F11 - 1.0 - Reasoning Review

Transitive Property:
$A = B, B = C \rightarrow A = C$

Statements:

Let: $Rains = p$ $Sky\ is\ Grey = q$

Conditional: An "if-then" statement.

Hypothesis Conclusion

If it "rains," then the "sky is grey." If "p" -> then "q" $p \rightarrow q$

Converse: A conditional statement where the hypothesis and the conclusion are switched.

If the "sky is grey," then it "rains." If "q" -> then "p" $q \rightarrow p$

Inverse: Formed by negating both the hypothesis and the conclusion of a conditional statement.

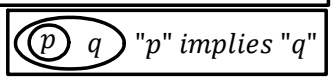
If it does "NOT rain," then the "sky is NOT grey." If NOT "p" -> then NOT "q" $\bar{p} \rightarrow \bar{q}$

Contrapositive: Formed by negating both the hypothesis and the conclusion of the converse.

If the "sky is NOT grey," then it does "NOT rain." If NOT "q" -> then NOT "p" $\bar{q} \rightarrow \bar{p}$

Biconditional: An "if and only if" statement.

It "rains" if and only if "the sky is grey" "p" if and only if "q" $p <-> q$



"p" *implies* "q"

Truth Tables

p	q	$p \rightarrow q$	$p \cap q$	$p \cup q$
T	T	T	T	T
F	F	T	F	F
F	T	T	F	T
T	F	F	F	T

A conditional statement is false only when :
- The hypothesis is true and
- The conclusion is false.

Otherwise :
- The conditional statement is true (even if the hypothesis is false.)

Conjecture: A opinion or conclusion but not yet proved. (A Hypothesis if Testable)

$2 + 2 = 4$ $2 \times 2 = 4$ Addition and Multiplication are the same thing

Counterexample: An example that invalidates a conjecture (Add and Multiply are the same thing.)

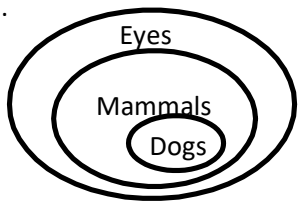
$2 + 3 = 5$ $2 \times 3 = 6$ False! Conclusion: The result of a hypothesis

Proof: A mathematical argument showing that a statement is valid in all cases. (Conjecture ->Proof)

Inductive Reasoning: Drawing a general conclusion by observing patterns and identifying properties in specific examples.

$1 + 1 = 2$ $1 + 2 = 3$ Adding two numbers equals a larger number. False!

Deductive Reasoning: Drawing a specific conclusion through logical reasoning by starting with general assumptions that are known to be valid.



All dogs are mammals
Mammals have eyes
Dogs have eyes. True!

EI/CPP/Fed/Prov Tax

https://www.canada.ca/en/revenue-agency/services/forms-publications/payroll/t4032-payroll-deductions-tables/t4032bc-jan.html

Federal/ Provincial Table/% Periods!

Gross Income $ 49566.56 *Yearly*

EI
1.63%, max $61500
Employer pays 1.4 times

EI
$49566.56 \times 0.0163 = 807.93$: *Yearly EI*
$807.93 \div 12 = 67.33$: *Monthly EI*

$1.63 \div 100 = 0.0163$

Changes every year

$5.95 \div 100 = 0.0595$

CPP
5.95% over 3500 (Yearly)
Employer matches amount
Max CPP=3754.45/Year
(Self employed pays double)

CPP
$49566.56 - 3500 = 46066.56$: *Amount to be taxed.*
$46066.56 \times 0.0595 = 2740.96$: *Yearly CPP amount*
$2740.96 \div 12 = 228.41$: *Monthly CPP amount*

Federal Tax (Table)

$49566.56 \div 12 = 4130.55$: *Monthly Income*

Claim code 1 : 369.40 : Federal monthly tax

So you don't have to use the table.

% *Rate*
2nd Bracket …
% *Rate*
1st Bracket
% *Rate*
$0

Changes every year

Provincial Tax (Table)

Claim code 1 : 150.85 : Provincial monthly tax

$228.41 + 67.33 + 369.40 + 150.85 = 815.99$: *Total monthly tax*

$815.99 \times 12 = 9791.88$: *Total yearly Tax*

$49566.56 - 9791.88 = \$39774.68$ *Yearly income after tax*

Make a Plan!
-

Decision Tree

F12 - 1.0 - Finance Review

LOGIC Algebra Check Answer Alpha Enter

Simple Interest

$I = Prt$

$I = F - P$

$F = P + I$

$F = P + Prt = P(1 + rt)$

Compound Interest

$F = P(1 \pm r)^t$

$F = P\left(1 \pm \frac{r}{n}\right)^{tn}$

I : *Interest Amount*
F : *Future Amount*
P : *Present Amount*
r : *Interest rate as decimal*
t : *time in years*
n : *# of compounding periods per year*

$\frac{72}{r} = t_{double}$

Find "F" of \$1000 "P", 1 year compounded yearly at 10%.

Ti-84 Calculator Instructions:

Simple Interest $I = Prt$

$I = 1000 \times 0.1 \times 1$

$I = 100$

$F = P + I$

$F = 1000 + 100$

$F = 1100$

Compound Interest $F = P(1 \pm r)^t$

$F = 1000(1 + 0.1)^1$

$F = 1100$

Apps | Finance | TVM Solver

N: 1× 1
I%: 10
PV: −1000
PMT: 0
FV: *1100*
P/Y: 1
C/Y: 1

$N = nt$ $n = \frac{comp}{year}$

Payment: End
Alpha Enter
C/Y Autocorrect!

P/Y=C/Y

Find "F" of a \$1000 "P", Investment in 24 months compounded monthly at 10%.

$I = Prt$

$I = 1000 \times 0.1 \times 2$

$I = 200$

$F = P + I$

$F = 1000 + 200$

$F = 1200$

$F = P\left(1 \pm \frac{r}{n}\right)^{tn}$

$F = 1000\left(1 + \frac{0.1}{12}\right)^{2\times12}$

$F = 1220.39$ Interest on Interest

N: Compounding Periods (PV) (Or Time in years (PV))
N: *Payments (PMT) payment periods per year* × *# years*
I%: 5% = 5
PV: −*ve*
PMT:
FV:
P/Y: *# of payment periods per year (P/Y*)*
C/Y: *# of compounding periods per year*

Payments: $N = \frac{\#payments}{year} \times \#years$

Money In : +*ve* Money Out : −*ve*

N: 12× 2
I%: 10
PV: −1000
PMT: 0
FV: *1220.39*
P/Y: 12
C/Y: 12

N: 2
I%: 10
PV: −1000
PMT: 0
FV: *1220.39*
P/Y: 1
C/Y: 12

$Rate\ of\ Return = \frac{FV - PV}{PV} = \frac{Interest}{Investment}$

$= \frac{1220.39 - 1000}{1000} = 0.22039 = 22.039\%$

$\#\ years = \frac{\#\ of\ months}{12} = \frac{24}{2} = 2$

Find "F" of 3 payments of \$100 at the end of each year for 3 Years at 10% compounded yearly.

$t = 0$ 1 2 3

Time Line

\$0

\$100 → \$121

\$100 → \$110

\$100

$F = \frac{P\left(\left(1 + \frac{r}{n}\right)^{tn} - 1\right)}{\frac{r}{n}}$ Payments Future Value

$F = \frac{100\left(\left(1 + \frac{0.1}{1}\right)^{3\times1} - 1\right)}{\frac{0.1}{1}} = 331$

P : *Payment*

N: 1×3
I%: 10
PV: 0
PMT: −100
FV: 331
P/Y: 1
C/Y: 1

Payments
P/Y=C/Y

$F = P(1 \pm r)^t$

$F = 100(1 + 0.1)^2$

$F = 121$

$F = P(1 \pm r)^t$

$F = 100(1 + 0.1)^1$

$F = 110$

$F = \$331$

OR

Excel

Years	0	1	2	3
	0	100		121
			100	110
				100
			FV	331

$Total = Payment \times \#\ Payments = 3 \times 100 = \$300$

$F = P\left(1 + \frac{r}{n}\right)^{tn}$

F12 - 1.0 - Simple/Compound Interest Notes $

Find the Future Value of a $1000 Investment in :

1 year at 10%.

Simple

$I = Prt$

$I = 1000 \times 0.1 \times 1$

$I = 100$

Two Step

$F = P + I$

$F = 1000 + 100$

$F = 1100$

One Step

$F = P + Prt$

$F = \$1100$

$F = 1000 + 1000(0.1)(1)$

$F = 1100$

$\frac{10\%}{100} = 0.1$

Compounded yearly

$F = P(1 \pm r)^t$

$F = 1000(1 + 0.1)^1$

$F = 1100$

$F = P + Prt$

$F = P(1 + rt)$

...

Ti84 Apps Fin TVM

N:	1× 1
$I\%$:	10
PV:	−1000
PMT:	0
FV:	1100
P/Y:	1
C/Y:	1

24 months at 10%.

Simple

$I = Prt$

$I = 1000 \times 0.1 \times 2$

$I = 200$

$F = P + I$

$F = 1000 + 200$

$F = 1200$

$Rate\ of\ Return = \frac{FV - PV}{PV}$

$R = \frac{1200 - 1000}{1000}$

$R = 0.2 = 20\%$

Compounded monthly $n = 12$

$F = P\left(1 \pm \frac{r}{n}\right)^{tn}$

$F = 1000\left(1 + \frac{0.1}{12}\right)^{2\times12}$

$F = 1220.39$

Interest on Interest!

Note:

N:	12× 2
$I\%$:	10
PV:	−1000
PMT:	0
FV:	1220.39
P/Y:	12
C/Y:	12

N:	2
FV:	1220.39
P/Y:	1

6 months at 10%.

$I = Prt$

$I = 1000 \times 0.1 \times \frac{1}{2}$

$I = 50$

$F = P + I$

$F = 1000 + 50$

$F = 1050$

Compounded quarterly $n = 4$

$F = P\left(1 \pm \frac{r}{n}\right)^{tn}$

$F = 1000\left(1 + \frac{0.1}{4}\right)^{\frac{1}{2}\times4}$

$F = 1000(1.050625)$

$F = 1050.63$

N:	12×1/2
$I\%$:	10
PV:	−1000
PMT:	0
FV:	1050.63
P/Y:	4
C/Y:	4

Find the Present Value of a $1000 Investment in

1 year at 10%

$F = P + Prt$

$1000 = P + P(0.1)(1)$

$1000 = P + .1P$

$1000 = 1.1P$

$\frac{1000}{1.1} = P$

$P = 909.09$

Compounded yearly

$F = P(1 \pm r)^t$

$1000 = P(1 + 0.1)^1$

$1000 = P(1.1)$

$P = 909.09$

N:	1× 1
$I\%$:	10
PV:	−909.09
PMT:	0
FV:	1000
P/Y:	1
C/Y:	1

2 years at 10%

$F = P + Prt$

$1000 = P + P(0.1)(2)$

$1000 = P + .2P$

$1000 = 1.2P$

$\frac{1000}{1.2} = P$

$P = 833.33$

$F = P(1 \pm r)^t$

$1000 = P(1 + 0.1)^2$

$1000 = P(1.21)$

$P = 826.45$

N:	1×2
$I\%$:	10
PV:	−826.45
PMT:	0
FV:	1000
P/Y:	1
C/Y:	1

F12 - 1.0 - Simple/Compound Interest Notes (Time!)

How long does it take for your money to double at 10%.

$F = P + Prt$
$2 = 1 + 1(0.1)t$
$t = 20$

Compounded yearly

$F = P(1 \pm r)^t$
$2 = 1(1 + 0.1)^t$
$2 = 1.1^t$
$y_1 = 2 \quad y_2 = 1.1^t$

Find Intersection

$t = 7.27\ years^*$

N:	7.27
$I\%$:	10
PV:	1
PMT:	0
FV:	2
P/Y:	1
C/Y:	1

Rule of 72:

$Doubling\ Time = \frac{72}{r} \quad \frac{72}{10} = 7.2$

Compounded quarterly

$F = P\left(1 \pm \frac{r}{n}\right)^{tn}$
$2 = 1\left(1 + \frac{0.1}{4}\right)^{t \times 4}$
$2 = 1.0825^{4t}$
$y = 2 \quad y = 1.015^{4t}$
$t = 7.02$

N:	28.07
$I\%$:	10
PV:	-1
PMT:	0
FV:	2
P/Y:	4
C/Y:	4

$N = nt$
$28.07 = 4t$
$t = 7.02$

$n = \frac{comp}{year}$

Find the Interest Rate of a $1000 PV Investment to a FV of $1500 in 1/2 Year/s.

$F = P + Prt$
$1500 = 1000 + 1000r(1)$
$500 = 1000r$
$1 = 0.5$
$r = 50\%$

$A = P(1 \pm r)^t$
$1500 = 1000(1 + r)^1$
$1.5 = 1 + r$
$r = 0.5$
$r = 50\%$

N:	1×1
$I\%$:	50
PV:	-1000
PMT:	0
FV:	1500
P/Y:	1
C/Y:	1

$F = P + Prt$
$1500 = 1000 + 1000r(2)$
$500 = 2000r$
$1 = 0.25$
$r = 25\%$

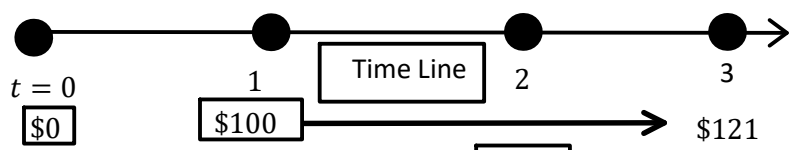
Reciprocal Exponent

$A = P(1 \pm r)^t$
$1500 = 1000(1 + r)^2$
$1.5 = (1 + r)^2$
$(1.5)^{\frac{1}{2}} = \left((1 + r)^2\right)^{\frac{1}{2}}$
$1.2247 = 1 + r$
$r = 0.2247$
$r = 22.47\%$

N:	1×2
$I\%$:	22.47
PV:	-1000
PMT:	0
FV:	1500
P/Y:	1
C/Y:	1

F12 - 1.0 - Payments Notes

Find the Future Value of 3 payments of $100 at the end of each year for 3 Years at 10% compounded yearly.



$$F = \frac{P\left(\left(1+\frac{r}{n}\right)^{tn}-1\right)}{\frac{r}{n}}$$ Payments Future Value

N: 1×3
$I\%$: 10
PV: 0
PMT: -100
FV: 331
P/Y: 1
C/Y: 1

Payments

P/Y=C/Y

$F = P(1 \pm r)^t$ $F = 100(1+0.1)^2$ $F = 121$

$F = P(1 \pm r)^t$ $F = 100(1+0.1)^1$ $F = 110$

$F = \$331$

$$F = \frac{100\left(\left(1+\frac{0.1}{1}\right)^{3\times 1}-1\right)}{\frac{0.1}{1}} = 331$$

$P : Payment$

OR

Excel

Years	0	1	2	3
	0	100		121
			100	110
				100
			FV	331

$Total = Payment \times \#\ Payments = 3 \times 100 = \$300$

$$F = P\left(1+\frac{r}{n}\right)^{tn}$$

Find the Future Value of $100 payments at the end of each month for 1 Year at 10% compounded monthly.

N: 12×1
$I\%$: 10
PV: 0
PMT: 100
FV: -1256.56
P/Y: 12
C/Y: 12

$$F = \frac{P\left(\left(1+\frac{r}{n}\right)^{tn}-1\right)}{\frac{r}{n}}$$

$$F = \frac{100\left(\left(1+\frac{0.1}{12}\right)^{1\times 12}-1\right)}{\frac{0.1}{12}}$$

$F = 1256.56$

Months	11	10	9	8	7	6	5	4	3	2	1	0
F=P(1+r)^t	109.56	108.65	107.75	106.86	105.98	105.11	104.24	103.38	102.52	101.67	100.83	100.00
r=0.1/12	0.008										FV=	1256.56

F12 - 1.0 - Payments Notes

How long to pay off a loan of \$2000 at 10% compounded weekly with payments of \$100 per month.

N: 21.97
$I\%$: 10
PV: -2000
PMT: 100
FV: 0
P/Y: 12
C/Y: 52

$N = nt$

$21.97 = 12t$

$t = 1.83$

$n = \frac{pay}{year}$

$1.83 \times 12 = 21.97$

$21.97 = 22\ Months$

Find the monthly payment to pay off a \$10000 loan in 4 years at 10% compounded semi-annually.

N: 12×4
$I\%$: 10
PV: -10000
PMT: 252.66
FV: 0
P/Y: 12
C/Y: 2

Find the monthly payment to pay off a \$10000 loan in 1 year at 10% compounded monthly.

N: 12×1
$I\%$: 10
PV: -10000
PMT: 879.16
FV: 0
P/Y: 12
C/Y: 12

Months	0	1	2	3	4	5	6	7	8	9	10	11	12
F=P(1+r)^t	10000	10080	9274	8462	7644	6819	5987	5149	4304	3452	2594	1728	856
r=0.1/12	Payment	879	879	879	879	879	879	879	879	879	879	879	879
0.008		9201	8395	7583	6765	5940	5108	4270	3425	2573	1714	849	-23