M9 - 3.1 - Add/Subtract Exponents Laws Notes

Base → 7^2 ← Exponent (power)

Remember:
-Never multiply the base by the exponent
-Must have same base to use laws.

Multiplying with the Same Base, Add Exponents

$2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2) = 2^5$

$5^2 \times 5^4 = (5 \times 5) \times (5 \times 5 \times 5 \times 5) = 5^6$

$2^3 \times 2^2 = 2^{3+2} = 2^5$ Add Exponents

$5^2 \times 5^4 = 5^6$

$2^3 \times 2^2 = 32 = 2^5$ ✓ Check Answer!

$3^2 \times 3^1 = 3^{2+1} = 3^3$

$3 = 3^1$

Dividing with the Same Base, Subtract Exponents.

$\frac{3^5}{3^2} = \frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3} = 3^3$

$\frac{3}{3} = 1$

$4^6 \div 4^3 = \frac{4 \times 4 \times 4 \times 4 \times 4 \times 4}{4 \times 4 \times 4} = 4^3$

$\frac{3^5}{3^2} = 3^{5-2} = 3^3$ Subtract Exponents

$\frac{4^6}{4^3} = 4^{6-3} = 4^3$

$\frac{3^5}{3^2} = 27 = 3^3$ ✓ Check Answer!

Ultimately you will either use: Exponent Laws OR Repeated Multiplication and Division Theory

M9 - 3.2 - Multiply Laws Notes

Exponents to exponents to exponents, Multiply exponents

$(2^2)^3 = (2 \times 2)^3 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) = 2^6$

$(2^2)^3 = 2^{2\times3} = 2^6$ Multiply Exponents

Check Answer!

$(2^2)^3 = 64 = 2^6$ ✓

$(5^4)^2 = (5 \times 5 \times 5 \times 5)^2 = (5 \times 5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5) = 5^8$

$(5^4)^2 = 5^{4\times2} = 5^8$

When Product/Quotients to Exponents, Multiply Exponents

Down the Page (Multistep)

$(3 \times 4)^2 =$
$(3^1 \times 4^1)^2$
$3^2 \times 4^2$

OR

Give it an Exponent of 1
Multiply Exponents

$3 = 3^1, 4 = 4^1$

$(3^1 \times 4^1)^2$ ×

$1 \times 2 = 2$
$1 \times 2 = 2$

$a^b \times c^b = (a \times c)^b$

$(3 \times 4)^2 =$
12^2

Multiply Inside Brackets

BEDMAS

$(3+4)^2 \neq 3^2 + 4^2 = 25$
$(3+4)^2 = (3+4)(3+4) = 7 \times 7 = 49$

Cannot distribute into a sum!

$(3 \times 4)^2 = 144 = 12^2$ ✓ Check Answer!

$\frac{12^3}{3^3} =$
$\left(\frac{12}{3}\right)^3$
$(4)^3$
4^3

$\frac{a^b}{c^b} = \left(\frac{a}{c}\right)^b$ OR

Divide

$\frac{12^3}{3^3} =$
$\frac{(3^1 \times 4^1)^3}{3^3}$
$\frac{3^3 \times 4^3}{3^3}$
4^3

Product

Simplify

Check Answer

$\frac{12^3}{3^3} = \frac{1728}{27} = 64 = 4^3$ ✓

$\left(\frac{3}{5}\right)^2 =$
$\left(\frac{3^1}{5^1}\right)^2$
$\frac{3^2}{5^2} = \frac{9}{25}$

Give it an Exponent of 1
Multiply Exponents

$\left(\frac{3^1}{5^1}\right)^2$ ×

$1 \times 2 = 2$
$1 \times 2 = 2$

$\left(\frac{3}{5}\right)^2 =$
$(0.6)^2$
0.6×0.6
0.36

$\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2} = \frac{9}{25} = 0.36$ ✓ Check Answer!

$(2x)^3 = (2x) \times (2x) \times (2x) = 8x^3$

$(2x)^3 =$
$(2^1x^1)^3$
2^3x^3
$8x^3$

Give it an Exponent of 1
Multiply Exponents

Check Answer!
Arbitrary Numbers!

$x = 3$

$(2x)^3$	Write Question/Answer	$8x^3$
$(2(3))^3$	Substitute Arbitrary #	$8(3)^3$
6^3	Solve	8×27
216	Compare	216

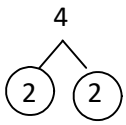
$216 = 216$ ✓

M9 - 3.3 - Change of Base Notes

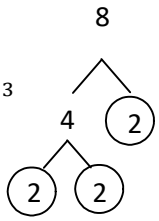
Change to Exponential Form (Change of Base)

$4 = 2^2$ $\quad 4 = 2 \times 2$

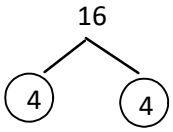
Base, Exponent



$8 = 2^3$ $\quad 8 = 2 \times 2 \times 2 = 2^3$

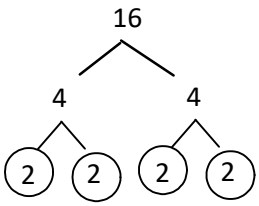


$16 = 2^4$ $\quad 16 = 2 \times 2 \times 2 \times 2$



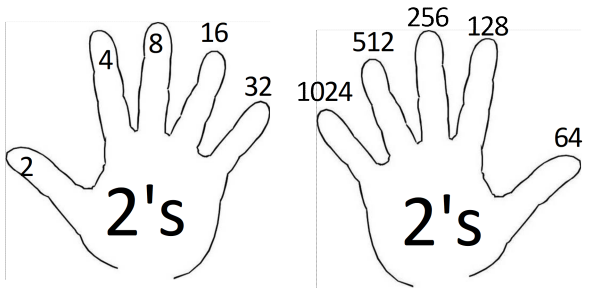
OR

$16 = 4^2$ $\quad 16 = 4 \times 4$



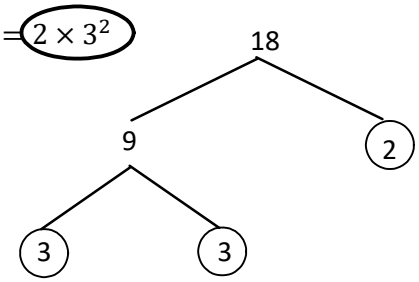
Check on Calculator!

Finger Counting

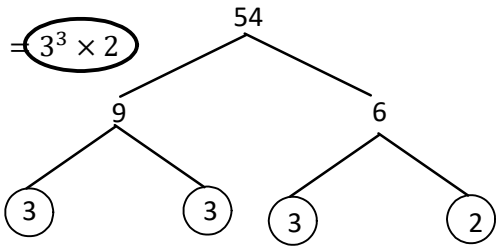


Change to Exponential Form with Lowest Bases

$18 = 2 \times 3 \times 3 = 2 \times 3^2$



$54 = 3 \times 3 \times 3 \times 2 = 3^3 \times 2$



Perfect Squares
1,4,9,16,25,36,49,64,81…

$\frac{18}{4} = 4.5$ $\quad \frac{18}{9} = 2$

OR Divide by Perfect Squares/Cubes

Perfect Cubes
1,8,27,64,125,216,343…

$\frac{54}{4} = 13.5$ $\quad \frac{54}{27} = 2$

Change to Exponential Form with Lowest Bases

4^3
$(4)^3$
$(2^2)^3$
2^6

$4 = 2^2$

Brackets Around Base
Change Base
Multiply Exponents

$4^3 = 64$
$2^6 = 64$ ✓

6^3
$(3 \times 2)^3$
$(3^1 \times 2^1)^3$
$3^3 \times 2^3$

$6 = 2 \times 3$
$3 = 3^1, 2 = 2^1$

Write as Product (×)
Write Exponents (1's)
Multiply Exponents

$4^3 \times 8^2$
$(2^2)^3 \times (2^3)^2$
$2^6 \times 2^6$
2^{12}

Change of Base
Multiply Exponents
Add Exponents

64^3 $\quad (8^2)^3$ $\quad 8^6$
64^3 $\quad (4^3)^3$ $\quad 4^9$
64^3 $\quad (2^6)^3$ $\quad 2^{18}$

Change to Certain Base
Multiply Exponents
Go Both Ways!

$8^6 = 4^9 = 2^{18} = 262144 = 64^3$

M9 - 3.3 - Negative Coefficient Laws Notes

Negative Coefficients

$-2^2 = -2^2 = -2 \times 2 = -4$

Negative numbers WITHOUT brackets stay NEGATIVE

Adding a Negative In Front

$-(-2^2) = 4$

Unnecessary brackets

$-(2)^2 = -4$

$(-2^2) = -4$

$(-2)^3 = (-2) \times (-2) \times (-2) = -8$

Negative numbers with brackets to ODD exponents stay NEGATIVE

$-(-2)^3 = 8$

$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16$

Negative numbers with brackets to EVEN exponents become POSITIVE

$-(-2)^4 = -16$

M9 - 3.4 - Negative Laws Notes

Negative Exponents

$5^{-2} = \left(\frac{1}{5^2}\right)$ *Bring to the bottom, make exponent positive*

$5^{-2} = 0.04 = \frac{1}{5^2}$ ✓ Check Answer

$\frac{1}{3^{-2}} = \left(\frac{3^2}{1}\right)$ *Bring to the top, make exponent positive*

$3a^{-2} = \left(\frac{3}{a^2}\right)$ *Bring to the bottom, make exponent positive*

$3^{-3}a^{-2} = \frac{1}{3^3a^2} = \left(\frac{1}{27a^2}\right)$ *Bring to the bottom, make exponent positive*

$(2x)^{-3} = \frac{1}{(2x)^3} = \frac{1}{2^3x^3} = \left(\frac{1}{8x^3}\right)$ *Bring to the bottom, make exponent positive*

$\frac{2}{(3x)^{-2}} =$

$2(3x)^2$ *Bring to the top, make exponent positive*

$2(3^2x^2)$ *Multiply Exponents*

$2(9x^2)$

$\left(18x^2\right)$ *Multiply Coefficients*

$x^{-a} = \frac{1}{x^a}$

Rules

$\frac{1}{x^{-a}} = x^a$

Notice the 3 doesn't come down

Theory

Theory on "Bring it to the Bottom" and Vice Versa

$3^3 = 27$

$\div 3$

$3^2 = 9$

$\div 3$

$3^1 = 3$

$\div 3$

$3^0 = 1$

$3^{-1} = \frac{1}{3^1} = \frac{1}{3}$ $\div 3$

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $\div 3$

The exponents on the left are going down by 1,

The numbers on the right are being divided by 3,

This pattern must continue

$\frac{3^2}{3^2} = 3^{2-2} = 2^0 = 1$ $\frac{3^2}{3^2} = \frac{8}{8} = 1$

$\frac{3}{9} = \frac{3 \div 3}{9 \div 3} = \frac{1}{3}$ $\frac{3}{3^2} = \frac{1\cancel{3}}{\cancel{3} \times 3} = \frac{1}{3}$

$\frac{3^1}{3^2} = 3^{-1} = \frac{1}{3^1} = \frac{1}{3}$ $\frac{\cancel{3}}{\cancel{3}} = 1$

Fractions Division Theory vs Exponents

M9 - 3.4 - Negative Laws Notes

Negative Exponents

$$\left(\frac{5^1}{3^1}\right)^{-2} = \frac{5^{-2}}{3^{-2}} = \text{\_\_\_\_} = \frac{3^2}{5^2}$$

Multiply Exponents
Start off with an "OVER"
Bring to the bottom, make exponent positive
Bring to the top, make exponent positive

When you can flip it!

$$\left(\frac{5}{3}\right)^{-2} = \left(\frac{3}{5}\right)^{2} = \frac{3^2}{5^2}$$

Flip it and make the exponent positive

Check Answer

$$\left(\frac{5}{3}\right)^{-2} = 0.36 = \frac{3^2}{5^2}\surd$$

Alternate Subtraction Methods

$$\frac{5^2}{5^5} = 5^{2-5} = 5^{-3} = \frac{1}{5^3}$$

Subtract from the top

OR

$$\frac{5^2}{5^5} = \frac{1 \quad \cancel{5 \times 5}}{\cancel{5 \times 5} \times 5 \times 5 \times 5} = \frac{1}{5^3}$$

Division Theory

OR

$$\frac{5^2}{5^5} = \frac{1}{5^{5-2}} = \frac{1}{5^3}$$

Subtract from the bottom

$$\frac{5^2}{5^5} = 0.008 = \frac{1}{5^3}\surd$$

Check Answer

$$\frac{5^2}{5^5} = \frac{25 \div 25}{3125 \div 25} = \frac{1}{125} = \frac{1}{5^3}$$

Division Theory

$$\frac{5^2}{5^{-3}} = \text{\_\_\_\_} = 5^2 5^3 = 5^{2+3} = 5^5$$

Bring Up, Add

OR

$$\frac{5^2}{5^{-3}} = 5^{2-(-3)} = 5^5$$

Subtract, Distribute Negative

$$\frac{5^{-2}}{5^3} = \text{\_\_\_\_} = \frac{1}{5^3 5^2} = \frac{1}{5^{3+2}} = \frac{1}{5^5}$$

Bring Down, Add

OR

$$\frac{5^{-2}}{5^3} = \frac{1}{5^{3-(-2)}} = \frac{1}{5^5}$$

Subtract From Bottom

Step 1

_____ ← Over

$$\frac{2x^5y^{-2}}{z^{-3}} = \text{\_\_\_\_\_} = \frac{2x^5z^3}{y^2}$$

When working with negative exponents:

Start with a fraction "Over" sign.
Put anything not moved!
Move whatever needs to be moved.
If nothing is left on the top, put a 1.

M9 - 3.5 - Combo Exponents Laws Notes

Simplify

$$\frac{2^3 \times 2^4}{2^5} =$$

$$\frac{2^{3+4}}{2^5}$$ Add Exponents

$$\frac{2^7}{2^5}$$ Subtract Exponents

$$2^{7-5}$$

$$2^2$$ Simplify

$$\boxed{4}$$

Check on Calculator!

$$\frac{(2^3 \times 2^4)}{(2^5)} = 4 \checkmark$$

$$\frac{3^4 \times 3^{-3}}{9} =$$

$$\frac{3^1}{3^2}$$ Add Exponents
Change Base

$$3^{1-2}$$ Subtract Exponents
Negative Exponents

$$3^{-1}$$

$$\frac{1}{3^1}$$

$$\boxed{\frac{1}{3}}$$ Simplify

$$\frac{4^2 \times 16^3}{128^2} =$$

$$\frac{((2^2)^2 \times (2^4)^3)}{(2^7)^2}$$ Change of base

$$\frac{2^4 \times 2^{12}}{2^{14}}$$ Multiply Exponents

$$\frac{2^{16}}{2^{14}}$$ Add Exponents

$$2^{(16-14)}$$ Subtract Exponents

$$2^2$$

$$\boxed{4}$$ Simplify

Simplify

$$\frac{(2x^3y^2)(6xy^4)}{(4x^3y)} =$$

$$\frac{12x^4y^6}{4x^3y}$$ Multiply Coefficients
Add Exponents
Divide
Subtract Exponents

$$\boxed{3xy^5}$$

$$\frac{(8x^3y^2)^2(6xy^4)^{-2}}{(4x^3y)} =$$

$$\frac{(8x^3y^2)^2}{(4x^3y)(6xy^4)^2}$$ Negative Exponents

$$\frac{64x^6y^4}{(4x^3y)(36x^2y^8)}$$ Multiply Exponents
Multiply Coefficients

$$\frac{64x^6y^4}{144x^5y^9}$$ Add Exponents
Subtract Exponents

$$\boxed{\frac{4x}{9y^5}}$$ Simplify

$$\frac{y^4}{y^9} = y^{4-9} = y^{-5} = \frac{1}{y^5}$$ Subtract from Bottom

$$\frac{y^4}{y^9} = \frac{1}{y^{9-4}} = \frac{1}{y^5}$$ Subtract from Top

M9 - 3.6 - Exponents Negative Brackets Notes

Simplify without Brackets

$(-2x)^2 =$
$((-2)^1x)^2$ Multiply Exponents
$(-2)^2x^2$ $(-2)^{even} = +ve$
$4x^2$ $(-2)^2 = 4$

Brackets

$(-2x)^3 =$
$((-2)^1x)^3$ Multiply Exponents
$(-2)^3x^3$ $(-2)^{odd} = -ve$
$-8x^3$ $(-2)^3 = -8$

Check Answer! Arbitrary Numbers! $x = 3$

$(-2x)^3$	Write Question/Answer	$-8x^3$
$(-2(3))^3$	Substitute Arbitrary #	$-8(3)^3$
$(-6)^3$	Solve	-8×27
-216	Compare	-216
	$-216 = -216$ ✓	

$(-2x^2y^3)^3$
$(-2)^3x^6y^9$ Multiply Exponents
$-8x^6y^9$ Simplify

Simplify without Brackets

$\left(\frac{6x^2}{2x^1}\right)^3 =$
$(3^1x^1)^3$
3^3x^3
$27x^3$

Simplify 1st

Divide, Subtract Exponents
Multiply Exponents
Solve

$\frac{6}{2} = 3^1$

$\frac{x^2}{x} = x^1$

OR

$\left(\frac{6^1x^2}{2^1x}\right)^3 =$
$\frac{6^3x^6}{2^3x^3}$
$\frac{216x^6}{8x^3}$
$27x^3$

Don't Expand 1st

$6^3 = 216$
$2^3 = 8$ $\frac{216}{8} = 27$

Simplify without Brackets

$(-2x)^{-2} =$
$\frac{1}{(-2x)^2}$ Negative Exponents
$\frac{1}{(-2)^2x^2}$ Multiply Exponents
$\frac{1}{4x^2}$ Simplify

$\left(\frac{5x}{-2x^2}\right)^{-2} =$
$\frac{5^{-2}x^{-2}}{(-2)^{-2}x^{-4}}$ Multiply Exponents
Start off with "OVER"
$\frac{(-2)^2x^4}{5^2x^2}$ Negative Exponents
$\frac{4x^4}{25x^2}$
$\frac{4x^2}{25}$ Subtract Exponents

$4 - 2 = 2$

$(-2)^2 = 4$

$\frac{x^4}{x^2}$
x^{4-2}
x^2

OR

$\left(\frac{5x}{-2x^2}\right)^{-2} =$
$\left(\frac{-2x^2}{5x}\right)^2$
$\frac{2^2x^4}{5^2x^2}$
$\frac{4x^4}{25x^2}$
$\frac{4x^2}{25}$

Flip it
Make Exponent Positive

$\left(\frac{a}{b}\right)^{-c} = \left(\frac{b}{c}\right)^c$