M10 - 9.0 - $ax + by = c$ Numbers Word Probs

One number is two more than another and their sum is 12. What are the numbers?

$Let\ x = 1st\ \#$
$Let\ x - 2 = 2nd\ \#$

One Variable!

$Let\ x = 1st\ \#$
$Let\ y = 2nd\ \#$

Two Variables!

$x + (x - 2) = 12$
$x + x - 2 = 12$
$2x - 2 = 12$
$+2 \quad +2$
$2x = 14$
$\dfrac{2x}{2} = \dfrac{14}{2}$
$x = 7$

OR

$Let\ x = 1st\ \#$
$Let\ x + 2 = 2nd\ \#$

$2nd\# = x - 2$
$= (7) - 2$
$2nd\# = 5$

$1st\ \# = 7$
$2nd\ \# = 5$

Check Answer
$5 + 2 = 7$ ✓ $\quad 5 + 7 = 12$ ✓

Equation #1   Equation #2

$x + y = 12$
$-x \qquad - x$
$y = (12 - x)$

| Isolate |
| Substitute |
| Solve (Algebra) |
| Substitute |
| Solve |

$x - y = 2$

$x - (12 - x) = 2$
$x - 12 + x = 2$
$2x - 12 = 2$
$+12 \quad + 12$
$\dfrac{2x}{2} = \dfrac{14}{2}$
$x = 7$

$y = 12 - x$
$y = 12 - (7)$
$y = 5$ Solve

(Substitute)

$1st\ \# = 7$
$2nd\ \# = 5$

$x - y = 2$
$y + 2 = x$
...
It Doesn't Matter*!

Find two numbers who's sum is 42 and difference is 18

$let\ a = 1st\#$
$let\ b = 2nd\#$

$a - b = 18$
$+b \quad + b$
$a = (18 + b)$

$a = 18 + 12$
$a = 30$

$30 - 12 = 18$ ✓

$a + b = 42$

$(18 + b) + b = 42$
$18 + 2b = 42$
$-18 \qquad - 18$
$\dfrac{2b}{2} = \dfrac{24}{2}$
$b = 12$

$1st\# = 30$
$2nd\# = 42$

Check Answer ✓ $30 + 12 = 42$

Adult bus tickets are $12 and children's tickets are $8. If the bus holds 50 seats and ticket sales are $416, how many of each are on the bus?

$let\ x = \#\ of\ adult\ tickets$
$let\ x = \#\ of\ child\ tickets$

$x + y = 50$
$-y \quad - y$
$x = 50 - y$

$x = 50 - 46$
$x = 4$

4 adult tickets
46 child tickets

$12x + 8y = 416$
$12(50 - y) + 8y = 416$
$600 - 12y + 8y = 416$
$600 - 4y = 416$
$-600 \qquad - 600$
$\dfrac{-4y}{-4} = \dfrac{-184}{-4}$
$y = 46$

Check Answer

$4 + 46 = 50$ ✓ $\quad 4 \times 12 + 46 \times 12 = 416$ ✓

Words Problems	Solve (Algebra)
Diagram	Substitute
Let Statements	Solve
Equation/s	Answer in English!
(Arbitrary#'s)	Check Answer!
Isolate	(Eliminate*)
Substitute	Explain it to a 10 year old!

M10 - 9.0 - $ax + by = c$ Coins/Inv/Mix Word Probs

A person has $2.30 in Dimes, How many Dimes do they have?

$let\ d = \#\ of\ Dimes$ $0.1d = 2.30$ Equation

Let Statement $\dfrac{0.1d}{0.1} = \dfrac{2.30}{0.1}$ Answer the Question

We have "d" Dimes!

$d = 23$ Solve They have 23 Dimes

$0.1 \times 23 = 2.30$ ✓ Check Answer

d	Value $	Calculation
0	0	$0 \times 0.1 = 0$
1	0.1	$1 \times 0.1 = 0.1$
2	0.2	$2 \times 0.1 = 0.2$
d	0.1d	$d \times 0.1 = 0.1d$

$Value\ of\ a\ Dime \times \#\ of\ Dimes$ 0.1d

Jay has 12 Total Coins of Quarters and Dimes worth $2.40. How many does he have of each?

$let\ d = \#\ dimes$ $d + q = 12$ $0.1d + 0.25q = 2.40$

$let\ q = \#\ quarters$ $-q\ \ \ \ -q$ $\boxed{Ax + By = C}$

$d = (12 - q)$

Check Answer

$4 + 8 = 12$ ✓ ✓

$0.1 \times 4 + 0.25 \times 8 = 2.40$

Answer the question

$0.1(12 - q) + 0.25q = 2.40$
$1.2 - 0.1q + 0.25q = 2.40$
$1.2 + 0.15q = 2.40$

$d = 12 - q$
$d = 12 - (8)$
$d = 4$

$-1.2 \quad\quad\quad\ - 1.2$
$\dfrac{0.15q}{0.15} = \dfrac{1.20}{0.15}$
$q = 8$

Jay has 4 dimes and 8 quarters worth $2.40.

2 equations
Isolate
Substitute
Distribute
Combne Like Terms
Subract Both Sides
Divide Both Sides
Solve
Substitute
Solve

Mark invests a total of $1500 in a 10% bond and an 8% bond earning $132. How much did he invest in each?

$let\ a = amount\ invested\ in\ 10\%$ $a + b = 1500$ $0.1a + 0.08b = 132$

$let\ b = amount\ invested\ in\ 8\%$ $-b\ \ \ \ -b$

$a = (1500 - b)$ $0.1(1500 - b) + 0.08b = 132$
$150 - 0.1b + 0.08b = 132$
$150 - 0.02b = 132$

$600\ at\ 10\%$
$900\ at\ 8\%$

$a = 1500 - 900$
$a = 600$

$-150 \quad\quad\quad - 150$

$\dfrac{-0.02b}{-0.02} = \dfrac{-18}{-0.02}$

$600 + 900 = 1500$

$b = 900$

$0.1 \times 600 + 0.08 \times 900 = 132$

As scientist wants to make 50 L of a 40% acid solution. They mixed together a 30% acid solution with the 70% acid solution. How many litres of each solution must the scientist mix?

$let\ a = litres\ of\ 30\%\ mix$ $\boxed{\% \times Amount + \% \times Amount = \% \times Amount}$

$let\ b = litres\ of\ 70\%\ mix$

$a + b = 50$ $0.3a + 0.7b = 0.4(50)$

$12.5\ L\ of\ 70\%\ Mix$
$37.5\ L\ of\ 30\%\ Mix$

$b = 50 - a$ $0.3a + 0.7(50 - b) = 20$

...   ...

$b = 12.5$ $a = 37.5$

M10 - 9.0 - Distance/Area Word Problems

A boat took 3 hours to travel 24 km with the Current. On the return trip, the boat took 5 hours to travel 24 km against the Current. Determine the speed of the Current.

$let\ x = speed\ of\ boat\ in\ still\ water$
$let\ c = speed\ of\ current$
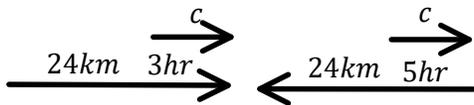
$$\xrightarrow{c} \qquad \xrightarrow{c}$$

$24km \quad 3hr \xrightarrow{} \quad \xleftarrow{} 24km \quad 5hr$

$$s = \frac{d}{t} \quad \boxed{\text{Diagrams}} \qquad s = \frac{d}{t}$$

$$x + c = \frac{24}{3} \qquad\qquad (x) - c = \frac{24}{5}$$

$$x + c = 8$$

$$x = (8 - c) \longrightarrow (8-c) - c = \frac{24}{5}$$

$$8 - 2c = 4.8 \qquad \text{Minus 4.8/Add } 2c \text{ to}$$

$$x = 8 - 1.6 \qquad 3.2 = 2c \qquad \text{Both Sides/Mirror}$$

$$\boxed{x = 6.4 \frac{km}{h}} \qquad \frac{3.2}{2} = \frac{2c}{2} \qquad \text{Divide}$$

Speed of boat in still water

$$\boxed{c = 1.6 \frac{km}{h}} \text{ Speed of current}$$

$$s = \frac{d}{t} \qquad 6.4 + 1.6 = \frac{24}{3} \qquad \checkmark \text{ Check Answer} \checkmark \qquad 6.4 - 1.6 = \frac{24}{5}$$

Find the Cost if Toska eats $3 burgers.

$let\ b = \#\ burgers$
$let\ c = total\ cost\ (\$)$

$c = 3b$

An airplane is flying at a height of 400 m and descending at 5 m/s.

$let\ h = height\ (m)$
$let\ t = time\ (s)$

$h = 400 - 5t$

$y = mx + b$

Jane's hair is 30 cm long and grows at 2 cm per month.

$let\ h = hair\ length\ (cm)$
$let\ t = time\ (months)$

$h = 30 + 2t$

Ben has $20 in the bank and spends $2 per day.

$let\ t = time\ (days)$
$let\ d = dollars\ in\ bank$

$d = 20 - 2t$

The total cost of a car is the selling price, plus sales tax of 12% and a $500 service fee. Find the equation.

$let\ c = total\ cost$
$let\ p = sellimg\ price$

$c = 0.12p + 500$

Gursavek has 10 dollars in the bank.

$let\ d = dollars\ in\ bank$

$d = 10$

A computer is worth $800 after two years and $600 after four years. Find the equation of its value, the purchase price, and value after 6 years and time till its worthless.

$let\ v = computer\ value$
$let\ t = time\ (years)$

(2,800)
(4,600)

V
(0,1000)
(2,800)
(4,600)
(6,400)
(10,0)
t
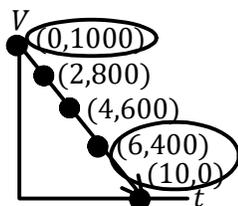
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m = \dfrac{600 - 800}{4 - 2}$

$m = -100$

$y = mx + b$

$v = -100t + b$
$600 = -100(4) + b$
$600 = -400 + b$
$+400 \quad +400$
$b = 1000$

$v = -100t + 1000$

$\$1000 = purchase\ price$

$v = -100t + 1000$
$v = -100(6) + 1000$
$v = 400$

Value $400 after 6 years.

$v = -100t + 1000$
$0 = -100t + 1000$
$-1000 \qquad -1000$
$\dfrac{-1000}{-100} = \dfrac{-100t}{-100}$
$t = 10$

Ten years till the computer is valueless

$$m = \dfrac{-100\$}{year}$$

M10 - 9.0 - $y = mx + b$ Cell Phone Intersection!

A cell phone company "Giga" Data Costs $40 per month plus $0.1 per Megabyte of Data.

Let c = cost $c = 40 + 0.1d$

Let d = # megabytes of data

If a person uses 100 megabytes of Data what will month bill cost? $d = 100$

If a person's bill is $70, How many Megabytes did the use? $c = 70$

$c = 40 + 0.1d$	Formula
$c = 40 + 0.1(100)$	Substitute
$c = 40 + 10$	
$c = \$50$	Solve

100 megabytes of Data will cost $50

$c = 40 + 0.1d$
$70 = 40 + 0.1d$
$-40 \quad -40$
$$\frac{30}{0.1} = \frac{0.1d}{0.1}$$
$300 = d$

$70 will buy 300 megabytes of data

"Mega" Cell Phone Company charges $20 per month plus $0.5 per megabyte of data. Which company would you choose?

Let c = cost $c = 20 + 0.5d$

Let d = # megabytes of data

$c = 0.5d + 20$

$c = 0.1d + 40$

$y = mx + b$

(50,45)

C O S T, 40, 20

MEGABYTES   40   100

d
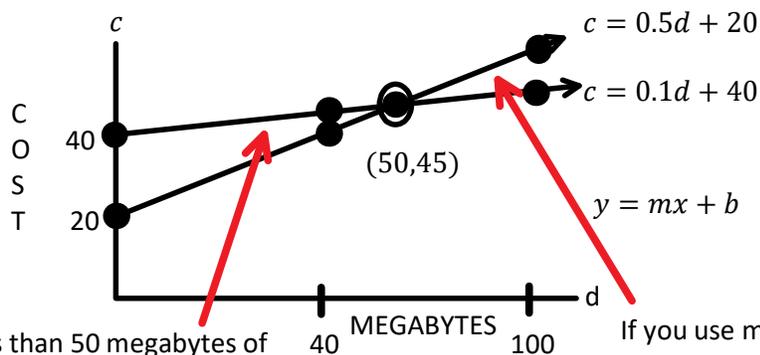
If you use less than 50 megabytes of data you would choose the Mega phone company. Because it's cheaper.

If you use more than 50 megabytes of data you would choose the Data phone company. Because it's cheaper.

$0.1d + 40 = 0.5d + 20$
$$\left(\frac{1}{10}d + 40 = \frac{1}{2}d + 20\right) \times 10$$
$d + 400 = 5d + 200$
$-d - 200 \quad -d - 200$
$$\frac{200}{4} = \frac{4d}{4}$$

$d = 50 \ megabytes \ od \ data$

$c = 0.5d + 20$
$$c = \frac{1}{2}(50) + 20$$
$c = \$45$

M10 - 9.0 - $y = mx + b$ Revenue Word Problems

(4,4)   (2,3)

Its 1954 and Benji rents out his lemonade stand for $2 a day and the Renter sells <u>2 lemonades for a total revenue for Benji of $3</u> and <u>4 for $4</u> respectively. (Benji farms lemons from a tree at no charge and collects water from the river, and the Renter works for free.)

Benji's Daily Lemonade Sales

R

1 lemonade
$0.5
2 lemonades
(x_2, y_2)
$(4,4)$
$1
(x_1, y_1)
$(2,3)$

(L, D)

let L = # of lemonades
let R = Revenue $ from sales

$ from lemonade sales

lemonades sold

L	D
0	2
2	3
4	4
5.50	7
6	8
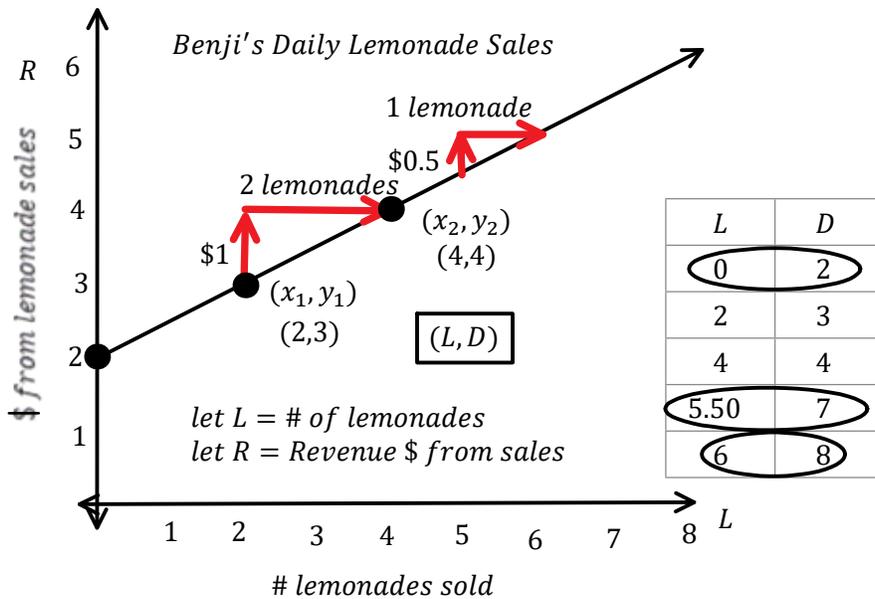
How much is made off each lemonade sale?

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{(4) - (3)}{(4) - (2)}$$

$$m = \frac{1}{2}$$

$$m = \frac{1 \ \$ \ in \ sales}{2 \ lemonade \ sold}$$

$$m = \frac{\$0.5}{1 \ lemonade}$$

Unit Slope is price per lemonade.

What is <u>Benji's equation</u> of sales/revenue from lemonade versus number of lemonade sold? What is the significance of the <u>Y-intercept?</u>

$$y = mx + b$$

$$R = \frac{1}{2}L + b$$

$$3 = \frac{1}{2}(2) + b$$ (L,D) $(2,3)$

$$3 = 1 + b$$

$$\frac{-1 \quad -1}{2 = b}$$

$$R = \frac{1}{2}L + 2$$ Equation of Sales

Daily rental revenue of lemonade stand = $2.

How much <u>money</u> would Benji make from lemonade sales if you <u>sold 8 lemonades?</u>

$$L = 8$$

$$D = \frac{1}{2}L + 2$$

$$D = \frac{1}{2}(8) + 2$$

$$D = 4 + 2$$ $\frac{1}{2} \times \frac{8}{1} = \frac{8}{2} = 4$

$$D = 6$$

Sell 8 lemonades to make $6 in sales.

How many <u>lemonades</u> would Benji have to sell to <u>make $5.50.</u>

$$D = 5.50$$

$$D = \frac{1}{2}L + 2$$

$$\frac{11}{2} = \frac{1}{2}L + 2$$

$$\frac{-2 \qquad -2}{\frac{7}{2} = \frac{1}{2}L}$$

$$2 \times \frac{7}{2} = \frac{1}{2}L \times 2$$

$$7 = L$$

$$5.50 = 5\frac{1}{2} = \frac{2 \times 5 + 1}{2} = \frac{11}{2}$$

$$\frac{11}{2} - \frac{2}{1}$$

$$\frac{11}{2} - \frac{2}{1} \times \frac{2}{2}$$

$$\frac{11}{2} - \frac{4}{2}$$

$$\frac{11 - 4}{2} = \frac{7}{2}$$

Sell 7 lemonades to make $5.50 in sales.

Decimals

$$D = \frac{1}{2}L + 2$$

$$D = \frac{1}{2}L + 2$$

$$\frac{11}{2} = \frac{1}{2}L + 2$$

$$\left(\frac{11}{2} = \frac{1}{2}L + 4\right) \times 2 \ ; LCD$$

$$11 = L + 4$$

$$\frac{-4 \qquad -4}{L = 7}$$

Multiply Both Sides by $LCD = 2$

$$D = \frac{1}{2}L + 2$$

$$5.50 = 0.5L + 2$$

$$\frac{-2 \qquad -2}{3.50 = 0.5L}$$

$$\frac{3.50}{0.5} = \frac{0.5L}{0.5}$$

$$7 = L$$

The Renter still rents the Lemonade stand for $2/day but now collects the $0.5 per lemonade sale in revenue and now must pay $0.25 per lemonade to the lemon tree owner themselves . Find The Renter's daily revenue, cost & profit equations of the Renter.

Revenue, Cost & Profit from Lemonade Sales

Let R = Revenue/Sales
Let C = Cost
Let P = Profit

$R = \frac{1}{2}L$

$Profit = \$1$

$C = \frac{1}{4}L + 2$

Break Even Point

Loss = \$1

$R = C$
$P = 0$

$P = \frac{1}{4}L - 2$

4 lemonades

\$1

lemonades sold

$ from lemonade sales
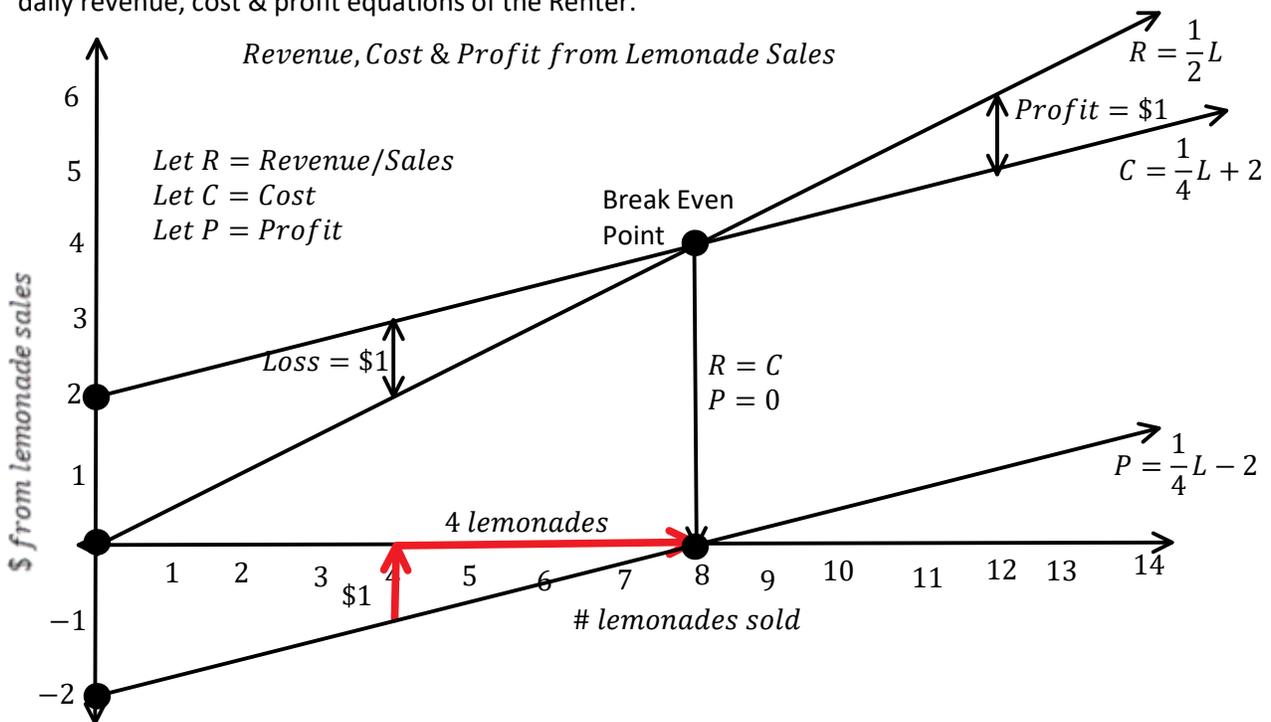
$y = mx + b$

$R = \frac{1}{2}L$

$y = mx + b$

$C = \frac{1}{4}L + 2$

$\boxed{P = R - C}$

$P = \frac{1}{2}L - (\frac{1}{4}L + 2)$

$P = \frac{1}{4}L - 2$ $\boxed{\frac{1}{2} - \frac{1}{4} = \frac{1}{4}}$

$y = mx + b$

Function Notation

$P(L) = \frac{1}{4}L - 2$

What is the break-even point?

$P = \frac{1}{4}L - 2$

$0 = \frac{1}{4}L - 2$ $\boxed{P = 0}$

$+2 \qquad + 2$

$2 = \frac{1}{4}L$

$4 \times 2 = \frac{1}{4}L \times 4$

$8 = L$ Sell 8 lemonades to Break Even.

$\boxed{R = C}$ **OR**

$\left(\frac{1}{2}L = \frac{1}{4}L + 2\right) \times 4$

$2L = L + 8$

$-L \quad -L$

$L = 8$

How much money would you profit from lemonade sales if you sold 0, 4, 12, 100 lemonades?

$P = \frac{1}{4}L - 2$

$P = \frac{1}{4}(0) - 2$

$P = 0 - 2$

$P = -2$

Loss of $2

$P = \frac{1}{4}L - 2$

$P = \frac{1}{4}(4) - 2$

$P = 1 - 2$

$P = -1$

Loss of $1

$P = \frac{1}{4}L - 2$

$P = \frac{1}{4}(12) - 2$

$P = 3 - 2$

$P = 1$

Profit of $1

$P = \frac{1}{4}L - 2$

$P = \frac{1}{4}(100) - 2$

$P = 25 - 2$

$P = 23$

Profit of $23

L	R	C	P
0	0	2	−2
1	0.5	2.25	−1.75
2	1	2.5	−1.5
3	1.5	2.75	−1.25
4	2	3	−1
5	2.5	3.25	−0.75
6	3	3.50	−0.5
7	3.5	3.75	−0.25
8	4	4	0
9	4.5	4.25	0.25
10	5	4.5	0.5
11	5.5	4.75	0.75
12	6	5	1
13	6.5	5.25	1.25
14	7	5.5	1.5