# M10 - 6.0 - Arithmetic Notes

| $n$ | $t_n$ |
|---|---|
| 1 | 4 |
| 2 | 7 |
| 3 | 10 |
| 4 | 13 |
| … | … |
| $n$ | $t_n$ |

$\underline{4}, \underline{7}, \underline{10}, \dots, \underline{31}, \dots, \underline{37}, \underline{\quad}$

$\underline{\quad}, \underline{\quad}, \underline{\quad}, \dots, \underline{\quad}, \dots, \underline{\quad}, \underline{\quad}$

$+3 \quad +3$

$\underset{t_1}{\underline{4}}, \underset{t_2}{\underline{7}}, \underset{t_3}{\underline{10}}, \dots, \underset{t_{10}}{\underline{31}}, \dots, \underset{t_{12}}{\underline{37}}, \underset{t_n}{\underline{\quad}}$

**Sequences**

4,7,10,13,16,19,22,25,28,31,34,37 …

General Term

$\boxed{t_n = t_1 + (n-1)d}$

$t_n = 4 + (n-1)(3)$

$t_n = 4 + 3n - 3$

$t_n = 3n + 1$

Difference

$d = 7 - 4$

$d = 3$

$d = 10 - 7$

$d = 3$

$\boxed{t_{10} = ?}$

$t_n = 3n + 1$

$t_{10} = 3(10) + 1$

$t_{10} = 31$

$\boxed{37 = t_n, n = ?}$

$t_n = 3n + 1$

$37 = 3n + 1$

$-1 \qquad -1$

$\frac{36}{3} = \frac{3n}{3}$

$n = 12$

How many toothpicks in 10 diagrams total?

$s_n = \frac{n}{2}(t_1 + t_n)$

$s_{10} = \frac{10}{2}(4 + 31)$

$s_{10} = 175$

How many toothpicks in 12 diagrams total?

$s_n = \frac{n}{2}(2t_1 + (n-1)d)$

$s_{12} = \frac{12}{2}\big(2(4) + (12-1)(3)\big)$

$s_{12} = 246$

There are 34 toothpicks in how many diagrams?

$s_n = \frac{n}{2}(2t_1 + (n-1)d)$

$34 = \frac{n}{2}\big(2(4) + (n-1)(3)\big)$

$34 = \frac{n}{2}(8 + 3n - 3)$

$34 = \frac{n}{2}(3n + 5)$

$2 \times 34 = \frac{n}{2}(3n + 5) \times 2$

$68 = n(3n + 5)$

$68 = 3n^2 + 5n$

$-68 \qquad -68$

$0 = 3n^2 + 5n - 68$

…

$0 = (3n + 17)(n - 4)$

…

$n = \cancel{-\frac{17}{3}}, n = 4$

**Series**

4 + 7 + 10 + 13 + 16 + 19 + 22 + 25 + 28 + 31 + 34 + 37 …

$t_2 = 2, t_5 = -4$

$\boxed{Logic}$

$-d \quad +d \quad +d \quad +d$

$\underset{t_1}{\underline{\quad}}, \underset{t_2}{\underline{2}}, \underset{t_3}{\underline{\quad}}, \underset{t_4}{\underline{\quad}}, \underset{t_5}{\underline{-4}}$

| $n$ | $t_n$ |
|---|---|
| 1 | |
| 2 | 2 |
| 3 | |
| 4 | |
| 5 | $-4$ |

$2 + 3d = -4$

$-2 \qquad -2$

$\frac{3d}{3} = -\frac{6}{3}$

$d = -2$

$\boxed{5 - 2 = 3}$

$2 - 2 = 0$

$0 - 2 = -2$

…

$2 + 2 = 4$

OR

$4, 2, 0, -2, -4$

$t_2 = 2, t_5 = -4$

Systems of Equations

$t_n = t_1 + (n-1)d$

$t_2 = t_1 + (2-1)d$

$2 = t_1 + d$

$t_1 = 2 - d \longrightarrow -4 = (2-d) + 4d$

$t_n = t_1 + (n-1)d$

$t_5 = t_1 + (5-1)d$

$-4 = t_1 + 4d$

$-4 = (2 - d) + 4d$

$-4 = 2 + 3d$

$d = -2$

$t_1 = 2 - (-2) \longleftarrow d = -2$

$t_1 = 4$

$s_2 = 9, s_3 = 21$, Find the first five arithmetic terms.

$s_n = \frac{n}{2}(t_1 + t_n)$

$s_2 = \frac{2}{2}(t_1 + t_2)$

$9 = \frac{2}{2}(t_1 + t_2)$

$9 = t_1 + t_2$

$9 = 2 + t_2$

$t_2 = 7$

$s_n = \frac{n}{2}(t_1 + t_n)$

$s_3 = \frac{3}{2}(t_1 + t_3)$

$21 = \frac{3}{2}(t_1 + t_3)$

$14 = t_1 + 12$

$t_1 = 2$

$d = 7 - 2$

$d = 5$

$2, 7, 12, 17, 22$

OR

$t_n = s_n - s_{n-1}$

$t_3 = s_3 - s_2$

$t_3 = 21 - 9$

$t_3 = 12$

# M10 - 6.0 - Arithmetic Polynomial Notes

$x+1, 3x-1, 2x+3$

$d=3x-1-(x+1)$ $\quad$ $d=2x+3-(3x-1)$

… $\quad$ …

$\boxed{d=2x-2}$ $\quad$ $\boxed{d=4-x}$

$d=d$

$2x-2=4-x$

$+x \qquad +x$

$3x-2=4$

$+2 \quad +2$

$3x=6$

$x=\frac{6}{3}$

$\boxed{x=2}$

$x+1, 3x-1, 2x+3$

$(2)+1, 3(2)-1, 2(2)+3$

$3, 5, 7$

$5-3=2$ ✓ $7-5=2$