

M8 - 11.1 - Probability Notation/Rules Notes

Probability Notation

Event	Sample Space	Notation
For a coin toss	Heads, Tails	$S = \{H, T\}$
Six-sided die?	1, 2, 3, 4, 5, 6	$S = \{1,2,3,4,5,6\}$

Sample Space:
The set of all possible outcomes.

$P(E)$ is the probability of event E taking place.

$$P(H) = \frac{1}{2} = 0.5 = 50\%$$

$$P(1) = \frac{1}{6} = 0.1\bar{6} = 16.67\%$$

Probabilities can be expressed: as decimals or fractions between 0 and 1
: as percentages between 0 and 100%.

$$0 \leq P(E) \leq 1$$

$$0\% \leq P(E) \leq 100\%$$

If an event can't happen it has a probability of 0.

The probability of rolling a 7 on a standard six-sided die has a probability of 0.

$$P(7) = 0$$

$$P(E) \neq > 1 \text{ or } 100\%$$

The probability can never be less than 0% or greater than 100%.

If an event will happen with certainty, it has a probability of 1.

The probability of getting a head or a tail when flipping a coin is 1.

$$P(H \cup T) = 1$$

\cup : OR

If the probability of an event occurring is $P(E)$, then the probability that it DOESN'T occur is:

The probability of NOT rolling a 6 is:

$$P(\bar{6}) = 1 - P(6)$$

\bar{E} : Not E

$$P(\bar{E}) = 1 - P(E)$$

Compliment

$$= 1 - \left(\frac{1}{6}\right)$$

$$P(\bar{6}) = \frac{5}{6}$$

The sum of probabilities of all outcomes in the sample space must sum to 1.

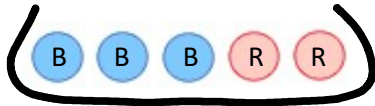
When rolling a dice the sample space is $S = \{1,2,3,4,5,6\}$ and the sum of probabilities of all possible outcomes is:

$$\begin{aligned}
 P(1,2,3,4,5 \text{ or } 6) &= P(1) + P(2) + P(3) + P(4) + P(5) + P(6) \\
 &= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \\
 &= \frac{6}{6} \\
 &= 1
 \end{aligned}$$

$$P(1,2,3,4,5 \text{ or } 6) = 1$$

M8 - 11.1 - Marbles Probability Notes

You have 3 blue marbles and 2 red marbles in a bag, a total of 5 marbles.



$$\text{Probability} = \frac{\text{number of desired outcomes}}{\text{total outcomes}}$$

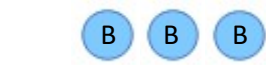
If you randomly take a marble out of the bag what is the probability that it will be:

A blue marble?

$$P(B) = ?$$

A red marble?

$$P(R) = ?$$



$$P(B) = \frac{3 \text{ blue}}{5 \text{ total}}$$



$$P(R) = \frac{2 \text{ red}}{5 \text{ total}}$$



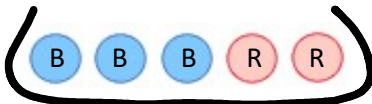
$$P(B) = 0.6$$



$$P(R) = 0.4$$

$$P(B) + P(R) = 1$$

You replace the marble. You now take a another marble out of the bag. Find the probability of:



With Replacement w/ rep

A blue marble (given blue)?

given: |

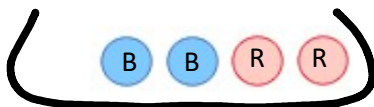
A red marble (given blue)?

$$P(B|B) = 0.6$$

$$P(R|B) = 0.4$$

$P(B|B) = P(B)$ **Independent** $P(R|B) = p(R)$
Probability does not depend

You now take a blue marble out of the bag.



Without Replacement w/o rep

What is now the probability that your next drawn marble will be:

A blue marble (given blue)?

A red marble (given blue)?



$$p(b|b) = \frac{2 \text{ blue}}{4 \text{ total}}$$



$$p(r|b) = \frac{2 \text{ blue}}{4 \text{ total}}$$



$$p(b|b) = 0.5$$



$$p(r|b) = 0.5$$

$p(b|b) \neq p(b^*)$ **Dependent** $p(r|b) \neq p(r^*)$
Probability depends

M8 - 11.1 - Coin Flip Probability Notes

What is the probability of flipping a Head?

$$P(H) = \frac{1}{2}$$

← 1 Head
← 2 possible outcomes (Heads or Tails)

What is the probability of flipping a Tail?

$$P(T) = \frac{1}{2}$$

← 1 Tail
← 2 possible outcomes (Heads or Tails)



If you flip a coin 2 times in a row:

Sample Space: HH, HT, TH, TT

Sample Space: Possible Outcomes

What is the probability of flipping?

Independent

$\cdot = \times$

$TH \neq HT$

Two Heads in a row?

Two Tails in a row?

A Tail THEN a Head?

A Head THEN a Tail?

$$P(HH) = P(H) \cdot P(H)$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$P(HH) = \frac{1}{4}$$

$$P(TT) = P(T) \cdot P(T)$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$P(TT) = \frac{1}{4}$$

$$P(TH) = P(T) \cdot P(H)$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$P(TH) = \frac{1}{4}$$

$$P(HT) = P(H) \cdot P(T)$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$P(HT) = \frac{1}{4}$$

$P(HH)$ or $P(2H)$

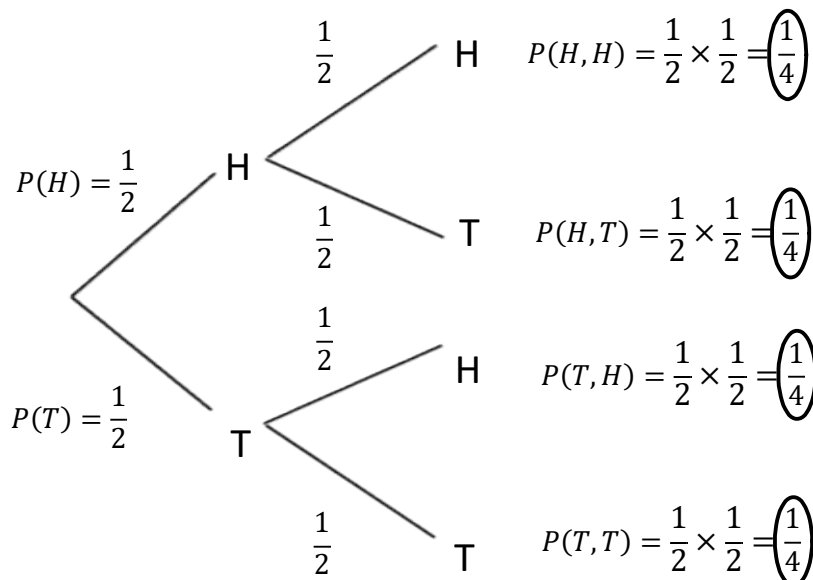
Method 1: Multiply

Method 2: Table

	H	T
H	H,H	H,T
T	T,H	T,T

$$P(TT) = \frac{1}{4}$$

Method 3: Tree



Multiply Branches

Add Leaves

$$P(H \cap T) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

A Head AND a Tail? \cap : And

$$P(H \cap T) = P(HT) + P(TH)$$

$$= P(H) \times P(T) + P(T) \times P(H)$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4} + \frac{1}{4}$$

$$P(H \cap T) = \frac{1}{2}$$

$P(H \cap T) = P(T \cap H)$

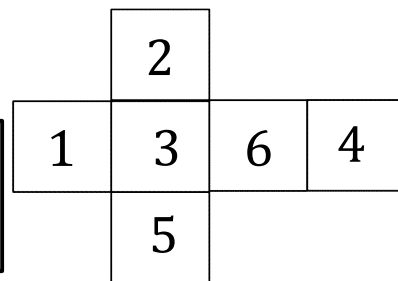
$P(HT \cup TH) = P(H \cap T)$ \cup : OR

M8 - 11.1 - Rolling a Dice Probability Notes

What is the probability of rolling a 6 with a die?

$$P(6) = \frac{1}{6}$$

← 1 Six
← 6 total numbers



What is the probability of rolling two 6's?

In a row = two separate dice

Method 1: Multiply

Independent

$$P(66) = P(6) \times P(6)$$

$$= \frac{1}{6} \times \frac{1}{6}$$

$$P(66) = \frac{1}{36}$$

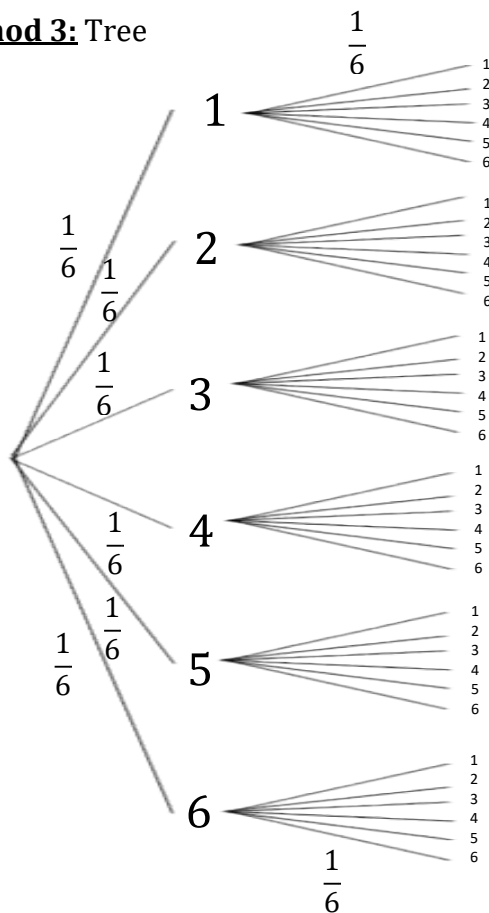


Method 2: Table

Sample Space

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Method 3: Tree



$$P(6,6) = \frac{1}{36}$$

$$P(6,6) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

M8 - 11.1 - Rolling a Dice and Coin Flip Probability Notes

What is the probability of flipping a Tail with a coin and rolling a 4 with a die?

Method 1: Multiply $P(T4) = P(T) \times P(4)$

$$= \frac{1}{2} \times \frac{1}{6}$$

$$P(T4) = \frac{1}{12}$$



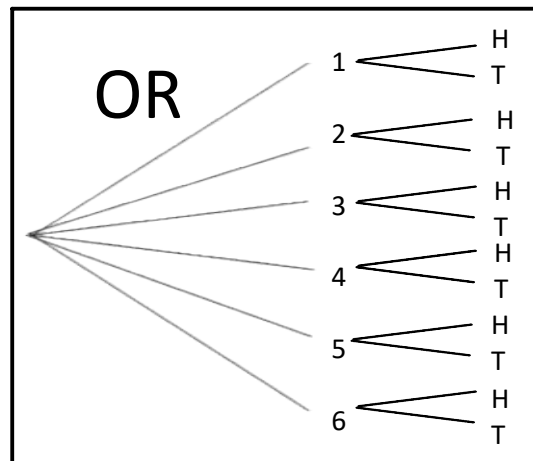
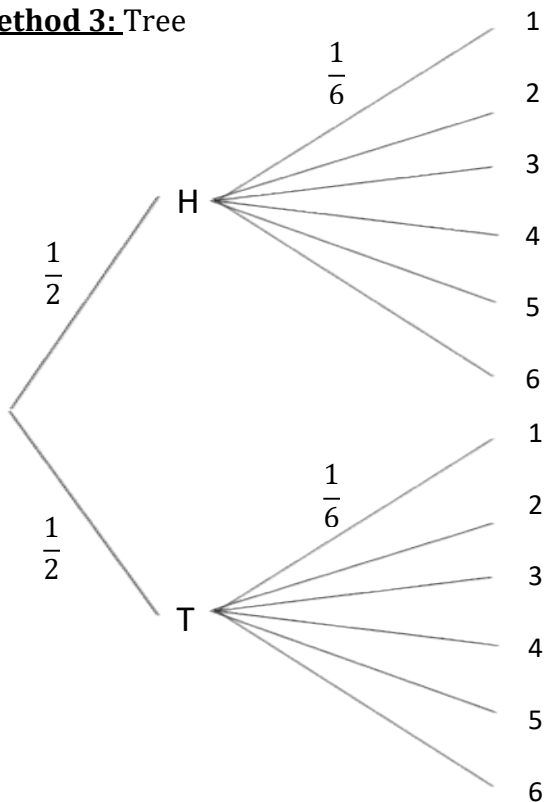
Method 2: Table

	1	2	3	4	5	6
H	H,1	H,2	H,3	H,4	H,5	H,6
T	T,1	T,2	T,3	T,4	T,5	T,6

Sample Space

$$P(T, 4) = \frac{1}{12}$$

Method 3: Tree



$$P(T, 4) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

M8 - 11.1 - Mean, Median, Mode, Range Notes

Mode: Most occurring number.

$$\text{Mode} = 3$$

0, 1, 1, 2, 2, 3, 3, 3, 4, 4, 10

$$\text{Range: } 10 - 0 = 10$$

Range: Top# - Bottom#

Median: Middle number

$$\text{Median} = 3$$

$$\text{Mean} = \frac{\text{All Numbers Added}}{\text{Number of Numbers}} \quad (\text{Average})$$

$$\text{Mean} = \frac{0 + 1 + 1 + 2 + 2 + 3 + 3 + 3 + 4 + 4 + 10}{11}$$

$$\text{Mean} = \frac{33}{11}$$

$$\text{Mean} = 3$$

No Mode

1, 3, 5, 7

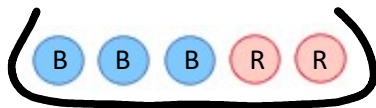
$$\text{Median} = \frac{3 + 5}{2}$$

$$= \frac{8}{2}$$

$$\text{Median} = 4$$

M8 - 11.1 - Odds Probability Notes

You have 3 blue marbles and 2 red marbles in a bag, a total of 5 marbles.



Odds: Odds in Favor : Odds Against

Choose a Marble. What are the odds?

Odds in favour Blue = Odds against Red

Odds in favour Red = Odds against Blue

3Blue: 2Red

3Red: 2Blue

$$3 : 2 \leftarrow 5 - 3 = 2$$

$$2 : 3 \leftarrow 5 - 2 = 3$$

Odds Against = Total - Odds in favour

Pick a Card.

	Hearts ♥	Diamonds ♦	Spades ♠	Clubs ♣
S	Ace ♥	Ace ♦	Ace ♠	Ace ♣
A	2 ♥	2 ♦	2 ♠	2 ♣
M	3 ♥	3 ♦	3 ♠	3 ♣
P	4 ♥	4 ♦	4 ♠	4 ♣
L	5 ♥	5 ♦	5 ♠	5 ♣
E	6 ♥	6 ♦	6 ♠	6 ♣
S	7 ♥	7 ♦	7 ♠	7 ♣
P	8 ♥	8 ♦	8 ♠	8 ♣
A	9 ♥	9 ♦	9 ♠	9 ♣
C	10 ♥	10 ♦	10 ♠	10 ♣
E	Jack ♥	Jack ♦	Jack ♠	Jack ♣
S	Queen ♥	Queen ♦	Queen ♠	Queen ♣
P	King ♥	King ♦	King ♠	King ♣

What are the odds of choosing an Ace?

4 Aces : 48 Other Cards

$$4 : 48 \leftarrow 52 - 4 = 48$$

Odds Against = Total - Odds in favour

What are the odds of choosing a Heart?

13 Hearts: 39 Other Cards

$$13 : 39 \leftarrow 52 - 13 = 39$$

Odds Against = Total - Odds in favour

(4 Suits/13 Cards per Suit/52 Cards)