The Golden Rule: Whatever you do to the right side of the equal sign, do to the left side.

What plus $1=4$ ?


Divide both sides by two

$$
x=4
$$

$\qquad$

Solve for $x$, by subtracting to both sides.


| Both sides: The Left Hand |
| :--- |
| Side and the Right Hand |
| Side of the Equal Sign |

$\begin{aligned} & x+\not p=9 \\ &-\neq \text { Cross it off }\end{aligned} \quad 5-5=0$

Short Forms
$x+5=9$
$-5=-5$
$x=4$
$x+5=9$
$x=9-5$
$x=4$

$$
x=9-5
$$

$x=4$ Circle Answer

Check Answer

$$
\begin{aligned}
x+5 & =9 \\
(4)+5 & =9 \\
9 & =9
\end{aligned}
$$

Question
Substitute with Brackets
Left Hand Side Must Equal Right Hand Side

## Solve for $\boldsymbol{x}$, by adding to both sides.

$x-3=7$
$x-3=7$
$+3+3$

## Solve for $\boldsymbol{x}$ If you

$x-\not \beta=7$
$+\not \beta+3$
$x=7+3$

$$
x=10
$$

Add 3 to both sides
$x-3=7 \quad$ accidentally get
$\begin{array}{ll}-7 & -7\end{array}$ it equal to zero
$x-10=0 \quad$ just keep going!
$+10+10$
$x=10$

Cross it off
We are always doing the opposite operation to both sides of the equation


$$
-x+2=5
$$

$$
-x+2^{2}=5
$$

$$
-8 \quad-2
$$

Subtract 2 from both sides

$$
-x=3
$$

$$
\frac{f^{x}}{-1}=\frac{3}{-1} \quad \begin{aligned}
& \text { Divide both sides by }-1 \\
& \text { Cross it off }
\end{aligned}
$$



$$
\begin{aligned}
& \text { Alternate Solution } \\
& \begin{array}{rll}
-x+2=5 & \\
+x & +x & \text { Add } x \text { to both sides } \\
2=5+x & \\
-5 \quad-5 & \text { Subtract } 5 \text { from both sides } \\
-3=x & \text { Divide both sides by }-1 \\
x=-3 & \text { Mirror } \\
x & \\
\hline 5+x=5+x & \text { Can't add unlike terms! } \\
\hline
\end{array}
\end{aligned}
$$

M8-10.2-" $a x=b " ~ " \frac{x}{a}=b " ~ " \frac{a x}{b}=c$ " Notes $\qquad$

Solve for $\boldsymbol{x}$, by dividing to both sides.
$2 x=4$
$\frac{2 x}{2}=\frac{4}{2}$
Divide both sides by 2
Divide both sides by
the coefficient on $x$.
$\frac{2 x}{2}=\frac{4}{2}$
Cross it off

$x=\frac{4}{2}$
$x=2$


Question
Substitute
Left Must Equal Right

Solve for $x$, by multiplying to both sides.

$$
\frac{x}{3}=6
$$

$3 \times \frac{x}{3}=6 \times 3 \quad$ Multiply both sides by 3
$3 \times \frac{x}{3}=6 \times 3 \quad$ Cross it off $\quad \frac{3}{3}=1$


Check Answer

$$
x=6 \times 3
$$

$$
\begin{aligned}
\frac{x}{3} & =6 \\
\frac{18}{3} & =6 \\
6 & =6
\end{aligned}
$$



$$
x=18
$$

Solve for $\boldsymbol{x}$

$$
\begin{aligned}
\frac{5}{4} x & =10 \\
4 \times \frac{5}{4} x & =10 \times 4 \quad \text { Multiply both sides by } 4 \\
4 \times \frac{5}{4} x & =10 \times 4 \\
\frac{5 x}{5} & =\frac{40}{5} \\
\frac{5 x}{5} & =\frac{40}{5} \\
x & =\frac{40}{5} \\
x & =8
\end{aligned} \quad \text { Divide both sides by } 5
$$

Check Answer

Short Form

$$
\begin{gathered}
4 \times \frac{5}{4} x=10 \times 4 \\
\frac{5 x}{5}=\frac{40}{5} \\
x=8
\end{gathered}
$$



M8-10.3- " $\frac{a}{x}=b " " \frac{a}{b x}=c "$ Notes

| Solve for $\boldsymbol{x}$ |  |  |
| :---: | :---: | :---: |
| $\frac{8}{x}=4$ |  |  |
| $x \times \frac{8}{x}=4 \times x$ | Multiply $x$ to both sides | Multiply both sides by th denominator |
| $x \times \frac{8}{x}=4 \times x$ | Cross it off |  |
|  |  | Check Answer |
| $8=4 x$ |  | $\underline{8}=4$ |
| $\frac{8}{4}=\frac{4 x}{\lambda}$ |  | $\begin{aligned} & x \\ & 8 \end{aligned}$ |
| $\overline{4}=\overleftarrow{4}$ | Divide both sides by 4 | $\frac{0}{2}=4$ |

Solve for $\boldsymbol{x}$

Short Form
$\frac{24}{2 x}=3$
$\frac{24}{2(3)}=x$
$x=4$

$$
\text { M8-10.4- " } \frac{a x}{b x}=\frac{c}{d} \text { " Cross Multiply Notes }
$$

Solve for $x$, by multiplying both sides by the opposite denominator.

$$
\begin{aligned}
\frac{x}{6} & =\frac{4}{3} \\
x & =\frac{4}{6} \\
3 \times x & =4 \times 6 \\
3 x & =24 \\
\frac{3 x}{3} & =\frac{24}{3}
\end{aligned}
$$

Denominators Multiply to Opposite Side Numerator

Divide both sides by 3
$x=8$

| Check Answer |
| :---: |
| $\frac{x}{6}=\frac{4}{3}$ |
| $\frac{8}{6}=\frac{4}{3}$ |
| $\frac{4}{3}=\frac{4}{3}$ |




Multiply Both Sides


M8-10.5-" $\pm a x+b=c, \frac{x}{a}+b=c "$ Notes

Solve for $x$

$$
\left.\begin{array}{rlr}
6 x+8 & =50 \\
6 x+8 & =50 \\
-\beta & -8 \\
6 x & =42 \\
\frac{6 x}{6} & =\frac{42}{6} & \text { Subtract } 8 \text { from both sides } \\
\frac{\phi x}{\phi} & =\frac{42}{6} & \text { Divide both sides by } 6 \\
x & =\frac{42}{6} & \text { Cross it off } \\
x & & \\
x
\end{array} \quad \begin{array}{rr}
\text { Check Answer } \\
6 x+8=50 \\
6(7)+8=50 \\
42+8=50 \\
50 & =50
\end{array}\right]
$$

$$
\begin{aligned}
& \text { Short Form } \\
& \begin{array}{l}
6 x+8=50 \\
6 x=50-8 \\
6 x=42 \\
x=7
\end{array}
\end{aligned}
$$

## Solve for $x$

$$
\begin{aligned}
\frac{x}{3}-8 & =-3 \\
\frac{x}{3}-8 & =-3 \quad \text { Add } 8 \text { to both sides } \\
+\phi & +8 \\
\frac{x}{3} & =5 \\
\frac{x}{3} \times 3 & =5 \times 3 \quad \text { Multiply both sides by } 3 \\
x & =5 \times 3 \\
x & =15
\end{aligned}
$$

| Check Answer |
| :--- |
| $\frac{x}{3}-8$ $=-3$ <br> $\frac{15}{3}-8$ $=-3$ <br> $5-8$ $=-3$ <br> -3 $=-3$ |

$$
\begin{aligned}
& \text { Short Form } \\
& \begin{aligned}
\frac{x}{3}-8 & =-3 \\
\frac{x}{3} & =-3+8 \\
\frac{x}{3} & =5 \\
x & =15
\end{aligned}
\end{aligned}
$$

M8-10.6-" $a(x+b)=c, \frac{a}{x+b}=c$ " Distribution Notes

Solve for $\boldsymbol{x}$, by Distributing a into $\boldsymbol{x}+\boldsymbol{b}$.


Short Forms

$$
-4(x-3)=-8
$$

$$
x-3=2
$$

$$
\begin{gathered}
4 x=20 \\
x=5
\end{gathered}
$$

$$
-4(x-3)=-8
$$

$$
-4 x+12=-8
$$

$$
\begin{aligned}
-4 x & =-20 \\
x & =5
\end{aligned}
$$

$-4 x=-20$
$x=5$


Solve for $\boldsymbol{x}$, by Distributing a into $\boldsymbol{x}+\boldsymbol{b}$.


Short Forms
$\frac{1}{2}(x+4)=6$
$x+4=12$
$x=8$
$\begin{aligned} \frac{1}{2}(x+4) & =6 \\ \frac{x}{2}+2 & =6 \\ \frac{x}{2} & =4 \\ x & =8\end{aligned}$

Solve for $x$, by multiplying to both sides by $x+b$.

$$
\frac{14}{x-3}=2
$$

$(x-3) \times \frac{14}{x-3}=2 \times(x-3) \quad$ Multiply $x-3$ to both sides $\begin{aligned}\left(x-3 \times \frac{14}{x-2}\right. & =2 \times(x-3) \\ 14 & =2 x-6 \\ +6 & +6 \\ 20 & =2 x \\ \frac{20}{2} & =\frac{2 x}{x} \\ 10 & =x \\ x & =10\end{aligned}$

$$
\begin{array}{l|l|}
\hline \text { Cross it off } \\
\text { Distribute } \\
\frac{14}{x-3} & =2 \\
\frac{14}{10-3} & =2 \\
\frac{14}{7} & =2 \\
2 & =2
\end{array}
$$

$$
\begin{array}{|l|}
\hline \text { Short Form } \\
\frac{14}{x-3}=2 \\
14=2(x-3) \\
14=2 x-6 \\
20=2 x \\
x=10
\end{array}
$$

M8-10.7-LCD " $\frac{x}{a}+\frac{b}{c}=\frac{d}{e}$ Notes
Solve for $\boldsymbol{x}$ by multiplying each term by the LCD

| $x-1=\frac{1}{2}$ | $L C D=2$ |  |
| :---: | :---: | :---: |
| $\bigcirc$ |  | Check Answer |
| $2 \times(x-1)=\frac{1}{2} \times 2$ | Multiply both sides by 2 | $x-1=\frac{1}{2}$ |
| $2 x-2 y=1$ | Distribute | $x-1=\frac{1}{2}$ |
| A2 +2 | Add 2 to both sides | 3 - 1 |
| 7x 3 |  | $\overline{2}-1=\frac{1}{2}$ |
| $\frac{1}{4}=\frac{3}{2}$ | Divide both sides by 2 | $\frac{3}{3}-\frac{2}{2}=\frac{1}{2}$ |
|  |  | $\begin{array}{llll}2 & 2 & 2 \\ & 1 & 1\end{array}$ |


| Short Form |
| :---: |
| $x-1=\frac{1}{2}$ |
| $2(x-1)=1$ |
| $2 x-2=1$ |
| $2 x=3$ |
| $x=\frac{3}{2}$ |

OR | Algebra | Add Fractions |
| :---: | :---: |
| $x-1=\frac{1}{2}$ | $\frac{1}{2}+1$ |
| +1 | Expand $\quad 1=\frac{1}{1}=\frac{1 \times 2}{1 \times 2}=\frac{2}{2}$ |
| $x=\frac{3}{2}$ | $\frac{1}{2}+\frac{2}{2}$ |
|  | $L C D=2$ |

## Solve for $\boldsymbol{x}$ by multiplying each term by the LCD



| Short Form |
| :---: |
| $x-\frac{1}{4}=\frac{1}{2}$ |
| $\left(x-\frac{1}{4}=\frac{1}{2}\right) \times 4$ |
| $4 x-1=2$ |
| $4 x=3$ |
| $x=\frac{3}{4}$ |

Instead of actually multiplying by the LCD we are going to multiply and simplify at the same time.

## Solve for $\boldsymbol{x}$ by multiplying each term by the LCD

| $\frac{x}{2}+\frac{1}{4}=\frac{1}{3}$ | $L C D=12$ | Check Answer | Fractions $\div+$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $\left(\frac{x}{2}+\frac{1}{4}=\frac{1}{3}\right) \times 12$ | Multiply | $\frac{x}{2}+\frac{1}{4}=\frac{1}{3}$ | $\left(\frac{1}{6}\right)$ | $\frac{1}{2}+\frac{1}{4}$ |
| $\frac{12 x}{2}+\frac{12}{4}=\frac{12}{3}$ | both sides by 12 Distribute | $\left(\frac{1}{6}\right) \quad 1$ |  | 12 1 12 |
| $\begin{gathered} 2 \\ 6 x+3=4 \end{gathered}$ |  | $\left.\frac{1}{6}\right)$ $\frac{1}{1}+\frac{1}{4}=$ 1 1 | $\begin{aligned} & \frac{1}{6} \div \\ & 1 \end{aligned}$ | $\frac{\overline{12}}{4}+\frac{\overline{12}}{12}$ |
| -3 -3 | Simplify | $\frac{1}{12}+\frac{1}{4}=\frac{1}{3}$ |  | $\overline{12}$ |
| $x=\frac{1}{6}$ | Algebra | $\frac{1}{3}=\frac{1}{3} \sqrt{ }$ | $\frac{1}{12}$ | $\frac{1}{3}$ |

Short Form
$\left(\frac{x}{2}+\frac{1}{4}=\frac{1}{3}\right) \times 12$
$6 x+3=4$
$6 x=1$
$x=\frac{1}{6}$

## M8-10.8-Combining Like Terms Notes

Combine the like terms: Add/Subtract like Terms

$$
x+x=2 x \quad x+2 x=3 x \quad 2 x+4 x=6 x \quad 6 x-4 x=2 x \quad 2 x-5 x=-3 x \quad x-x=0
$$

## Solve for $\boldsymbol{x}$



Solve for $x$, by combining like terms by adding and subtracting to both sides


| $3 x+2=2 x+6$ |  |  |
| :---: | :---: | :---: |
| $3 x+2=2 x+6$ |  |  |
| $-2 \quad-2$ | Subtract 2 from both sides | Check Answer |
| $3 x=2 x+4$ |  | $3 x+2=2 x+6$ |
| $-2 x-2 x$ | Subtract $2 \times$ from both sides | $3(4)+2=2(4)+6$ |
| $x=4$ |  | $\begin{aligned} 12+2 & =8+6 \\ 14 & =14 \end{aligned}$ |


| Short Form |
| :---: |
| $3 x+2=2 x+6$ |
| $x=4$ |

Solve for $x$, by combining like terms

$$
\begin{array}{rlrl}
3 x-1+4 x & =x+11 & \\
3 x+4 x-1 & =x+11 \\
7 x-1 & =x+11 \\
+1 & & & \\
7 x & =x+12 \\
-x & -x \\
6 x & =12 & & \text { Rearrange Order of Terms (Signs!!!) } \\
\frac{6 x}{6} & =\frac{12}{6} & & \text { Combine Like Terms } \\
x & =2 & & \begin{aligned}
& \text { Check Answer } \\
& 3 x-1+4 x=x+11 \\
& 1+4(2)=(2)+11 \\
& 3(2)-1+8=2+11 \\
& 13=13
\end{aligned} \\
\hline
\end{array}
$$

M8-10.9-Creating/Solving Equations Notes

Pick a Number.
Let $x=$ the number Let Statements

| Word | Meaning |
| :---: | :---: |
| Sum, More, Add, Increased | + |
| Difference, Less, Subtract, <br> Decreased, Take away | - |
| Product, Times, Multiplied | $\times$ |
| Quotient, Divide, Split | $\div$ |

Words Problems

Let Statements
Equation
Isolate
Solve (Algebra)
Answer!
Check Answer!


| A third <br> of a <br> number | Eight divided <br> by a number | Twice the sum <br> of a number <br> and three | A number plus <br> four "ALL" <br> divided by two |
| :--- | :---: | :--- | :--- |
| $\frac{1}{3} x$ | $\frac{8}{x}$ | $2(x+3)$ | $\frac{x+4}{2}$ |

Create and Solve the following:

Five more than a number is 8 . What is the number?

$$
\begin{aligned}
& \text { Let } x=\text { the } \# \quad \text { Let Statements } \\
& x+5=8 \quad \text { Create Equation } \\
& \text { Check Answer } \\
& x+5=8 \\
& \begin{array}{r}
-5 \quad-5 \\
x=3
\end{array} \\
& \text { Solve } \\
& x+5=8 \\
& (3)+5=8 \\
& 8=8 \mathrm{~V}
\end{aligned}
$$

The number is 3 Answer the question

Twice the "SUM" of a number and three is 12. What is the number?

$$
\text { Let } x=\text { the number }
$$

$$
\begin{aligned}
& 2(x+3)=12 \\
& 2(x+3)=12
\end{aligned}
$$

$$
2 x+6=12
$$

$$
\begin{aligned}
& -6=-6 \\
& 2 x=6
\end{aligned}
$$

$$
\frac{2 x}{2}=\frac{6}{2}
$$

$$
\begin{aligned}
& \hline \text { Check Answer } \\
& 2(x+3)=12 \\
& 2((3)+3)=12 \\
& 2(6)=12
\end{aligned}
$$

$$
x=3
$$

The number is 3

Three less than twice a number is 7. What is the number?

Let $x=\#$

$$
2 x-3=7
$$

$$
2 x-3=7
$$

$$
+3+3
$$

$$
2 x=10
$$

$$
\frac{2 x}{2}=\frac{10}{2}
$$



$$
\begin{array}{rl|}
\hline \text { Check Answer } \\
2 x-3 & =7 \\
2(5)-3 & =7 \\
10-3 & =7 \\
7 & =7
\end{array}
$$

Five times a number plus three "ALL" divided by two equals triple the number. What is the number?

Let $x=\#$
$\frac{(5 x+3)}{2}=3 x$

| $2 \times \frac{5 x+3}{2}=3 x \times 2$ |
| :---: | :---: |
| $5 x+3=6 x$ |
| $-5 x \quad-5 x$ |
| $x=3$ |
| The number is 3 |\(\quad \begin{array}{r}\frac{5(3)+3}{2}=3(3) <br>

\frac{18}{2}=9 <br>
9=9 \mathrm{~V}\end{array}\)

## M8-10.9 - One vs Two Variable Equations Notes

Create and Solve the following:
One number is two more than another and their sum is 12 . What are the numbers?

| $\begin{aligned} \text { Let } x & =1 \text { st } \# \\ \text { Let } x-2 & =2 n d \# \end{aligned}$ | One Variable! | $\begin{aligned} \text { Let } x & =1 s t \# \\ \text { Let } x+2 & =2 n d \end{aligned}$ |  |
| :---: | :---: | :---: | :---: |
| $x+(x-2)=12$ |  | $x+(x+2)=12$ |  |
| $x+x-2=12$ |  | $x+x+2=12$ |  |
| $2 x-2=12$ |  | $\begin{array}{r} 2 x+2=12 \\ -2 \end{array} \quad \text { OR }$ | Words Problems |
| $2 x=14$ |  | $2 x=10$ | Let Statements |
| $2 x \quad 14$ |  | $2 x \quad 10$ | Equation |
| $\overline{2}=\frac{1}{2}$ |  | $\overline{2}=\frac{1}{2}$ | Solve (Algebra) |
| $x=7$ | $\begin{aligned} 2 n d \# & =x-2 \\ & =(7)-2 \end{aligned}$ | $x=5 \quad \begin{aligned} 2 n d \# & =x+2 \\ & =(5)+2 \end{aligned}$ | Answer! <br> Check Answer! |

One number is two more than another and their sum is 12 . What are the numbers?

Let $x=1$ st $\#$
Let $y=2 n d \quad$ Two Variable! $\quad$ R


## M8-10.9-2/3 Number/Consecutive Equations Notes

Create and Solve the following:
The sum of three numbers is 67 . The 2 nd number one less than is twice the 1 st. The 3 rd number is four more than the 1st.

```
    Let \(x=1\) st \(\#\)
Let \(2 x-1=2 n d \#\)
    Let \(x+4=3 r d\) \#
    \(x+2 x-1+x+4=67\)
    \(x+2 x-1+x+4=67\)
        \(4 x+3=67\)
            \(-3-3\)
            \(4 x=64\)
                \(\frac{4 x}{4}=\frac{64}{4}\)
1 st \(\#=16 \quad x=16\)
\[
\begin{array}{rlrl}
2 n d \# & =2 x-1 & 3 r d \# & =x+4 \\
& =2(16)-1 & & =(16)+4 \\
& =32-1 & & 3 r d \#=20 \\
2 n d \# & =31 & &
\end{array}
\]
\(1 s t \#=16 \quad x=16\)
```



```

> Check Answer
> \(16+31+20=67 \mathrm{~V}\)
```

The sum of three consecutive integers is 24 .

Let $x=1$ st $\#$
Let $x+1=2 n d$ \#
Let $x+2=3 r d$ \#

Consecutive Integers: ie. -2,-1,0,1,2,3,4,5,6 Consecutive Even Integers: ie. -2,0,2,4,6
Consecutive Odd Integers: ie. -1,1,3,5,7
$x+x+1+x+2=24$
$x+x+1+x+2=24$

$$
3 x+3=24
$$

$1 s t \#=7 \quad x=7$


$$
\begin{array}{ll}
-3 & -3
\end{array}
$$

$$
3 x=21
$$

$$
\frac{3 x}{3}=\frac{21}{3}
$$

Find three consecutive odd integers where five less than triple the 2nd is quadruple the 1st.

$$
\text { Let } x=1 s t \#
$$

Let $x+2=2 n d \#$
Let $x+4=3 r d \#$


## M8-10.9-Age/Now-Then Equations Notes

Create and Solve the following:
Four years less than triple Mark's age equals fourteen years more than double his age. How old is Mark?
Let $m=M a r k ' s$ age

$$
3 m-4=2 m+14
$$

$$
3 m-4=2 m+14
$$

$$
-2 m \quad-2 m
$$

$$
m-4=14
$$

$$
+4 \quad+4
$$



If Nicole were triple her age she was three years ago she would be twice her current age. How old is Nicole now?

$$
\begin{aligned}
\text { Let } n & =\text { Nicole's age } \\
\text { Let } n-3 & =\text { Nicole's age } 3 \text { years ago } \\
\text { Let } 2 n & =\text { Twice Nicole's age }
\end{aligned}
$$

$$
3(n-3)=2 n
$$

$$
3(n-3)=2 n
$$

$$
3 n-9=2 n
$$

$$
-3 n \quad-3 n
$$

$$
-9=-n
$$

$$
\frac{-9}{-1}=\frac{-n}{-1}
$$

$$
9=n
$$

$$
\begin{array}{|c|}
\hline \text { Check Answer } \\
3(9-3)=2(9) \\
3(6)=2(9) \\
18=18 \\
\hline
\end{array}
$$

Answer
$n=9$


