

M10 - 5.2 - Factoring GCF Notes

Remove Greatest Common Factor "GCF."

1) $12x + 8$ $GCF = 4$

$4(3x + 2)$

↑ ↙

4 times 4 times

what is 12x what is 8

2) $2x^2 + 3x =$ $GCF = x$
 $x(2x + 3)$

3) $12x^2 + 8x =$ $GCF = 4x$
 $4x(3x + 2)$

4) $x^2 + x^3 =$ $GCF = x^2$
 $x^2(1 + x)$

5) $8x^2y + 4xy$ $GCF = 4xy$
 $4xy(2x + 1)$

6) $-2x + 8$ $GCF = -2$
 $-2(x - 4)$

8) $(2 - x) =$
 $-1(-2 + x) =$ $GCF = -1$
 $-(x - 2)$ Rearrange order of the terms

9) $(-x - 2) =$ $GCF = -1$
 $-1(x + 2) =$
 $-(x + 2)$

10) $x(x + 2) + 4(x + 2) =$ $GCF = (x + 2)$
 $(x + 2)(x - 4)$

11) $2x - \frac{1}{2}$
 $2(x - \frac{1}{4})$

Divide both terms by GCF

$$\frac{12x}{4} + \frac{8}{2} = 3x + 2$$

Answer goes in brackets

Check your answer by Distribution

$$4(3x + 2)$$

$$12x + 8$$

The answer should be the same as the original question.

Remove Greatest Common Factor "GCF."

$2x - 1$
 $2(x - \frac{1}{2})$

M10 - 5.3 - Labelling "a", "b" and "c" in Polynomials Notes

Identifying "a", "b", and "c" in: $ax^2 + bx + c$

"a" is the number to the left of the x^2 term.

"b" is the number to the left of the x term.

"c" is the number by itself.

a = 1

1) $y = 1x^2 + 2x + 3$

$$a = 1, b = 2, c = 3$$

2) $y = x^2 + 3x + 4$

$$a = 1, b = 3, c = 4$$

3) $y = x^2 - 4x + 2$

$$a = 1, b = -4, c = 2$$

4) $y = x^2 + 4x - 6$

$$a = 1, b = 4, c = -6$$

$y = kx^2 + mx + c$

$a = k, b = m, c = c$

a ≠ 1

1) $y = 2x^2 + 3x + 4$

$$a = 2, b = 3, c = 4$$

3) $y = 2x^2 - 3x - 9$

$$a = 2, b = -3, c = -9$$

2) $y = 3x^2 + 5x + 7$

$$a = 3, b = 5, c = 7$$

4) $y = -3x^2 + 7x - 1$

$$a = -3, b = 7, c = -1$$

b = 0

1) $y = 2x^2 + 3$

$$a = 2, b = 0, c = 3$$

2) $y = 2x^2 - 5$

$$a = 2, b = 0, c = -5$$

c = 0

1) $y = x^2 + 4x$

$$a = 1, b = 4, c = 0$$

2) $y = -3x^2 - 7x$

$$a = -3, b = -7, c = 0$$

M10 - 5.3 - Factoring Polynomials $ax^2 + bx + c$ "a = 1" Notes

$$\begin{array}{l}
 1) \quad x^2 + 5x + 6 \\
 \quad \quad \quad \diagup \quad \diagdown \\
 x^2 + 2x + 3x + 6 \\
 (x^2 + 2x)(+3x + 6) \\
 x(x + 2) + 3(x + 2) \\
 (x + 2)(x + 3)
 \end{array}$$

$$a = 1$$

Decompose

$$\underline{\quad} \times \underline{\quad} = c$$

$$\underline{2} \times \underline{3} = \cancel{c} 6$$

Group

GCF

Switch

$$\underline{\quad} + \underline{\quad} = b$$

$$\underline{2} + \underline{3} = \cancel{b} 5$$

1,6
2,3

$$\begin{array}{l}
 x^2 + 5x + 6 \\
 (x + 2)(x + 3)
 \end{array}$$

What are two numbers that: multiply to get "c", the last number, and add together to get "b", the middle number. The numbers above on the right go in the brackets on the left.

$$\begin{array}{l}
 (x + 2)(x + 3) \\
 x^2 + 3x + 2x + 6 \\
 x^2 + 5x + 6
 \end{array}$$

Don't forget to check by multiplying out: FOIL. The answer should be the same as the original question.

$$\begin{array}{l}
 2) \quad x^2 + 6x + 8 \\
 \quad \quad (x + 2)(x + 4)
 \end{array}$$

$$a = 1$$

$$\underline{2} \times \underline{4} = \cancel{c} 8$$

$$\underline{2} + \underline{4} = \cancel{b} 6$$

$$\begin{array}{l}
 3) \quad x^2 - 3x - 10 \\
 \quad \quad (x - 5)(x + 2)
 \end{array}$$

$$a = 1$$

$$\underline{-5} \times \underline{2} = \cancel{c} -10$$

$$\underline{-5} + \underline{2} = \cancel{b} -3$$

Remember the sign of the numbers you choose goes in the bracket along with the number.

$$6) \quad x^2 + 4x + 15$$

Cannot factor

$$\begin{array}{l}
 (x - 4) \\
 (\sqrt{x} + \sqrt{4})(\sqrt{x} - \sqrt{4})
 \end{array}$$

M10 - 5.3 - Factor by Decomposition $ax^2 + bx + c$ ($a \neq 1$) Notes

1) $2x^2 + 7x + 6$ $a \neq 1$

$2x^2 + 3x + 4x + 6$
 $(2x^2 + 3x) | (+4x + 6)$

$x(2x + 3) + 2(2x + 3)$
 $(2x + 3)(x + 2)$

$2x + 2y$
 $2(x + y)$

<i>Decompose</i>	Step 1	$\frac{3}{\cancel{ac}} \times \frac{4}{\cancel{12}} = 12$	1,12
<i>Group</i>	Step 2	$\frac{3}{\cancel{b}} + \frac{4}{\cancel{7}} = 7$	2,6 3,4
<i>GCF</i>	Step 3		
<i>GCF</i>	Step 4		

$2x^2 + 7x + 6$
 $(2x + 3)(x + 2)$

Step 1 Decompose: What are two numbers that: multiply to get "a X c" and add to get "b." "b" gets split into the two numbers above on the right.

Step 2 Group: Place brackets around the first two terms, and the second two terms.

Step 3 GCF: Remove a GCF from each set of brackets.

Step 4 Switch: The GCFs go in a set of brackets together, the identical pair of brackets become one set of brackets below

$$(2x + 3)(x + 2)$$

$$2x^2 + 4x + 3x + 6$$

$$2x^2 + 7x + 6$$

Don't forget to check by multiplying out: FOIL. The answer should be the same as the original question.

2) $2x^2 + 3x - 2$ $a \neq 1$

$2x^2 + 4x - x - 2$
 $(2x^2 + 4x)(-x - 2)$
 $2x(x + 2) - 1(x + 2)$

Don't Cut off a negative!

<i>Decompose</i>		$\frac{4}{\cancel{ac}} \times \frac{-1}{\cancel{-4}} = -4$	
<i>Group</i>		$\frac{4}{\cancel{b}} + \frac{-1}{\cancel{3}} = 3$	
<i>GCF</i>	Factor GCF out each set of brackets		

3) $2x^2 + 5x + 2$ $a \neq 1$ REARRANGE

$2x^2 + 4x + x + 2$
 $(2x^2 + 4x)(+x + 2)$
 $2x(x + 2) + 1(x + 2)$
 $(2x + 2)(x + 2)$

$2x^2 + 2 + x + 4x$
 $2x^2 + 4x + x + 2$

<i>Decompose</i>		$\frac{4}{\cancel{ac}} \times \frac{1}{\cancel{4}} = 4$	
<i>Group</i>		$\frac{4}{\cancel{b}} + \frac{1}{\cancel{5}} = 5$	
<i>GCF</i>	Factor GCF out each set of brackets		

M10 - 5.4 - Differences of Squares Notes

1) $x^2 - 9$
 $(x + 3)(x - 3)$

What squared is x^2 ? That answer goes first in each set of brackets.
 What squared is 9? That number goes second in each set of brackets.
 Put a plus sign in one set of brackets and a minus sign in the other.

$(x + 3)(x - 3)$
 $x^2 - 3x + 3x - 9$
 $x^2 - 9$

Don't forget to check by multiplying out: FOIL. (5.1) The answer should be the same as the original question.

2) $4x^2 - 36$
 $(2x)^2 - 6^2$
 $(2x + 6)(2x - 6)$

$(2x + 6)(2x - 6)$
 $4x^2 - 12x + 12x - 36$
 $4x^2 - 36$

Check by "FOIL"

3) $9x^2 - y^2$
 $(3x)^2 - y^2$
 $(3x + y)(3x - y)$

Change of base

$(3x + y)(3x - y)$
 $9x^2 - 3xy + 3xy - y^2$
 $9x^2 - y^2$

Check by "FOIL"

4) $-x^2 + 49$
 $49 - x^2$
 $(7 + x)(7 - x)$

Rearrange order of terms

$49 - x^2$
 $-(x^2 - 49)$ GCF=-1
 $-(x - 7)(x + 7)$ Factor

$(7 + x)(7 - x)$
 $49 - 7x + 7x - x^2$
 $49 - x^2$

Check by "FOIL"

$x^2 + 4$

Cannot Factor

$x^4 - 1$
 $(x^2 - 1)(x^2 + 1)$
 $(x + 1)(x - 1)(x^2 + 1)$

$x^2 - 81$
 $(x^2 - 9)(x^2 + 9)$
 $(x + 3)(x - 3)(x^2 + 9)$

$(x - y)$
 $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$

M10 - 5.5 - Factoring Special/Multi-Step Trinomials Notes

$$1) x^2 - 3xy - 10y^2$$

$$(x - 5y)(x + 2y)$$

$$a = 1$$

$$\underline{-5} \times \underline{2} = \cancel{c} -10$$

$$\underline{-5} + \underline{2} = \cancel{b} -3$$

Factor as you would in $a = 1$ and put a y after the last number in both sets of brackets, or do decomposition

$$x^2 - 3xy - 10y^2$$

$$x^2 - 5xy + 2xy - 10y^2$$

$$(x^2 - 5xy) + (2xy - 10y^2)$$

$$x(x - 5y) + 2y(x - 5y)$$

$$(x + 2y)(x - 5y)$$

$$(x + 2y)(x - 5y)$$

$$x^2 - 5xy + 2xy - 10y^2$$

$$x^2 - 3xy - 10y^2$$

Check by "FOIL"

$$2) 2x^2 + 10x + 12$$

$$2(x^2 + 5x + 6)$$

$$2(x + 2)(x + 3)$$

Remove GCF
 $a = 1$

OR

$$2x^2 + 10x + 12$$

$$\textcircled{2}(x^2 + 5x + 6)$$

$$x^2 + 2x + 3x + 6$$

$$(x^2 + 2x)(+3x + 6)$$

$$x(x + 2) + 3(x + 2)$$

$$2(x + 2)(x + 3)$$

$$2.5) x^3 + 5x^2 + 6x$$

$$x(x^2 + 5x + 6)$$

$$x(x + 2)(x + 3)$$

$$3) -x^2 - 5x - 6$$

$$-(x^2 + 5x + 6)$$

$$-(x + 2)(x + 3)$$

Remove GCF
 $a = -1$

$$4) 3x^2 - 27$$

$$3(x^2 - 9)$$

$$3(x + 3)(x - 3)$$

Remove GCF
Differences of squares

$$5) x^4 - 1$$

$$(x^2 + 1)(x^2 - 1)$$

$$(x^2 + 1)(x + 1)(x - 1)$$

Differences of squares
Differences of squares

$$x^4 + 5x^2 + 6$$

$$(x^2 + 3)(x^2 + 2)$$

M10 - 5.5 - Factoring Substitution Let $x = m+1$ Notes

$$(m + 1)^2 + 5(m + 1) + 6$$

$$x^2 + 5x + 6$$

$$(x + 2)(x + 3)$$

$$((m + 1) + 2)((m + 1) + 3)$$

$$(m + 3)(m + 4)$$

Let $x = m + 1$

OR

$$(m + 1)^2 + 5(m + 1) + 6$$

$$(m + 1)(m + 1) \dots$$

$$m^2 + 2m + 1 + 5m + 5 + 6$$

$$m^2 + 7m + 12$$

...

$$(m + 3)(m + 4)$$

$$4x^2 - (x + 2)^2$$

$$a^2 - b^2$$

$$(a + b)(a - b)$$

$$(2x + (x + 2))(2x - (x + 2))$$

$$(3x + 2)(x - 2)$$

let $a = 2x$

let $b = (x + 2)$

Substitute $2x$ and $x + 2$ back in with brackets

$$9(x + 2)^2 - 16(x - 1)^2$$

$$9a^2 - 16b^2$$

$$(3a + 4b)(3a - 4b)$$

$$(3(x + 2) + 4(x - 1))(3(x + 2) - 4(x - 1))$$

$$(3x + 6 + 4x - 4)(3x + 6 - 4x + 4)$$

$$(7x + 2)(-x + 10)$$

Let $a = x + 2$

Let $b = x - 1$

Substitute $x + 2$ and $x - 1$ back in with brackets

$$9x^4 - 9x^2 + 6xy - y^2$$

$$9x^4 - (9x^2 - 6xy + y^2)$$

$$9x^4 - (3x - 1)^2$$

$$(3x^2)^2 - (3x - 1)^2$$

...