# M10 - 4.1 - Entire to Mixed Radicals Notes

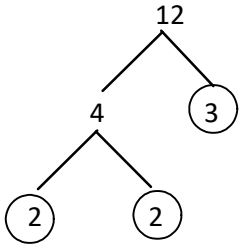
**Simplify** **Prime Factorization**

$\sqrt[2]{12} = \sqrt[2]{2 \times 2 \times 3}$

$= 2\sqrt[2]{3}$

Check Answer

$3.46 = 3.46$ ✓




$\sqrt[2]{2 \times 2} = \sqrt[2]{4} = 2$

Two Identical Numbers Under a Square Root: One on Outside

**Perfect Squares**

$\sqrt[2]{12} = \sqrt[2]{4 \times 3}$

$= \sqrt[2]{4} \times \sqrt[2]{3}$

$\frac{12}{4} = 3$

$= 2\sqrt[2]{3}$

Find Two Numbers that Multiply to the Number Underneath the Square Toot such that you know the Square Root of One of them.

**Perfect Squares**

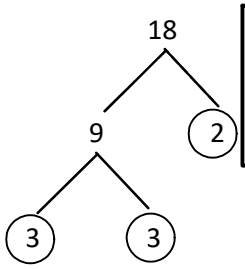
1,4,9,16,25,36,49 …

$\sqrt[2]{18} = \sqrt[2]{3 \times 3 \times 2}$

$= 3\sqrt[2]{2}$

Check Answer

$4.25 = 4.24$ ✓




Check on Calculator

2nd $\sqrt[2]{\square}$ $\sqrt[3]{\square}$

Ti84 Math $\sqrt[x]{\square}$

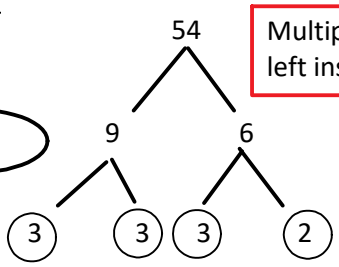
$\sqrt[2]{54} = \sqrt[2]{3 \times 3 \times 3 \times 2}$

$= 3\sqrt[2]{3 \times 2}$

$= 3\sqrt[2]{6}$

Check Answer

$7.35 = 7.35$ ✓




Multiply what's left inside.

**Cant Even Root a Negative**

$\sqrt[2]{-9} =$ No Solution

**Can Odd Root a Negative**

$\sqrt[3]{-27} = \sqrt[3]{-3 \times -3 \times -3}$

$= -3$

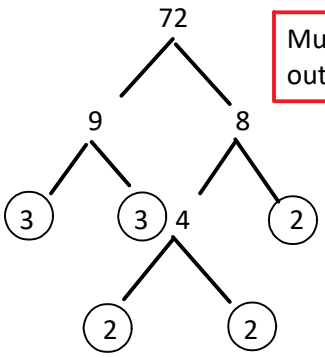
$\sqrt[2]{72} = \sqrt[2]{3 \times 3 \times 2 \times 2 \times 2}$

$= 3 \times 2\sqrt[2]{2}$

$= 6\sqrt[2]{2}$

Check Answer

$8.49 = 8.49$ ✓




Multiply numbers outside of root.

$5\sqrt[2]{12} = 5\sqrt[2]{2 \times 2 \times 3}$

$= 5 \times 2\sqrt[2]{3}$

$= 10\sqrt[2]{3}$

Check Answer

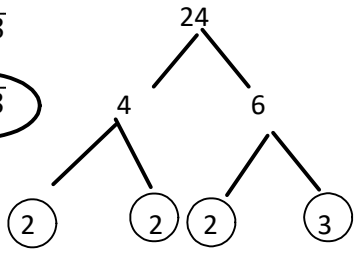
$17.32 = 17.32$ ✓

$\sqrt[3]{24} = \sqrt[3]{2 \times 2 \times 2 \times 3}$

$= 2\sqrt[3]{3}$

Check Answer

$2.88 = 2.88$ ✓




$\sqrt[3]{2 \times 2 \times 2} = \sqrt[3]{8} = 2$

Three Identical Numbers Under a Cube Root: One on Outside.

$\sqrt[3]{54} = \sqrt[3]{3 \times 3 \times 3 \times 2}$

$= 3\sqrt[3]{2}$ **See Above**

Check Answer

$3.78 = 3.78$ ✓

**Perfect Cubes**

$\sqrt[3]{24} = \sqrt[3]{8 \times 3}$

$= \sqrt[3]{8} \times \sqrt[3]{3}$

$\frac{24}{8} = 3$

$= 2\sqrt[3]{3}$

**Perfect Cubes**

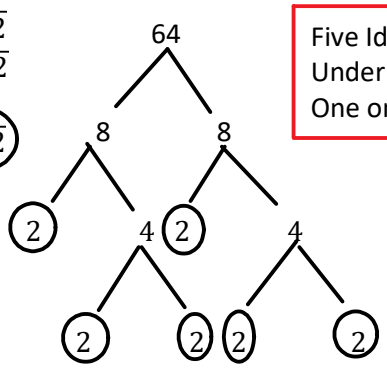
1,8,27,64,125,216 …

What are Two Numbers that Multiply to the Number Underneath the Cube Root that you know the Cube Root of One of them.

# M10 - 4.2 - Mixed to Entire/Variables Radicals Notes

**Simplify**

$\sqrt[5]{64} = \sqrt[5]{2\times2\times2\times2\times2\times2}$
$= \sqrt[5]{2\times2\times2\times2\times2\times2}$
$= 2\sqrt[5]{2}$




Five Identical Numbers Under a Fifth Root: One on Outside.

$5 = \sqrt{5^2}$
$5x = \sqrt{5^2x^2}$
$5 = \sqrt[3]{5^3}$

Check Answer
$2.30 = 2.30$ ✓

Check on Calculator OR

| | | | |
|---|---|---|---|
| 2nd | 5 | $\sqrt[x]{\square}$ | $x^{\frac{1}{y}}$ |
| Ti84 | Math | 5 | $\sqrt[x]{\square}$ |

$\sqrt[2]{x^2} = \sqrt[2]{x\times x}$
$= x$

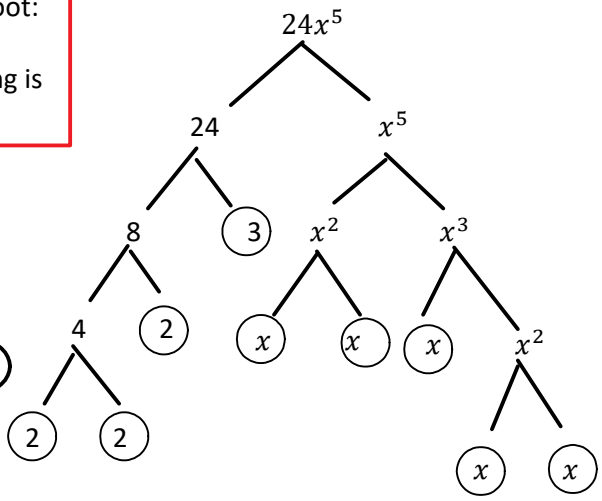
$x^2$

$x$ $x$

Two Identical Variables Under a Square Root: One Comes Out. Nothing is left.

Check Answer
$\sqrt[2]{5^2} = 5$
Arbitrary Number ✓

$\sqrt[3]{24x^5} = \sqrt[3]{2\times2\times2\times3\times x\times x\times x\times x\times x}$
$= 2x\sqrt[3]{3x^2}$




**Expand**

$5\sqrt[2]{2} = \sqrt[2]{5\times5\times2}$
$= \sqrt[2]{25\times2}$
$= \sqrt[2]{50}$

One number Outside of a Square Root: Two Inside.

Check Answer
$7.08 = 7.07$ ✓

$5\sqrt[3]{2} = \sqrt[3]{5\times5\times5\times2}$
$= \sqrt[3]{125\times2}$
$= \sqrt[3]{250}$

One Number Outside of a Cube root: Three Inside.

Check Answer
$8.55 = 8.55$ ✓

$-7\sqrt[2]{3} = -\sqrt[2]{7\times7\times3}$
$= -\sqrt[2]{49\times3}$
$= -\sqrt[2]{147}$

A Negative may Not go Inside an Even Root

Check Answer
$-12.12 = -12.12$ ✓

$-4\sqrt[5]{5} = \sqrt[5]{-4\times4\times4\times4\times4\times5}$
$= \sqrt[5]{-4^5\times5}$
$= \sqrt[5]{-5120}$

One Number Outside of a Fifth Root: Five Inside.

Check Answer
$-5.52 = -5.52$ ✓

A Negative may go Inside an Odd Root

# M10 - 4.3 - Add/Sub/Multiply Exponents Laws Notes

Coefficient → $4x^2$ ← Exponent

Base

**Remember:**
**-Never multiply the base by the exponent**
**-Must have same base to use laws.**

**Multiplying with the Same Base, Add Exponents.**

$x^3 \times x^2 = (x \times x \times x) \times (x \times x) = x^5$

$x^3 \times x^2 = x^{3+2} = x^5$ Add Exponents

Check Answer

$2^3 \times 2^2 = 32 = 2^5$ ✓

Arbitrary Numbers

**Dividing with the Same Base, Subtract Exponents.**

$\frac{x^5}{x^2} = \frac{x \times x \times x \times x \times x}{x \times x} = x^3$

$\frac{x^5}{x^2} = x^{5-2} = x^3$ Subtract Exponents

Check Answer

$\frac{3^5}{3^2} = 27 = 3^3$ ✓ Arbitrary Numbers

**Exponents to Exponents, Multiply Exponents**

$(x^2)^3 = (x \times x)^3 = (x \times x) \times (x \times x) \times (x \times x) = x^6$

$(x^2)^3 = x^{2\times3} = 2^6$ Multiply Exponents

Check Answer

$(5^2)^3 = 15625 = 5^6$ ✓

Arbitrary Numbers

Ultimately you will either use:

Exponent Laws

OR

Repeated Multiplication/Division Theory

**Product/Quotients to Exponents, Multiply Exponents**

$(x^1 \times y^1)^2 = x^2y^2$ Multiply Exponents

$(2x)^3 = (2x) \times (2x) \times (2x) = 8x^3$

$(2x)^3 = 2^3x^3 = 8x^3$

$\left(\frac{2^1x^1}{y^3}\right)^2 = \frac{2^2x^2}{y^{2\times3}} = \frac{4x^2}{y^6}$

Cannot distribute into a sum!

$(3+4)^2 \neq 3^2 + 4^2 = 25$

$(3+4)^2 = (3+4)(3+4) = 7 \times 7 = 7^2 = 49$

$\left(\frac{6mn^3}{4m^2n}\right)^3$ OR $\left(\frac{6mn^3}{4m^2n}\right)^3$

$\left(\frac{3n^2}{2m}\right)^3$ Simplify 1st

$\frac{3^3n^6}{2^3m^3}$

$\frac{27n^6}{8m^3}$

$\frac{6^3m^3n^9}{4^3m^6n^3}$ Multiply Exponents 1st

$\frac{216n^6}{64m^3}$

$\frac{27n^6}{8m^3}$

# M10 - 4.4 - Negative Exponents Laws Notes

Negative Exponents

$x^{-2} = \frac{1}{x^2}$ — *Bring to the bottom, make exponent positive*

$x^{-a} = \frac{1}{x^a}$

$\frac{1}{x^{-2}} = \frac{x^2}{1}$ — *Bring to the top, make exponent positive*

$\frac{1}{x^{-a}} = x^a$

$3a^{-2} = \frac{3}{a^2}$ — *Bring to the bottom, make exponent positive*

Notice the 3 doesn't come down

$3^{-3}a^{-2} = \frac{1}{3^3a^2}$ — *Bring to the bottom, make exponent positive*

$(2x)^{-3} = \frac{1}{(2x)^3} = \frac{1}{8x^3}$ — *Bring to the bottom, make exponent positive*

$\frac{x^{-2}+5}{3} \neq \frac{5}{3x^2}$

# Step 1

Over ←

$\frac{2x^5y^{-2}}{z^{-3}} = \frac{2x^5z^3}{y^2}$

When working with negative exponents:

Start with a fraction "Over" sign.
Put anything not moved!
Move whatever needs to be moved.
If nothing is left on the top, put a 1.

**When you can flip it!**

$\left(\frac{x}{y}\right)^{-2} = \frac{x^{-2}}{y^{-2}} = \frac{y^2}{x^2}$

Distribute Exponents
*Bring to the bottom, make exponent positive*
*Bring to the top, make exponent positive*

OR

$\left(\frac{x}{y}\right)^{-2} = \left(\frac{y}{x}\right)^{2} = \frac{y^2}{x^2}$ — Flip it and make the exponent positive

**Alternate Subtraction Methods**

$\frac{x^2}{x^5} = x^{2-5} = x^{-3} = \frac{1}{x^3}$

Subtract from the top

Theory

$\frac{x^2}{x^5} = \frac{\cancel{x \times x}\ 1}{\cancel{x \times x} \times x \times x \times x} = \frac{1}{x^3}$

$\frac{x^2}{x^5} = \frac{1}{x^{5-2}} = \frac{1}{x^3}$

Subtract from Bottom

$\frac{x^2}{x^{-3}} = x^2x^3 = x^5$

$\frac{x^2}{x^{-3}} = x^{2-(-3)} = x^5$

Bring Up, Add

OR

Subtract

$\frac{x^{-2}}{x^3} = \frac{1}{x^3x^2} = \frac{1}{x^5}$

$\frac{x^{-2}}{x^3} = \frac{1}{x^{3-(-2)}} = \frac{1}{x^5}$

Bring Down, Add

OR

Subtract From Bottom

# M10 - 4.5 - Fraction Exponents/Radical/Root Form Notes

**Change from exponential form to radical/root form. Simplify if necessary.**

$5^{\frac{3}{4}} = \sqrt[4]{5^3}$

Check on Calculator

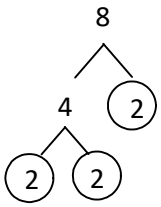
$5^{\frac{3}{4}} = 3.34 = \sqrt[4]{5^3}$ ✓

$x^{\frac{2}{3}} = \sqrt[3]{x^2}$

$8^{\left(\frac{1}{3}\right)} = \sqrt[3]{8^1} = 2$

Check on Calculator

$8^{\frac{1}{3}} = 2 = \sqrt[3]{8^1}$ ✓




$\sqrt[3]{8}$

$\sqrt[3]{2 \times 2 \times 2}$

2

$\sqrt[3]{8} = 2$

$$x^{\frac{m}{n}} = \sqrt[n]{x^m}$$

$8^{\frac{2}{3}} =$

$\sqrt[3]{8^2}$

$2^2$

4

Change to Radical/Root Form
Cube Root 1st
Square 2nd

$\sqrt[3]{8} = 2$

Check on Calculator $8^{\frac{2}{3}} = 4$ ✓

OR

$8^{\frac{2}{3}} =$

$\sqrt[3]{8^2}$

$\sqrt[3]{64}$

4

Change to Radical/Root Form
Square 1st
Cube Root 2nd

Easier to Root 1st

$8^2 = 64$

$\sqrt[3]{64} = 4$

$(-27)^{\frac{4}{3}}$

$\sqrt[3]{(-27)^4}$

$(-3)^4$

81

Change to Radical/Root Form
Cube Root 1st
Square 2nd

$\sqrt[3]{-27} = -3$

Check on Calculator

$(-27)^{\frac{4}{3}} = 81$ ✓

**Simplify by exponents laws. Answer in root form.**

$(2^{\frac{1}{2}})(2^{\frac{1}{4}}) = 2^{\frac{3}{4}} = \sqrt[4]{2^3} = \sqrt[4]{8}$

Add Exponents

$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$(3)^{\frac{3}{2}} \div (3)^{\frac{3}{5}} = (3)^{\frac{9}{10}} = \sqrt[10]{3^9}$

Subtract Exponents

$\frac{3}{2} - \frac{3}{5} = \frac{9}{10}$

$\left(\sqrt[2]{2^3}\right)^{\frac{1}{4}} =$

$\left(2^{\frac{3}{2}}\right)^{\frac{1}{4}}$

$2^{\frac{3}{8}}$

$\sqrt[8]{2^3} = \sqrt[8]{8}$

$\frac{3}{2} \times \frac{1}{4} = \frac{3}{8}$

Multiply Exponents

Check Answer

$\left(\sqrt[2]{2^3}\right)^{\frac{1}{4}} = 1.30 = \sqrt[8]{8}$

Check on Calculator

$(2^{\frac{1}{2}})(2^{\frac{1}{4}}) = 1.68 = \sqrt[4]{8}$ ✓

$(3)^{\frac{3}{2}} \div (3)^{\frac{3}{5}} = 2.69 = \sqrt[10]{3^9}$ ✓

$\left(\sqrt[2]{2^3}\right)^{\frac{1}{4}} = 1.30 = \sqrt[8]{2^3}$ ✓

$(-27x^9y^{-3})^{\frac{4}{3}}$

$\sqrt[3]{(-27)^4 x^{12} y^{-4}}$

$\frac{81x^{12}}{y^4}$

$9 \times \frac{4}{3} = 12$

$-3 \times \frac{4}{3} = -4$

$\sqrt[3]{(-27)^4} = (-3)^4 = 81$

$\left(\frac{-27a^3}{8}\right)^{\frac{1}{3}}$

$\frac{(-27)^{\frac{1}{3}} a^{3 \times \frac{1}{3}}}{8^{\frac{1}{3}}}$

$\frac{-3a}{2}$

$(5x^3)^{\frac{1}{2}}$

$\sqrt[2]{5^1}\sqrt[2]{x^3}$

$\sqrt{5}x\sqrt{x}$

$x\sqrt{5x}$