$



Find Intersections

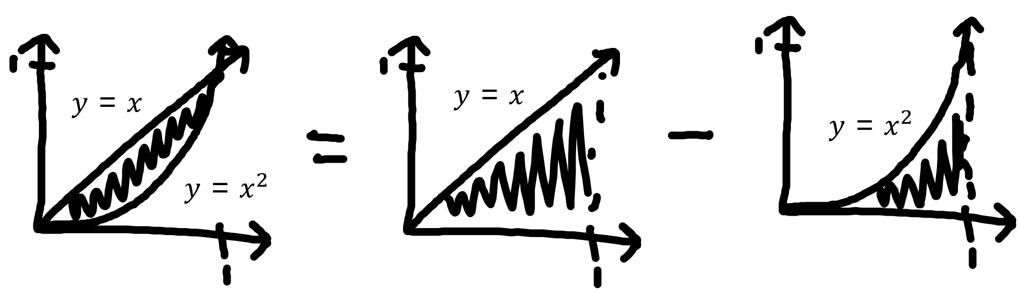
$x = x^2$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0 \qquad x = 1$

$\therefore \int_0^1 f(x)$



$$\int_0^1 (f_{upper} - f_{lower})dx = \int_0^1 (x - x^2)dx$$

$$= \frac{x^2}{2} - \frac{x^3}{3} \Big|_0^1$$

$$= \frac{1^2}{2} - \frac{1^3}{3} - \left(\frac{0^2}{2} - \frac{0^3}{3}\right)$$

$$= \frac{1}{2} - \frac{1}{3}$$

$$= \frac{1}{6}$$

$$\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$$

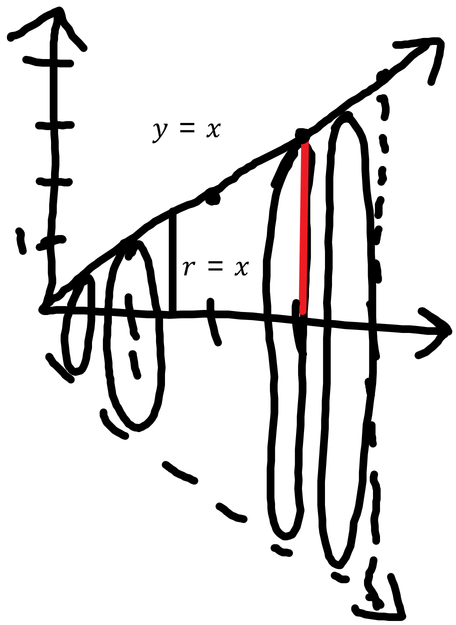
If either function is below the x-axis, subtracting a negative area adds the area. Don’t forget to distribute any negatives.

X-LAND

C12 - 5.4 - Int Volume Notes

Find the Volume of revolution by Integration. Confirm the Volume by geometry if possible.

$y = x$ $\qquad 0 \le x \le 4$



$$V = \int_a^b A(x)dx = \int_a^b \pi r^2 dx$$

$$= \pi \int_0^4 x^2 dx$$

$$= \pi \frac{x^3}{3} \Big|_0^4$$

$$= \pi\left(\frac{4^3}{3} - \frac{0^3}{3}\right)$$

$$= \frac{64\pi}{3}$$

Volume

$$V = \int_a^b A(x)dx$$

$A(x) = \pi r^2$

$r = y = x$

radius is the y height

Check by Geometry

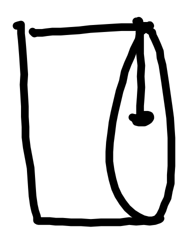
$$V_{cone} = \frac{1}{3}\pi r^2 h$$

$$V_{cone} = \frac{1}{3}\pi 4^2 4$$

$$V_{cone} = \frac{64\pi}{3}$$

SHELLS $\quad$ f h $\quad$ YLAND

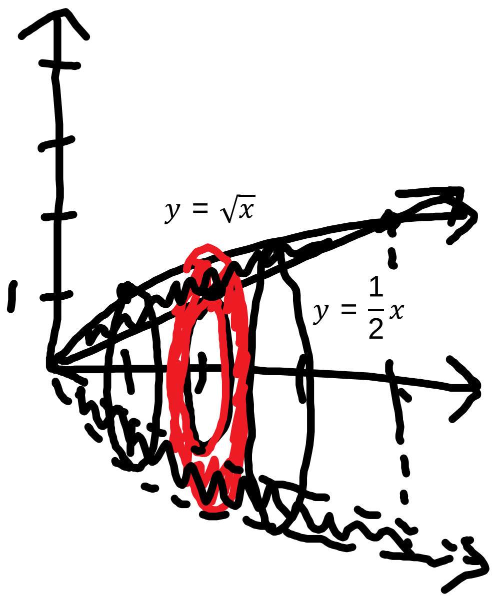
$$V = 2\pi \int_0^4 y \cdot y \, dy$$

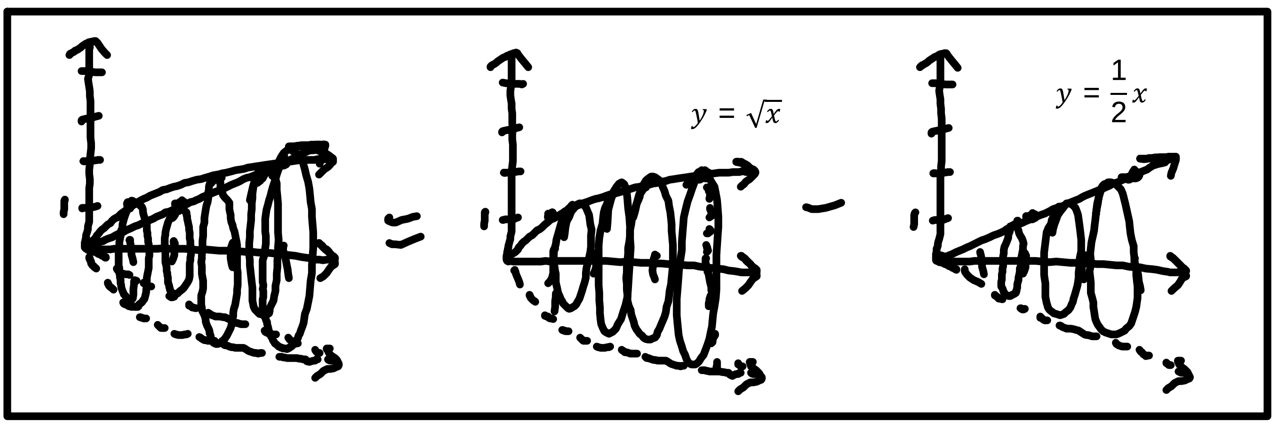


C11 - 5.4 - Int Volume Discs Notes

Find the Volume of revolution between the two functions around the x-axis by Integration.

$y = \sqrt{x}$ $\qquad y = \frac{1}{2}x$ $\qquad 0 \le x \le 4$





$$V = \pi\int_0^4\left((r_{outer})^2 - (r_{inner})^2\right)dx = \pi\int_0^4\left((\sqrt{x})^2 - \left(\frac{1}{2}x\right)^2\right)dx$$

$$= \pi\int_0^4\left(x - \frac{1}{4}x^2\right)dx$$

$$= \pi\left(\frac{x^2}{2} - \frac{x^3}{12}\right)\Big|_0^4$$

$$= \pi\left(\frac{4^2}{2} - \frac{4^3}{12} - \left(\frac{0^2}{2} - \frac{0^3}{12}\right)\right)$$

$$= \frac{8\pi}{3}$$