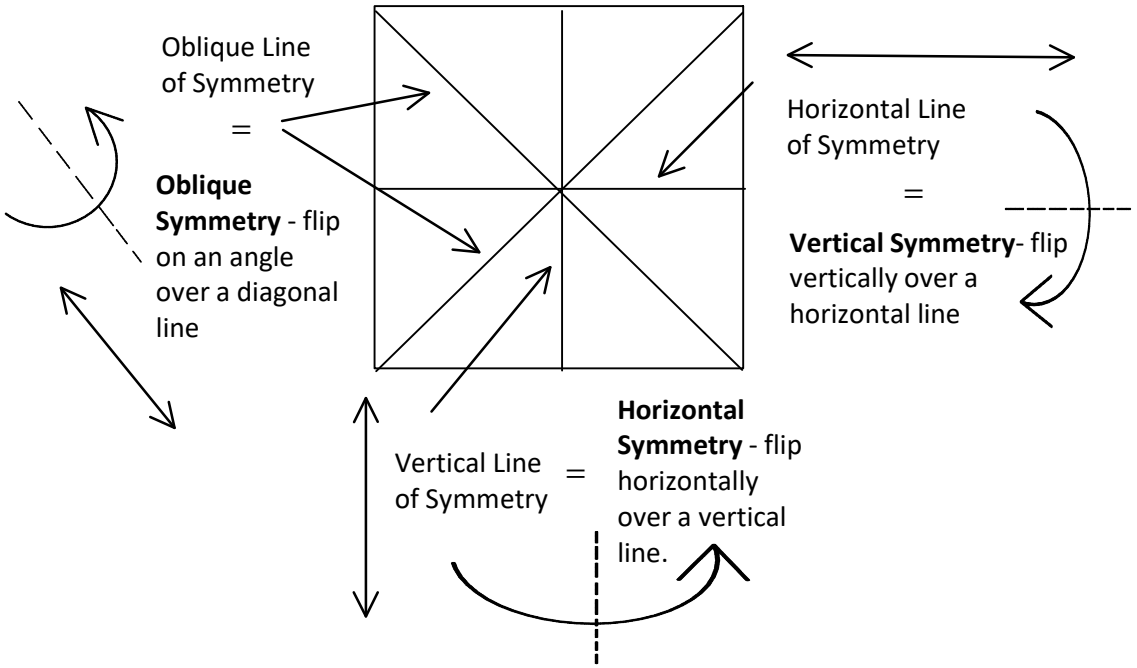
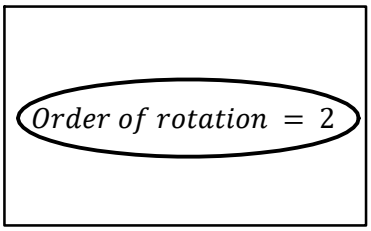
M9 - 1.1 - Symmetry/Rotational Notes

Types of Symmetry



Order of Rotation: The number of times you can rotate the shape to be identical to its original orientation in one circle of rotation 360^o.

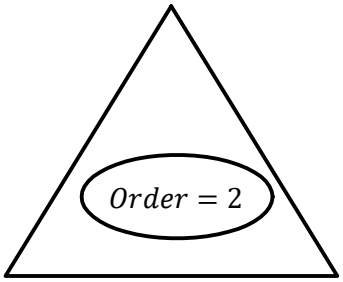
$$Order\ of\ Rotation = \frac{360^o}{Angle\ of\ rotation} \qquad Angle\ of\ Rotation = \frac{360^o}{Order\ of\ rotation}$$



$$Angle\ of\ Rotation = \frac{360^o}{2}$$

$$Angle\ of\ Rotation = 180^o$$

If you rotate a rectangle 180^O, it is in the same orientation it started.



$$Angle\ of\ Rotation = \frac{360^o}{3}$$

$$Angle\ of\ Rotation = 120^o$$

If you rotate an equilateral triangle 120^O, it is in the same orientation it started.

M9 - 2.1 - Rounding Notes

Round the following to the Hundreds Place

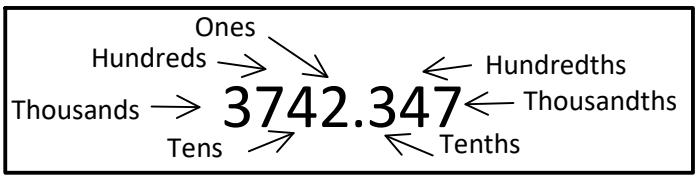
$1754 = 1800$

Round Up

$1734 = 1700$

Round Down

Look at Tens



Look at Place to Right
Round Place Up if 5 or More
Round Place Down if Less than 5

Round the following to the Tens Place

$87 = 90$ $84 = 80$

$8 = 10$ $2 = 0$

$186 = 190$ $11.3 = 10$

Round of the following to the Ones Place

$31.56 = 32$ $123.2 = 123$

$39.5 = 40$

Round Twice

Round the following to the Tenths Place

$0.172 = 0.2$

$0.146 = 0.1$

Round the following to the Hundredths Place

$0.176 = 0.18$

$0.172 = 0.17$

M9 - 2.2 - Scientific Notation Notes

Check on Calculator/Reverse

Write in Standard Form (Normal)

$5.0 \times 10^2 = 500.$ Move the Decimal 2 to the Right $10^2 = 100$

$8.43 \times 10^5 = 843000.$ Move the Decimal 5 to the Right $10^5 = 100000$

Positive Exponent : Decimal to Right

$243.\times 10^{-4} = 0.0243$ Move the Decimal 4 to the Left $10^{-4} = 0.0001$

Negative Exponent : Decimal to Left

Write in Scientific Notation

$\#.\#\# \ldots \times 10^{\#}$ $1\ \#\ (1-9)$ in front of decimal

$9624. = 9.624 \times 10^3$ Move the Decimal 3 to the Left $10^3 = 1000$

$5000000. = 5.0 \times 10^6$ Move the Decimal 6 to the Left $10^6 = 1000000$

$0.000000367 = 3.67 \times 10^{-7}$ Move the Decimal 7 to the Right $10^{-7} = 0.0000001$

Write in Scientific Notation

$0.00367 \times 10^5 = 367. = 3.67 \times 10^2$

Move the Decimal 5 to the Right
Write in Standard Form
Move the Decimal 2 to the Left

OR

$0.00367 \times 10^5 = 3.67 \times 10^2$

Move the Decimal 3 to the Right
Subtract 3 from Exponent

$5234.\times 10^{-2} = 52.34 = 5.234 \times 10^1$

Move the Decimal 2 to the Left
Write in Standard Form
Move the Decimal 1 to the Left

OR

$5234.\times 10^{-2} = 5.234 \times 10^1$

Move the Decimal 3 to the Left
Add 3 to Exponent

Decimal Right <-> Exponent Down Decimal Left <-> Exponent Up

$5 \times 10^0 = 5$ $10^0 = 1$

M9 - 3.1 - Add/Subtract Exponents Laws Notes

Base → 7^2 ← Exponent (power)

Remember:
-Never multiply the base by the exponent
-Must have same base to use laws.

Multiplying with the Same Base, Add Exponents

$2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2) = 2^5$

$2^3 \times 2^2 = 2^{3+2} = 2^5$ Add Exponents

$2^3 \times 2^2 = 32 = 2^5$ ✓ Check Answer!

$5^2 \times 5^4 = (5 \times 5) \times (5 \times 5 \times 5 \times 5) = 5^6$

$5^2 \times 5^4 = 5^6$

$3^2 \times 3^1 = 3^{2+1} = 3^3$

$3 = 3^1$

Dividing with the Same Base, Subtract Exponents.

$\frac{3^5}{3^2} = \frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3} = 3^3$

$\frac{3}{3} = 1$

$4^6 \div 4^3 = \frac{4 \times 4 \times 4 \times 4 \times 4 \times 4}{4 \times 4 \times 4} = 4^3$

$\frac{3^5}{3^2} = 3^{5-2} = 3^3$ Subtract Exponents

$\frac{4^6}{4^3} = 4^{6-3} = 4^3$

$\frac{3^5}{3^2} = 27 = 3^3$ ✓ Check Answer!

Ultimately you will either use: Exponent Laws OR Repeated Multiplication and Division Theory

M9 - 3.2 - Multiply Laws Notes

Exponents to exponents to exponents, Multiply exponents

$(2^2)^3 = (2 \times 2)^3 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) = 2^6$

$(2^2)^3 = 2^{2\times3} = 2^6$ Multiply Exponents

Check Answer!

$(2^2)^3 = 64 = 2^6$ ✓

$(5^4)^2 = (5 \times 5 \times 5 \times 5)^2 = (5 \times 5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5) = 5^8$

$(5^4)^2 = 5^{4\times2} = 5^8$

When Product/Quotients to Exponents, Multiply Exponents

Down the Page (Multistep)

$(3 \times 4)^2 =$
$(3^1 \times 4^1)^2$
$3^2 \times 4^2$

OR

Give it an Exponent of 1
Multiply Exponents

$3 = 3^1, 4 = 4^1$

$(3^1 \times 4^1)^2$ ×

$1 \times 2 = 2$
$1 \times 2 = 2$

$a^b \times c^b = (a \times c)^b$

$(3 \times 4)^2 =$
12^2

Multiply Inside Brackets

BEDMAS

$(3+4)^2 \neq 3^2 + 4^2 = 25$
$(3+4)^2 = (3+4)(3+4) = 7 \times 7 = 49$

Cannot distribute into a sum!

$(3 \times 4)^2 = 144 = 12^2$ ✓ Check Answer!

$\frac{12^3}{3^3} =$
$\left(\frac{12}{3}\right)^3$
$(4)^3$
4^3

$\frac{a^b}{c^b} = \left(\frac{a}{c}\right)^b$ OR

Divide

$\frac{12^3}{3^3} =$
$\frac{(3^1 \times 4^1)^3}{3^3}$
$\frac{3^3 \times 4^3}{3^3}$
4^3

Product

Simplify

Check Answer

$\frac{12^3}{3^3} = \frac{1728}{27} = 64 = 4^3$ ✓

$\left(\frac{3}{5}\right)^2 =$
$\left(\frac{3^1}{5^1}\right)^2$
$\frac{3^2}{5^2} = \frac{9}{25}$

Give it an Exponent of 1
Multiply Exponents

$\left(\frac{3^1}{5^1}\right)^2$ ×

$1 \times 2 = 2$
$1 \times 2 = 2$

$\left(\frac{3}{5}\right)^2 =$
$(0.6)^2$
0.6×0.6
0.36

$\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2} = \frac{9}{25} = 0.36$ ✓ Check Answer!

$(2x)^3 = (2x) \times (2x) \times (2x) = 8x^3$

$(2x)^3 =$
$(2^1x^1)^3$
2^3x^3
$8x^3$

Give it an Exponent of 1
Multiply Exponents

Check Answer!
Arbitrary Numbers!

$x = 3$

$(2x)^3$	Write Question/Answer	$8x^3$
$(2(3))^3$	Substitute Arbitrary #	$8(3)^3$
6^3	Solve	8×27
216	Compare	216

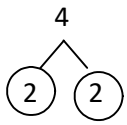
$216 = 216$ ✓

M9 - 3.3 - Change of Base Notes

Change to Exponential Form (Change of Base)

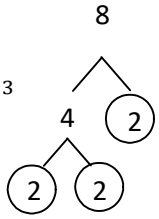
$4 = 2^2$ (Base: 2, Exponent: 2)

$4 = 2 \times 2$



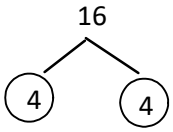
$8 = 2^3$

$8 = 2 \times 2 \times 2 = 2^3$



$16 = 2^4$

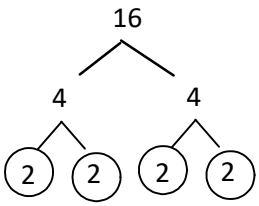
$16 = 2 \times 2 \times 2 \times 2$



OR

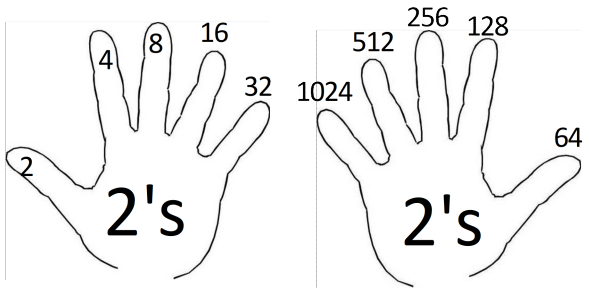
$16 = 4^2$

$16 = 4 \times 4$



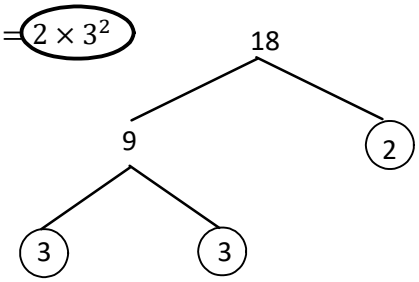
Check on Calculator!

Finger Counting



Change to Exponential Form with Lowest Bases

$18 = 2 \times 3 \times 3 = 2 \times 3^2$

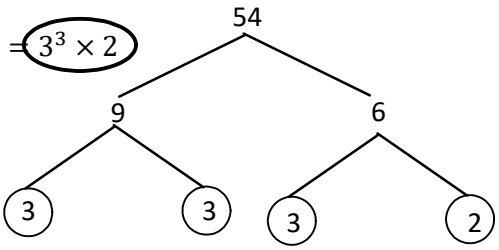


Perfect Squares
1,4,9,16,25,36,49,64,81…

$\frac{18}{4} = 4.5$ $\quad \frac{18}{9} = 2$

OR Divide by Perfect Squares/Cubes

$54 = 3 \times 3 \times 3 \times 2 = 3^3 \times 2$



Perfect Cubes
1,8,27,64,125,216,343…

$\frac{54}{4} = 13.5$ $\quad \frac{54}{27} = 2$

Change to Exponential Form with Lowest Bases

4^3
$(4)^3$
$(2^2)^3$
2^6

$4 = 2^2$

Brackets Around Base
Change Base
Multiply Exponents

$4^3 = 64$
$2^6 = 64$ ✓

6^3
$(3 \times 2)^3$
$(3^1 \times 2^1)^3$
$3^3 \times 2^3$

$6 = 2 \times 3$
$3 = 3^1, 2 = 2^1$

Write as Product (×)
Write Exponents (1's)
Multiply Exponents

$4^3 \times 8^2$
$(2^2)^3 \times (2^3)^2$
$2^6 \times 2^6$
2^{12}

Change of Base
Multiply Exponents
Add Exponents

64^3 $\quad (8^2)^3$ $\quad 8^6$ $\quad 8^6 = 262144 = 64^3$

64^3 $\quad (4^3)^3$ $\quad 4^9$ $\quad 4^9 = 262144 = 64^3$

64^3 $\quad (2^6)^3$ $\quad 2^{18}$ $\quad 2^{18} = 262144 = 64^3$

Change to Certain Base
Multiply Exponents
Go Both Ways!

M9 - 3.3 - Negative Coefficient Laws Notes

Negative Coefficients

$-2^2 = -2^2 = -2 \times 2 = -4$

Negative numbers WITHOUT brackets stay NEGATIVE

Adding a Negative In Front

$-(-2^2) = 4$

Unnecessary brackets

$-(2)^2 = -4$

$(-2^2) = -4$

$(-2)^3 = (-2) \times (-2) \times (-2) = -8$

Negative numbers with brackets to ODD exponents stay NEGATIVE

$-(-2)^3 = 8$

$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16$

Negative numbers with brackets to EVEN exponents become POSITIVE

$-(-2)^4 = -16$

M9 - 3.4 - Negative Laws Notes

Negative Exponents

$5^{-2} = \left(\frac{1}{5^2}\right)$ *Bring to the bottom, make exponent positive*

$5^{-2} = 0.04 = \frac{1}{5^2}$ ✓ Check Answer

$\frac{1}{3^{-2}} = \left(\frac{3^2}{1}\right)$ *Bring to the top, make exponent positive*

$3a^{-2} = \left(\frac{3}{a^2}\right)$ *Bring to the bottom, make exponent positive*

$3^{-3}a^{-2} = \frac{1}{3^3a^2} = \left(\frac{1}{27a^2}\right)$ *Bring to the bottom, make exponent positive*

$(2x)^{-3} = \frac{1}{(2x)^3} = \frac{1}{2^3x^3} = \left(\frac{1}{8x^3}\right)$ *Bring to the bottom, make exponent positive*

$\frac{2}{(3x)^{-2}} =$

$2(3x)^2$ *Bring to the top, make exponent positive*

$2(3^2x^2)$ *Multiply Exponents*

$2(9x^2)$

$\left(18x^2\right)$ *Multiply Coefficients*

$x^{-a} = \frac{1}{x^a}$

Rules

$\frac{1}{x^{-a}} = x^a$

Notice the 3 doesn't come down

Theory

Theory on "Bring it to the Bottom" and Vice Versa

$3^3 = 27$

$\div 3$

$3^2 = 9$

$\div 3$

$3^1 = 3$

$\div 3$

$3^0 = 1$

$3^{-1} = \frac{1}{3^1} = \frac{1}{3}$ $\div 3$

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $\div 3$

The exponents on the left are going down by 1,

The numbers on the right are being divided by 3,

This pattern must continue

$\frac{3^2}{3^2} = 3^{2-2} = 2^0 = 1$ $\frac{3^2}{3^2} = \frac{8}{8} = 1$

$\frac{3}{9} = \frac{3 \div 3}{9 \div 3} = \frac{1}{3}$ $\frac{3}{3^2} = \frac{1\cancel{3}}{\cancel{3} \times 3} = \frac{1}{3}$

$\frac{3^1}{3^2} = 3^{-1} = \frac{1}{3^1} = \frac{1}{3}$ $\frac{\cancel{3}}{\cancel{3}} = 1$

Fractions Division Theory vs Exponents

M9 - 3.4 - Negative Laws Notes

Negative Exponents

$$\left(\frac{5^1}{3^1}\right)^{-2} = \frac{5^{-2}}{3^{-2}} = \text{\_\_\_\_} = \frac{3^2}{5^2}$$

Multiply Exponents
Start off with an "OVER"
Bring to the bottom, make exponent positive
Bring to the top, make exponent positive

When you can flip it!

$$\left(\frac{5}{3}\right)^{-2} = \left(\frac{3}{5}\right)^{2} = \frac{3^2}{5^2}$$

Flip it and make the exponent positive

Check Answer

$$\left(\frac{5}{3}\right)^{-2} = 0.36 = \frac{3^2}{5^2}\surd$$

Alternate Subtraction Methods

$$\frac{5^2}{5^5} = 5^{2-5} = 5^{-3} = \frac{1}{5^3}$$

Subtract from the top

OR

$$\frac{5^2}{5^5} = \frac{1 \quad \cancel{5 \times 5}}{\cancel{5 \times 5} \times 5 \times 5 \times 5} = \frac{1}{5^3}$$

Division Theory

OR

$$\frac{5^2}{5^5} = \frac{1}{5^{5-2}} = \frac{1}{5^3}$$

Subtract from the bottom

$$\frac{5^2}{5^5} = 0.008 = \frac{1}{5^3}\surd$$

Check Answer

$$\frac{5^2}{5^5} = \frac{25 \div 25}{3125 \div 25} = \frac{1}{125} = \frac{1}{5^3}$$

Division Theory

$$\frac{5^2}{5^{-3}} = \text{\_\_\_\_} = 5^2 5^3 = 5^{2+3} = 5^5$$

Bring Up, Add

OR

$$\frac{5^2}{5^{-3}} = 5^{2-(-3)} = 5^5$$

Subtract, Distribute Negative

$$\frac{5^{-2}}{5^3} = \text{\_\_\_\_} = \frac{1}{5^3 5^2} = \frac{1}{5^{3+2}} = \frac{1}{5^5}$$

Bring Down, Add

OR

$$\frac{5^{-2}}{5^3} = \frac{1}{5^{3-(-2)}} = \frac{1}{5^5}$$

Subtract From Bottom

Step 1

_____ ← Over

$$\frac{2x^5y^{-2}}{z^{-3}} = \text{\_\_\_\_\_} = \frac{2x^5z^3}{y^2}$$

When working with negative exponents:

Start with a fraction "Over" sign.
Put anything not moved!
Move whatever needs to be moved.
If nothing is left on the top, put a 1.

M9 - 3.5 - Combo Exponents Laws Notes

Simplify

$$\frac{2^3 \times 2^4}{2^5} =$$

$$\frac{2^{3+4}}{2^5}$$ Add Exponents

$$\frac{2^7}{2^5}$$ Subtract Exponents

$$2^{7-5}$$

$$2^2$$ Simplify

$$\boxed{4}$$

Check on Calculator!

$$\frac{(2^3 \times 2^4)}{(2^5)} = 4 \checkmark$$

$$\frac{3^4 \times 3^{-3}}{9} =$$

$$\frac{3^1}{3^2}$$ Add Exponents
Change Base

$$3^{1-2}$$ Subtract Exponents
Negative Exponents

$$3^{-1}$$

$$\frac{1}{3^1}$$

$$\boxed{\frac{1}{3}}$$ Simplify

$$\frac{4^2 \times 16^3}{128^2} =$$

$$\frac{((2^2)^2 \times (2^4)^3)}{(2^7)^2}$$ Change of base

$$\frac{2^4 \times 2^{12}}{2^{14}}$$ Multiply Exponents

$$\frac{2^{16}}{2^{14}}$$ Add Exponents

$$2^{(16-14)}$$ Subtract Exponents

$$2^2$$

$$\boxed{4}$$ Simplify

Simplify

$$\frac{(2x^3y^2)(6xy^4)}{(4x^3y)} =$$

$$\frac{12x^4y^6}{4x^3y}$$ Multiply Coefficients
Add Exponents
Divide
Subtract Exponents

$$\boxed{3xy^5}$$

$$\frac{(8x^3y^2)^2(6xy^4)^{-2}}{(4x^3y)} =$$

$$\frac{(8x^3y^2)^2}{(4x^3y)(6xy^4)^2}$$ Negative Exponents

$$\frac{64x^6y^4}{(4x^3y)(36x^2y^8)}$$ Multiply Exponents
Multiply Coefficients

$$\frac{64x^6y^4}{144x^5y^9}$$ Add Exponents
Subtract Exponents

$$\boxed{\frac{4x}{9y^5}}$$ Simplify

$$\frac{y^4}{y^9} = y^{4-9} = y^{-5} = \frac{1}{y^5}$$ Subtract from Bottom

$$\frac{y^4}{y^9} = \frac{1}{y^{9-4}} = \frac{1}{y^5}$$ Subtract from Top

M9 - 3.6 - Exponents Negative Brackets Notes

Simplify without Brackets

$(-2x)^2 =$
$((-2)^1x)^2$ Multiply Exponents
$(-2)^2x^2$ $(-2)^{even} = +ve$
$4x^2$ $(-2)^2 = 4$

Brackets

$(-2x)^3 =$
$((-2)^1x)^3$ Multiply Exponents
$(-2)^3x^3$ $(-2)^{odd} = -ve$
$-8x^3$ $(-2)^3 = -8$

Check Answer!
Arbitrary Numbers!
$x = 3$

$(-2x)^3$	Write Question/Answer	$-8x^3$
$(-2(3))^3$	Substitute Arbitrary #	$-8(3)^3$
$(-6)^3$	Solve	-8×27
-216	Compare	-216
	$-216 = -216$ ✓	

$(-2x^2y^3)^3$
$(-2)^3x^6y^9$ Multiply Exponents
$-8x^6y^9$ Simplify

Simplify without Brackets

$\left(\frac{6x^2}{2x^1}\right)^3 =$
$(3^1x^1)^3$
3^3x^3
$27x^3$

Simplify 1st

Divide, Subtract Exponents
Multiply Exponents
Solve

$\frac{6}{2} = 3^1$
$\frac{x^2}{x} = x^1$

OR

$\left(\frac{6^1x^2}{2^1x}\right)^3 =$
$\frac{6^3x^6}{2^3x^3}$
$\frac{216x^6}{8x^3}$
$27x^3$

Don't Expand 1st

$6^3 = 216$
$2^3 = 8$ $\frac{216}{8} = 27$

Simplify without Brackets

$(-2x)^{-2} =$
$\frac{1}{(-2x)^2}$ Negative Exponents
$\frac{1}{(-2)^2x^2}$ Multiply Exponents
$\frac{1}{4x^2}$ Simplify

$\left(\frac{5x}{-2x^2}\right)^{-2} =$
$\frac{5^{-2}x^{-2}}{(-2)^{-2}x^{-4}}$ Multiply Exponents
Start off with "OVER"
$\frac{(-2)^2x^4}{5^2x^2}$ Negative Exponents
$\frac{4x^4}{25x^2}$
$\frac{4x^2}{25}$ Subtract Exponents

$4 - 2 = 2$

$(-2)^2 = 4$

$\frac{x^4}{x^2}$
x^{4-2}
x^2

OR

$\left(\frac{5x}{-2x^2}\right)^{-2} =$
$\left(\frac{-2x^2}{5x}\right)^2$
$\frac{2^2x^4}{5^2x^2}$
$\frac{4x^4}{25x^2}$
$\frac{4x^2}{25}$

Flip it
Make Exponent Positive

$\left(\frac{a}{b}\right)^{-c} = \left(\frac{b}{c}\right)^c$

M9 - 5.1 - Algebraic Expressions Notes

Coefficient: a number in front of (multiplying) a variable

Coefficient $= 4$ $\rightarrow$ $4x^2$ $\leftarrow$ Exponent/Power $= 2$

Letter/Variable/Base $= x$

Exponent: $3^2 = 3 \times 3$

$5^3 = 5 \times 5 \times 5$

Variable: a letter

Like term: Same Letter(s), Same Exponent(s).

Term:	**Like Terms:**
2	$\ldots\ldots\ldots\ldots 1, 5, -12, 4, 5, -6, \ldots.$
a	$\ldots\ldots\ldots\ldots 4a, 2a, -5a, 4a, \ldots.$
xy	$\ldots\ldots\ldots\ldots 2xy, -3xy, 4xy, \ldots$
a^2	$\ldots\ldots\ldots\ldots a^2, -2a^2, 3a^2, 4a^2, \ldots$
x^2y	$\ldots\ldots\ldots\ldots 2x^2y, -3x^2y, , \ldots$

$a + 1 = a + 1$	You can only add and subtract like terms.
$x + x^2 = x + x^2$	You cannot add or subtract unlike terms.

Degree of term: The Variable Exponent or Sum of Variable Exponents.

Degree of polynomial: Degree of Leading term.

Leading Term: The Term with the Highest Degree.

Leading Coefficient: Coefficient of Highest Degree Term

Term:	**Degree:**
x^2	$\ldots\ldots\ldots\ldots$ (2)
$x = x^1$	$\ldots\ldots\ldots\ldots$ (1)
$x^2(y^3)$	$\ldots\ldots\ldots\ldots$ (5)
$8 = 8x^0$	$\ldots\ldots\ldots\ldots$ (0)

Numbers have a degree of "0"

Polynomial:	**Leading Term:**	**Degree of Polynomial:**	
$x^2 - 4$	$\ldots\ldots$ (x^2)	$\ldots\ldots\ldots$ (2)	
$2x^2 - 5x^3$	$\ldots\ldots$ ($-5x^3$)	$\ldots\ldots\ldots$ (3)	
$\sqrt{3}x + 2$	$\ldots\ldots$ ($\sqrt{3}x^1$)	$\ldots\ldots\ldots$ (1)	$\sqrt{3} = 1.73$
$2^{-3}x^2y + 2x + 2$	$\ldots\ldots$ ($2^{-3}x^2y^1$)	$\ldots\ldots\ldots$ (3)	$2^{-3} = \frac{1}{8}$

Polynomial: Terms with Variables with Whole Number Exponents. (ie. 0,1,2,3…)

Examples:

Monomial: One term. $2, x, x^2, 2xy, 5z, 10$

Binomial: Two terms. $x + 2, x^2 - 4, xy + 5, 3x^2 + y^2, 2x^2 + x$

Trinomial: Three terms. $x^2 + 5x + 6, a + b + c$

Polynomial: Any # $2, x + 2, x^2 + 5x + 6, a + b + c + d + e$

Polynomial: Monomials, Binomials, Trinomials and more than three terms.

Not Polynomial

$x^{-2}, x^{\pi}, 2^x, \frac{1}{x}, \sqrt{x}, logx, sinx$

M9 - 5.2 - Combining Like Terms Notes

Adding and Subtracting Like Terms:

$x + x = 2x$ $\quad 3y + 2y = 5y$ $\quad x^2 + x^2 = 2x^2$ $\quad -9xy + 7xy = -2xy$

Add/Subtract Coefficients.

Combine Like Terms

$2 + x + 3 =$
$x + 2 + 3$ Rearrange Order of Terms

$x + 5$ Combine Like Terms

$3x + 1 - x =$
$3x - x + 1$ Rearrange Coefficient! Combine

$2x + 1$

Subtract Coefficients

$3x - 1x = 2x$

$3 - 1 = 2$

$3 + x^2 + 2x - 1 + 3x^2 + x =$
$x^2 + 3x^2 + 2x + x + 3 - 1$ Rearrange Order of Terms

Combine Like Terms

$4x^2 + 3x + 2$

Highest to Lowest Degree
ie. $x^2 + \#x + \# \ldots$

$x + 3x^2 = 4x^2 \quad -2x + x = -1x \quad 3 - 1 = 2$

Combine Like Terms

$5 - x + 2 =$ Circle Like Terms

$7 - x$ $\quad 5 + 2 = 7$

Remember to Circle the Sign!

$2x - 3 + 3x =$

$5x - 3$

Do like term addition and subtraction off to the right.

$2x + 3x = 5x \quad 2 + 3 = 5$

Add Coefficients

$-2x + 3 - x =$

$-3x + 3$

$-2x - x = -3x$

$-2 - 1 = -3$

$5x - 2 - 2x + 3 =$

$3x + 1$

Square Like Terms

$5 - 2 = 3$
$5x - 2x = 3x$

$-3 - 2x + 1 + 6x =$

$4x - 2$

$-2x + 6x = 4x$

$-2 + 6 = 4$

$x^2 + 3x - 2x^2 - 1 - 2x =$

$-x^2 + x - 1$

Cloud Like Terms

$x^2 - 2x^2 = -x^2$
$1 - 2 = -1$

$3x - 2x = 1x$
$3 - 2 = 1$

$-1 = -1$

Remember to cross off terms you have dealt with.

$5xy + 2yx = 7xy$
$5 + 2 = 7$

$xy = yx$ They are the same $x^2y^3 = y^3x^2$

$3x^2y^3 - 5y^3x^2 = -2x^2y^3$
$3 - 5 = -2$

M9 - 5.3 - Multiplying/Dividing Polynomials Notes

Multiplying

$a \times a = a^2$ $\quad 2a \times 3a = 6a^2$ $\quad -3x^2y \times 5x^3 = -15x^5y$

Multiply Coefficients
Add Exponents

$2x \times 3x^2 = 6x^3$ $\quad abcd \times efg = abcdefg$

Dividing

$20x^3 \div -5x^2 = -4x$ $\quad 30a^4 \div 6a^2 = 5a^2$ $\quad \frac{12x^2}{6x} = 2x$

Divide Coefficients
Subtract Exponents

$\frac{6x}{2} = 3x$ $\quad \frac{8x}{2x} = 4$ $\quad \boxed{\frac{x}{x} = 1}$ $\quad \frac{4x}{2x^2} = \frac{2}{x}$

$\frac{8x+4}{2} =$
$\frac{8x}{2} + \frac{4}{2}$
Separate into two fractions
Divide
$4x + 2$

$\boxed{\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}}$

$\frac{1}{2}(8x+4) =$
$\frac{8x+4}{2}$
Distribute
…

$\frac{9x^2+6x}{3x} =$
$\frac{9x^2}{3x} + \frac{6x}{3x}$
$3x + 2$

$-\frac{2x+4}{2} =$
$-\left(\frac{2x+4}{2}\right)$
Separate into two fractions
$-\left(\frac{2x}{2} + \frac{4}{2}\right)$
Divide
$-(x+2)$
$-x-2$
Distribute

$\frac{x^3}{x^2} = \frac{x \times x \times x}{x \times x} = x$
$\frac{x^2}{x} = \frac{x \times x}{x} = x$
$\frac{x^3}{x} = \frac{x \times x \times x}{x} = x^2$
$\frac{x}{x} = 1$
$\frac{x}{x^2} = \frac{1}{x \times x} = \frac{1}{x}$

$\frac{x^3}{x^2} = x$
$\frac{x^2}{x} = x$
$\frac{x^3}{x} = x^2$
$\frac{x}{x} = 1$
$\frac{x}{x^2} = \frac{1}{x}$

M9 - 5.4 - Distribution "FOIL" Notes

Expand and Simplify

$2(x+3)$
$2x+6$ Distribute/Multiply

Negative Distribution

$+(x+3)$
$+1(x+3)$
$x+3$

$-(x-2)$
$-1(x-2)$
$-x+2$

Backwards Distribution

$(x+2)(3)$
$3x+6$

$5-(x-2)$	Don't Distribute
$5-x+2$	the 5

Expand and Simplify

$(x+2)(x+3) =$ "FOIL" Method

$(x+2)(x+3) =$

$x^2+3x+2x+6 =$
x^2+5x+6

F - multiply **First** numbers in brackets
O - multiply **Outside** numbers in brackets
I - multiply **Inside** numbers in brackets
L - multiply **Last** numbers in brackets

Combine like terms.

Alternative Method:
$(x+2)(x+3) =$
$x(x+3)+2(x+3)$
$x^2+3x+2x+6$
x^2+5x+6

Quick Method
$(x+2)(x+3) =$
x^2+5x+6
Multiply and combine like terms in the same step.

$(x+2)^2$
$(x+2)(x+2)$
$x^2+2x+2x+4$
FOIL
Combine Like Terms
x^2+4x+4

$-(x+2)(x+4)$
$-(x^2+4x+2x+8)$
$-(x^2+6x+8)$
FOIL
Combine Like Terms
$-x^2-6x-8$

$2(x+4)(x-1)$
$2(x^2-x+4x-4)$
$2(x^2+3x-4)$
FOIL
Combine Like Terms
Distribute
$2x^2+6x-8$

Expand and Simplify

$(x+3)(x^2-2x+8)$

$(x+3)(x^2-2x+8)$
Triple FOIL

$(x+1)(x+2)(x-3)$
$(x^2+2x+1x+2)(x-3)$
$(x^2+3x+2)(x-3)$
$\cdots$
FOIL
Combine Like Terms
Then Triple FOIL

$x^3-2x^2+8x+3x^2-6x+24$
$x^3+x^2+2x+24$
Combine Like Terms

$(x+4)+2(x-1)$
$x+4+2x-2$
$3x+2$
Distribute
Combine Like Terms

$(x-2)-(x+1)(x-3)$
$x-2-(x^2-3x+x-3)$
$x-2-(x^2-2x-3)$
$x-2-x^2+2x+3$
$-x^2+3x+1$
FOIL
Distribute
Combine Like Terms

Algebra Tiles

$(2x-1)(x+2)$
Check by FOIL

$2x-1$

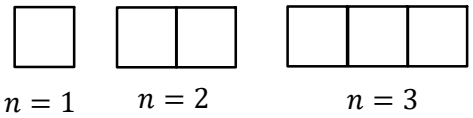
$x+2$

$2x^2+4x-x-2$
$2x^2+3x-2$

Legend
$= x^2$
$= -x^2$
$= x$
$= -x$
$= 1$
$= -1$

M9 - 6.1 - Find Equation Toothpick Notes

The following diagrams are made out of toothpicks. Create a Table of Values.



$n = 1$ $n = 2$ $n = 3$ ...

n	t
1	4
2	7
3	10

Let Statements

Let $n =$ the diagram number
Let $t =$ the number of toothpicks

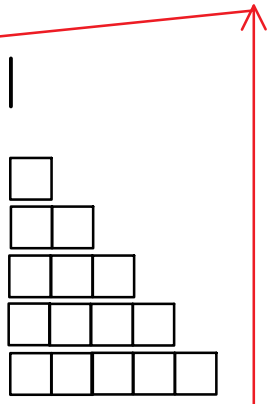
$t \neq +$

Words Problems
1) Let Statements
2) Table of Values
3) Equation (Logic)
4) Substitute
5) Solve (Algebra)
6) Answer in English!
7) Check Answer!

Find the number of toothpicks of the 4th and 5th diagram. ($n = 4, 5$) Then $n = 0$.

0	1
n	t
1	4
2	7
3	10
4	13
5	16

$+1$ $+1$ $+1$ $+1$ -3 $+3$ $+3$ $+3$ $+3$



$t = \frac{3}{1}n + 1$ $t = 1, n = 0$

Find the Equation of the Table (TOV)

$t = n + 3$ Try for $n = 1$

~~$t = n + 3$~~

~~$t = 4n$~~ See if pattern works for $n = 2,3$ ...

~~$t = 2n + 2$~~ If not cross it off

$t = 3n + 1$ Repeat until works for all n.

How many toothpicks in the 10th diagram?

$n = 10$

$t = 3n + 1$ Start with Equation
$t = 3(10) + 1$ Substitute
$t = 30 + 1$ Solve

$t = 31$

The 50th diagram has 31 toothpicks.

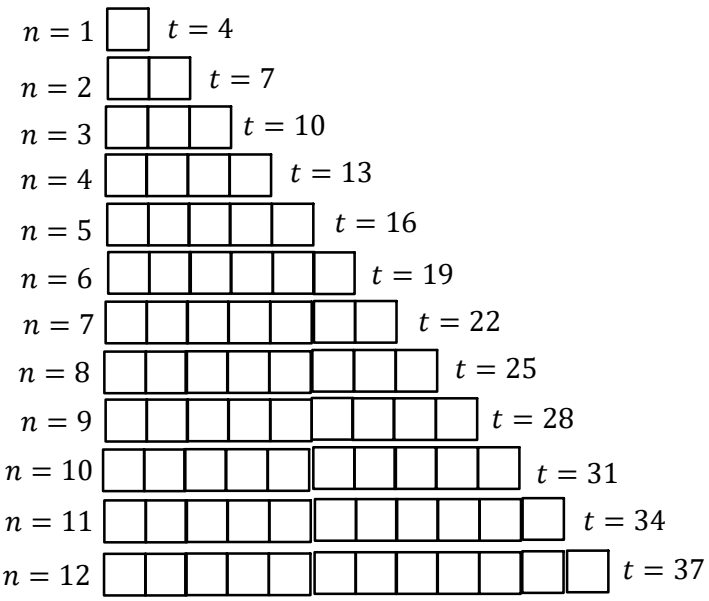
Which diagram has 37 toothpicks?

$t = 37$

$t = 3n + 1$ Start with Equation
$(37) = 3n + 1$ Substitute
$-1 \quad -1$ Solve
$36 = 3n$
$\frac{36}{3} = \frac{3n}{3}$
$12 = n$

$n = 12$

The 12th diagram has 37 toothpicks.



M9 - 6.2 - Equation TOV Notes

Find the equation from the Table of Values (TOV).

Right Letter = 1,2,3 Letter

n	t
1	3
2	4
3	5

~~$t = 3n$~~

$t = n + 2$

1) Try for $n = 1$ (Logic)
2) See if pattern works for $n = 2,3$ … by Substitution
3) If not cross it off
4) Repeat until works for all n.

Check Answer

Do this in your head!

Substitute with Brackets

~~$t = 3n$~~
$(3) = 3(1)$
$3 = 3$ ✓

~~$t = 3n$~~
$(4) \neq 3(2)$
$3 \neq 3$ ✗

$t = n + 2$
$(3) = (1) + 2$
$3 = 3$ ✓

$t = n + 2$
$(4) = (2) + 2$
$4 = 4$ ✓

$t = n + 2$
$(5) = (3) + 2$
$5 = 5$ ✓

Simple Patterns

0	2
n	t
1	3
2	4
3	5

-1, $+1$, $+1$ (left) and -1, $+1$, $+1$ (right)

$t = \frac{1}{1}n + 2$ → $t = 2, n = 0$

Sideways Tables

n	1	2	3
t	3	4	5

As Blanks

$\frac{3}{\text{Term } 1}, \frac{4}{\text{Term } 2}, \frac{5}{\text{Term } 3}$

$\frac{t = 3}{n = 1}, \frac{t = 4}{n = 2}, \frac{t = 5}{n = 3}$

Right Letter $= \frac{\text{Change on Right letter}}{\text{Change in Left letter}}$ Left Letter $\pm$ Value of Right Letter when Left Letter $= 0$

Simple Patterns (Lines)

n	t
1	2
2	5
3	8

$t = 3n - 1$

n	t
1	0
2	-1
3	-2

$t = -n + 1$

0	1
n	t
1	$\frac{5}{2}$
2	4
3	$\frac{11}{2}$
4	7
5	$\frac{17}{2}$
6	10

-1, $+1$, $+1$, $+1$, $+1$ (left) and $-\frac{3}{2}$, $+\frac{3}{2}$, $+\frac{3}{2}$, $+\frac{3}{2}$, $+\frac{3}{2}$ (right)

$t = \frac{3}{2}x + 1$

0	1
n	t
2	4
4	7
6	10

-2, $+2$, $+2$ (left) and -3, $+3$, $+3$ (right)

$t = \frac{3}{2}x + 1$

Complex Patterns (Curves)

n	t
1	2
2	5
3	10

$t = n^2 + 1$

n	t
1	1
2	8
3	27

$t = n^3$

n	t_n
1	0
2	2
3	6

$t = n^2 - n$

n	t_n
1	1
4	2
9	3

$t = \sqrt{n}$

M9 - 9.1 - Inequalities Notes

Greater than: $>$
Greater than or equal to: $\geq$
Less than: $<$
Less than or equal to: $\leq$
Does not equal: $\neq$

The Alligator Eats the Bigger Thing

$5 < 8$

5 is less than 8

5 is less than 8

$5 < 8$

8 is greater than 5

$8 > 5$

7 is less than or equal to 7. $7 \leq 7$

Equal To $\geq\leq$

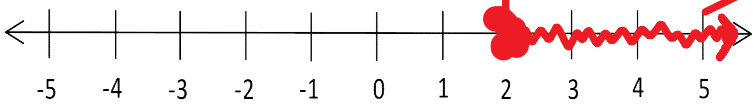
9 is greater than or equal to 7. $9 \geq 7$

Sketching Inequalities

$\leq, \geq$ Equal To — Closed Dot

$x \geq 2$

Arrow Above



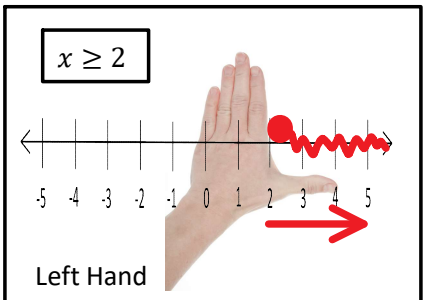
OR Shade the Line

Steps:

Put a **Closed Dot** at 2 on the Number Line $x = 2$

Draw a Line with an Arrow to the Right $x > 2$

$x \geq 2$



Left Hand
Thumb Points Greater Than

$x \geq 2$ Interval Notation $[2, \infty)$

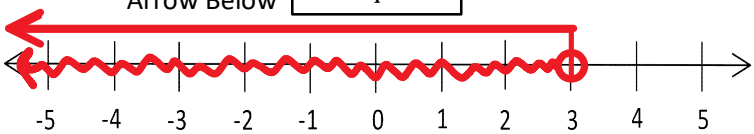
Equal To [ ] [Square Brackets]

$(-\infty, \infty)$

$x < 3$

<, > Not Equal to — Open Dot

Arrow Below



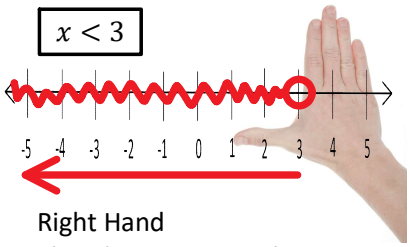
Shade the Line

Steps:

Put an **Open Dot** at 3 on the Number Line $x < 3$

Draw a Line with an Arrow to the Left $x < 3$

$x < 3$



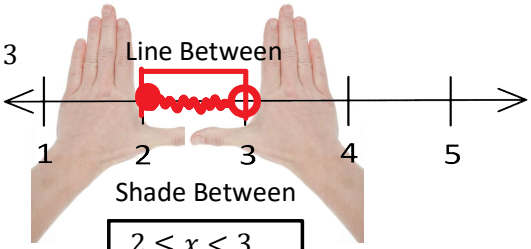
Right Hand
Thumb Points Less Than

$x < 3$ Interval Notation $(-\infty, 3)$

Not Equal to ( ) (Round Brackets)

Between

$x \geq 2$ $x < 3$



Shade Between

$2 \leq x < 3$

Interval Notation $[2,3)$

Side by Side in Order* $x \geq 2$ $x < 3$

Mirror Left $2 \leq x$ $x < 3$

Bring Together $2 \leq x < 3$

Smaller #, Less Than, Variable, Less Than, Bigger #

$-x \leq 4$

$-x \leq 4$ Divide by a Negative
$x \geq -4$ Change Direction of Sign

Proofs

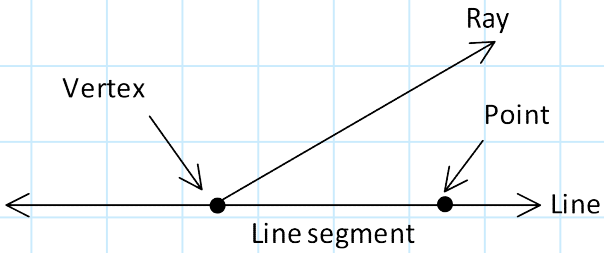
$-x \leq 4$
$+x \quad +x$
$0 \leq 4 + x$
$-4 \quad -4$ Add x
$-4 \leq x$ Subtract 4
$x \geq -4$ Mirror

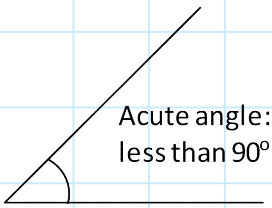
$-4 \leq x$
$x \geq -4$ Bring it Over Mirror

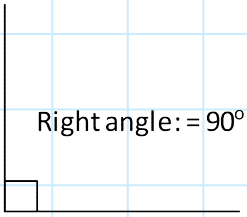
Mirror

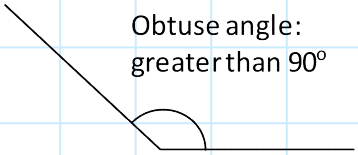
$a \rightarrow b$ Points to b
$b \leftarrow a$ Points to b

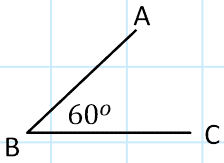
M9 - 10.0 - Angles/Triangles/Parallel/Lines Notes





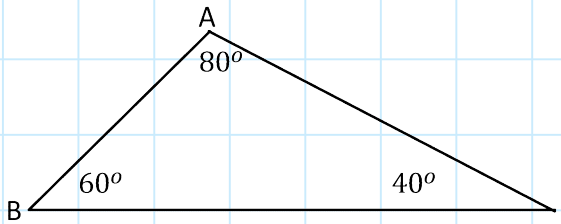






$\angle ABC = \angle B = 60^o$

$\angle C = \angle BAC = 80^o$

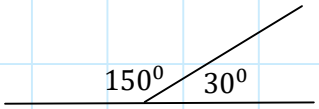


$\angle B = \angle ABC = 60^o$

$\angle C = \angle BCA = 40^o$

M9 - 10.1 - Lines/Angles/Parallel Review Notes

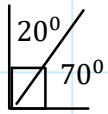
Supplementary



Angles on a line sum to 180^o.

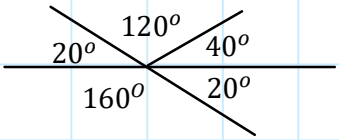
$\angle 1 + \angle 2 = 180^o$

Complementary



Angles sum to 90^o.

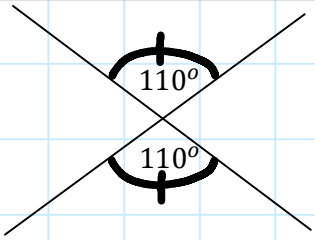
$\angle 1 + \angle 2 = 90^o$



$\angle's$ on a point add to 360^o

$180^o + 180^o = 360^o$

Opposite Angles are Equal.

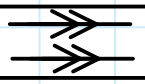


X

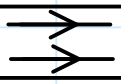
OPP $\angle$'s =

$\angle 1 = \angle 2$

Tick Equal Angles

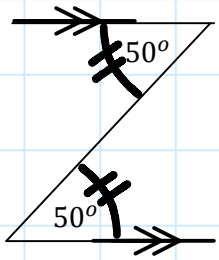


Parallel lines: Lines never touch



Tick/Double Arrow Parallel Lines

Alternate Interior Angles Equal.



Alternate: Across a Transversal.

Interior: Inside Parallel Lines.

Transversal: a line through Parallel Lines.

Z

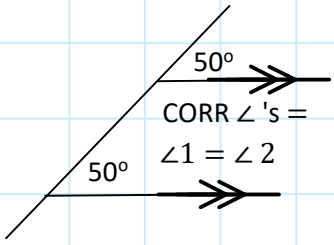
Alt Int. $\angle$'s =

$\angle 1 = \angle 2$

Rotate the Page

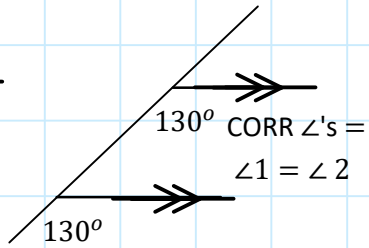
Double Tick Equal Angles

Corresponding Angles Equal.



CORR $\angle$'s =

$\angle 1 = \angle 2$



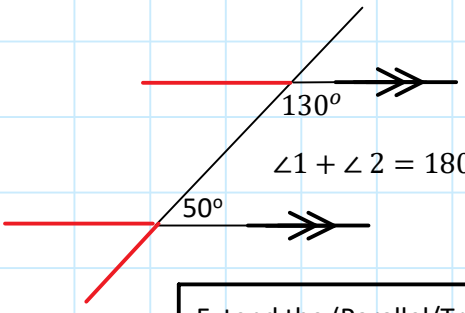
CORR $\angle$'s =

$\angle 1 = \angle 2$

C

Co-Interiors $\angle's$ add to 180°

Co-Interior: Same side of a Transversal



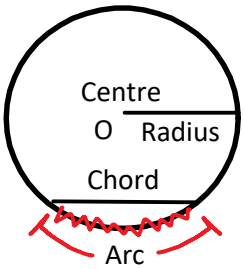
$\angle 1 + \angle 2 = 180^o$

C

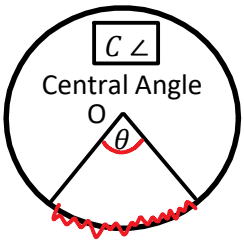
Extend the (Parallel/Transversal) Lines

M9 - 10.2 - Circles/Inscribed/Central Angles/Arc/Chords Notes

O : Centre of Circle



Chord: Edge to Edge

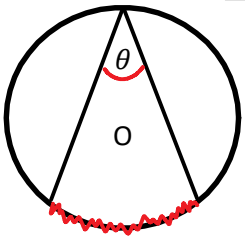


Central $\angle$: on the Centre

$\angle$: Angle

θ is a Greek symbols for an Angle

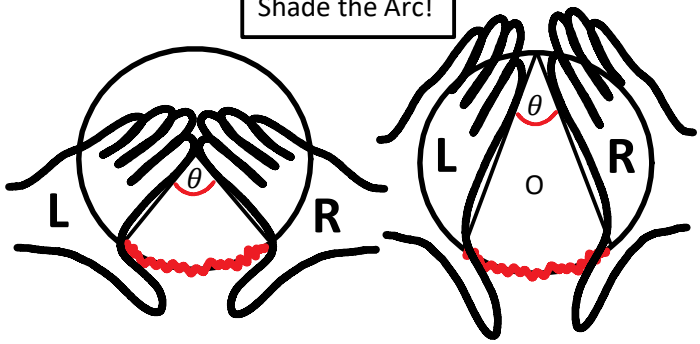
Inscribed Angle $I\angle$



Inscribed $\angle$: on the Edge

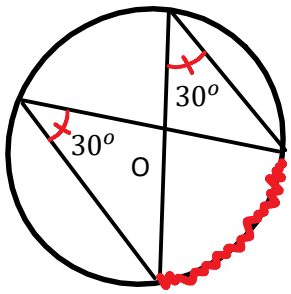
1. Make a Slice of Pie with your Left and Right Hand.
2. Central/Inscribed Angle is between your Index Fingers.
3. Arc is crust of piece of pie.

Shade the Arc!

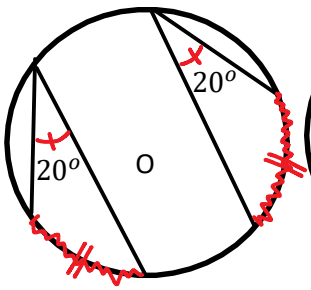


Rotate the Page

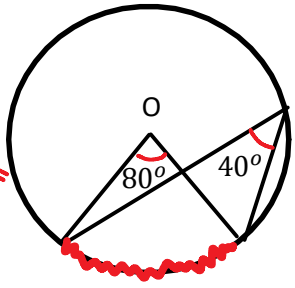
$I\angle = I\angle$



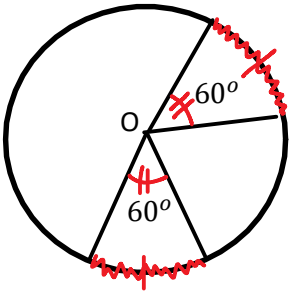
$I\angle = I\angle$



$C\angle = 2 \times I\angle$



$C\angle = C\angle$



Inscribed Angles from Same/Equal Arc are Equal.

Equal Things

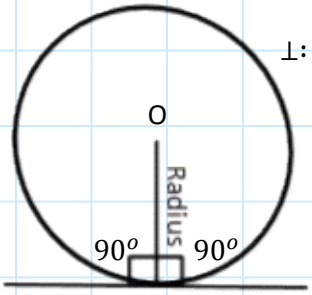
Other Equal Things

Central <u>Angles are Twice</u> Inscribed Angles from Same/Equal Arc.

Central Angles from Equal Arc are Equal.

$I\angle = \frac{1}{2} \times C\angle$ Inscribed <u>Angles are Half</u> Central Angles from Same/Equal Arc.

M9 - 10.3 - Circles/Semi/Tangents/Polygons Notes



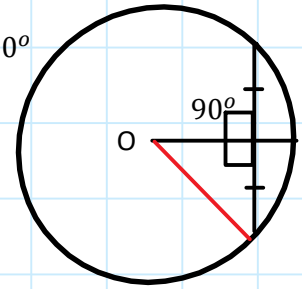
Tangent

Radius ⊥ to Tangent

⊥: Perpendicular 90^o

Tangent: Line meets Circe Edge

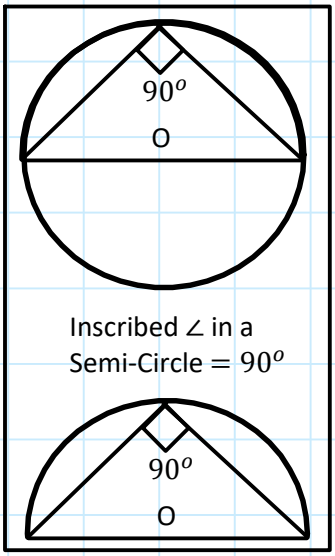
Draw a Radius



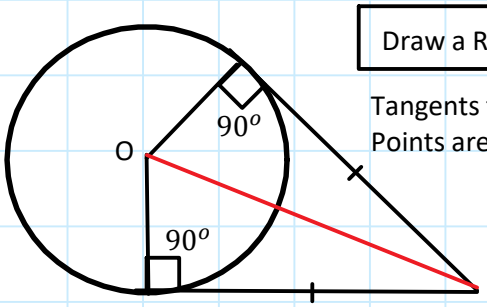
Radius ⊥ to Chord

⊥ Bisects Chord & goes through Centre

(Bisects: Cuts in Half)



Inscribed ∠ in a Semi-Circle $= 90^o$

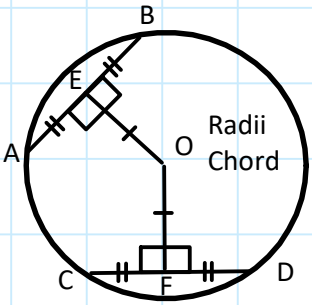


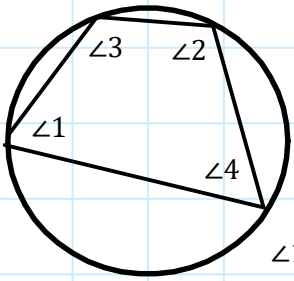
Draw a Radius to Ext Pt.

Tangents to Exterior Points are Equal.

Exterior Point (Ext Pt)

If: $OE = OF$
Then: $AB = CD$





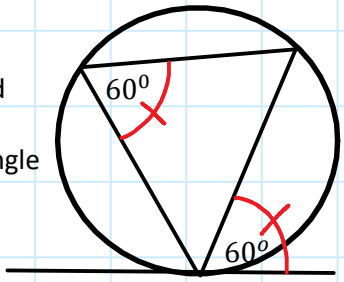
$\angle 1 + \angle 2 = 180^o$

$\angle 3 + \angle 4 = 180^o$

Interior Angles in a Cyclic Quadrilateral sum to 360^o.

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^o$

∠ Between Tangent and Chord = Inscribed Angle Opposite of Chord.



Int : Interior
Ext : Exterior
n : # of Sides
$\sum$: Sum

$$Int\angle' = \frac{\#\Delta' s \times 180^o}{n}$$

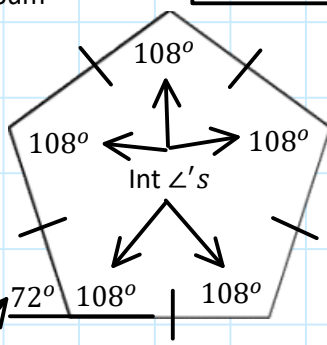
OR

$$\sum \text{Int}\angle = (n-2) \times 180^o$$

$\sum \text{Int}\angle = (5-2) \times 180^o$

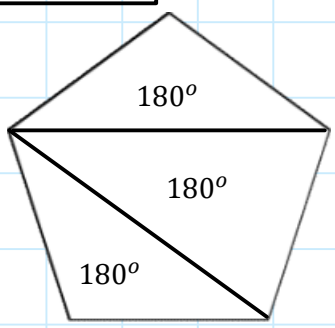
$\sum \text{Int}\angle = 3 \times 180^o$

$\sum \text{Int}\angle = 540^o$



Ext ∠

Pentagon: 5 Sides



Draw Triangles to Vertices (Without Overlap)

$$\text{Int}\angle = \frac{\sum \text{Int } \angle' s}{n} = \frac{(n-2)\times 180^o}{n}$$

$\text{Int}\angle = \frac{540^o}{5}$

$\text{Int}\angle = 108^o$

$\text{Ext}\angle' s \sum = 360^o$

M9 - 10.4 - Triangles/Similar/Congruent Notes

Congruent (Equal) Triangle's Triangles are Congruent if:

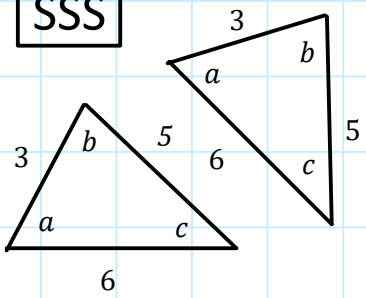
Like : Like

IN ORDER!

Side Side Side

SSS

$\angle a = \angle a$
$\angle b = \angle b$
$\angle c = \angle c$

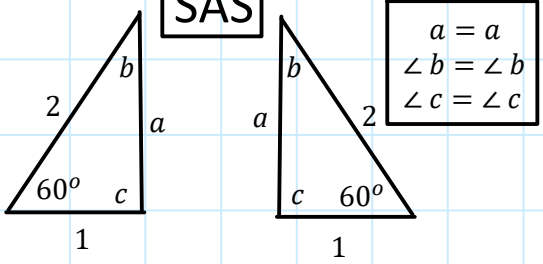


A Side then a Side then a Side

Side Angle Side

SAS

$a = a$
$\angle b = \angle b$
$\angle c = \angle c$

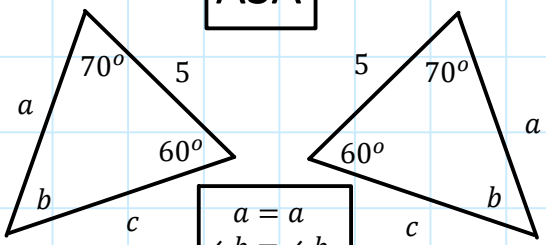


A Side then an Angle then a Side

Angle Side Angle

ASA

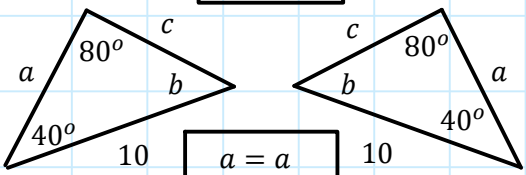
$a = a$
$\angle b = \angle b$
$c = c$



An Angle then a Side then an Angle.

Angle Angle Side

AAS



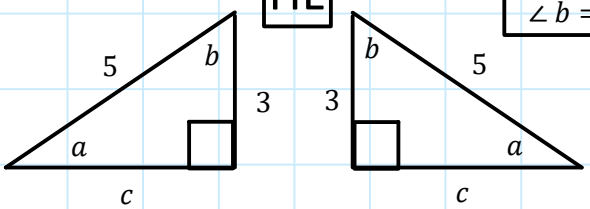
$a = a$
$\angle b = \angle b$
$c = c$

An Angle then an Angle then a Side.

Hypotenuse Leg

HL

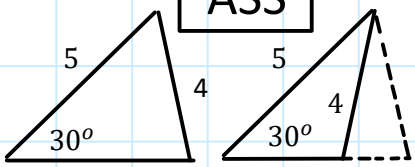
$c = c$
$\angle a = \angle a$
$\angle b = \angle b$



A Hypotenuse and a Leg

Angle Side Side

ASS



Neither!

Unless they are!

Similar Triangles

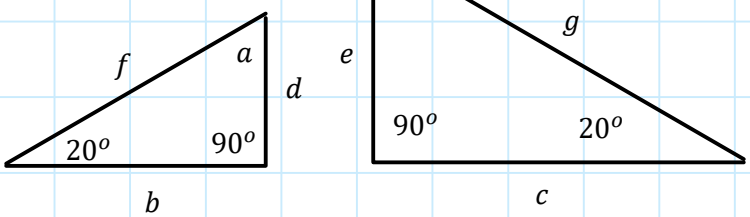
Equal Fractions

$$\frac{b}{c} = \frac{d}{e} = \frac{f}{g}$$

Can be used for all Congruent Triangles as well (for sides*)!

Angle Angle Angle

AAA



3rd Angle in a Triangle

$\angle a = \angle a$

$180^o - 90^o - 20^o = 70^o$