M8 - 10.0 - Solving Equations Notes

Solve for x

$x + 5 = 9$
$x + 5 = 9$
$-5 \quad -5$
✓ $\boxed{x = 4}$

Check Answer
$x + 5 = 9$
$(4) + 5 = 9$
$9 = 9$

In your head!

$x + 5 = 9$
$x = 9 - 5$
$\boxed{x = 4}$

$x - 3 = 7$
$x - 3 = 7$
$+3 \quad +3$
$x = 7 + 3$
$\boxed{x = 10}$

$x - 3 = 7$
$x = 7 + 3$
$\boxed{x = 10}$

$x - 3 = 7$
$-7 \quad -7$
$x - 10 = 0$
$+10 \quad +10$
$\boxed{x = 10}$

$-x + 2 = 5$
$-x + 2 = 5$
$-2 \quad -2$
$-x = 3$
$\dfrac{-x}{-1} = \dfrac{3}{-1}$
$\boxed{x = -3}$

OR

$-x + 2 = 5$
$+x \quad +x$
$2 = 5 + x$
$-5 \quad -5$
$-3 = x$
$\boxed{x = -3}$

$2x = 4$
$2x = 4$
$\dfrac{2x}{2} = \dfrac{4}{2}$
$\dfrac{2x}{2} = \dfrac{4}{2}$
$x = \dfrac{4}{2}$
$\boxed{x = 2}$

$2x = \dfrac{4}{2}$
$\boxed{x = 2}$

$2x = 4$
$2(2) = 4$
$4 = 4$ ✓

$\dfrac{x}{3} = 6$
$3 \times \dfrac{x}{3} = 6 \times 3$
$x = 6 \times 3$
$\boxed{x = 18}$

$\dfrac{x}{3} = 6 \times 3$
$\boxed{x = 18}$

$2x + 1 = 5$
$-1 \quad -1$
$2x = 4$
$\dfrac{2x}{2} = \dfrac{4}{2}$
$\boxed{x = 2}$

$2x + 1 = 5 - 1$
$x = \dfrac{4}{2}$
$\boxed{x = 2}$

$2x + 1 = 5$
$2(2) + 1 = 5$
$4 + 1 = 5$
$5 = 5$

$\dfrac{x - 24}{3} = -3$
$3 \times \dfrac{x - 24}{3} = -3 \times 3$
$x - 24 = -9$
$+24 \quad +24$
$\boxed{x = 15}$

$\dfrac{x - 24}{3} = -3 \times 3 + 24$
$\boxed{x = 15}$

$\dfrac{x - 24}{3} = -3$
$\dfrac{(15) - 24}{3} = -3$
$\dfrac{-9}{3} = -3$
$-3 = -3$

$\dfrac{5}{4}x = 10$
$4 \times \dfrac{5}{4}x = 10 \times 4$
$\dfrac{5x}{5} = \dfrac{40}{5}$
$\boxed{x = 8}$

$\dfrac{5}{4}x = 10 \times \dfrac{4}{5}$
$\boxed{x = 8}$

$\dfrac{8}{x} = 4$
$x \times \dfrac{8}{x} = 4 \times x$
$\dfrac{8}{4} = \dfrac{4x}{4}$
$\boxed{2 = x}$

$\dfrac{8}{x} = 4$
$\dfrac{8}{4} = x$
$\boxed{x = 2}$

$\dfrac{24}{2x} = 3$
$2x \times \dfrac{24}{2x} = 3 \times 2x$
$\dfrac{24}{6} = \dfrac{6x}{6}$
$\boxed{4 = x}$

$\dfrac{24}{2x} = 3$
$\dfrac{24}{2(3)} = x$
$\boxed{x = 4}$

$\dfrac{x}{6} = \dfrac{4}{3}$
$\dfrac{x}{6} = \dfrac{4}{3}$
$3 \times x = 4 \times 6$
$3x = 24$
$\dfrac{3x}{3} = \dfrac{24}{3}$
$\boxed{x = 8}$

$\dfrac{x}{6} = \dfrac{4}{3}$
$3 \times x = 4 \times 6$
$\boxed{\cdots}$

$6x + 8 = 50$
$6x + 8 = 50$
$-8 \quad -8$
$\dfrac{6x}{6} = \dfrac{42}{6}$
$x = \dfrac{42}{6}$
$\boxed{x = 7}$

$6x + 8 = 50$
$6x = 50 - 8$
$6x = 42$
$\boxed{x = 7}$

$\dfrac{x}{3} - 8 = -3$
$\dfrac{x}{3} - 8 = -3$
$+8 \quad +8$
$\dfrac{x}{3} = 5$
$3 \times \dfrac{x}{3} = 5 \times 3$
$x = 5 \times 3$
$\boxed{x = 15}$

$\dfrac{x}{3} - 8 = -3$
$\dfrac{x}{3} = -3 + 8$
$\dfrac{x}{3} = 5$
$\boxed{x = 15}$

$\dfrac{1}{2}(x + 4) = 6$
$\dfrac{x}{2} + \dfrac{4}{2} = 6$
$\dfrac{x}{2} + 2 = 6$
$-2 \quad -2$
$\dfrac{x}{2} = 4$
$2 \times \dfrac{x}{2} = 4 \times 2$
$\boxed{x = 8}$

$\dfrac{1}{2}(x + 4) = 6$
$2 \times \dfrac{1}{2}(x + 4) = 6 \times 2$
$x + 4 = 12$
$-4 \quad -4$
$\boxed{x = 8}$

$-4(x - 3) = -8$
$-4x + 12 = -8$
$-12 \quad -12$
$\dfrac{-4x}{-4} = \dfrac{-20}{-4}$
$\boxed{x = 5}$

$\dfrac{-4(x - 3)}{-4} = \dfrac{-8}{-4}$
$x - 3 = 2$
$+3 \quad +3$
$\boxed{x = 5}$

M8 - 10.0 - Solving Equations Notes

$x = 1 + 2$
$\boxed{x = 3}$

$x + x = 4$
$2x = 4$
$\dfrac{2x}{2} = \dfrac{4}{2}$
$\boxed{x = 2}$

$3x + 3x = 4 + 8$
$6x = 12$
$\dfrac{6x}{6} = \dfrac{12}{6}$
$\boxed{x = 2}$

$2x = 4 + x$
$-x \qquad -x$
$\boxed{x = 4}$

$2x = 4 + x$
$-4 \quad -4$
$2x - 4 = x$
$-2x \qquad -2x$
$-4 = -x$
$\dfrac{-4}{-1} = \dfrac{-x}{-1}$
$\boxed{x = 4}$

$3x + 2 = 2x + 6$
$-2 \qquad -2$
$3x = 2x + 4$
$-2x \quad -2x$
$\boxed{x = 4}$

$3x + 4x - 1 = x + 11$
$7x - 1 = x + 11$
$+1 \qquad +1$
$7x = x + 12$
$-x \qquad -x$
$6x = 12$
$\dfrac{6x}{6} = \dfrac{12}{6}$
$\boxed{x = 2}$

Multiply Both Sides by the LCD

$\dfrac{14}{x - 3} = 2$

$(x - 3) \times \dfrac{14}{x - 3} = 2 \times (x - 3)$

$(x - 3) \times \dfrac{14}{x - 3} = 2 \times (x - 3)$

$14 = 2x - 6$
$+6 \qquad +6$
$20 = 2x$
$\dfrac{20}{2} = \dfrac{2x}{x}$
$10 = x$
$\boxed{x = 10}$

$\boxed{\begin{array}{l} \dfrac{14}{x - 3} = 2 \\[4pt] 14 = 2(x - 3) \\ 14 = 2x - 6 \\ 20 = 2x \\ \boxed{x = 10} \end{array}}$

$x - 1 = \dfrac{1}{2}$
$2 \times (x - 1) = \dfrac{1}{2} \times 2$
$2x - 2 = 1$
$+2 \qquad +2$
$\dfrac{2x}{2} = \dfrac{3}{2}$
$\boxed{x = \dfrac{3}{2}}$

$x - 1 = \dfrac{1}{2}$
$+1 \quad +1$
$x = \dfrac{1}{2} + 1$
$x = \dfrac{1}{2} + 1 \times \dfrac{2}{2}$
$x = \dfrac{1}{2} + \dfrac{2}{2}$
$\boxed{x = \dfrac{3}{2}}$

$x - \dfrac{1}{4} = \dfrac{1}{2}$
$4 \times \left(x - \dfrac{1}{4} \right) = \dfrac{1}{2} \times 4$
$4x - \dfrac{4}{4} = \dfrac{4}{2}$
$4x - 1 = 2$
$+1 \quad +1$
$4x = 3$
$\dfrac{4x}{4} = \dfrac{3}{4}$
$\boxed{x = \dfrac{3}{4}}$

$\dfrac{12}{x - 1} = \dfrac{4}{x + 1}$
$12(x + 1) = 4(x - 1)$
$12x + 12 = 4x - 4$
$-4x \qquad -4x$
$8x + 12 = -4$
$-12 \quad -12$
$8x = -16$
$\dfrac{8x}{8} = \dfrac{-16}{8}$
$\boxed{x = -2}$

$2x = \dfrac{1}{4}$
$\dfrac{2x}{2} = \dfrac{\frac{1}{4}}{2}$
$x = \dfrac{1}{4} \div \dfrac{2}{1}$
$x = \dfrac{1}{4} \times \dfrac{1}{2}$
$\boxed{x = \dfrac{1}{8}}$

$2x = \dfrac{1}{4}$
$4 \times 2x = \dfrac{1}{4} \times 4$
$8x = 1$
$\dfrac{8x}{8} = \dfrac{1}{8}$
$\boxed{x = \dfrac{1}{8}}$

$\dfrac{x}{2} + \dfrac{1}{4} = \dfrac{1}{3}$
$\left(\dfrac{x}{2} + \dfrac{1}{4} = \dfrac{1}{3} \right) \times 12$
$\dfrac{12x}{2} + \dfrac{12}{4} = \dfrac{12}{3}$
$6x + 3 = 4$
$-3 \quad -3$
$6x = 1$
$\boxed{x = \dfrac{1}{6}}$

$\dfrac{x}{2} + \dfrac{1}{4} = \dfrac{1}{3}$
$-\dfrac{1}{4} \quad -\dfrac{1}{4}$
$\dfrac{x}{2} = \dfrac{1}{3} - \dfrac{1}{4}$
$\dfrac{x}{2} = \dfrac{4}{12} - \dfrac{3}{12}$
$\dfrac{x}{2} = \dfrac{1}{12}$
$2 \times \dfrac{x}{2} = \dfrac{1}{12} \times 2$
$\boxed{x = \dfrac{1}{6}}$

M8 - 10.0 - Solving Equations Notes

Five more than a number is 8. What is the number?

$Let\ x = the\ \#$ | Let Statements

$x + 5 = 8$
$\quad -5 \quad -5$
✓ $x = 3$ The number is 3

Check Answer
$x + 5 = 8$
$(3) + 5 = 8$
$8 = 8$

Twice the "SUM" of a number and three is 12. What is the number?

$Let\ x = the\ number$

$2(x + 3) = 12$
$2x + 6 = 12$
$\quad\quad -6 \quad -6$
$2x = 6$
$\dfrac{2x}{2} = \dfrac{6}{2}$
$x = 3$ The number is 3

Check Answer
$2(x + 3) = 12$
$2((3) + 3) = 12$
$2(6) = 12$ ✓

Three less than twice a number is 7. What is the number?

$Let\ x = \#$

$2x - 3 = 7$
$\quad +3 \quad +3$
$2x = 10$
$\dfrac{2x}{2} = \dfrac{10}{2}$
$x = 5$ The number is 5

Check Answer
$2x - 3 = 7$
$2(5) - 3 = 7$
$10 - 3 = 7$
$7 = 7$ ✓

Five times a number plus three "ALL" divided by two equals triple the number. What is the number?

$Let\ x = \#$

$\dfrac{(5x + 3)}{2} = 3x$
$2 \times \dfrac{5x + 3}{2} = 3x \times 2$
$5x + 3 = 6x$
$-5x \quad\quad -5x$
$x = 3$ The number is 3

Check Answer
$\dfrac{5x + 3}{2} = 3x$
$\dfrac{5(3) + 3}{2} = 3(3)$
$\dfrac{18}{2} = 9$
$9 = 9$ ✓

One number is two more than another and their sum is 12. What are the numbers?

$Let\ x = 1st\ \#$
$Let\ x - 2 = 2nd\ \#$ | One Variable!

$x + (x - 2) = 12$
$x + x - 2 = 12$
$2x - 2 = 12$
$\quad +2 \quad +2$
$2x = 14$
$\dfrac{2x}{2} = \dfrac{14}{2}$
$x = 7$

$1st\ \# = 7$
$2nd\ \# = 5$

OR
$Let\ x = 1st\ \#$
$Let\ x + 2 = 2nd\ \#$

$2nd\# = x - 2$
$= (7) - 2$
$2nd\# = 5$

Check Answer
$5 + 2 = 7$ ✓
$5 + 7 = 12$ ✓

Words Problems | Solve (Algebra)
Diagram | Substitute
Let Statements | Solve
Equation/s | Answer in English!
(Arbitrary#'s) | Check Answer!
Isolate | (Eliminate*)
Substitute | Explain it to a 10 year old!

$Let\ x = 1st\ \#$
$Let\ y = 2nd\ \#$ | Two Variables!

Equation #1
$x + y = 12$
$-x \quad\quad -x$
$y = (12 - x)$

Isolate
Substitute
Solve (Algebra)
Substitute
Solve

$y = 12 - x$
$y = 12 - (7)$
$y = 5$ Solve

$1st\ \# = 7$
$2nd\ \# = 5$

Equation #2
$x - y = 2$
$x - (12 - x) = 2$
$x - 12 + x = 2$
$2x - 12 = 2$
$\quad +12 \quad +12$
$\dfrac{2x}{2} = \dfrac{14}{2}$
$x = 7$

(Substitute)

$x - y = 2$
$y + 2 = x$
...
It Doesn't Matter*!

84m

x | $x + 18$

$let\ x = shorter\ length$
$let\ x + 18 = longer\ length$

$x + x + 18 = 84$
$\dfrac{2x}{2} = \dfrac{66}{2}$
$x = 33$

$shorter\ length = 33m$
$longer\ length = x + 18 = 51m$

M8 - 10.0 - Solving Equations Notes

The sum of three numbers is 67. The 2nd number one less than is twice the 1st. The 3rd number is four more than the 1st.

$$Let\ x = 1st\ \#$$
$$Let\ 2x - 1 = 2nd\ \#$$
$$Let\ x + 4 = 3rd\ \#$$

$$1st\ \# = 16$$
$$2nd\ \# = 31$$
$$3rd\ \# = 20$$

$$x + 2x - 1 + x + 4 = 67$$
$$4x + 3 = 67$$
$$-3 \quad -3$$
$$4x = 64$$
$$\frac{4x}{4} = \frac{64}{4}$$
$$1st\ \# = 16 \quad x = 16$$

$$2nd\# = 2x - 1$$
$$= 2(16) - 1$$
$$= 32 - 1$$
$$2nd\# = 31$$

$$3rd\# = x + 4$$
$$= (16) + 4$$
$$3rd\# = 20$$

$$16 + 31 + 20 = 67 \checkmark$$

The sum of three consecutive integers is 24.

$$Let\ x = 1st\ \#$$
$$Let\ x + 1 = 2nd\ \#$$
$$Let\ x + 2 = 3rd\ \#$$

$$x + x + 1 + x + 2 = 24$$
$$3x + 3 = 24$$
$$-3 \quad -3$$
$$3x = 21$$
$$\frac{3x}{3} = \frac{21}{3}$$
$$x = 7$$

$$2nd\# = x + 1$$
$$= (7) + 1$$
$$2nd\# = 8$$

$$3rd\# = x + 2$$
$$= (7) + 2$$
$$3rd\# = 9$$

$$7 + 8 + 9 = 24 \checkmark \quad 1st\ \# = 7$$

| Consecutive Integers: ie. -2,-1,0,1,2,3,4,5,6 |
| Consecutive Even Integers: ie. -2,0,2,4,6 |
| Consecutive Odd Integers: ie. -1,1,3,5,7 |

Find three consecutive odd integers where five less than triple the 2nd is quadruple the 1st.

$$Let\ x = 1st\ \#$$
$$Let\ x + 2 = 2nd\ \#$$
$$Let\ x + 4 = 3rd\#$$

$$3(x + 2) - 5 = 4x$$
$$3x + 6 - 5 = 4x$$
$$3x + 1 = 4x$$
$$-3x \qquad -3x$$
$$1 = x$$
$$x = 1$$

$$2nd\# = x + 2$$
$$= (1) + 2$$
$$2nd\# = 3$$

$$3rd\# = x + 5$$
$$= (1) + 4$$
$$3rd\# = 5$$

$$1st\ \# = 1$$

$$3(3) - 5 = 4(1)$$
$$9 - 5 = 4$$
$$4 = 4 \checkmark$$

Find a number where it equals its square.

$$let\ x = \#$$

$$x = x^2 \qquad \text{Trial and Error}$$
$$...$$
$$x = 0,1 \qquad \text{See Grade 11 Quadratics}$$

Find a number where itself plus 2 equals its square.

$$let\ x = \#$$

$$x + 2 = x^2$$
$$...$$
$$x = 2, -1$$

Four years less than triple Mark's age equals fourteen years more than double his age. How old is Mark?

$$Let\ m = Mark's\ age$$

$$3m - 4 = 2m + 14$$
$$-2m \qquad -2m$$
$$m - 4 = 14$$
$$+4 \quad +4$$
$$m = 18 \qquad \text{Mark is 18 years old}$$

$$3(18) - 4 = 2(18) + 14 \checkmark$$

If Nicole were triple her age she was three years ago she would be twice her current age. How old is Nicole now?

$$Let\ n = Nicole's\ age$$
$$Let\ n - 3 = Nicole's\ age\ 3\ years\ ago$$
$$Let\ 2n = Twice\ Nicole's\ age$$

$$3(n - 3) = 2n$$
$$3n - 9 = 2n$$
$$-3n \qquad -3n$$
$$-9 = -n$$
$$\frac{-9}{-1} = \frac{-n}{-1}$$
$$9 = n$$
$$n = 9 \qquad \text{Nicole is 9 years old now}$$

$$3(9 - 3) = 2(9)$$
$$3(6) = 2(9)$$
$$18 = 18 \checkmark$$

$\boxed{a + b = c}$ *Solve for "a"* $\boxed{+ = -}$
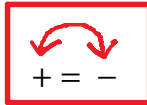
$a + b = c$
$\underline{-b \quad -b}$
$\boxed{a = c - b}$

Subtract "b" from both sides

$a + b = c$
$\boxed{a = c - b}$ *Bring "b" over, change sign*

$\boxed{v = \dfrac{d}{t}}$

$v = \dfrac{d}{t}$ *Solve for d*

$t \times v = \dfrac{d}{t} \times t$ × *both sides by "t"*
$tv = d$ *Simplify*
$\boxed{d = vt}$ *Mirror*

$\boxed{\begin{array}{l} v = \dfrac{d}{t} \quad \text{Bring t up} \\ tv = d \quad \text{Mirror} \\ \boxed{d = vt} \end{array}}$

Just cross multiply!

$v = \dfrac{d}{t}$ *Solve for t*

$t \times v = \dfrac{d}{t} \times t$
$tv = d$
$\dfrac{tv}{v} = \dfrac{d}{v}$
$\boxed{t = \dfrac{d}{v}}$

× *both sides by "t"*
÷ *both sides by "v"*

$\boxed{\begin{array}{l} \text{Bring } t \text{ up} \\ \text{Bring } v \text{ down} \end{array}}$
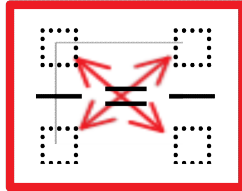
$\boxed{\begin{array}{l} v = \dfrac{d}{t} \\ \boxed{t = \dfrac{d}{v}} \quad \text{Switch } v \text{ and } t \end{array}}$

$\boxed{\dfrac{a}{b} = \dfrac{c}{d}}$

Solve for "a"
$\dfrac{a}{b} = \dfrac{c}{d}$
$\boxed{a = \dfrac{cb}{d}}$
Bring "b" up

Solve for c
$\dfrac{a}{b} = \dfrac{c}{d}$
$\dfrac{ad}{b} = c$
$\boxed{c = \dfrac{ad}{b}}$
Bring "d" up
Mirror

Solve for b
$\dfrac{a}{b} = \dfrac{c}{d}$
$a = \dfrac{cb}{d}$
$ad = cb$
$\dfrac{ad}{c} = b$
$\boxed{b = \dfrac{ad}{c}}$

Bring b up
Bring d up
Bring c down
Mirror

Solve for "d"
$\dfrac{a}{b} = \dfrac{c}{d}$
$\dfrac{ad}{b} = c$
$ad = cb$
$\boxed{d = \dfrac{cb}{a}}$

Bring d up
Bring b up
Bring "a" down

Solve for a.

$ab + c = d$
$\underline{\quad -c \quad\quad -c}$
$ab = d - c$
$\dfrac{ab}{b} = \dfrac{d - c}{b}$
$\boxed{a = \dfrac{d - c}{b}}$

$\overset{1}{\boxed{ab + c = \dfrac{d - c}{b}}}\underset{2}{}$