Math 11 Notes

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C11 - 1.1 - Arithmetic Means Notes

Write the first terms 5 of the sequence

$t_1 = 2, d = 3$

$+3$

$\underline{\ 2\ }, \underline{\quad}, \underline{\quad}, \underline{\quad}, \underline{\quad}$

$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5$

$n = 1 \quad n = 2 \quad n = 3 \quad n = 4 \quad n = 5$

2,5,8,11,14

$2 + 3 = 5$
$5 + 3 = 8$
...

$\boxed{\begin{array}{l} t_1 = 1st\ term\ (aka: \text{"a or } u_1\text{"}) \\ \text{d} = common\ difference \\ t_n = term\ n, every\ term \\ n = Term\ \#, or\ \#\ of\ terms \end{array}}$

OR $\boxed{t_n = t_1 + (n-1)d}$

$t_2 = 2, t_5 = -4$ *Logic*

$-d \quad +d \quad +d \quad +d$

$\underline{\quad}, \underline{\ 2\ }, \underline{\quad}, \underline{\quad}, \underline{-4}$

$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5$

$$\begin{aligned} 2 + 3d &= -4 \\ -2 \qquad & \quad -2 \\ 3d &= -6 \\ \frac{3d}{3} &= -\frac{6}{3} \end{aligned}$$

$4 - 1 = 3$

$d = -2$

$\underline{\ 4\ }, \underline{\ 2\ }, \underline{\ 0\ }, \underline{-2}. \underline{-4}$

$4, 2, 0, -2, -4$

Systems of Equations

$t_2 = 2, t_5 = -4$

$$\begin{aligned} t_n &= t_1 + (n-1)d \\ t_2 &= t_1 + (2-1)d \\ 2 &= t_1 + d \\ t_1 &= 2 - d \end{aligned}$$

$$\begin{aligned} t_n &= t_1 + (n-1)d \\ t_5 &= t_1 + (5-1)d \\ -4 &= t_1 + 4d \\ -4 &= (2-d) + 4d \\ -4 &= 2 + 3d \\ d &= -2 \end{aligned}$$

$t_1 = 2 - (-2)$

$t_1 = 4$

$2 - 2 = 0$
$0 - 2 = -2$
...
$2 + 2 = 4$

$t_7 = 26, t_{95} = 378$ *Logic*

$$\begin{aligned} 26 + 88d &= 378 \\ -26 \qquad & \quad -26 \\ 88d &= 352 \\ \frac{88d}{88} &= \frac{352}{88} \\ d &= 4 \end{aligned}$$

$95 - 7 = 88$

OR

$$\begin{aligned} t_n &= t_1 + (n-1)d \\ t_7 &= t_1 + (7-1)(4) \\ 26 &= t_1 + 24 \\ t_1 &= 2 \end{aligned}$$

$$\begin{aligned} t_n &= t_1 + (n-1)d \\ t_2 &= 2 + (2-1)(4) \\ t_2 &= 6 \end{aligned}$$

$\underline{\ 2\ }, \underline{\ 6\ }, \underline{\ 10\ }, \underline{\ 14\ }. \underline{\ 18\ }$

2,6,10,14,18

$26 - 4 = 22$
$22 - 4 = 18$
$18 - 4 = 14$
$14 - 4 = 10$
...

C11 - 1.1 - Arithmetic Sequences Notes

2,5,8 …

$d = ?$ $t_n = ?$ $t_{10} = ?$ $t_n = 53, n = ?$

	$+3$	$+3$			
2,	5,	8,	? …	? …..	53
t_1	t_2	t_3	t_4	t_{10}	t_n
$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 10$	$n = ?$

$\boxed{\_\_\_\_, \_\_\_\_, \_\_\_\_ \cdots \_\_\_\_ \cdots \_\_\_\_}$

$t_1 = 2$

$d = t_n - t_{n-1}$
$d = 8 - 5$
$d = 3$

$d = t_n - t_{n-1}$
$d = 5 - 2$
$d = 3$

Difference

$\boxed{d = t_n - t_{n-1}}$

A term subtracted by the term before it

$t_{n-1} = term\ before\ t_n$

Arithmetic: d must always be the same

Find the General term $\boldsymbol{t_n} = ?$

$t_n = t_1 + (n-1)d$
$t_n = 2 + (n-1)3$
$t_n = 2 + 3n - 3$
$t_n = 3n - 1$

General term formula

$\boxed{t_n = t_1 + (n-1)d}$

The first term
plus'n − 1' *differences*

What is the tenth term $\boldsymbol{t_{10}}$?

$t_n = 3n - 1$
$t_{10} = 3(10) - 1$
$t_{10} = 29$

Check your answer:
2,5,8,11,14,17,20,23,26,29

Or, Start from beginning

$t_n = t_1 + (n-1)d$
$t_{10} = 2 + (10-1)3$
$t_{10} = 2 + 27$
$t_{10} = 29$

Remember: You could have also added the common difference repeatedly

53 *is what term*, $\boldsymbol{t_n} = 53, \boldsymbol{n} = ?$

$t_n = 3n - 1$
$53 = 3n \cancel{-1}$
$+1 \qquad \cancel{+1}$
$54 = 3n$
$\frac{54}{3} = \frac{\cancel{3}n}{\cancel{3}}$
$n = 18$

Check your answer:
2,5,8,11,14,17,20,23,26,29,32,35,28,41,44,47,50,53

C11 - 1.2 - Arithmetic Series Notes

2,5,8 ... $s_{12} = ?$

$\boxed{s_n = sum\ of\ n\ terms}$

$$\underset{\substack{t_1 \\ n=1}}{\underline{2}} \xrightarrow{+3} \underset{\substack{t_2 \\ n=2}}{\underline{5}} \xrightarrow{+3} \underset{\substack{t_3 \\ n=3}}{\underline{8}},\ \underset{\substack{t_4 \\ n=4}}{\underline{?}}\ \ldots\ \underset{\substack{t_{12} \\ n=12}}{\underline{?}}$$

$t_1 = 2$

$d = t_n - t_{n-1}$
$d = 8 - 5$
$d = 3$

$d = t_n - t_{n-1}$
$d = 5 - 2$
$d = 3$

What is the sum of the first twelve terms $\boldsymbol{s_{12}}$? $\boldsymbol{s_{12} = ?, n = 12}$.

$$s_n = \frac{n}{2}(2t_1 + (n-1)d)$$
$$s_{12} = \frac{12}{2}(2(2) + (12-1)3)$$
$$s_{12} = 6(4 + (11)3)$$
$$s_{12} = 6(4 + 33)$$
$$s_{12} = 6(37)$$
$$s_{12} = 222$$

$$\boxed{s_n = \frac{n}{2}(2t_1 + (n-1)d)}$$

Sum of "n" terms formula: if t_n is not known.

Check your answer:
$2 + 5 + 8 + 11 + 14 + 17 + 20 + 23 + 26 + 29 + 32 + 35 = 222$ ✓

OR

$$s_n = \frac{n}{2}(t_1 + t_n)$$
$$s_{12} = \frac{12}{2}(2 + t_{12})$$
$$s_{12} = 6(2 + 35)$$
$$s_{12} = 222$$

$t_n = 3n - 1$
$t_{12} = 3(12) - 1$
$t_{12} = 35$

$$\boxed{s_n = \frac{n}{2}(t_1 + t_n)}$$

Sum of "n" terms formula: if t_n is known.

C11 - 1.3 - Geometric Means Notes

Write the first terms 5 of the sequence

$t_1 = 2, r = 3$

$\times 3$

$\underline{\ 2\ }, \underline{\quad}, \underline{\quad}, \underline{\quad}, \underline{\quad}$

$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5$

$\times 3$

$\underline{\ 2\ }, \underline{\ 6\ }, \underline{\ 18\ }, \underline{\ 54\ }, \underline{\ 162\ }$

2,6,18,54,162

t_1= $1st\ term$ (aka: "a or u_1")
$r = common\ ratio$
t_n= $term\ n, every\ term$
$n = Term\ \#, or\ \#\ of\ terms$

$t_2 = 4, t_4 = 16$

$\div r \quad \times r \quad \times r \quad \times r$

$\underline{\quad}, \underline{\ 4\ }, \underline{\quad}, \underline{\ 16\ }, \underline{\quad}$

$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5$

$4r^2 = 16$

$r^2 = 4$

$\sqrt{r^2} = \sqrt{4}$

$r = \pm 2$

$4 - 2 = 2$

$\div +2 \quad \times +2 \quad \times +2 \quad \times +2$

$\underline{\ 2\ }, \underline{\ 4\ }, \underline{\ 8\ }, \underline{\ 16\ }, \underline{\ 32\ }$

$\div +2 \quad \times +2 \quad \times +2 \quad \times +2$

$\underline{\ -2\ }, \underline{\ 4\ }, \underline{\ -8\ }, \underline{\ 16\ }, \underline{\ -32\ }$

2,4,8,16,32 $\qquad -2,4,-8,16,-32$

$t_2 = 9, t_5 = 243$

$9r^3 = 243$

$r^3 = 27$

$\sqrt{r^3} = \sqrt{27}$

$r = 3$

$5 - 2 = 3$

$3, 9, 27, 81, 243$

$t_1 = 2, t_5 = 162$

$2r^4 = 162$

$r^4 = 81$

$r = \pm 3$

$5 - 1 = 4$

$2, 6, 18, 54, 162 \qquad 2, -6, 18, -54, 162$

C11 - 1.3 - Geometric Sequences Notes

3,6,12 …

$r = ?$ $t_n = ?$ $t_5 = ?$ $t_n = 768, n = ?$

$$\underset{t_1}{\underline{\ 3\ }} \xrightarrow{\times 2} \underset{t_2}{\underline{\ 6\ }} \xrightarrow{\times 2} \underset{t_3}{\underline{\ 12\ }}, \ \underset{t_4}{\underline{\ ?\ }} \ldots \underset{t_{10}}{\underline{\ ?\ }} \ldots\ldots \underset{t_n}{\underline{\ 768\ }}$$

$n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$ $n = ?$

$\_\_\_\_, \_\_\_\_, \_\_\_\_ \ldots \_\_\_\_ \ldots \_\_\_\_$

$t_1 = 3$

$r = \frac{t_n}{t_{n-1}}$

$r = \frac{6}{3}$

$r = 2$

$r = \frac{t_n}{t_{n-1}}$

$r = \frac{12}{6}$

$r = 2$

Ratio

$r = \frac{t_n}{t_{n-1}}$

A term divided by the term before it

$t_{n-1} = term\ before\ t_n$

Geometric: r must always be the same

$\boldsymbol{Find\ the\ General\ term\ t_n} = ?$

$t_n = t_1 r^{n-1}$

$t_n = 3(2)^{n-1}$

General term formula

$\boxed{t_n = t_1 r^{n-1}}$

The first term times 'r − 1' differences

$\boldsymbol{What\ is\ the\ fifth\ term\ t_5?\ t_5 = ?, n = 5.}$

$t_n = 3(2)^{n-1}$

$t_5 = 3(2)^{n-1}$

$t_5 = 3(2)^{5-1}$

$t_5 = 3(2)^4$

$t_5 = 48$

Check your answer: 3,6,12,24,48 ✓

Or, Start from beginning

$t_n = t_1 r^{n-1}$

$t_5 = 3(2)^{5-1}$

$t_5 = 48$

Remember: You could have also multiplied by the common ratio repeatedly

$\boldsymbol{The\ number\ 768\ is\ what\ term?\ t_n = 768, n} = ?$

$t_n = 3(2)^{n-1}$

$768 = 3(2)^{n-1}$ *divide both sides by 3*

$256 = 2^{n-1}$

$2^8 = 2^{n-1}$ *Change of base*: $256 = 2^8$

$8 = n - 1$ *Same Base, exponents are equal*

$n = 9$

Check your answer: 3,6,12,24,48,96,192,384,768 ✓

C11 - 1.4 - Geometric Series Notes

3,6,12 …

$s_8 = ?$ $s_\infty = ?$

$\boxed{s_n = sum\ of\ n\ terms}$

$$\underset{t_1}{\underline{3}},\ \xrightarrow{\times 2}\ \underset{t_2}{\underline{6}},\ \xrightarrow{\times 2}\ \underset{t_3}{\underline{12}},\ \underset{t_4}{\underline{?}}\ \ldots\ \underset{t_{10}}{\underline{384}}$$

$n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 8$

$t_1 = 3$

$$r = \frac{t_n}{t_{n-1}} \qquad r = \frac{t_n}{t_{n-1}}$$

$$r = \frac{6}{3} \qquad r = \frac{12}{6}$$

$$r = 2 \qquad r = 2$$

What is the sum of the first eight terms $\boldsymbol{s_8}$? $\boldsymbol{s_8} = ?, \boldsymbol{n = 8}$.

$$s_n = \frac{t_1(1 - r^n)}{1 - r}$$

$$s_8 = \frac{3(1 - 2^8)}{1 - 2}$$

$$s_8 = 765$$

$$\boxed{s_n = \frac{t_1(1 - r^n)}{1 - r}}$$

Sum of "n" terms formula (if number of terms is known)

Check your answer: $3 + 6 + 12 + 24 + 48 + 96 + 192 + 384 = 765$ √

OR

$$s_n = \frac{t_1 - rt_n}{1 - r}$$

$$s_8 = \frac{3 - 2(t_8)}{1 - 2}$$

$$s_8 = \frac{3 - 2(384)}{1 - 2}$$

$$s_8 = 756$$

$$t_n = 3(2)^{n-1}$$

$$t_8 = 3(2)^{8-1}$$

$$t_8 = 3(2)^7$$

$$t_8 = 3(128)$$

$$t_8 = 384$$

$$\boxed{s_n = \frac{t_1 - rt_n}{1 - r}}$$

Sum of "n" terms formula (if last term t_n is known)

What is the sum of an infinite number of terms?

$r = 2$

$r > 1, \therefore$ *no sum*

Check your answer: $3 + 6 + 12 + 24 + 48 + 96 + 192 + 384 + 768 + 1536 + 3072 + \cdots = \infty$ √

C11 - 1.5 - Infinite Geometric Sequences Notes

8,4,2 ...

$s_\infty = ?$

What is the sum of the infinite sequence?

$\times\frac{1}{2}$ $\times\frac{1}{2}$

$\underset{t_1}{\underline{8}}, \underset{t_2}{\underline{4}}, \underset{t_3}{\underline{2}}, \underset{t_4}{\underline{1}}, \underset{t_5}{\underline{\tfrac{1}{2}}}, \underset{t_6}{\underline{\tfrac{1}{4}}}, \ldots$

$t_1 = 8$

$r = \frac{t_n}{t_{n-1}}$ $\quad r = \frac{t_n}{t_{n-1}}$

$r = \frac{4}{8}$ $\quad r = \frac{2}{4}$

$r = \frac{1}{2}$ $\quad r = \frac{1}{2}$

$$\boxed{\begin{array}{l} -1 < r < 1 \\ -1 < \frac{1}{2} < 1 \\ \therefore \text{Convergent, has sum} \end{array}}$$

$s_\infty = \frac{t_1}{1-r}$

$s_\infty = \frac{8}{1-\frac{1}{2}}$

$s_\infty = \frac{8}{\frac{1}{2}}$

$s_\infty = 16$

$$\boxed{s_\infty = \frac{t_1}{1-r}}$$

Sum of "n" terms formula (infinite number of terms)

$\textit{Check your answer}: 8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 15.9375 \approx 16$ ✓

8,16,32 ...

$s_\infty = ?$

What is the sum of the infinite sequence?

$\times 2$ $\times 2$

$\underset{t_1}{\underline{8}}, \underset{t_2}{\underline{16}}, \underset{t_3}{\underline{32}}, \underset{t_4}{\underline{64}}, \underset{t_5}{\underline{128}}, \underset{t_6}{\underline{256}}, \ldots$

$t_1 = 8$

$r = \frac{t_n}{t_{n-1}}$ $\quad r = \frac{t_n}{t_{n-1}}$

$r = \frac{16}{8}$ $\quad r = \frac{32}{16}$

$r = 2$ $\quad r = 2$

$$\boxed{\begin{array}{l} r > 1 \\ \therefore \text{Divergent} \\ \therefore \text{No sum} \end{array}}$$

$\textit{Check your answer}: 8 + 16 + 64 + 128 + 256 + 512 + 1024 + 2048 + \cdots = \infty$ ✓

C11 - 1.6 - Sigma Notation - Notes

Find the sum of the terms

Arithmetic

$$\sum_{k=1}^{4} 2k = ?$$

d

2, 4, 6, 8,

k=1 k=2 k=3 k=4

$2k = 2(1) = 2$ $2k = 2(2) = 4$ $2k = 2(3) = 6$ $2k = 2(4) = 8$

$s_4 = 2+4+6+8 = 20$

Steps
Put in k = bottom number the equation
Put in $k+1$ (bottom # plus 1)
Repeat until k = top number

k	$2k$
1	2
2	4
3	6
4	8

Arithmetic

$$\sum_{k=1}^{100} 2k = ?$$

$$s_n = \frac{n}{2}(2t_1 + (n-1)d)$$

$$s_{100} = \frac{100}{2}(2(2) + (100-1)2)$$

$$s_{100} = 10100$$

$$\boxed{s_n = \frac{n}{2}(2t_1 + (n-1)d)}$$

$\# \ of \ terms = n - k + 1$
= Top # minus Bottom # + 1

$d = 4 - 2$ $d = 6 - 4$
$d = 2$ $d = 2$

$n = 100 - 1 + 1$
$n = 100$

Geometric

$$\sum_{k=2}^{6} 8\left(\frac{1}{2}\right)^{k-1} = ?$$

4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$,

k=2 k=3 k=4 k=5 k=6

Always r

Not Always t1

$3(\frac{1}{2})^{k-1} = 8\left(\frac{1}{2}\right)^{2-1} = 4$ $3(\frac{1}{2})^{k-1} = 8\left(\frac{1}{2}\right)^{3-1} = 2$ ... $3(\frac{1}{2})^{k-1} = 8\left(\frac{1}{2}\right)^{6-1} = \frac{1}{4}$

$4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} = 7.75$

$n = 6 - 2 + 1$
$n = 5$

$$\boxed{s_n = \frac{t_1(1-r^n)}{1-r}}$$

$$s_n = \frac{t_1(1-r^n)}{1-r}$$

$$s_5 = \frac{4\left(1-\left(\frac{1}{2}\right)^5\right)}{1-\left(\frac{1}{2}\right)}$$

$$s_5 = 7.75$$

Infinite Geometric

$$\sum_{k=2}^{\infty} 3(2)^{k-1} = ?$$

4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$ ...

$r = \frac{2}{4}$
$r = \frac{1}{2}$

$r = \frac{1}{2}$

$-1 < r < 1$
$-1 < \frac{1}{2} < 1$
$\therefore$ Convergent, has sum

$$s_\infty = \frac{t_1}{1-r}$$

$$s_\infty = \frac{4}{1-\left(\frac{1}{2}\right)}$$

$$s_\infty = \frac{4}{\frac{1}{2}}$$

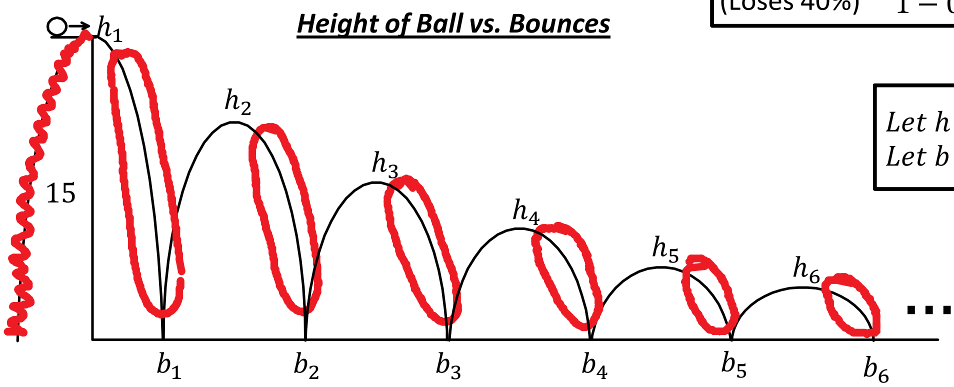
$$s_\infty = 4 \times \frac{2}{1}$$

$$s_\infty = 8$$

C11 - 1.8 - Bouncing Ball Notes (up 60%)

A ball rolls off a building 15 m tall. After each bounce, it rises to 60% of the previous height.

(Loses 40%) $1 - 0.4 = 0.6$



$Let\ h = Fall\ height$
$Let\ b = Bounce\ Number$

$r = 60\%$
$r = 0.6$

$\times 0.6 \quad \times 0.6 \quad \times 0.6 \quad \times 0.6 \quad \times 0.6$

$\underset{t_1}{15}, \underset{t_2}{9}, \underset{t_3}{5.4}, \underset{t_4}{3.24}, \underset{t_5}{1.94}, \underset{t_6}{1.17}$

Terms are Fall Heights!

$r = \frac{9}{15}$ $\quad r = .6$

$r = \frac{5.4}{9}$ $\quad r = .6$

How high does the ball bounce after the 1st, 2nd bounce?

Height After 1st Bounce

$15 \times 0.6 = 9\ m$

Height After 2nd Bounce

$9 \times 0.6 = 5.4\ m$

$1 \rightarrow 2!$
$2 \rightarrow 3!$

After $1st = t_2$
After $2nd = t_3$

How high does the ball bounce after the nth bounce? (Find the general formula)

$t_n = t_1 r^{n-1}$

$t_n = t_1 r^{n-1}$

$t_n = 15(0.6)^{n-1}$

How high does the ball bounce after the 4th bounce. $\boxed{t_5}$

$t_n = t_1(r)^{n-1}$

$t_5 = 15(0.6)^{5-1}$

$t_5 = 15(0.6)^4$

$t_5 = 1.94\ m$

$4 \rightarrow 5!$

After $4th$ bounce $= t_5$

How high does the ball bounce after the 10th bounce. $\boxed{t_{11}}$

$t_n = t_1 r^{n-1}$

$t_{11} = 15(0.6)^{11-1}$

$t_{11} = 15(0.6)^{10}$

$t_{11} = 0.09\ m$

$10 \rightarrow 11!$

After $10th$ bounce $= t_{11}$

What is the total vertical distance the ball has travelled when it hits the ground for the 5th bounce? $\boxed{s_5 = ?}$

$$s_n = \frac{t_1(1 - r^n)}{1 - r}$$

$$s_5 = \frac{15(1 - (.6)^5)}{1 - .6}$$

$$s_5 = \frac{15(0.87)}{.4}$$

$$s_5 = 34.6\ m$$

$34.6 \times 2 - 15 = 54.2\ m$

Count it

15
$+9 \times 2$
$+5.4 \times 2$
$+3.24 \times 2$
$+1.94 \times 2$
54.2

If it bounces forever, what is the total vertical distance travelled? $\boxed{s_\infty = ?}$

$$s_\infty = \frac{t_1}{1 - r}$$

$$h_\infty = \frac{h_1}{1 - r}$$

$$h_\infty = \frac{15}{1 - 0.6}$$

$$h_\infty = \frac{15}{0.4}$$

$$h_\infty = 37.5\ m$$

$r = 0.6 \quad r < 1$

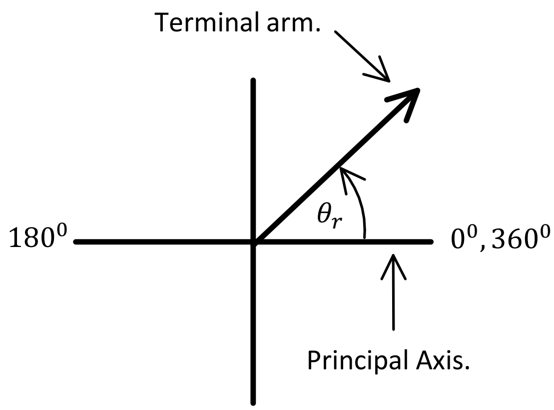
$37.5 \times 2 - 15 = 60m$

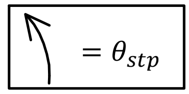
Double it to account for rise heights and subtract the initial height (double counted)

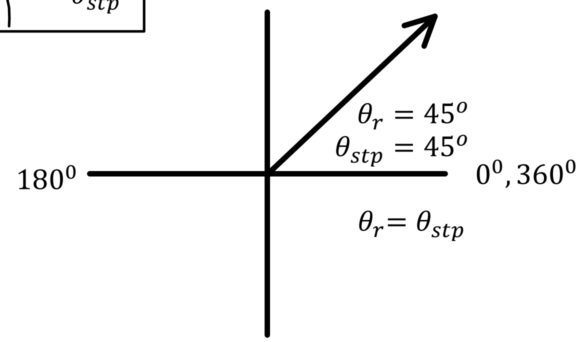
C11 - 2.1 - θ_r, θ_{stp} Notes

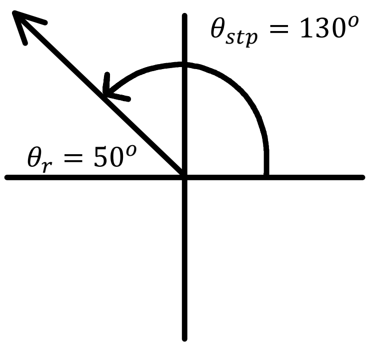
θ_r: the "reference angle"is the angle between the terminal arm and the x-axis ($0^o \le \theta \le 90^o$).

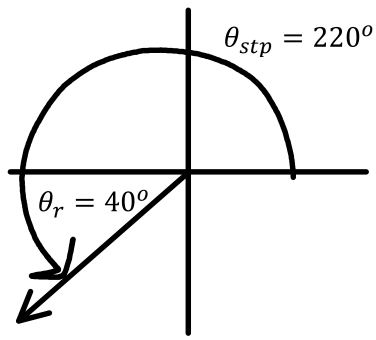
θ_{stp}: the "angle in standard position" from the principal axis (+ x-axis) to the terminal arm.

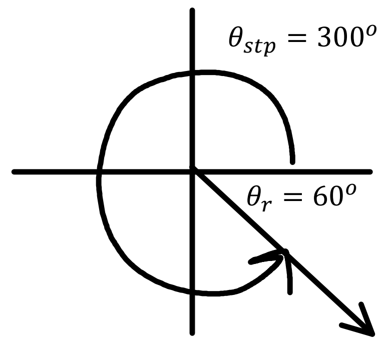












$\theta_{stp} = 180^O - 50^o$
$\theta_{stp} = 130^O$

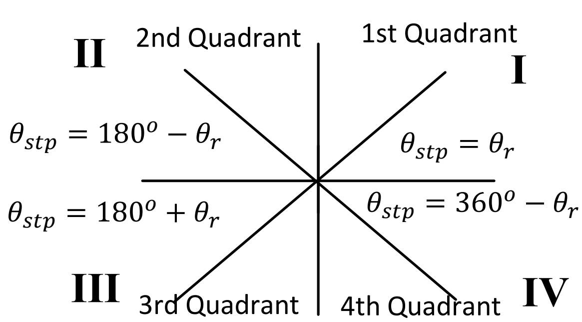
$\theta_r = 180^O - 130^o$
$\theta_r = 50^O$

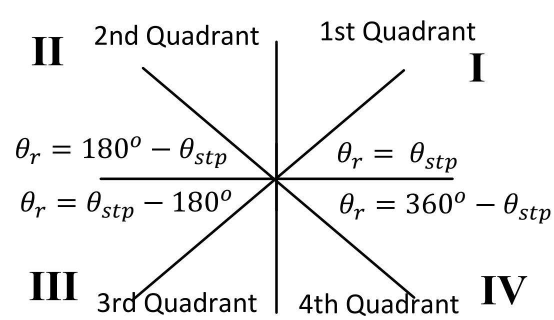
$\theta_{stp} = 180^O + 40^o$
$\theta_{stp} = 220^O$

$\theta_r = 220^o - 180^O$
$\theta_r = 40^O$

$\theta_{stp} = 360^O - 50^o$
$\theta_{stp} = 310^O$

$\theta_r = 360^O - 310^o$
$\theta_r = 50^O$

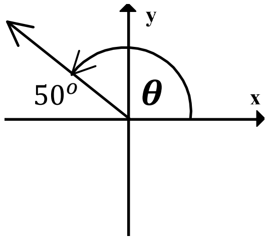




Basic logic will calculate θ_{stp} *and* θ_r much more easily than using these formulas.

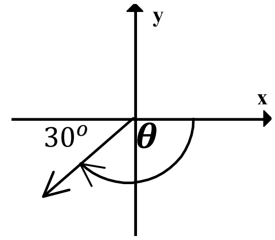
C11 - 2.1 - $\pm\,\theta_{stp}, \theta_{cot}, \theta_{pri}$ Notes

Counter-clockwise rotation is a positive θ_{stp}



$\theta_{stp} = 180^o - 50^o$
$\theta_{stp} = 130^o$

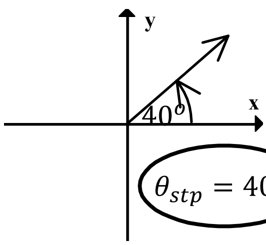
Clockwise rotation is a negative θ_{stp}



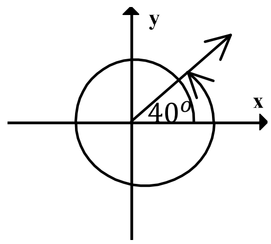
$\theta_{stp} = -(180^o - 30^o)$
$\theta_{stp} = -150^o$

Positive Co-terminal Angles (θ_{cot})

$$\boxed{\theta_{cot} = \theta_{stp} \pm 360^o}$$



$\theta_{stp} = 40^o$

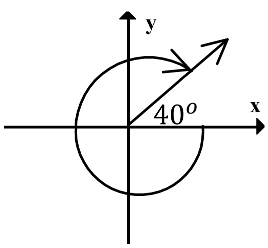


$\theta_{cot} = \theta_{stp} \pm 360^o$
$\theta_{cot} = 40^o + 360^o$
$\theta_{cot} = 400^o$

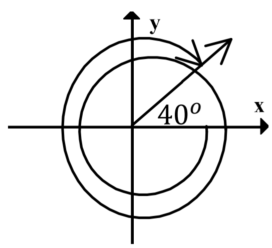
$\theta_{stp} = 40^o, \theta_{stp} = 400^o$

$\theta_{\text{cot}} = 40^o, 400^o, 760^o, 1120^o, 1480^o, \ldots$

Negative Co-terminal Angles (θ_{cot})



$\theta_{cot} = \theta_{stp} \pm 360$
$\theta_{cot} = 40 - 360$
$\theta_{cot} = -320^o$



$\theta_{cot} = \theta_{stp} \pm 360$
$\theta_{cot} = -320 - 360$
$\theta_{cot} = -680^o$

$\theta_{\text{cot}} = 40^o, -320^o, -680^o, -1040^o, -1400^o, \ldots$

$\theta_{principle} = smallest + ve\ \theta_{stp}\ coterminal.$

$\theta_{pri} = 0 \le \theta_{cot} < 360$

$\theta_{stp} = 1000^0$

$\theta_{pri} = 1000^0 - 360^0 = 640^0$
$= 640^0 - 360^0 = 280^0$

OR

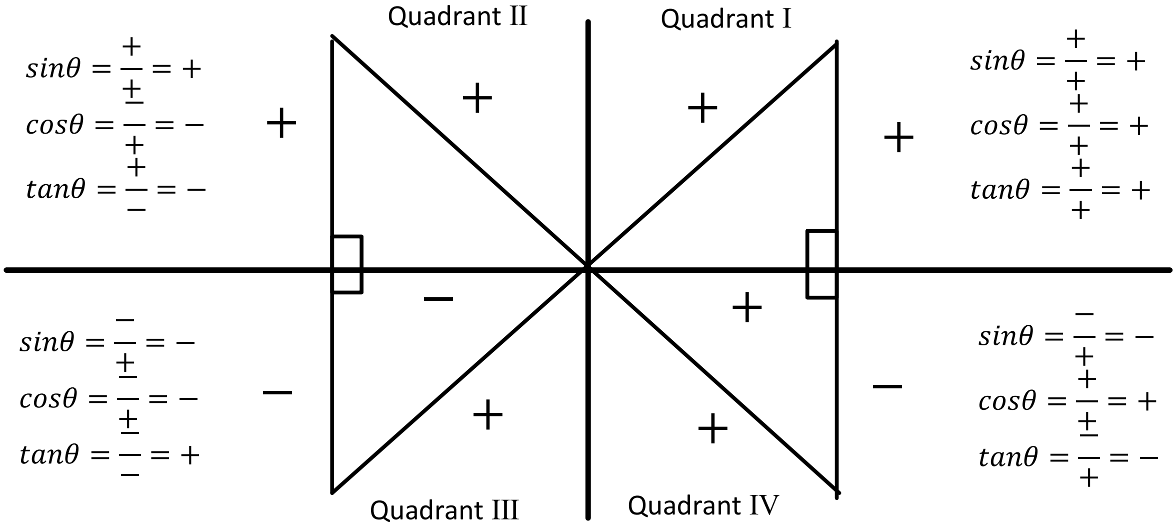
$1000^0 - 2(360^0) = 280^0$

$\frac{1000^0}{360^0} = 2.777\ldots$

OR

$0.777\ldots \times 360^0 = 280^0$

You may need to add or subtract 360^0 more than once.



Quadrant II:

$sin\theta = \frac{+}{+} = +$

$cos\theta = \frac{-}{+} = -$

$tan\theta = \frac{+}{-} = -$

Quadrant I:

$sin\theta = \frac{+}{+} = +$

$cos\theta = \frac{+}{+} = +$

$tan\theta = \frac{+}{+} = +$

Quadrant III:

$sin\theta = \frac{-}{+} = -$

$cos\theta = \frac{-}{+} = -$

$tan\theta = \frac{-}{-} = +$

Quadrant IV:

$sin\theta = \frac{-}{+} = -$

$cos\theta = \frac{+}{+} = +$

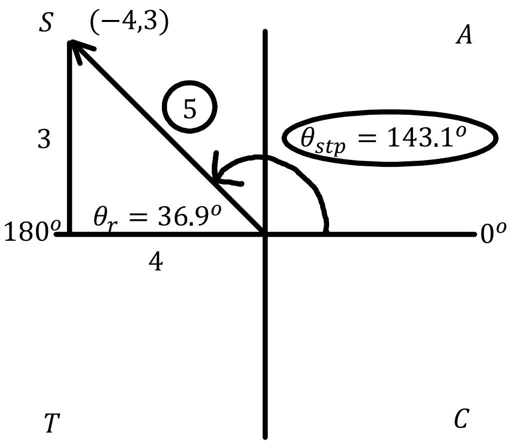
$tan\theta = \frac{-}{+} = -$

$(+)^2 + (-)^2 = +$ $\sqrt{+} = +$	Remember: the hypotenuse is always positive.

S **Students** Only **S**in positive.	**A** **All** **A**ll (sin, cos, tan) positive
Only **T**an positive. **Take** **T**	**Calculus** Only **C**os positive. **C**

C11 - 2.3 - *Trig Ratios* Notes

Find sinx, cox, and tanx for the following point. Find θ_{stp}. SOH CAH TOA



$a^2 + b^2 = c^2$
$3^2 + 4^2 = c^2$
$9 + 16 = c^2$
$25 = c^2$
$\sqrt{25} = \sqrt{c^2}$
$5 = c$

$sin\theta = +\frac{3}{5}$

$cos\theta = -\frac{4}{5}$

$tan\theta = -\frac{3}{4}$

$tan\theta = -\frac{3}{4}$
$\theta = \tan^{-1}(+0.75)$
$\theta = 36.9^o$

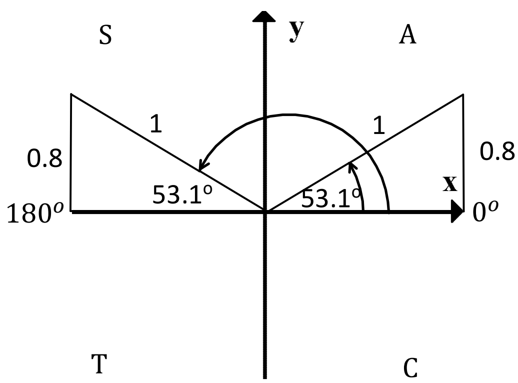
$180^o - 36.9^o = 143.1^o$

$\theta_{stp} = 143.1$

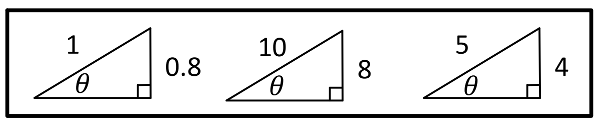
Check Answer
$sin143.1 = +0.6 = +\frac{3}{5}$ ✓

$\theta = \sin^{-1}(-\frac{3}{5})$ $\theta = 36.9$	$\theta = \cos^{-1}(-\frac{4}{5})$ $\theta = 143.1^o$	$\theta = \tan^{-1}(-\frac{3}{4})$ $\theta = -36.9^o$
$\theta = \sin^{-1}(+\frac{3}{5})$ $\theta = 36.9$	$\theta = \cos^{-1}(+\frac{4}{5})$ $\theta = 36.9^o$	$\theta = \tan^{-1}(+\frac{3}{4})$ $\theta = 36.9^o$

$\sin\theta = 0.8$ **Solve for $\theta, 0^o \le \theta < 360^o$ *and general solution***



$sin\theta = \frac{0.8}{1} = \frac{8}{10} = \frac{4}{5}$ *θ is the same*!



Draw two $\Delta's$ where $\sin\theta$ is positive: ASTC Quadrant I, II

Label the triangles according to SOH CAH TOA

Solve for θ_r: $\boxed{\theta_r = \sin^{-1}(+\frac{O}{H})}$

Draw an arrow from the principal axis:
To the first and second terminal arm

Solve for the arrows θ_{stp}

Check your answer:

$sin53.1^o = 0.8$ ✓

$sin126.9^o = 0.8$ ✓

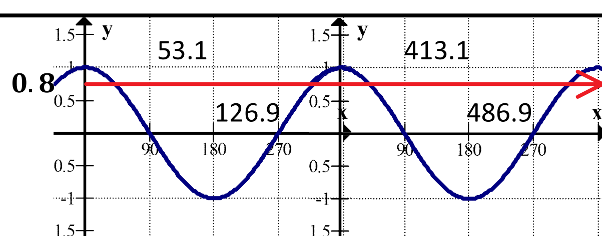
$sin\theta = \frac{0.8}{1}$
$\theta_r = \sin^{-1}\left(+\frac{0.8}{1}\right)$
$\theta_r = 53.1^o$

$\theta_{stp} = 53.1^o$ $\theta_{stp} = 180^o - 53.1^o = 126.9^o$

$\theta_{stp} = 53.1^o, 126.9^o$

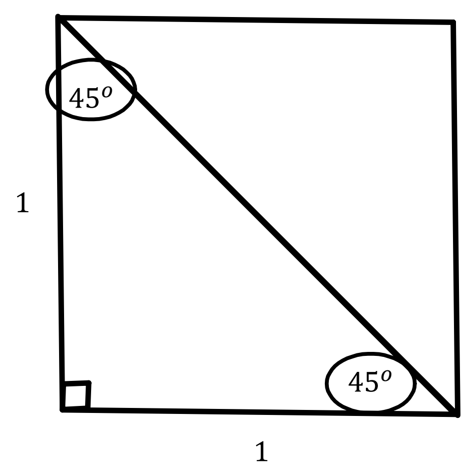
General Solution:
$\theta = \theta_{stp} \pm pn, n \in I$

$\theta = 53.1^o \pm 360^o n, n \in I$
$\theta = 126.9^o \pm 360^o n, n \in I$

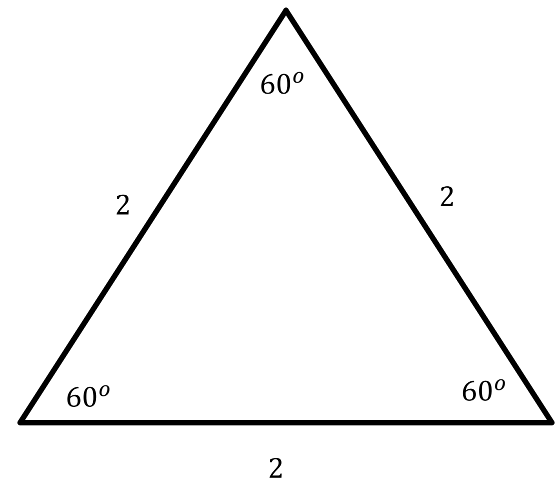


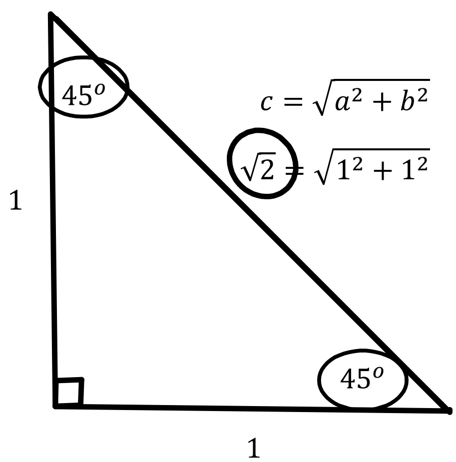
C11 - 2.4 - Special Triangles 30,45,60 sin/cos/tan Notes

Diagonal of a square with sides lengths of 1



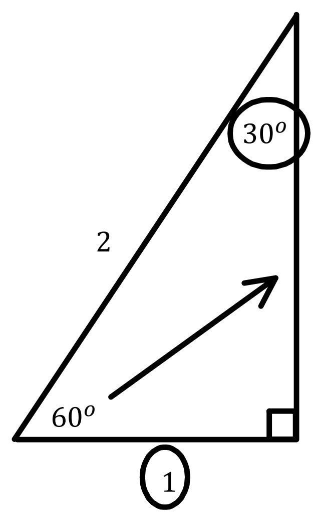
Half an equilateral with sides 2





$$c = \sqrt{a^2 + b^2}$$

$$\sqrt{2} = \sqrt{1^2 + 1^2}$$



$$b = \sqrt{c^2 - a^2}$$

$$\sqrt{3} = \sqrt{2^2 - 1^2}$$

$60 > 30$

$\sqrt{3} > 1$

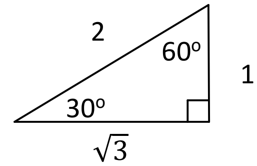
60 must open up to the root 3. And Vice Versa

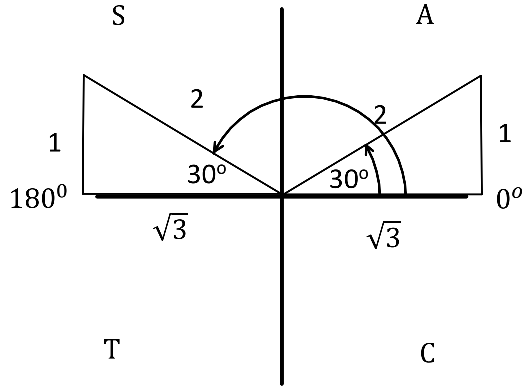
$\sin 45 = \frac{1}{\sqrt{2}}$	$\sin 60 = \frac{\sqrt{3}}{2}$	$\sin 30 = \frac{1}{2}$
$\cos 45 = \frac{1}{\sqrt{2}}$	$\cos 60 = \frac{1}{2}$	$\cos 30 = \frac{\sqrt{3}}{2}$
$\tan 45 = \frac{1}{1}$	$\tan 60 = \frac{\sqrt{3}}{1}$	$\tan 30 = \frac{1}{\sqrt{3}}$

C11 - 2.5 - $sin\theta = \frac{1}{2}$ Notes

$sin\theta = \frac{1}{2}$ **Solve for $\boldsymbol{\theta, 0^o \le \theta < 360^o}$.**

Between 0 and 360 degrees





Draw Two $\Delta's$ where $\sin\theta$ is +ve: ASTC Quadrant I, II

Label the $\Delta's$ according to SOH CAH TOA
Label the reference angle according to special Δ's.

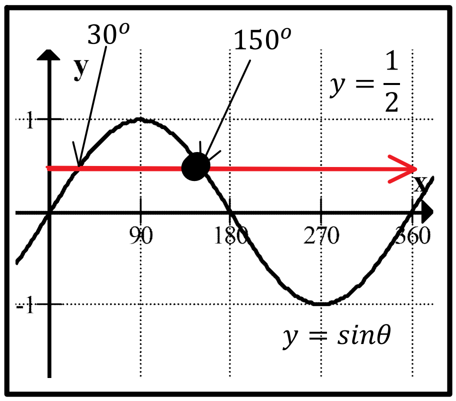
Draw an arrow from the principal axis:
To the first terminal arm and the second terminal arm.

Solve for the arrows θ_{stp}

Check on Calculator

$\theta_{stp} = 30^o$ $\quad \theta_{stp} = 180^o - 30^o = 150^o$

$\theta_{stp} = 30^o, 150^o$

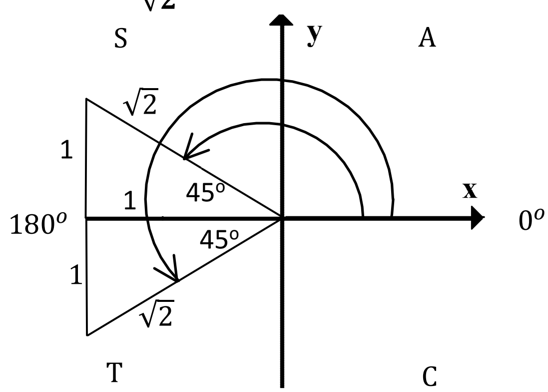


Graphing Calculator
$y = sinx$
$y = \frac{1}{2}$
Zoom 7:
$-360 \le x \le 360$
Window = Domain
Find Intersections

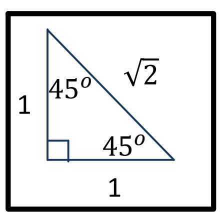
Check your answer: $sin30^o = \frac{1}{2}$ ✓

$\sin\theta = \frac{1}{2}$ $\quad sin150^o = \frac{1}{2}$ ✓

$\boldsymbol{cos\theta = -\frac{1}{\sqrt{2}}}$ **Solve for $\boldsymbol{\theta, 0^o \le \theta < 360^o}$ *and general solution*.**



Draw two triangles where $\cos\theta$ is -ve...



$\theta_{stp} = 180^o + 45^o = 225^o$ $\quad \theta_{stp} = 180^o - 45^o = 135^o$

$\boldsymbol{cos\theta = -\frac{1}{\sqrt{2}} = -0.707}$

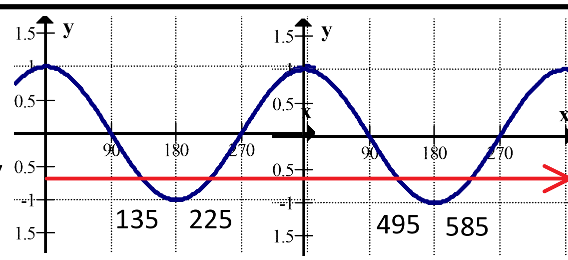
$\boldsymbol{cos135 = -\frac{1}{\sqrt{2}}}$ ✓ $\quad \boldsymbol{cos225 = -\frac{1}{\sqrt{2}}}$ ✓

$\theta_{stp} = 225^o, 135^o$

General Solution:
$\theta = \theta_{stp} \pm pn, n \in I$

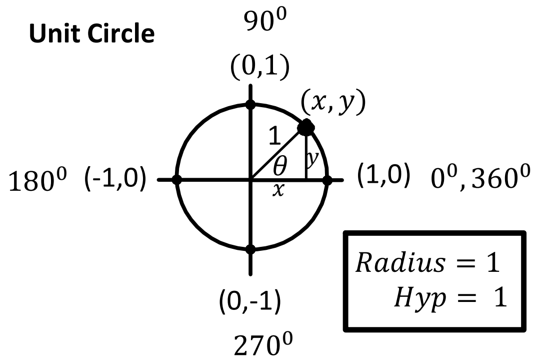
$\theta = 225^o \pm 360^o n, n \in I$
$\theta = 135^o \pm 360^o n, n \in I$

$\boldsymbol{-0.707}$

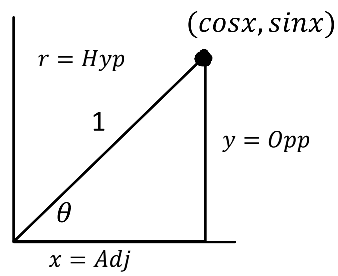


C11 - 2.6 - Unit Circle sin/cos/tan 90, 180, 270, 360 Notes

Unit Circle



$Radius = 1$
$Hyp = 1$



(x, y)

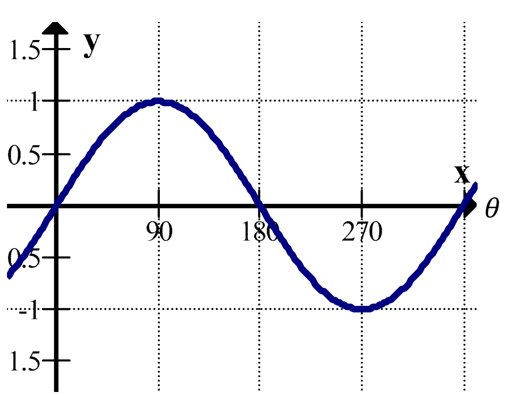
$(cos\theta, sin\theta)$

$sin\theta = y$

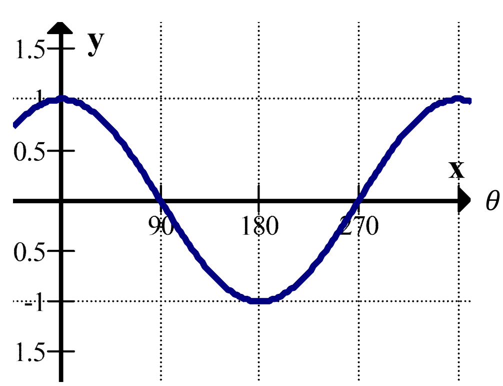
$cos\theta = x$

$tan\theta = \frac{y}{x}$

$\boldsymbol{sin\theta = \frac{Opp}{Hyp}}$ $sin\theta = \frac{y}{1}$ $sin\theta = y$	$\boldsymbol{cos\theta = \frac{Adj}{Hyp}}$ $cos\theta = \frac{x}{1}$ $cos\theta = x$	$\boldsymbol{tan\theta = \frac{Opp}{Adj}}$ $tan\theta = \frac{y}{x}$
$sin0^o = \frac{0}{1}$ $sin0^o = 0$	$cos0^o = \frac{1}{1}$ $cos0^o = 1$	$tan0^o = \frac{0}{1}$ $tan0^o = 0$
$sin90^o = \frac{1}{1}$ $sin90^o = 1$	$cos90^o = \frac{0}{1}$ $cos90^o = 0$	$tan90^o = \frac{1}{0}$ $tan90^o = UND$
$sin180^o = \frac{0}{1}$ $sin180^o = 0$	$\cos 180^o = -\frac{1}{1}$ $cos180^o = -1$	$tan180^o = \frac{0}{-1}$ $tan180^o = 0$
$sin270^o = \frac{-1}{1}$ $sin270^o = -1$	$cos270^o = \frac{0}{1}$ $cos270^o = 0$	$tan270^o = \frac{-1}{0}$ $tan270^o = UND$
$sin360^o = \frac{0}{1}$ $sin360^o = 0$	$cos360^o = \frac{1}{1}$ $cos360^o = 1$	$tan360^o = \frac{0}{1}$ $tan360^o = 0$



$y = sinx$

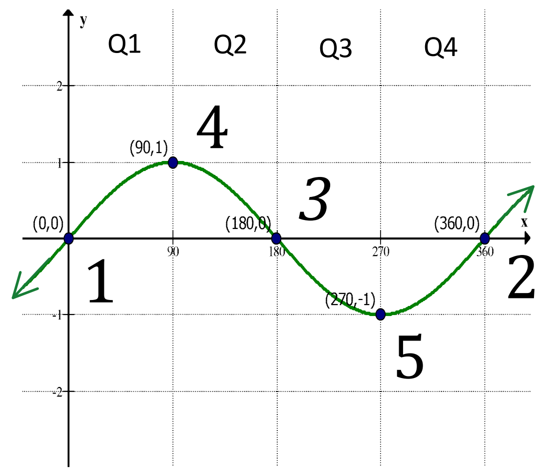


$y = cosx$

$y = \sin x$

x	y
0°	0
90°	1
180°	0
270°	-1
360°	0

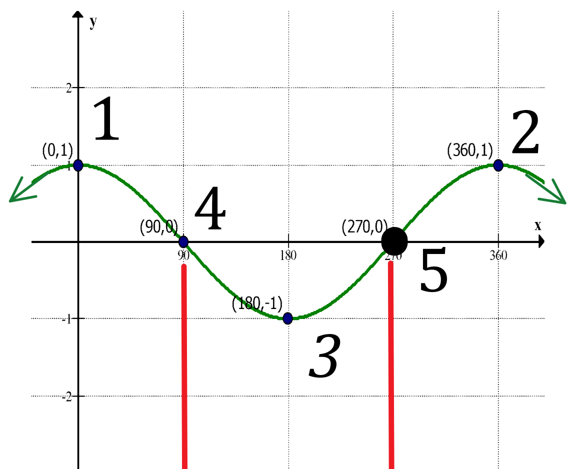
Pt.
(0,0)
(90,1)
(180,0)
(270,-1)
(360,0)



$y = \cos x$

x	y
0°	1
90°	0
180°	-1
270°	0
360°	1

Pt.
(0,1)
(90,0)
(180,-1)
(270,0)
(360,1)

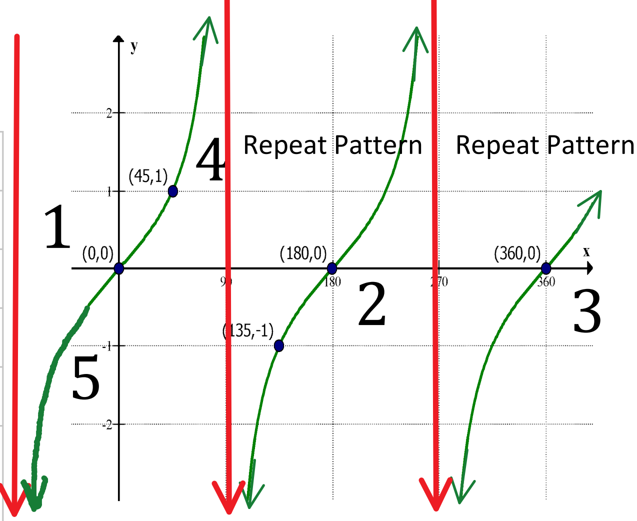


$y = \tan x$

x	y
0°	0
45°	1
90°	und
135°	-1
180°	0

Pt.
(-45,-1)
(0,0)
(45,1)
(90,und)
(135,-1)
(180,0)

ASTC
Special Triangles



Tan is Zero when sin is zero
Tan is UND when cos is zero

$$tanx = \frac{sinx}{cosx}$$

C11 - 2.8 - $\sin 2\theta$ Notes

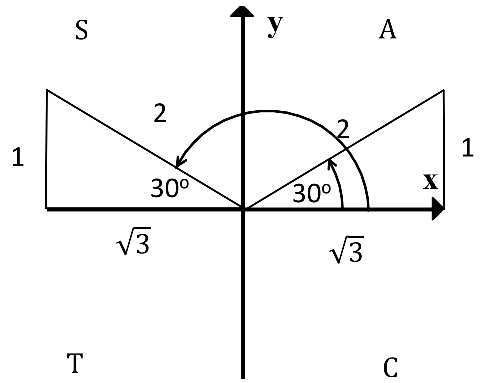
$sin2\theta = \frac{1}{2}$

Solve for θ $0^o \le \theta < 360^o$, and the general solution.

$\sin m = \frac{1}{2}$

$\boxed{Let\ m = 2\theta}$

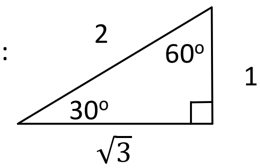
Draw two $\Delta's$ where $\sin m$ is +ve: ASTC Quadrant I, II



Label the triangles according to SOH CAH TOA

Label the reference angle according to special $\Delta's$.

Draw an arrow from the principal axis:
To the first and second terminal arm



Solve for the arrows m_{stp}

Check your answer: $sin2\theta = \frac{1}{2}$

$sinm = \frac{1}{2}$

$\sin(2(15)) = \frac{1}{2}$ ✓ $\sin(2(75)) = \frac{1}{2}$ ✓

$m_{stp} = 30^o$

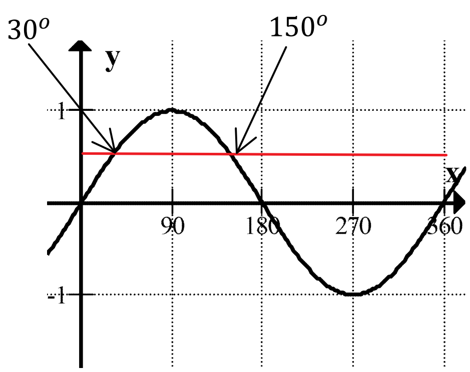
$m_{stp} = 180^o - 30^o$
$= 150^o$

$m = 30^o$
$2\theta = 30^o$
$\frac{2\theta}{2} = \frac{30^o}{2}$
$\theta = 15^o$

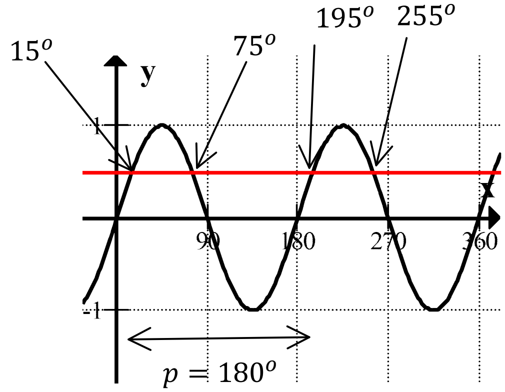
$m = 150^o$
$2\theta = 150^o$
$\frac{2\theta}{2} = \frac{150^o}{2}$
$\theta = 75^o$

$\boxed{\text{Substitute } 2\theta \text{ back in for } m.}$

$y = sin\theta$



$y = sin2\theta$



$HC = \frac{1}{2}$

$p = \frac{360^0}{b}$

$p = \frac{360^o}{2}$

$= 180^o$

$\theta = \theta_{stp} \pm p$
$\theta = 15^o + 180^o$
$\theta = 195^o$

$\cancel{\theta = \theta_{stp} \pm p}$
$\cancel{\theta = 195^o + 180^o}$
$\cancel{\theta = 375^o}$

$\theta = \theta_{stp} \pm p$
$\theta = 75^o + 180^o$
$\theta = 255^o$

$0 \le \theta \le 360^o$
$\theta = 15^o, 75^o, 195^o, 225^o$

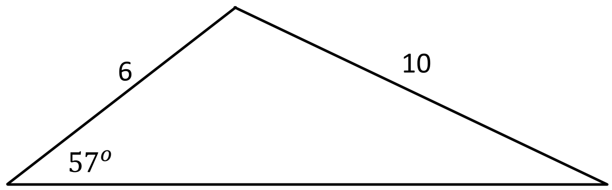
$\sin(2(195)) = \frac{1}{2}$ ✓

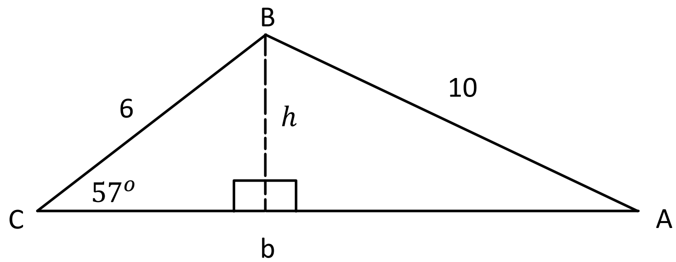
$\sin(2(225)) = \frac{1}{2}$ ✓

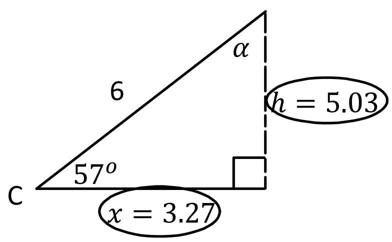
$General\ Solution: \theta_{gen} = \theta_{stp} \pm pn, n \in I$ $\theta_{gen} = 15^o \pm 180^o\ n, n \in I$	$\theta_{gen} = \theta_{stp} \pm pn, n \in I$ $\theta_{gen} = 75^o \pm 180^o n, n \in I$

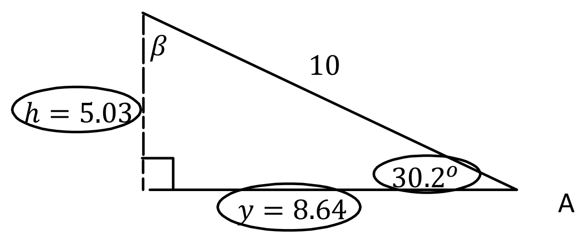
C11 - 2.9 - Solve ASS Triangle Without Sine Law Notes

Solve the triangle with side lengths of 6 m and 10 m, and an angle of 57^o.









$$sin\theta = \frac{O}{H}$$
$$sin57^o = \frac{h}{6}$$
$$6 \times sin57^o = \frac{h}{\cancel{6}} \cancel{\times 6}$$
$$6sin57^o = h$$
$$5.03 = h$$
$$h = 5.03$$

$$cos\theta = \frac{A}{H}$$
$$cos57^o = \frac{x}{6}$$
$$6 \times cos57^o = \frac{x}{\cancel{6}} \cancel{\times 6}$$
$$6cos57^o = x$$
$$3.27 = x$$
$$x = 3.27$$

$$\alpha = 180^o - (57^o + 90^o)$$
$$\alpha = 180^o - 147^o$$
$$\alpha = 33^o$$

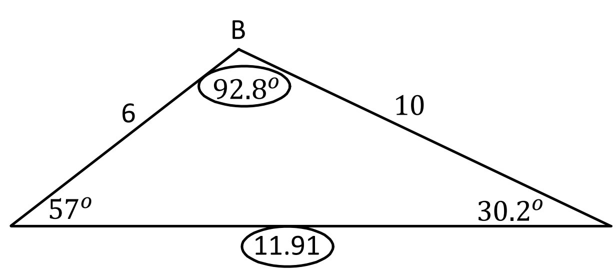
$$sin\theta = \frac{O}{H}$$
$$sin\theta = \frac{5.03}{10}$$
$$sin\theta = 0.503$$
$$\theta = \sin^{-1} 0.503$$
$$\theta = 30.2^o$$

$$cos\theta = \frac{A}{H}$$
$$\cos 30.2^o = \frac{y}{10}$$
$$0.864 = \frac{y}{10}$$
$$10 \times 0.864 = \frac{y}{\cancel{10}} \cancel{\times 10}$$
$$8.64 = y$$
$$y = 8.64$$

$$\beta = 180^o - (30.2^o + 90^o)$$
$$\beta = 180^o - 120.2^o$$
$$\beta = 59.8^o$$

$$B = \alpha + \beta$$
$$= 33^o + 59.8^o$$
$$= 92.8^o$$

$$b = x + y$$
$$b = 3.27 + 8.64$$
$$b = 11.91$$



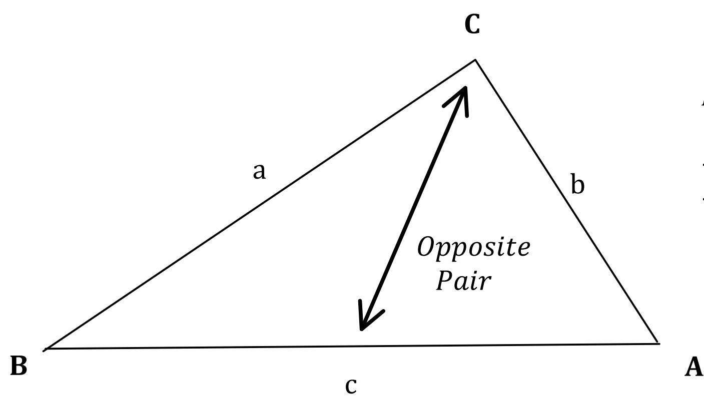
Or: 180 Minus

C11 - 2.9 - Sine Law Notes

Sine Law: $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$ **OR** $\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$

$(to\ find\ a\ side)$ $(to\ find\ an\ angle)$

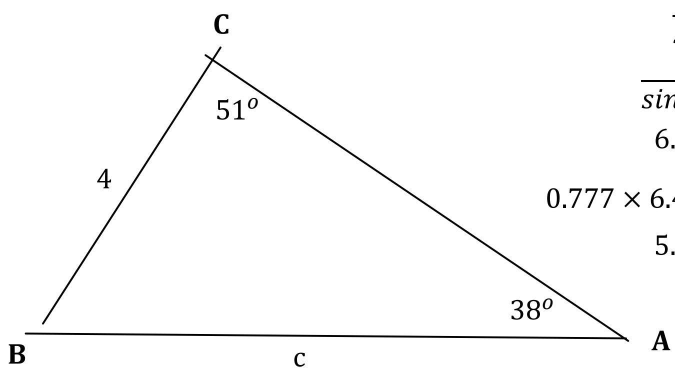
$What\ you\ are\ looking\ for\ goes\ on\ top\ but\ algebra\ allows\ you\ to\ do\ either$



$Notice$: $Use\ the\ Sine\ Law\ if\ you\ have$:

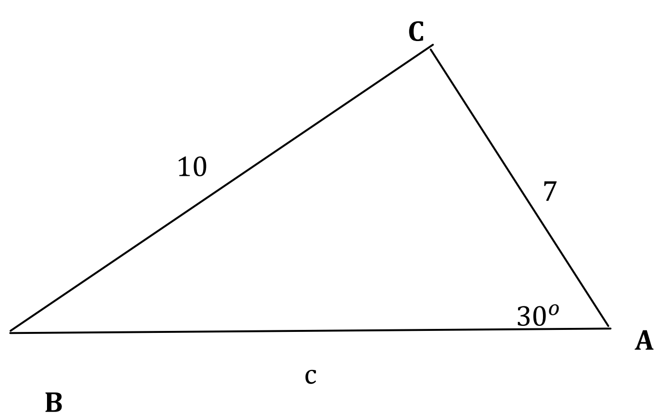
$-An\ opposite\ pair$
$-And\ one\ other\ piece\ of\ information$

Remember: We only sin angles.
$180^o\ in\ a\ triangle$



$$\frac{a}{sinA} = \frac{c}{sinC}$$
$$\frac{4}{sin38^o} = \frac{c}{sin51^o}$$
$$6.497 = \frac{c}{0.777}$$
$$0.777 \times 6.497 = \cancel{\frac{c}{0.777} \times 0.777}$$
$$5.048 = c$$
$$c = 5.048$$

$$\frac{a}{sinA} = \frac{c}{sinC}$$
$$\frac{asinC}{sinA} = c$$

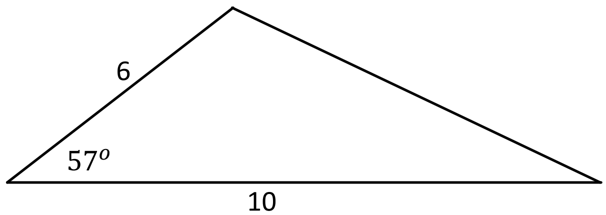


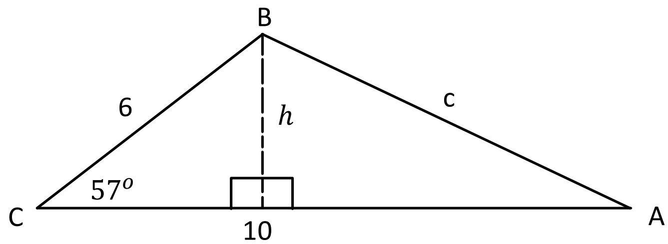
$$\frac{sinA}{a} = \frac{sinB}{b}$$
$$\frac{\sin(30)}{10} = \frac{sinB}{7}$$
$$0.05 = \frac{sinB}{7}$$
$$7 \times .05 = \cancel{\frac{sinB}{7} \times 7}$$
$$0.35 = \text{sinB}$$
$$\text{sinB} = 0.35$$
$$\text{B} = \sin^{-1}(0.35)$$
$$B = 20.5^o$$

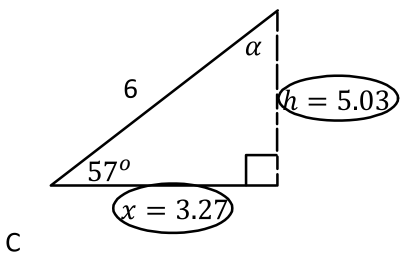
Remember: If you have 2 angles without either opposite side, use 180^o in a triangle.

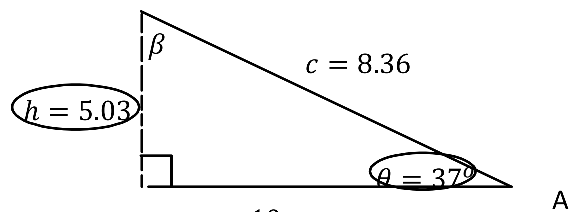
C11 - 2.10 - Solve SAS Triangle Without Cosine Law Notes

Solve the triangle with side lengths of 6 m and 10 m, and an angle between the two given sides of 57^o.









$10 - x =$
$10 - 3.27 = 6.73$

$$\begin{aligned} sin\theta &= \frac{O}{H} \\ sin57^o &= \frac{h}{6} \\ 6 \times sin57^o &= \frac{h}{\cancel{6}} \cancel{\times 6} \\ 6sin57^o &= h \\ 5.03 &= h \\ h &= 5.03 \end{aligned}$$

$$\begin{aligned} cos\theta &= \frac{A}{H} \\ cos57^o &= \frac{x}{6} \\ 6 \times cos57^o &= \frac{x}{\cancel{6}} \cancel{\times 6} \\ 6cos57^o &= x \\ 3.27 &= x \\ x &= 3.27 \end{aligned}$$

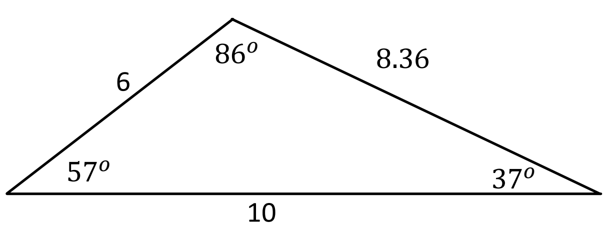
$$\begin{aligned} 57^o + 90^o + \alpha &= 180^o \\ 147^o + \alpha &= 180^o \\ -147^o \qquad &\quad - 147^o \\ \alpha &= 33^o \end{aligned}$$

$$\begin{aligned} tan\theta &= \frac{O}{A} \\ tan\theta &= \frac{5.03}{6.73} \\ tan\theta &= 0.7474 \\ \theta &= \tan^{-1}(0.7474) \\ \theta &= 36.77^o \\ \theta &= 37^o \end{aligned}$$

$$\begin{aligned} sin\theta &= \frac{O}{H} \\ sin37 &= \frac{5.03}{c} \\ c \times sin37 &= \frac{5.03}{\cancel{c}} \cancel{\times c} \\ csin37 &= 5.03 \\ \frac{csin\cancel{37}}{\cancel{sin}37} &= \frac{5.03}{sin37} \\ c &= \frac{5.03}{sin37} \\ c &= 8.36 \end{aligned}$$

$$\begin{aligned} 37^o + 90^o + \beta &= 180^o \\ 127^o + \beta &= 180^O \\ -127^o \qquad &\quad - 127^o \\ \beta &= 53^o \end{aligned}$$

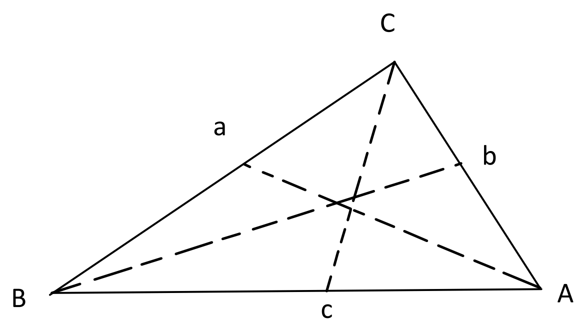
$$\begin{aligned} B &= \alpha + \beta \\ &= 33^o + 53^o \\ &= 86^o \end{aligned}$$



Remember: Find the smallest angle first, and/or 180 minus

C11 - 2.10 - Cosine Law Notes

Cosine Law



Cosine Law:

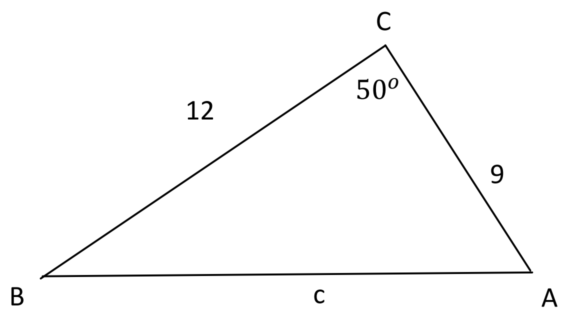
$$c^2 = b^2 + a^2 - 2abcosC$$

Notice: This pattern should occur.

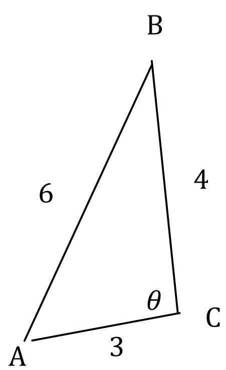
Cosine Law: SSS (hard) and SAS (easy)

Remember: Only one angle in the formula

Remember: We only cos angles.



$$c^2 = b^2 + a^2 - 2abCosC$$
$$c^2 = 9^2 + 12^2 - 2(12)(9)cos50$$
$$c^2 = 86.2$$
$$\sqrt{c^2} = \sqrt{86.2}$$ Plug into calculator
$$c = 9.3$$ Square root both sides



$$c^2 = b^2 + a^2 - 2abcosC$$

$$6^2 = 3^2 + 4^2 - 2(4)(3)cosC$$ Substitute values in

$$36 = 9 + 16 - 24cosC$$ Calculate the squares, multiply

$$36 = 25 - 24cosC$$ Add

$$36 = 25 - 24cosC$$
$$-25 \quad -25$$ Subtract from both sides

$$11 = -24cosC$$
$$\frac{11}{-24} = \frac{-24cosC}{-24}$$ Divide both sides
$$-\frac{11}{24} = cosC$$
$$cosC = -\frac{11}{24}$$
$$C = \cos^{-1}\left(-\frac{11}{24}\right)$$ Inverse cos
$$C = 117.3^o$$

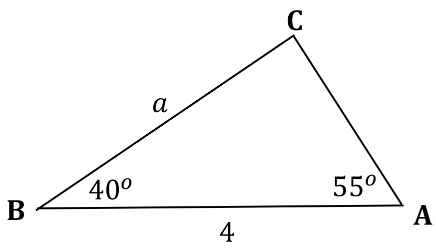
$$\boxed{C = \cos^{-1}\left(\frac{(a^2+b^2-c^2)}{(2ab)}\right)}$$

$$c^2 = b^2 + a^2 - 2abcosC$$
$$b^2 = c^2 + a^2 - 2cacosB$$
$$a^2 = b^2 + c^2 - 2cbcosA$$

See edit below

C11 - 2.11 - Sine/Cosine Law Notes Solve the Triangle

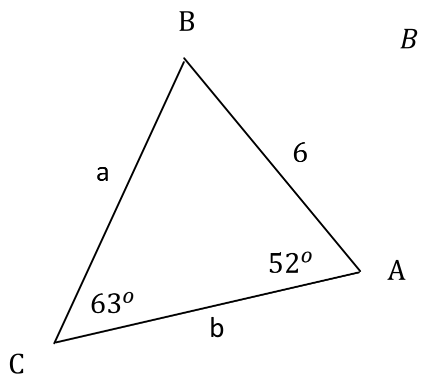
Solve for a.



$$C = 180^o - 40^o - 55^o$$
$$= 85^o$$

$$\frac{a}{sinA} = \frac{c}{sinC}$$
$$\frac{a}{sin55^o} = \frac{4}{sin85^o}$$
$$\frac{a}{0.819} = 4.015$$
$$\cancel{0.819} \times \frac{a}{\cancel{0.819}} = 4.015 \times 0.819$$
$$a = 3.289$$

Solve the triangle.

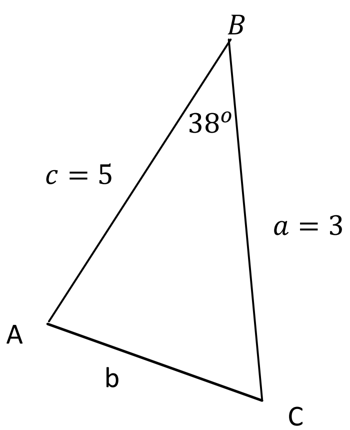


$$B = 180^o - 63^o - 52^o$$
$$= 65^o$$

$$\frac{a}{sinA} = \frac{c}{sinC}$$
$$\frac{a}{sin52^o} = \frac{6}{sin63^o}$$
$$\frac{a}{0.788} = 6.734$$
$$\cancel{0.788} \times \frac{a}{\cancel{0.788}} = 6.734 \times 0.788$$
$$a = 6.734 \times 0.788$$
$$a = 5.306$$

$$\frac{b}{sinB} = \frac{c}{sinC}$$
$$\frac{b}{sin65^o} = \frac{6}{sin63^o}$$
$$\frac{b}{0.906} = 6.734$$
$$\cancel{0.906} \times \frac{b}{\cancel{0.906}} = 6.734 \times 0.906$$
$$b = 6.101$$

Solve the triangle *Find the angle opposite of the smaller side 1st.



Cosine Law: Switched b and c

$$c^2 = a^2 + b^2 - 2abcosC$$
$$b^2 = a^2 + c^2 - 2ac \cdot cosB$$
$$b^2 = 3^2 + 5^2 - 2(3)(5) \cdot \cos(38^o)$$
$$b^2 = 9 + 25 - 30\cos(38^o)$$
$$b^2 = 34 - 23.64$$
$$b^2 = 10.36$$
$$\sqrt{b^2} = \sqrt{10.36}$$
$$b = 3.22$$

Sine Law:

$$\frac{sinA}{a} = \frac{sinB}{b}$$
$$\frac{sinA}{3} = \frac{sin38^o}{3.22}$$
$$\frac{sinA}{3} = 0.19$$
$$3 \times \frac{sinA}{3} = 0.19 \times 3$$
$$sinA = 0.57$$
$$A = 35^o$$

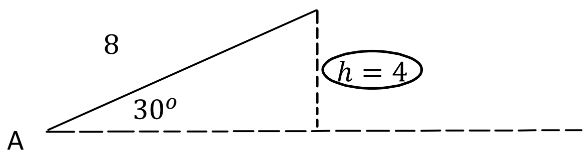
180^o in a triangle:

$$C = 180^o - 38^o - 35^o$$
$$= 107^o$$

C11 - 2.12 - Ambiguous Case of Sine (ASS) Notes

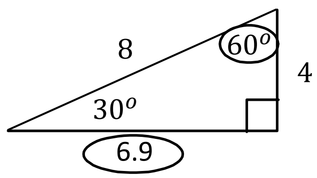
How many triangles? Solve the triangles. **Remember: Always find the height first.**

$\angle A = 30^o, b = 8, a = 4$



$$\sin\theta = \frac{O}{H}$$
$$\sin 30^o = \frac{h}{8}$$
$$8\sin 30^o = h$$
$$4 = h$$
$$h = 4$$

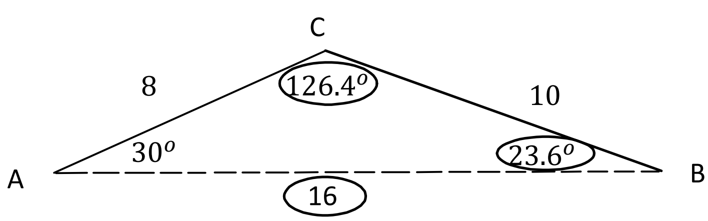
$$cos\theta = \frac{A}{H}$$
$$cos30^o = \frac{A}{8}$$
$$8\cos 30^o = A$$
$$6.9 = A$$
$$A = 6.9$$



$a = h$
One triangle

$$\theta = 180^o - 30^o - 90^o$$
$$\theta = 60^o$$

$\angle A = 30^o, b = 8, a = 10$



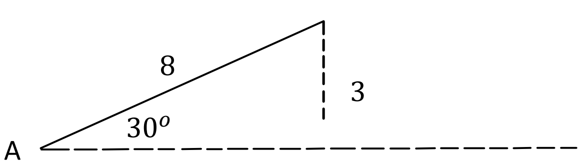
$10 > 8$
$a > b$
One triangle

$$\frac{sinB}{b} = \frac{sinA}{a}$$
$$\frac{sinB}{8} = \frac{\sin 30^O}{10}$$
$$\frac{sinB}{8} = 0.05$$
$$8 \times \frac{sinB}{8} = 0.05 \times 8$$
$$sinB = 0.4$$
$$B = \sin^{-1} 0.4$$
$$B = 23.6^o$$

$$\theta = 180^o - 23.6^o - 30^o$$
$$\theta = 126.4^o$$

$$\frac{c}{sinC} = \frac{a}{sinA}$$
$$\frac{c}{\sin 126.4^o} = \frac{10}{\sin 30^o}$$
$$\frac{c}{0.8} = 20$$
$$0.8 \times \frac{c}{0.8} = 20 \times 0.8$$
$$c = 16$$

$\angle A = 30^o, b = 8, a = 3$



$3 < 4$
$a < H$
no triangle

No triangle, can't solve.

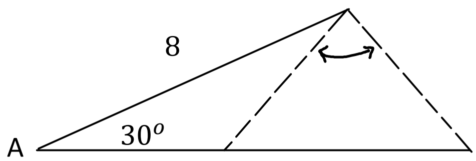
C11 - 2.12 - Ambiguous Case of Sine (ASS) Notes

How many triangles? Solve the triangles.

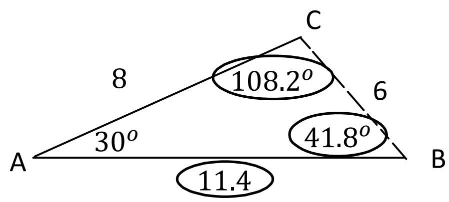
Remember: Always find the height first.

$\angle A = 30^o, b = 8, a = 6$

$4 < 6 < 8$
$H < a < B$
$\underline{Two\ triangles}$



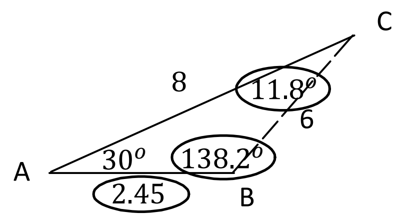
Draw both triangles together and separately.



$$\frac{\sin 30^o}{6} = \frac{\sin B}{8}$$
$$0.08\bar{3} = \frac{sinB}{8}$$
$$8 \times 0.08\bar{3} = \frac{sinB}{8} \times 8$$
$$0.\bar{6} = sinB$$
$$sinB = 0.\bar{6}$$
$$B = \sin^{-1} 0.\bar{6}$$
$$B = 41.8^o$$

$$\theta = 180^o - 30^o - 41.8^o$$
$$\theta = 108.2^o$$

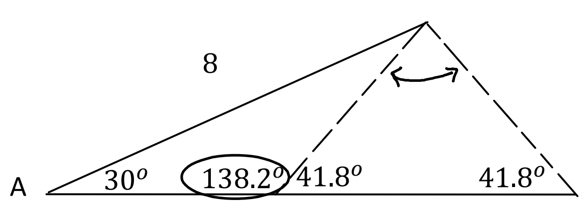
$$\frac{c}{sinC} = \frac{a}{sinA}$$
$$\frac{c}{\sin 108.2^o} = \frac{6}{\sin 30^o}$$
$$\frac{c}{0.95} = 12$$
$$0.95 \times \frac{c}{0.95} = 12 \times 0.95$$
$$c = 11.4$$



$$\theta = 180^o - 41.8^o$$
$$\theta = 138.2^o$$

$$\theta = 180^o - 30^o - 138.2^o$$
$$\theta = 11.8^o$$

$$\frac{c}{sinC} = \frac{a}{sinA}$$
$$\frac{c}{\sin 11.8^o} = \frac{6}{\sin 30^o}$$
$$\frac{c}{0.204} = 12$$
$$0.204 \times \frac{c}{0.204} = 12 \times 0.204$$
$$c = 2.45$$



Notice: Both triangles have an angle of 30^o, a side going up of 8, and a side opposite to 30^o of 6.

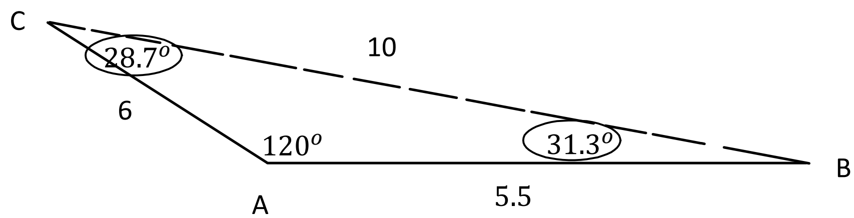
Notice: The isosceles triangle.

C11 - 2.12 - Ambiguous Case of Sine (ASS) Notes

How many triangles? Solve the triangles.

$10 > 6$
$a > b$
One triangle

$\angle A = 120^o, b = 6, a = 10$



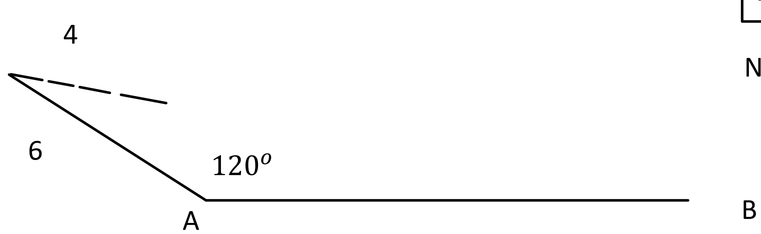
$$\frac{sinB}{b} = \frac{sinA}{a}$$
$$\frac{sinB}{6} = \frac{\sin 120^o}{10}$$
$$\frac{sinB}{6} = 0.0866$$
$$6 \times \frac{sinB}{6} = 0.0866 \times 6$$
$$sinB = 0.52$$
$$B = \sin^{-1} 0.52$$
$$B = 31.3^o$$

$$\theta = 180^o - 31.3^o - 120^o$$
$$\theta = 28.7^o$$

$$\frac{c}{sinC} = \frac{a}{sinA}$$
$$\frac{c}{\sin 28.7^o} = \frac{10}{\sin 120^o}$$
$$\frac{c}{0.48} = 11.55$$
$$0.48 \times \frac{c}{0.48} = 11.55 \times 0.48$$
$$c = 5.5$$

$\angle A = 120^o, b = 6, a = 4$

$4 < 6$
$a < b$
No triangle



No triangle. Can't solve.

C11 - 3.1 - Quadratics Graphing x^2 TOV Notes

Graphing: $y = x^2$

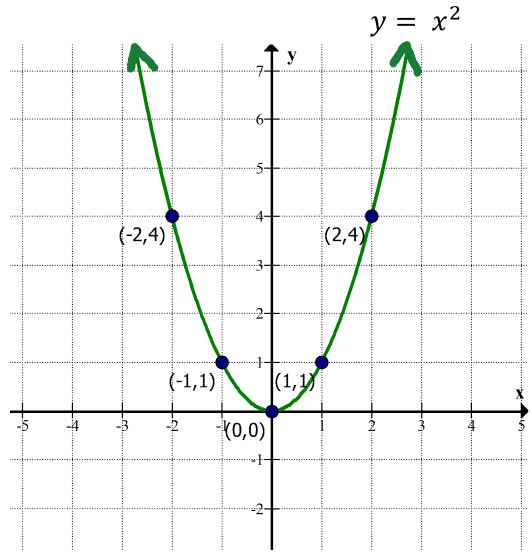


Table of Values

	x	y	Pt.
	-2	4	(-2,4)
	-1	1	(-1,1)
Vertex:	0	0	(0,0)
	1	1	(1,1)
	2	4	(2,4)

$y = x^2$
$y = (-2)^2$
$y = 4$

$y = x^2$
$y = (-1)^2$
$y = 1$

$y = x^2$
$y = (0)^2$
$y = 0$

$y = x^2$
$y = (1)^2$
$y = 1$

$y = x^2$
$y = (2)^2$
$y = 4$

Notice: the pattern from the vertex (0,0) is **symmetrical** on both sides.

Over 1, 1 squared = 1, up 1. Back to the vertex. Over 2, 2 squared = 4, up 4.

$V: (0,0)$

$AOS: x = 0$ $\quad$ $R: y \geq 0$ $\quad$ Domain xER

Range

C11 - 3.1 - Quadratic Vertical Translation Notes $y = x^2 + q$

Graphing: $y = x^2 + c$

$y = x^2 + 1$

$y = x^2 + 1$

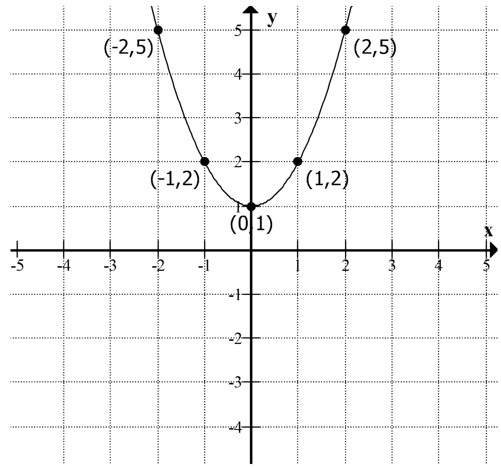


Table of Values

x	y
-2	5
-1	2
0	1
1	2
2	5

Pt.
(-2,5)
(-1,2)
(0,1)
(1,2)
(2,5)

$y = x^2 + 1$
$y = (-2)^2 + 1$
$y = 4 + 1$
$y = 5$

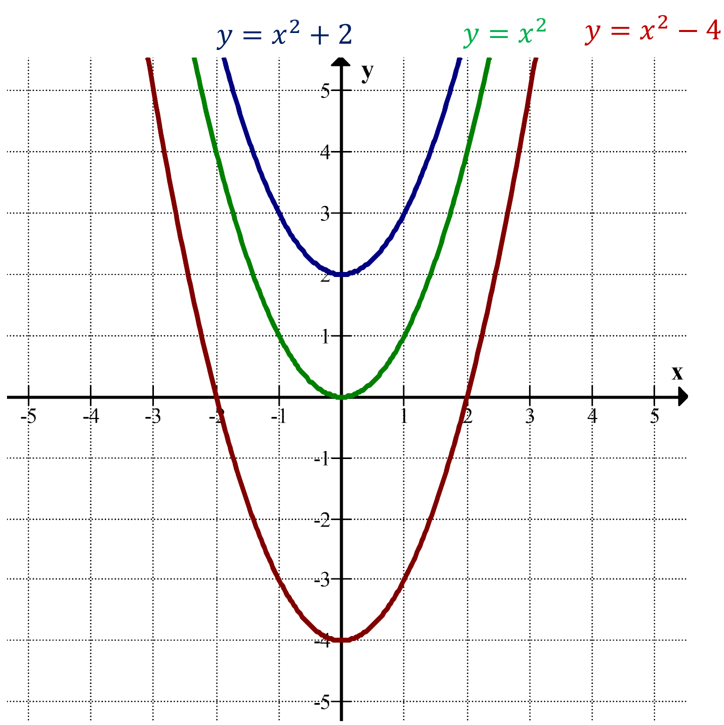
$y = x^2 + 1$
$y = (-1)^2 + 1$
$y = 1 + 1$
$y = 2$

$y = x^2 + 1$
$y = (0)^2 + 1$
$y = 0 + 1$
$y = 5$

$y = x^2 + 1$
$y = (1)^2 + 1$
$y = 1 + 1$
$y = 2$

$y = x^2 + 1$
$y = (2)^2 + 1$
$y = 4 + 1$
$y = 5$

Notice: the graph of $y = x^2 + 1$ is the graph $y = x^2$ shifted up 1. "c" is the y intercept. "c" is only the vertex if there is no "b".



C11 - 3.1 - Quadratics Horizontal Translation Notes $(x-p)^2$

Graphing: $y = (x-p)^2$

$y = (x-2)^2$

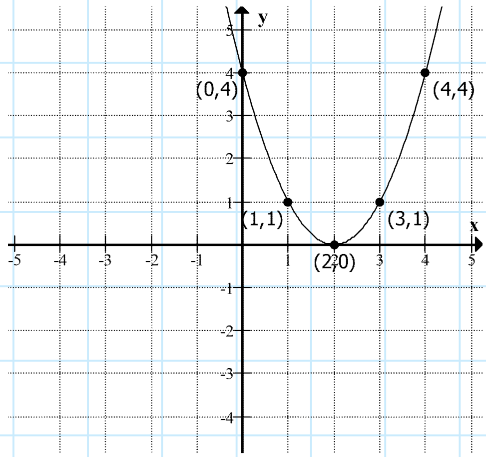


Table of Values

x	y
0	4
1	1
2	0
3	1
4	4

Pt.
(0,4)
(1,1)
(2,0)
(3,1)
(4,4)

$y = (x-2)^2$
$y = ((0)-2)^2$
$y = (0-2)^2$
$y = (-2)^2$
$y = 4$

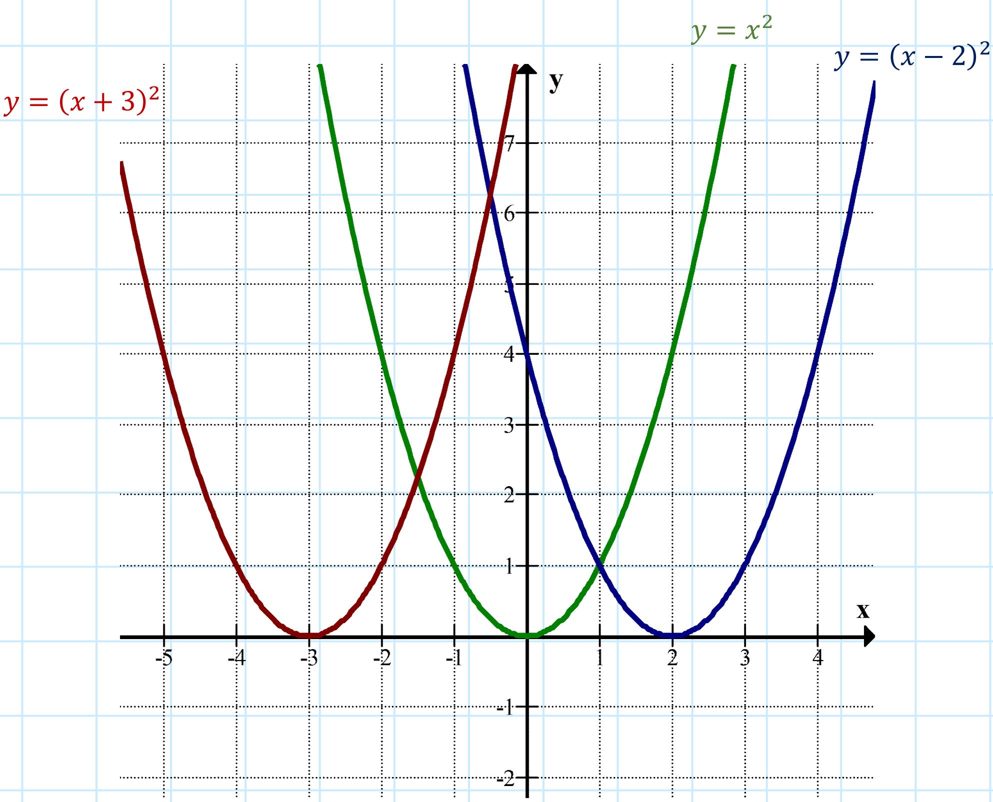
$y = (x-2)^2$
$y = ((1)-2)^2$
$y = (1-2)^2$
$y = (-1)^2$
$y = 1$

$y = (x-2)^2$
$y = ((2)-2)^2$
$y = (2-2)^2$
$y = (0)^2$
$y = 0$

$y = (x-2)^2$
$y = ((3)-2)^2$
$y = (3-2)^2$
$y = (-1)^2$
$y = 1$

$y = (x-2)^2$
$y = ((4)-2)^2$
$y = (4-2)^2$
$y = (2)^2$
$y = 4$

Notice: the graph of $y = (x-p)^2$ is the graph $y = x^2$ shifted right 2.
Notice we shift the opposite of "p".



C11 - 3.1 - Quadratics Reflection Notes $-x^2$

Graphing: $y = -x^2$

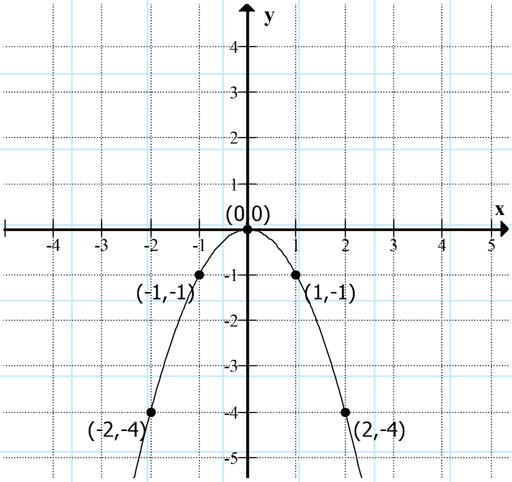
$y = -x^2$

Table of Values

x	y
-2	-4
-1	-1
0	0
1	-1
2	-4

Pt.
(-2,-4)
(-1,-1)
(0,0)
(1,-1)
(2,-4)

$y = -x^2$



$y = -x^2$
$y = -(-2)^2$
$y = -4$

$y = -x^2$
$y = -(-1)^2$
$y = -1$

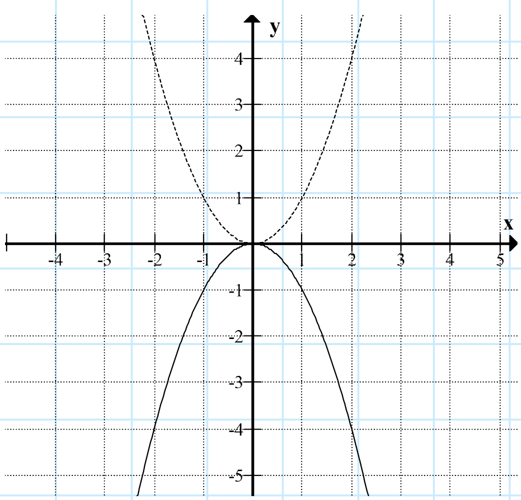
$y = -x^2$
$y = -(0)^2$
$y = -4$

$y = -x^2$
$y = -(1)^2$
$y = -1$

$y = -x^2$
$y = -(2)^2$
$y = -4$

Notice: The graph of $y = -x^2$ is the graph of $y = x^2$ opening downwards.
Over 1, 1 squared = 1, down 1. Back to the vertex. Over 2, 2 squared = 4, down 4.

$y = x^2$



$y = -x^2$

C11 - 3.2 - Quadratics Vertical Exp Notes $(2x^2, \frac{1}{2}x^2)$

Graphing: $y = ax^2$

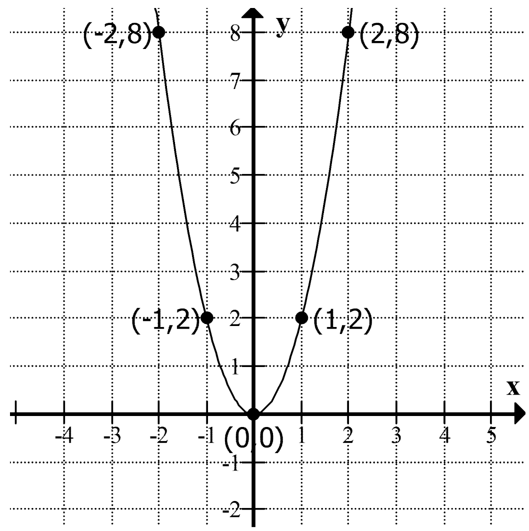
$\boldsymbol{y = 2x^2}$

Table of Values

x	y
-2	8
-1	2
0	0
1	2
2	8

Pt.
$(-2,8)$
$(-1,2)$
$(0,0)$
$(1,2)$
$(2,8)$

$y = 2x^2$



$y = 2x^2$
$y = 2(-2)^2$
$y = 2(4)$
$y = 8$

$y = 2x^2$
$y = 2(-1)^2$
$y = 2(1)$
$y = 2$

$y = 2x^2$
$y = 2(0)^2$
$y = 2(0)$
$y = 0$

$y = 2x^2$
$y = 2(1)^2$
$y = 2(1)$
$y = 2$

$y = 2x^2$
$y = 2(2)^2$
$y = 2(4)$
$y = 8$

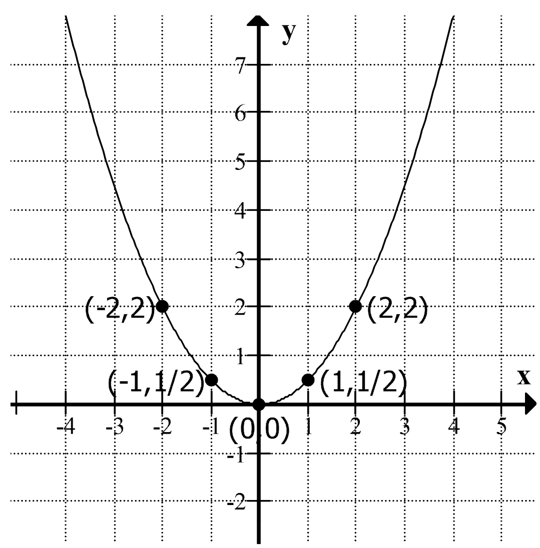
Notice: the pattern from the vertex (0,0) is symmetrical on both sides.
Over 1, 1 squared = 1, 1 times 2 = 2, up 2. Back to the vertex. Over 2, 2 squared = 4, 4 times 2 = 8, up 8.
In the last two steps, we are multiplying by 2 because $a = 2$.

$\boldsymbol{y = \frac{1}{2}x^2}$

Table of Values

x	y
-2	2
-1	$\frac{1}{2}$
0	0
1	$\frac{1}{2}$
2	2

Pt.
$(-2,2)$
$(-1,\frac{1}{2})$
$(0,0)$
$(1,\frac{1}{2})$
$(2,2)$



$y = \frac{1}{2}x^2$
$y = \frac{1}{2}(-2)^2$
$y = \frac{1}{2}(4)$
$y = 2$

$y = \frac{1}{2}x^2$
$y = \frac{1}{2}(-1)^2$
$y = \frac{1}{2}(1)$
$y = \frac{1}{2}$

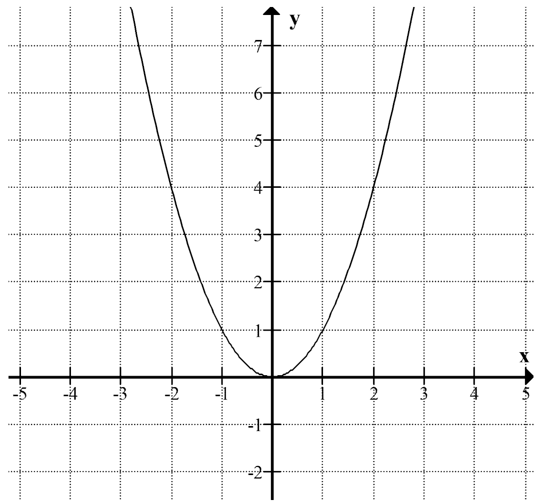
$y = \frac{1}{2}x^2$
$y = \frac{1}{2}(0)^2$
$y = \frac{1}{2}(0)$
$y = 0$

$y = \frac{1}{2}x^2$
$y = \frac{1}{2}(1)^2$
$y = \frac{1}{2}(1)$
$y = \frac{1}{2}$

$y = \frac{1}{2}x^2$
$y = \frac{1}{2}(2)^2$
$y = \frac{1}{2}(4)$
$y = 2$

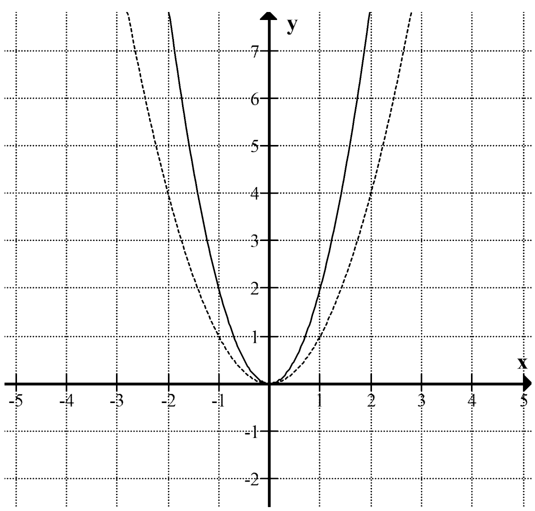
C11 - 3.2 - Quadratics Compression/Expansion Summary

$y = x^2$



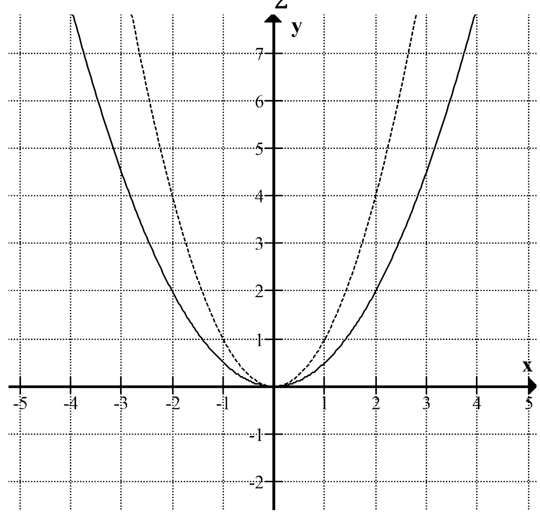
Expand

$y = 2x^2$

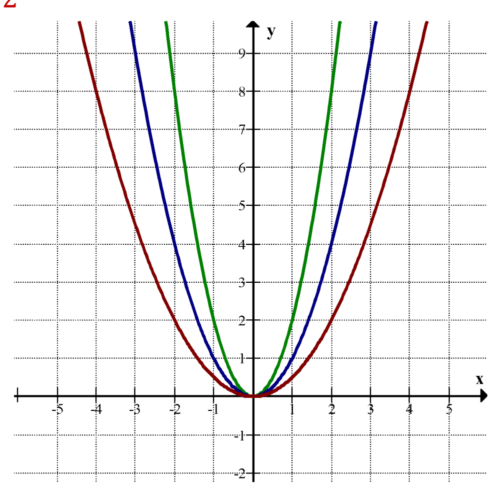


Compress

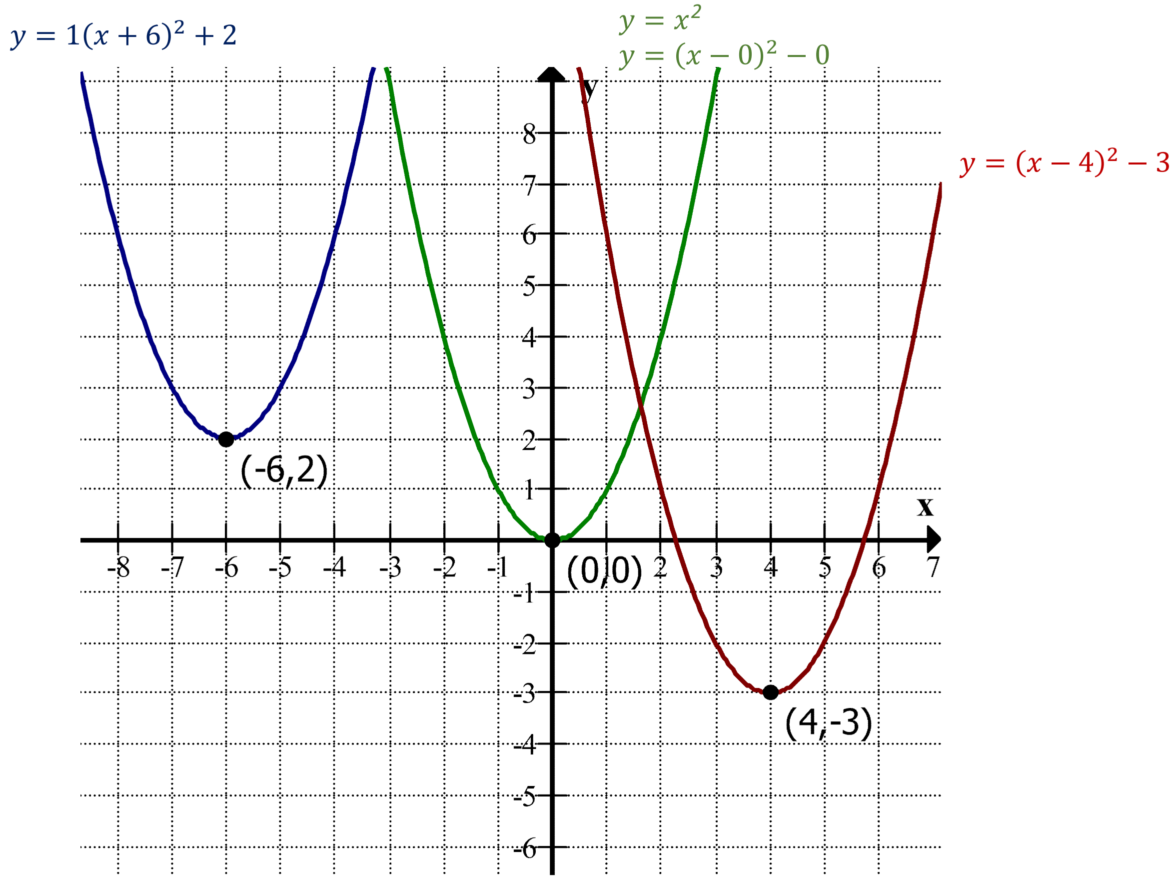
$y = \frac{1}{2}x^2$

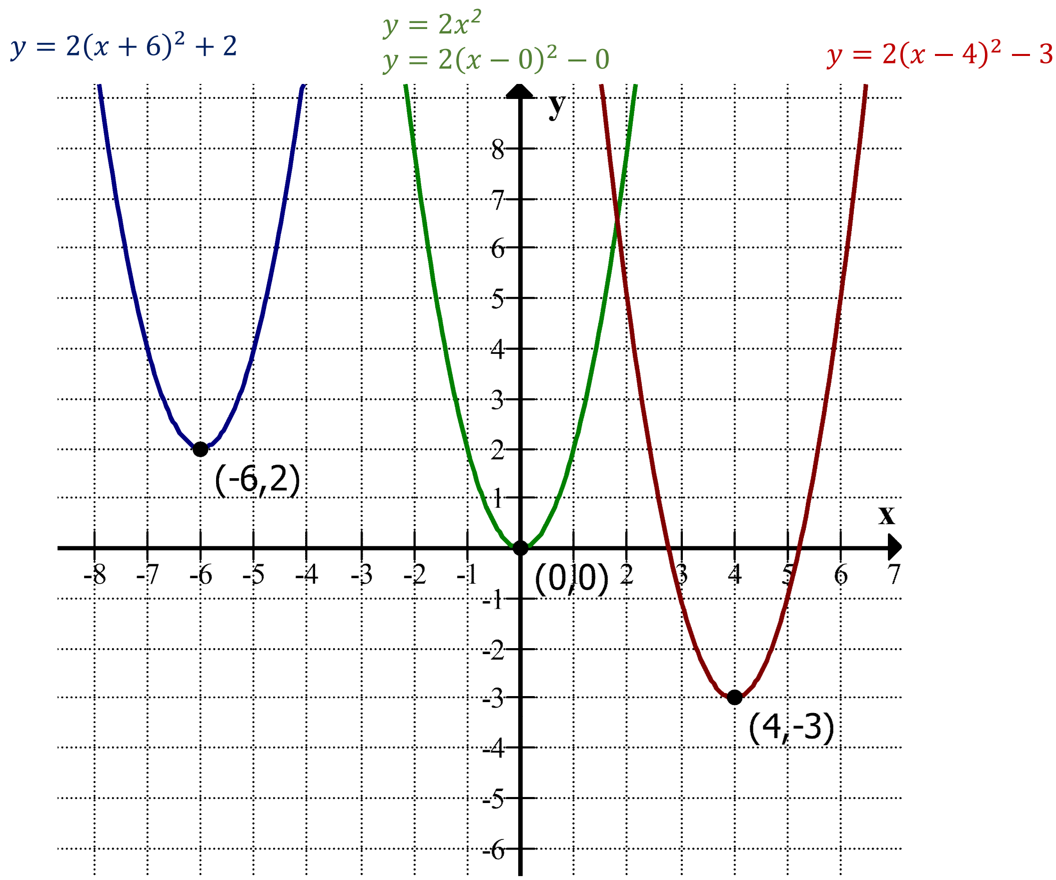


$y = \frac{1}{2}x^2$ $y = 2x^2$ $y = x^2$



C11 - 3.2 - Quadratics Vertical/Horizontal Combo Notes





C11 - 3.3 - Completing the Square Notes

$Standard\ form \longrightarrow Vertex\ form$

$y = ax^2 + bx + c \longrightarrow y = a(x-p)^2 + q \qquad Vertex = (p, q)$

$y = x^2 + 6x + c$

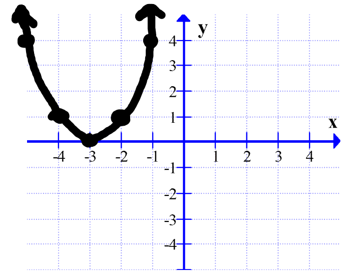
$\boxed{\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = (3)^2 = 9}$ "b" divided by 2 all squared:

$y = x^2 + 6x + 9$

Factor

$y = (x+3)(x+3)$
$y = (x+3)^2$

Vertex form: Vertex = $(-3, 0)$



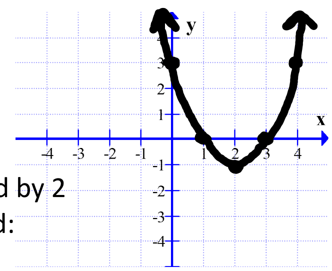
---

a = 1

$y = x^2 - 4x + 3$
$y = (x^2 - 4x) + 3$

Group x terms

$\boxed{\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4}$ "b" divided by 2 all squared:



$y = (x^2 - 4x + 4 - 4) + 3$ Add and subtract inside brackets

$y = (x^2 - 4x + 4) - 4 + 3$ Remove number not contributing to perfect square (-ve)

$y = (x-2)(x-2) - 1$ Factor brackets, simplify outside

$y = (x-2)^2 - 1$ Vertex form: Vertex = $(2, -1)$

$a \neq 1$

$y = 2x^2 - 8x + 3$
$y = (2x^2 - 8x) + 3$

Group x terms

$y = 2(x^2 - 4x) + 3$

Factor out coefficient of x^2

$\boxed{\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4}$ New "x" coefficient divided by 2 all squared:

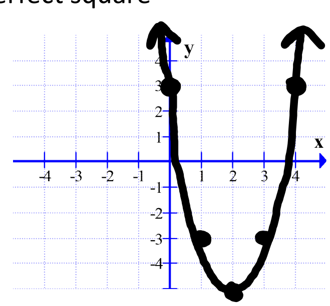
x2

$y = 2(x^2 - 4x + 4 - 4) + 3$ Add and subtract inside brackets

$y = 2(x^2 - 4x + 4) - 8 + 3$ Remove number not contributing to perfect square
Don't forget to multiply by "a"

$y = 2(x-2)(x-2) - 5$ Factor brackets, simplify outside

$y = 2(x-2)^2 - 5$ Vertex form: Vertex = $(2, -5)$



Remember: $\frac{b}{2a}$ or $\frac{\text{"new } b\text{"}}{2}$ is the number that goes inside the brackets with x

C11 - 3.4 - Find Vertex Form Vertex Point Notes

Using the vertex and a point on the parabola, find the equation in Vertex Form.

$\boldsymbol{Vertex}: (\mathbf{-1}, \mathbf{-4})$ $\boldsymbol{and}$ $\boldsymbol{Point}: (\mathbf{2}, \mathbf{-3})$

$\boxed{y = a(x-p)^2 + q}$

$y = a(x-p)^2 + q$ — Write Vertex Form

$y = a\big(x-(-1)\big)^2 - 4$ — Substitute Vertex for (p, q)

$y = a(x+1)^2 - 4$ — $(\mathbf{-1}, \mathbf{-4})$

$-3 = a(-2+1)^2 - 4$ — Substitute (x, y)

$-3 = a(1)^2 - 4$ — $(\mathbf{-2}, \mathbf{-3})$

$-3 = 1a - 4$

$+4 \qquad +4$

$1 = 1a$

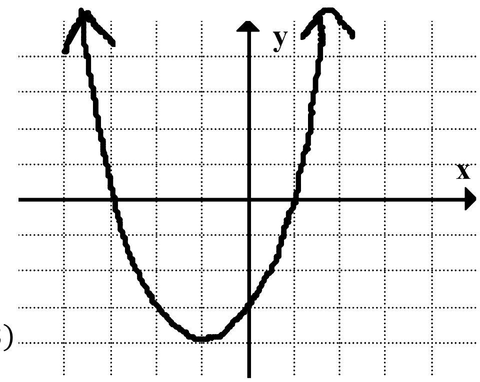
$\frac{1}{1} = \frac{1a}{1}$

$1 = a$

$a = 1$ — Solve for a.

$y = 1(x+1)^2 - 4$ — Substitute 'a' and Vertex into Vertex Form

Draw on Graph



(x, y)
$(\mathbf{-2}, \mathbf{-3})$

(p, q)
$(\mathbf{-1}, \mathbf{-4})$

$\boldsymbol{Vertex}: (\mathbf{3}, \mathbf{-2})$ $\boldsymbol{and}$ $\boldsymbol{x - intercept = 4}$ $(4,0)$

$y = a(x-p)^2 + q$

$y = a\big(x-(3)\big)^2 - 2$

$y = a(x-3)^2 - 2$

$0 = a(4-3)^2 - 2$

$0 = a(1)^2 - 2$

$0 = 1a - 2$

$+2 \qquad +2$

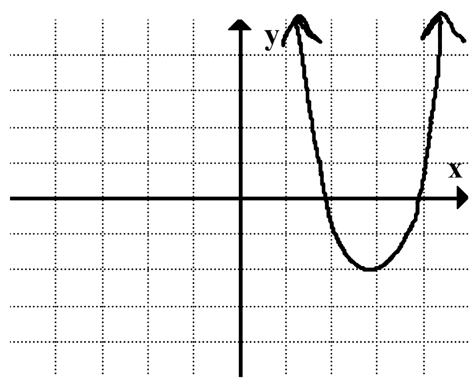
$2 = a$

$a = 2$

$y = 2(x-3)^2 - 2$

Draw on Graph

Check on Graphing Calculator Table of Values



(x, y)
$(\mathbf{0}, \mathbf{0})$

(p, q)
$(\mathbf{3}, \mathbf{-2})$

C11 - 3.5 - Vertex:$\left(-\frac{b}{2a}, y\right)$ Quadratics in Standard Form Notes

$y = x^2 - 6x + 5$

$$Vertex = \left(\frac{-b}{2a}, y\right)$$
$$Vertex = \left(\frac{-(-6)}{2(1)}, y\right)$$
$$Vertex = \left(\frac{6}{2}, y\right)$$
$$Vertex = (3, y)$$

$$\boxed{Vertex = \left(\frac{-b}{2a}, y\right)}$$

$$\boxed{\left(\frac{-b}{2a}, c - \frac{b^2}{4a}\right)}$$

$y = x^2 - 6x + 5$
$y = (3)^2 - 6(3) + 5$
$y = 9 - 18 + 5$
$y = -4$

Substitute 3 in for x and solve for y

$Vertex = (3, -4)$

$y = x^2 - 6x + 5$

$Vertex = (3, -4)$

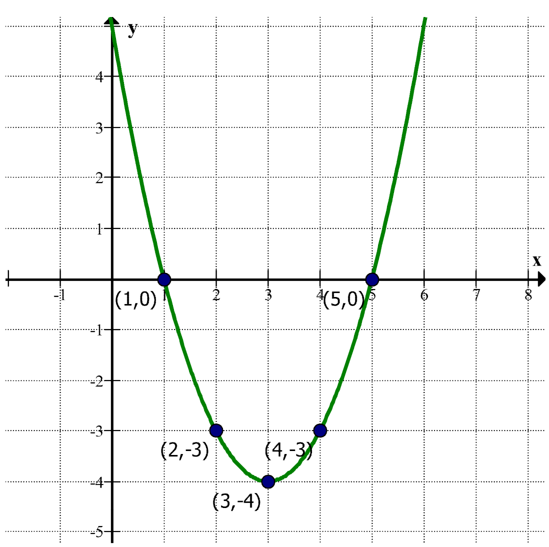
	x	y
	1	0
	2	-3
Vertex:	3	-4
	4	-3
	5	0

$y = x^2 - 6x + 5$
$y = (1)^2 - 6(1) + 5$
$y = 1 - 6 + 5$
$y = 0$

$y = x^2 - 6x + 5$
$y = (2)^2 - 6(2) + 5$
$y = 4 - 12 + 5$
$y = -3$

$y = x^2 - 6x + 5$
$y = (4)^2 - 6(4) + 5$
$y = 16 - 24 + 5$
$y = -3$

$y = x^2 - 6x + 5$
$y = (5)^2 - 6(5) + 5$
$y = 25 - 30 + 5$
$y = 0$



AOS: Average Two Horizontal Points ($x - int's$)

$$x = \frac{1+5}{2}$$
$$x = 3$$

C11 - 3.6 - Product of Numbers is a Min Notes

The difference between two numbers is 10. Their product is a minimum.

Let $a = 1st\ \#$
Let $b = 2nd\ \#$

Let statements: get used to using variables other than x and y

1 $a - b = 10$

2 $a \times b = minimum$
$a \times b = \cancel{minimum}\ y$
$y = a \times b$

Equation 1, equation 2.
The minimum or maximum will be y.

$a - b = 10$
$+b \qquad + b$
$a = (10 + b)$

Equation #1
Isolate a variable

$y = a \times b$
$y = (10 + b) \times b$
$y = 10b + b^2$
$y = b^2 + 10b$

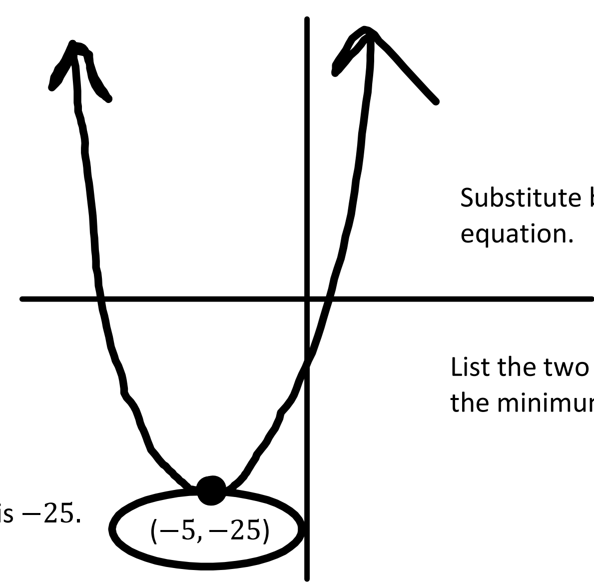
Equation #2
Substitute the isolated variable

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

$y = b^2 + 10b$
$y = (b^2 + 10b + 25 - 25)$
$y = (b^2 + 10b + 25) - 25$
$y = (b + 5)^2 - 25$

Vertex = $(-5, -25)$

b Minimum



$a = 10 + b$
$a = 10 - 5$
$a = 5$

Substitute b into the other equation.

$a = 5$
$b = -5$

List the two numbers and the minimum.

The minimum product is -25.

(x, y)
(b, min)

C11 - 3.6 - Product of Numbers is a Min Notes

Two numbers differ by 10. The product of the larger number and twice the smaller number is a minimum. What are the numbers?

Let $a = 1st\ \#$
Let $b = 2nd\ \#$

Let statements:

1 $a - b = 10$

2 $a \times 2b = minimum$
$a \times 2b = \cancel{minimum}\ y$
$y = a \times 2b$

Equation 1, equation 2.
The minimum or maximum will be y.

$a - b = 10$
$a = 10 + b$

Equation #1
Isolate a variable

$y = a \times 2b$
$y = (10 + b) \times 2b$
$y = 20b + 2b^2$
$y = 2b^2 + 20b$

Equation #2
Substitute the isolated variable

×2

$y = 2b^2 + 20b$
$y = 2(b^2 + 10b + 25 - 25)$
$y = 2(b^2 + 10b + 25) - 50$
$y = 2(b + 5)^2 - 50$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

Vertex = (-5, -50)

b Minimum

$a = 10 + b$
$a = 10 - 5$
$a = 5$

Substitute b into the other equation.

$a = 5$
$b = -5$

List the two numbers and the minimum.

The minimum product is -50.

C11 - 3.6 - Sum of Squares is a Min Notes

Two numbers sum to 8. The sum of their squares is a minimum.

Let $a = 1st\ \#$
Let $b = 2nd\ \#$

Let statements:

① $a + b = 8$

② $a^2 + b^2 = minimum$
$a^2 + b^2 = \cancel{minimum}\ y$
$y = a^2 + b^2$

Equation 1, equation 2.
The minimum or maximum will be y.

$$\begin{aligned} a + b &= 8 \\ -b \quad &\ \ -b \\ a &= 8 - b \\ a &= (8 - b) \end{aligned}$$

Equation #1
Isolate a variable

$$\begin{aligned} y &= a^2 + b^2 \\ y &= (8 - b)^2 + b^2 \\ y &= 64 - 16b + b^2 + b^2 \\ y &= 2b^2 - 16b + 64 \end{aligned}$$

Equation #2
Substitute the isolated variable

$$\begin{aligned} y &= 2b^2 - 16b + 64 \\ y &= 2(b^2 - 8b) + 64 \\ y &= 2(b^2 - 8b + 16 - 16) + 64 \\ y &= 2(b^2 - 8b + 16) + 64 - 32 \\ y &= 2(b - 4)^2 + 32 \end{aligned}$$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{8}{2}\right)^2 = (4)^2 = 16$

Vertex = (4, 32)

b

Minimum

$$\begin{aligned} a &= 8 - b \\ a &= 8 - (4) \\ a &= 4 \end{aligned}$$

Substitute b into the other equation.

$a = 4$
$b = 4$

List the two numbers and the maximum.

The minimum product is 32.

C11 - 3.6 - Product of Numbers is a Max Notes

The sum of two times one number and six times another is sixty. Find the numbers if their product is a maximum.

Let $a = 1st\ \#$
Let $b = 2nd\ \#$

Let statements:

1 $2a + 6b = 60$

2 $a \times b = maximum$
$a \times b = \cancel{maximum}\ y$
$y = a \times b$

Equation 1, equation 2.
The minimum or maximum will be y.

$$\frac{2a}{2} + \frac{6b}{2} = \frac{60}{2}$$
$$a + 3b = 30$$
$$a = 30 - 3b$$

Equation #1
Isolate a variable

$$y = a \times b$$
$$y = (30 - 3b) \times b$$
$$y = 30b - 3b^2$$
$$y = -3b^2 + 30b$$

Equation #2
Substitute the isolated variable

$$y = -3b^2 + 30b$$
$$y = -3(b^2 - 10b + 25 - 25)$$
$$y = -3(b^2 - 10b + 25) + 75$$
$$y = -3(b - 5)^2 + 75$$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

Vertex = (5, 75)

b Maximum

$$a = 30 - 3b$$
$$a = 30 - 3(5)$$
$$a = 15$$

Substitute b into the other equation.

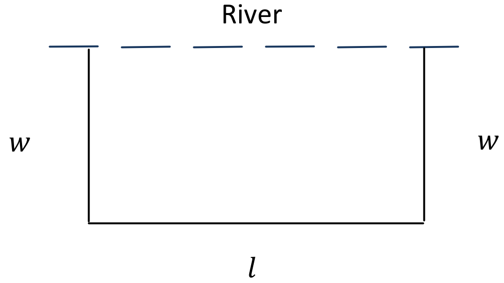
$a = 15$
$b = 5$

List the two numbers and the maximum.

The maximum product is 75

C11 - 3.7 - Fence w/ River Notes (p = 8m)

A rectangular enclosure is bounded on the side of a river. 3 sides total 8m of fencing. Find the dimensions of the largest possible enclosure.



Let $w = width$
Let $l = length$

Let statements:

1 $2w + l = P$
$2w + l = 8$

2 $A = l \times w$

Equation 1, equation 2.
The minimum or maximum will be y.

$$\begin{aligned} 2w + l &= 8 \\ -2w \quad &\quad - 2w \\ l &= 8 - 2w \end{aligned}$$

Equation #1
Isolate a variable

$$\begin{aligned} A &= l \times w \\ A &= (8 - 2w) \times w \\ A &= 8w - 2w^2 \\ A &= -2w^2 + 8w \end{aligned}$$

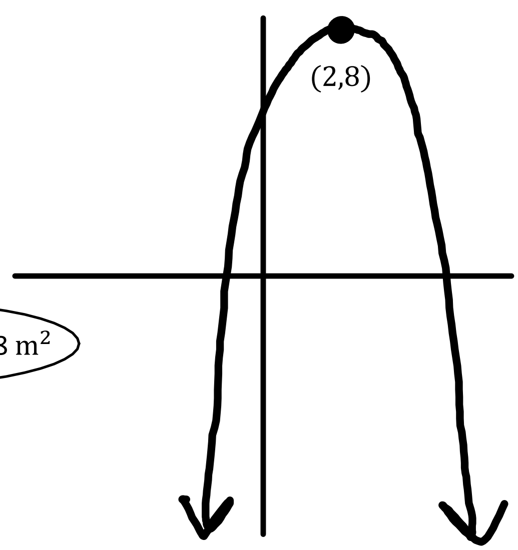
Equation #2
Substitute the isolated variable

$$\begin{aligned} A &= -2w^2 + 8w \\ A &= -2(w^2 - 8w) \\ A &= -2(w^2 - 4w + 4 - 4) \\ A &= -2(w^2 - 4w + 4) + 8 \\ A &= -2(w - 2)^2 + 8 \end{aligned}$$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

$Vertex = (2,8)$

w $\quad$ $Maximum\ Area$



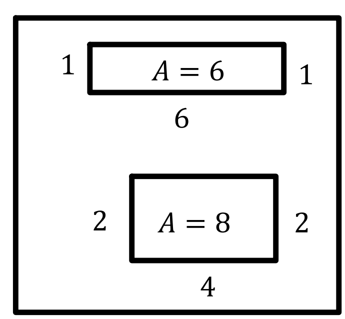
Substitute w into the other equation.

$$\begin{aligned} l &= 8 - 2w \\ l &= 8 - 2(2) \\ l &= 4 \end{aligned}$$

List the length and width and the maximum area.

$width = 2$ m
$length = 4$ m

The maximum area is 8 m^2

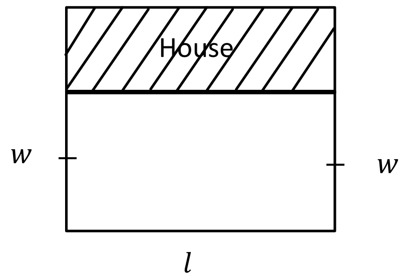


Or, factor, solve, average solutions, substitute.

C11 - 3.7 - Fence w/ River Notes (p = 60m)

Jack has 60m of fencing to build a three sided fence on the side of his house. Determine the maximum possible area of the fenced area, and the dimensions of the fence.

Let $w = width$
Let $l = length$



Let statements:

1 $P = 2w + \cancel{2}l$
$60 = 2w + l$

2 $\cancel{a} = l \times w$
$\cancel{max} = l \times w$
$y = l \times w$

Equation 1, equation 2.
The minimum or maximum will be y.

$$60 = 2w + l$$
$$-2w \quad -2w$$
$$60 - 2w = l$$
$$l = 60 - 2w$$

Equation #1
Isolate a variable

$$y = l \times w$$
$$y = (60 - 2w)w$$
$$y = 60w - 2w^2$$
$$y = -2w^2 + 60w$$

Equation #2
Substitute the isolated variable

$$y = -2(w^2 + 30w)$$
$$y = -2(w^2 - 30w + 225 - 225)$$
$$y = -2(w^2 - 30w + 225) + 450$$
$$y = -2(w - 15)^2 + 450$$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

$$Vertex = (\underbrace{15}_{w},\underbrace{450}_{Maximum})$$

$$l = 60 - 2w$$
$$l = 60 - 2(15)$$
$$l = 60 - 30$$
$$l = 30$$

Substitute w into the other equation.

$width = 15\text{m}$
$length = 30\text{ m}$

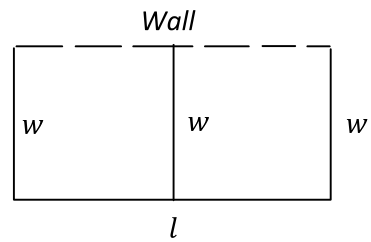
The maximum area is 450 m^2

List the length and width and the maximum area.

C11 - 3.7 - Fence w/ wall Split in Two

A rectangular fence that is split in half is against a wall. The total fencing length is 42 m. What is the max area of the fence?

Let $w = width$
Let $l = length$



Let statements:

$$F = l + 3w$$

$$A = l \times w$$
$$max = l \times w$$
$$y = l \times w$$

Equation 1, equation 2.
The minimum or maximum will be y.

$$P = l + 3w$$
$$42 = l + 3w$$
$$-3w \quad -3w$$
$$42 - 3w = l$$
$$l = 42 - 3w$$

Equation #1
Isolate a variable

$$A = l \times w$$
$$y = (42 - 3w) \times w$$
$$y = 42w - 3w^2$$
$$y = -3w^2 + 42w$$
$$y = -3(w^2 - 14w)$$
$$y = -3(w^2 - 14w + 49 - 49)$$
$$y = -3(w^2 - 14w + 49) + 147$$
$$y = -3(w - 7)^2 + 147$$

Equation #2
Substitute the isolated variable

Complete the square.
$$\left(\frac{b}{2}\right)^2 = \left(\frac{-14}{2}\right)^2 = (7)^2 = 49$$

$Vertex: (7,147)$

w $\quad$ $Maximum$

The maximum is the y value.

$$l = 42 - 3w$$
$$l = 42 - 3(7)$$
$$l = 21$$

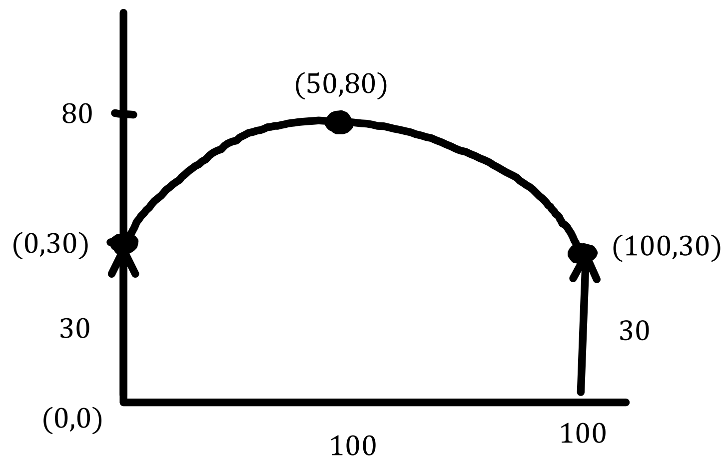
$length = 21m$
$width = 7m$

List the length and width and the maximum area.

$Max\ area = 147\ m^2$

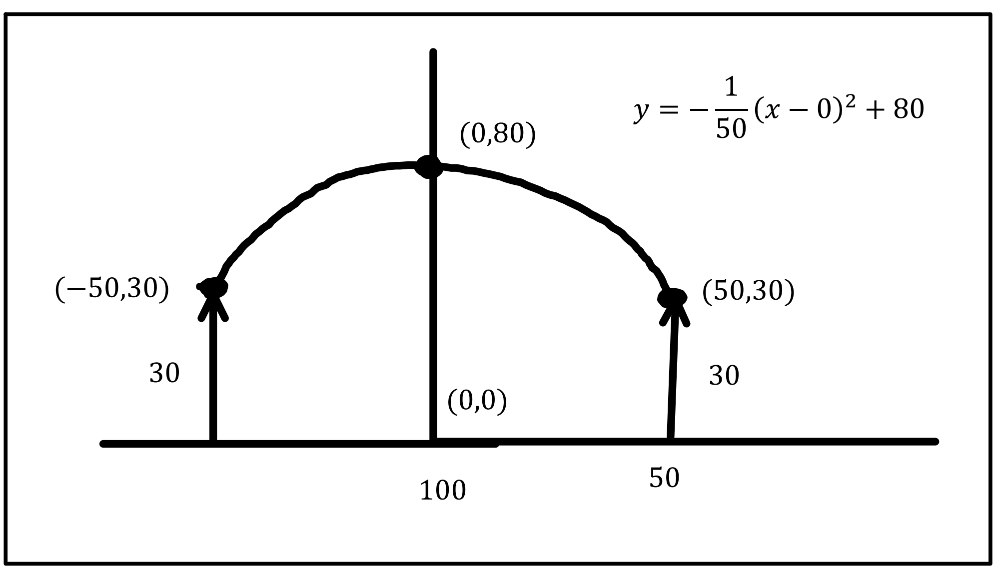
C11 - 3.8 - Bridge Find Equation Notes

A bridge has pillars 30 m tall and are 100 m apart. The maximum at the center of the bridge is 80 m tall. Find the equation of the parabolic bridge. What is the height 5 m away from each pillar.



$$y = a(x-p)^2 + q$$
$$y = a(x-50)^2 + 80$$
$$30 = a(0-50)^2 + 80$$
$$30 = a(50)^2 + 80$$
$$-80 \qquad\qquad -80$$
$$-\frac{50}{2500} = \frac{2500a}{-2500}$$
$$a = -\frac{1}{50}$$

$$y = -\frac{1}{50}(x-50)^2 + 80$$



C11 - 3.9 - Set Up Maximize Candy Sales Notes

A student sells candy to all of his friends. Each candy costs 6 dollars, and he has 10 friends who buy the candy each day. Every time he increases the cost by 1 dollar, 1 of his friends decides not to buy the candy. Set up how this question will look.

$Let\ p = price$
$Let\ q = quantity$
$Let\ r = revenue$
$\boxed{Let\ x = \#\ of\ price\ increases}$

$Revenue = price \times quantity$
If $p = 6, \quad q = 10 \quad r = 6 \times 10$
$r = 60$

$p = 6 + 1x \longrightarrow$ Raising the price by 1 dollar x times.

$q = 10 - 1x \longrightarrow$ Each x times he raises the price, 1 less friend will buy the candy.

$r = p \times q$
$r = (6 + 1x) \times (10 - 1x)$

Price

x	p
-2	4
-1	5
0	6
1	7
2	8

Starting Price and Quantity (zero price increase)

Quantity

x	q
-2	12
-1	11
0	10
1	9
2	8

C11 - 3.9 - Maximize Candy Sales Notes

A student sells candy to all of his friends. Each candy costs 6 dollars, and he has 10 friends who buy the candy each day. Every time he increases the cost by 1 dollar, 1 of his friends decides not to buy the candy. What is the price that will maximize revenue?

$Let\ p = price$
$Let\ q = quantity$
$Let\ r = revenue$
$\boxed{Let\ x = \#\ of\ price\ increases}$

$Revenue = price \times quantity$	$r = p \times q$
If $p = 6, \quad q = 10$	$r = 6 \times 10$
$r = 60$	$r = \$60$

$p = 6 + 1x$ → If he decides to raise the price by 1 dollar x times.

$q = 10 - 1x$ → One less friend will buy the candy each time he increases the price.

$r = p \times q$
$r = (6 + x)(10 - x)$
$r = 60 - 6x + 10x - x^2$
$r = 60 + 4x - x^2$
$r = -x^2 + 4x + 60$
$r = -(x^2 - 4x) + 60$ $\times (-1)$
$r = -(x^2 - 4x + 4 - 4) + 60$
$r = -(x^2 - 4x + 4) + 60 + 4$
$r = -(x - 2)^2 + 64$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(-\frac{4}{2}\right)^2 = (-2)^2 = 4$

$\mathrm{y} = \max\ revenue = \$64$

Vertex: $(2, 64)$

$x = 2\ price\ increases$

$p = 6 + 1x$
$p = 6 + 1(2)$
$p = 6 + 2$
$p = 8$
$\boxed{price = 8}$

$q = 10 - 1x$
$q = 10 - 1(2)$
$q = 10 - 2$
$q = 8$
$\boxed{quantity = 8}$

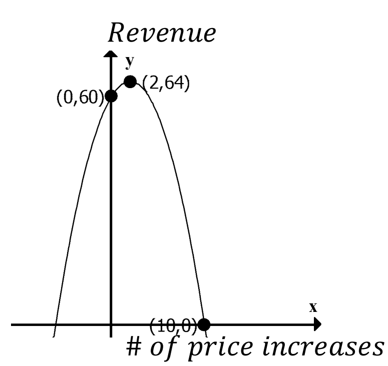
Check with Table of Values

1st increase

2nd increase

Price	Quantity	(x)	Revenue (y)
6	10	0	60
7	9	1	63
8	8	2	64
9	7	3	63
10	6	4	60
11	5	5	55

Max revenue



C11 - 3.9 - Maximize Car Sales Notes

A car salesman sells a car for \$4000, with 20 people buying the car. For every \$200 he takes off the price, 2 more people buy a car. What is the price that will maximize revenue?

$Let\ p = price$
$Let\ q = quantity$
$Let\ r = revenue$
$\boxed{Let\ x = \#\ of\ price\ decreases}$

$Revenue = price \times quantity$
$If\ p = \$4000, \quad q = 20$
$r = \$80{,}000$

If they sell 20 cars at \$4000, revenue is \$80,000.

$p = 4000 - 200x \longrightarrow$ If he decides to decrease the price by \$200 x times.

$q = 20 + 2x \longrightarrow$ Two more people will buy the car each time he decreases the price.

$r = p \times q$
$r = (4000 - 200x)(20 + 2x)$
$r = 80000 + 8000x - 4000x - 400x^2$
$r = -400x^2 + 4000x + 80000$
$r = -400(x^2 - 10x) + 80000$
$r = -400(x^2 - 10x + 25 - 25) + 80000$
$r = -400(x^2 - 10x - 25) + 80000 + 10000$
$r = -400(x - 5)^2 + 90000$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(-\frac{10}{2}\right)^2 = (-5)^2 = 25$

Vertex: $(5, 90000)$

$x = 5\ price\ decreases$

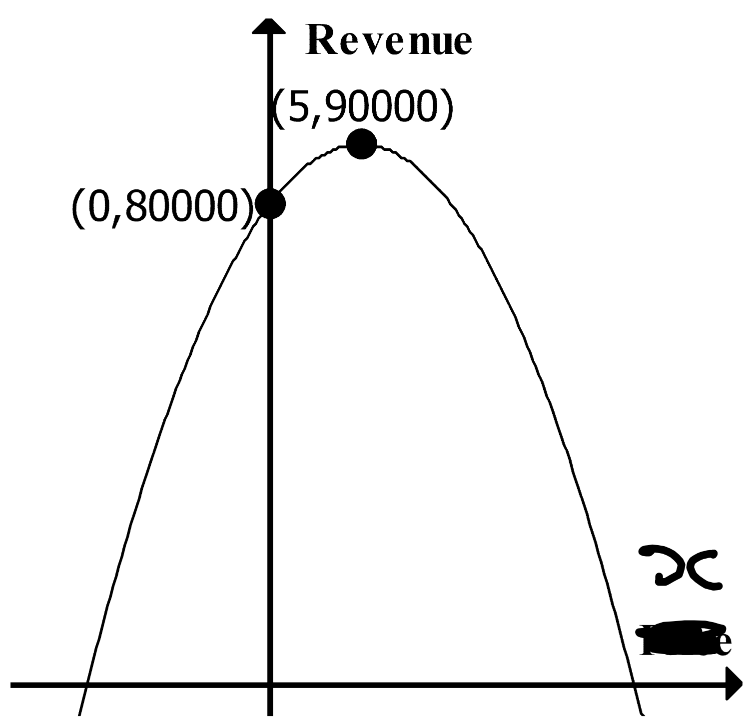
$\mathrm{y = max}\ \ revenue = \$90000$

$p = 4000 - 200x$
$p = 4000 - 200(5)$
$p = 4000 - 1000$
$p = 3000$

$\boxed{price = \$3000}$

$q = 20 + 2x$
$q = 20 + 2(5)$
$q = 20 + 10$
$q = 30$

$\boxed{quantity = 30\ people}$



C11 - 3.9 - Maximize Car Sales Notes (No Price Increases)

A car salesman sells a car for \$2000, with 20 people buying the car. For every \$200 he takes off the price, 2 more people buy a car. What is the price that will maximize revenue?

$Let\ p = price$
$Let\ q = quantity$
$Let\ r = revenue$
$\boxed{Let\ x = \#\ of\ price\ decreases}$

$Revenue = price \times quantity$
$If\ p = \$2000,\quad q = 20$
$r = \$40{,}000$

If they sell 20 cars at \$8000, revenue is \$40,000.

$p = 2000$
$p = 2000 - 200x$ → If he decides to decrease the price by \$200 x times.

$q = 20$
$q = 20 + 2x$ → Two more people will buy the car each time he decreases the price.

$r = p \times q$
$r = (2000 - 200x)(20 + 2x)$
$r = 40000 + 4000x - 4000x - 400x^2$
$r = -400x^2 + 40000$
$r = -400(x + 0)^2 + 40000$

Vertex: $(0, 40000)$

$x = 0\ price\ decreases$

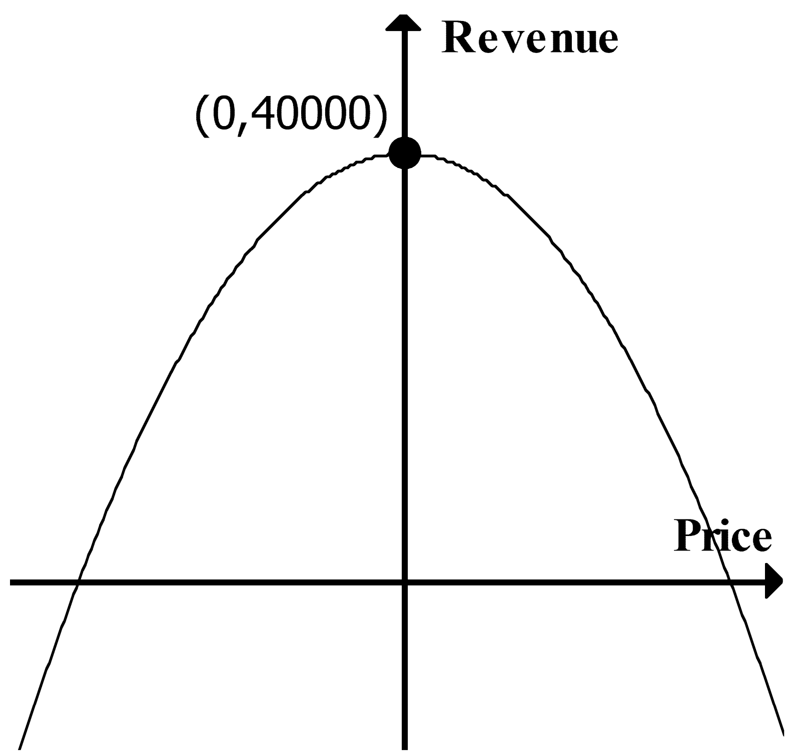
$\mathrm{y = max}\ \ revenue = \$30000$

$p = 2000 - 200x$
$p = 2000 - 200(0)$
$p = 2000$

$\boxed{price = \$2000}$

$q = 20 + 2x$
$q = 20 + 2(0)$
$q = 20 - 0$
$q = 20$

$\boxed{quantity = 20\ people}$



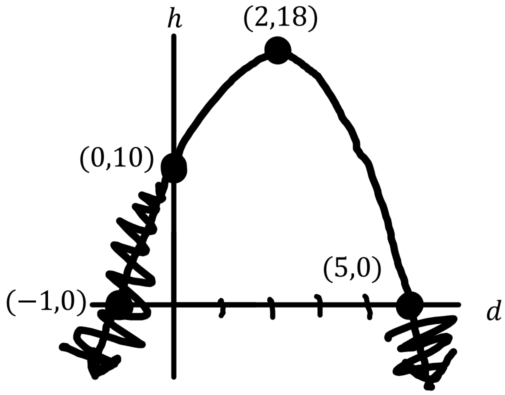
C11 - 3.10 - Max Height/Total Distance

Or 2nd Calc

The height vs distance of a bow and arrow shot off a cliff is represented by following equation:

$$h = -2d^2 + 8d + 10$$

What is the maximum height and the distance it took to get there? Draw on a graph.



$h = -2d^2 + 8d + 10$

Complete the Square

$$h = -2d^2 + 8d + 10$$
$$h = (-2d^2 + 8d) + 10$$
$$h = -2(d^2 - 4d) + 10$$
$$h = -2(d^2 - 4d + 4 - 4) + 10$$
$$h = -2(d^2 - 4d + 4) + 8 + 10$$
$$h = -2(d - 2)^2 + 18$$

$$\boxed{\begin{array}{c} \left(\frac{b}{2}\right)^2 \\ \left(-\frac{4}{2}\right)^2 \\ (-2)^2 \\ 4 \end{array}}$$

$V: (2,18)$

(d, h)

$d = 2 \quad h = 18$

What was the height of the cliff?

$h - int$

$d = 0$

$$h = -2d^2 + 8d + 10$$
$$h = -2(0)^2 + 8(0) + 10$$

$h = 10$

How far did the arrow go before it hit the ground?

$h = 0$

$$h = -2(d^2 - 4d - 5)$$
$$0 = -2(d - 5)(d + 4)$$

Factor

$\cancel{d + 1 = 0}$ $\quad d - 5 = 0$

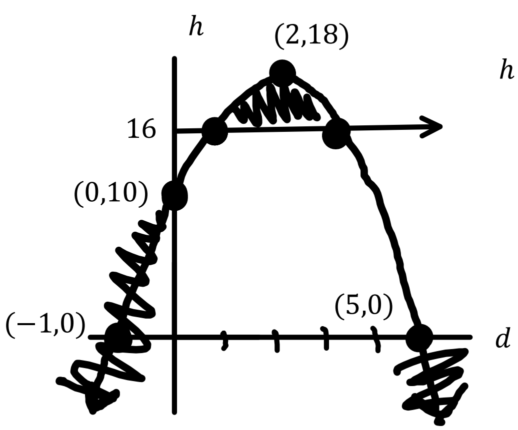
$\cancel{d = -1}$ $\quad d = 5$

Reject

Find Domain and Range

$D: [0,5]\ or\ 0 \le x \le 5 \qquad R: [0,18]\ or\ 0 \le y \le 18$

At what distance is the height 16 m (CH8)? At what distance is the height greater than 16m (CH9)?

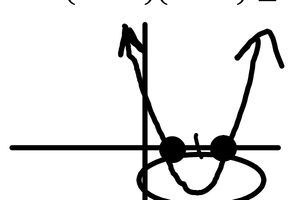


$h = 16 \qquad h = -2d^2 + 8d + 10$

$$h = -2d^2 + 8d + 10$$
$$16 = -2d^2 + 8d + 10$$
$$-16 \qquad\qquad -16$$
$$0 = -2d^2 + 8d - 6$$
$$\frac{0}{-2} = \frac{-2d^2 + 8d - 6}{-2}$$
$$0 = d^2 - 4d + 3$$
$$0 = (d - 3)(d - 1)$$

$d = 3 \quad d = 1$

$$-2d^2 + 8d + 10 \ge 16$$
$$-16 \quad -16$$
$$-2d^2 + 8d - 6 \ge 0$$
$$\frac{-2d^2 + 8d - 6}{-2} \ge \frac{0}{-2}$$
$$d^2 - 4d + 3 \le 0$$
$$(d - 3)(d - 1) \le 0$$



$0 \le d \le 3$

C11 - 4.1 - Solving $x - intercepts$ Notes

Solve for $x - intercepts$.

$y = x^2 - 4x + 3$
$y = (x-1)(x-3)$
$0 = (x-1)(x-3)$

$\underline{\quad 1 \quad} \times \underline{\quad 5 \quad} = 5$
$\underline{\quad 1 \quad} + \underline{\quad 5 \quad} = 6$

Factor
$x - int$: *Set y equal to zero*, $(y = 0)$

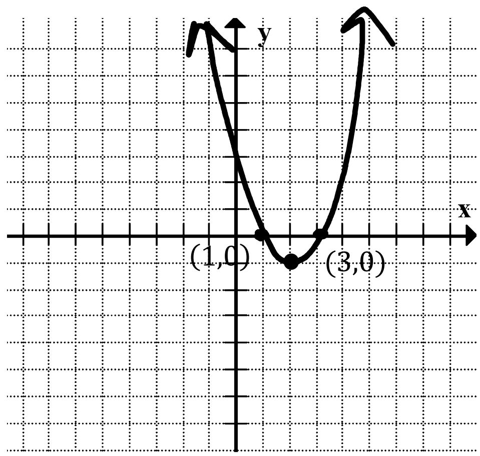
Set the brackets equal to zero seperately

$x - 1 = 0 \qquad x - 3 = 0$
$+1 \quad +1 \qquad +3 \quad +3$
$x = +1 \qquad x = +3$

Solve

$(1,0) \quad x - int: \quad (3,0)$

State $x - intercepts$ $(x, 0)$



Draw a graph and label $x - intercepts$.

$$\boxed{\begin{array}{cc} & (a)(b) = 0 \\ a = 0 & b = 0 \end{array}}$$

$y = 2x^2 - 3x - 2$
$y = 2x^2 - 4x + 1x - 2$
$y = (2x^2 - 4x)(+1x - 2)$
$y = 2x(x-2) + 1(x-2)$
$y = (x-2)(2x+1)$
$0 = (x-2)(2x+1)$

$\underline{\quad -4 \quad} \times \underline{\quad 1 \quad} = -4$
$\underline{\quad -4 \quad} + \underline{\quad 1 \quad} = -3$

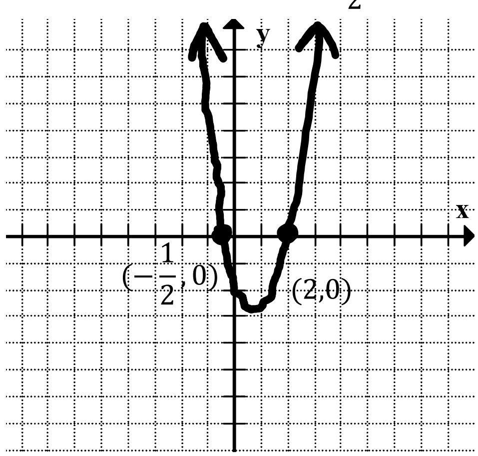
Factor
Decompose
Group
GCF
Switch
$x - int$: *Set y equal to zero*, $(y = 0)$

$x - 2 = 0 \qquad 2x + 1 = 0$
$+2 \quad +2 \qquad -1 \quad -1$
$x = 2 \qquad \frac{2x}{2} = -\frac{1}{2}$
$x = -\frac{1}{2}$

State $x - intercepts$ $(x, 0)$

$x - int$: $\quad (2,0) \quad (-\frac{1}{2}, 0)$

Draw a graph and label $x - intercepts$.



Set the brackets equal to zero seperately

Solve

C11 - 4.1 - Solving $x - intercepts$ Notes

Set $y = 0$ $and\ factor\ to\ find\ x - intercepts.$ $(x, 0)$

$y = x^2 - 6x + 5$
$0 = x^2 - 6x + 5$
$0 = (x - 5)(x - 1)$

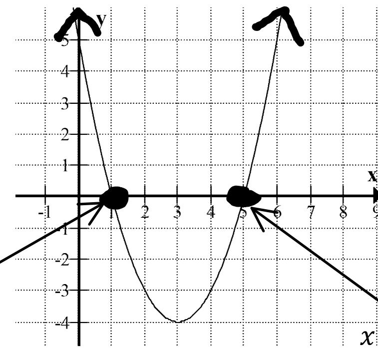
$x - 5 = 0$ $\quad$ $x - 1 = 0$
$+5 \quad +5$ $\quad$ $+1 \quad +1$
$x = 5$ $\quad$ $x = 1$

$(5,0)$ $\quad$ $(1,0)$

$x\ intercepts$: set $y = 0$
Factor.

Set brackets equal to 0 separately and solve.

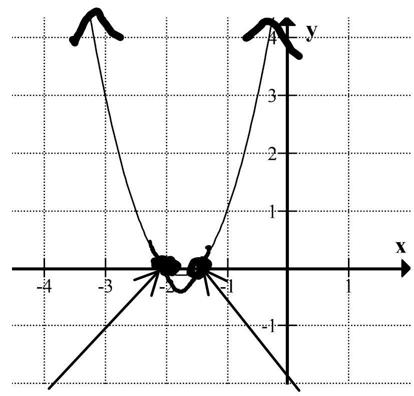
$x - intercepts$



$x\ int = (1,0)$ $\quad$ $x\ int = (5,0)$

$y = 2x^2 + 7x + 6$
$0 = 2x^2 + 7x + 6$
$0 = 2x^2 + 4x + 3x + 6$
$0 = 2x(x + 2) + 3(x + 2)$
$0 = (2x + 3)(x + 2)$

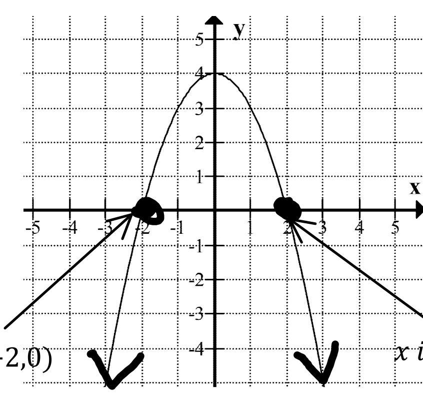
$2x + 3 = 0$ $\quad$ $x + 2 = 0$
$-3 \quad -3$ $\quad$ $-2 \quad -2$
$2x = -3$ $\quad$ $x = -2$
$\frac{2x}{2} = \frac{-3}{2}$
$x = -\frac{3}{2}$



$x\ int = (-2,0)$ $\quad$ $x\ int = (-\frac{3}{2}, 0)$

$y = -x^2 + 4$
$0 = -x^2 + 4$
$0 = -(x^2 - 4)$ $\quad$ GCF: -1
$0 = -(x + 2)(x - 2)$ $\quad$ Factor.

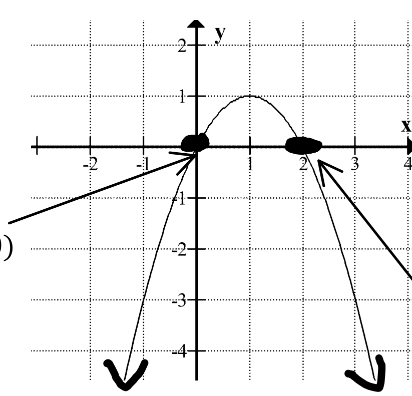
$x + 2 = 0$ $\quad$ $x - 2 = 0$
$-2 \quad -2$ $\quad$ $+2 \quad +2$
$x = -2$ $\quad$ $x = 2$



$x\ int = (-2,0)$ $\quad$ $x\ int = (2,0)$

$y = -x^2 + 2x$
$0 = -x^2 + 2x$
$0 = -x(x - 2)$

$x = 0$ $\quad$ $x - 2 = 0$
$+2 \quad +2$
$x = 2$



$x\ int = (0,0)$ $\quad$ $x\ int = (2,0)$

C11 - 4.2 - $x - int$/Standard Form Notes

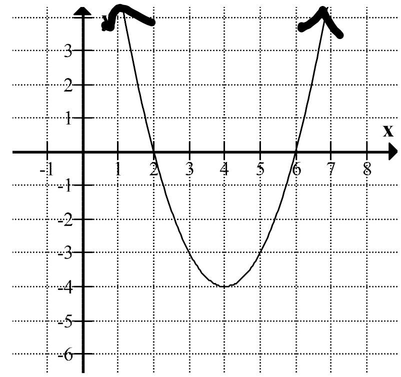
$x\ int = (2,0)\,,(6,0)$

$x = 2$
$-2 \quad -2$
$x - 2 = 0$

$x = 6$
$-6 \quad -6$
$x - 6 = 0$

Write down the x values.

Add or subtract to both sides to make = 0



$y = (x-2)(x-6)$

Factored Form

$y = x^2 - 8x + 12$

Standard Form

$x\ int = \left(\frac{1}{2},0\right),(4,0)$

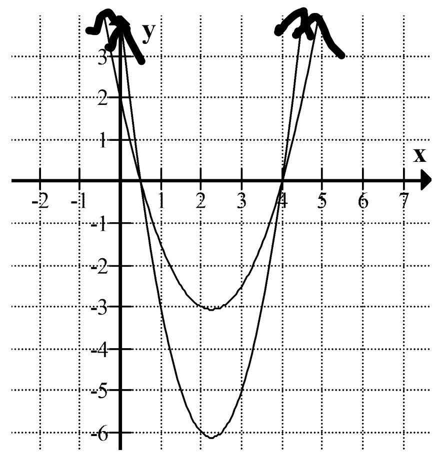
$x = \frac{1}{2}$
$2 \times x = \frac{1}{2} \times 2$
$2x = 1$
$-1 \quad -1$
$2x - 1 = 0$

$x = 4$
$-4 \quad -4$
$x - 4 = 0$

Multiply and Add or subtract to both sides to make = 0

$y = x^2 - \frac{9}{2}x + 2$
$0 = x^2 - \frac{9}{2}x + 2$

$y = (2x-1)(x-4)$
$y = 2x^2 - 9x + 4$



$y = 2x^2 - 9x + 4$
$0 = 2x^2 - 9x + 4$

$x\ int = \left(\frac{1}{2},0\right),(4,0)$

$x = \frac{1}{2}$
$-\frac{1}{2} \quad -\frac{1}{2}$
$x - \frac{1}{2} = 0$

$x = 4$
$-4 \quad -4$
$x - 4 = 0$

$y = \left(x - \frac{1}{2}\right)(x-4)$
$y = x^2 - 4x - \frac{1}{2}x + 2$
$y = x^2 - \frac{9}{2}x + 2$

Notice: two different graphs in standard form can have the same x-intercepts.

C11 - 4.2 - Find Standard Form x-int "a" and a Point Notes

Find equation in Standard Form using x − intercepts and "a" $\boxed{y = a(x+\#)(x+\#)}$

$x - int = 2$ and 6
$a = 1$

$$\begin{aligned} x &= 2 \\ -2 &\quad -2 \\ x - 2 &= 0 \end{aligned} \qquad \begin{aligned} x &= 6 \\ -6 &\quad -6 \\ x - 6 &= 0 \end{aligned}$$

Set x − int = # and make equal to zero

$$\begin{aligned} y &= a(x+\#)(x+\#) && \textit{Write Factored Form} \\ y &= 1(x-2)(x-6) && \textit{Substitute Factors} \\ y &= (x-2)(x-6) \\ y &= x^2 - 8x + 12 && \textit{Foil} \end{aligned}$$

$x - int = 2$ and -2
$a = 2$

$$\begin{aligned} x &= 2 \\ -2 &\quad -2 \\ x - 2 &= 0 \end{aligned} \qquad \begin{aligned} x &= -2 \\ +2 &\quad +2 \\ x + 2 &= 0 \end{aligned}$$

$$\begin{aligned} y &= a(x+\#)(x+\#) \\ y &= 2(x-2)(x+2) \\ y &= 2(x^2 + 2x - 2x - 4) \\ y &= 2(x^2 - 4) \\ y &= 2x^2 - 8 \end{aligned}$$

$x - int = \frac{3}{2}$ and $-\frac{7}{2}$

$$\begin{aligned} x &= \frac{3}{2} \\ 2 \times x &= \frac{3}{2} \times 2 \\ 2x &= 3 \\ -3 &\quad -3 \\ 2x - 3 &= 0 \end{aligned} \qquad \begin{aligned} x &= -\frac{7}{2} \\ 2 \times x &= \frac{3}{2} \times 2 \\ 2x &= -7 \\ +7 &\quad +7 \\ 2x + 7 &= 0 \end{aligned}$$

$$\begin{aligned} y &= a(x+\#)(x+\#) \\ y &= (2x-3)(2x+7) \\ y &= 4x^2 + 14x - 6x - 21 \\ y &= 4x^2 + 8x - 21 \end{aligned}$$

$x - int = -1$ and 3
$(2, -6)$

$$\begin{aligned} y &= a(x+1)(x-3) \\ -6 &= a(2+1)(2-3) \\ -6 &= a(3)(-1) \\ -6 &= -3a \\ a &= 2 \end{aligned}$$

$$y = 2(x+1)(x-3)$$

C11 - 4.3 - x –Intercepts/Vertex/AOS Form

$y = x^2 - 2x - 8$
$y = (x-2)(x+4)$

$x - 2 = 0$ $\quad$ $x + 4 = 0$
$+2 \quad +2$ $\quad$ $-4 \quad -4$
$x = 2$ $\quad$ $x = -4$

$x - int:$ $\quad$ $(2,0)$ $\quad$ $(-4,0)$

The x coordinate of the vertex is always halfway between the two x-intercepts.

$$x = \frac{(2)+(-4)}{2} = \frac{-2}{2}$$
$x = -1$

Find the average between the two x-intercept values.
(Or any two horizontal x-values)

Vertex: $(-1, y)$

$Axis\ of\ Symmetry: x = -1$

$y = (x-2)(x+4)$
$y = ((-1)-2)((-1)+4)$
$y = (-3)(3)$
$y = -9$

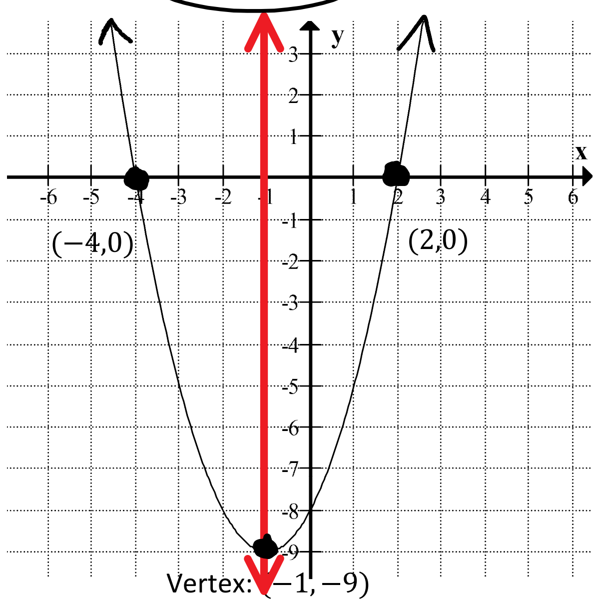
Find the y value of the vertex by putting in the x value of the vertex

Vertex: $(-1,-9)$

$AOS: x = -1$

x	y
-3	-5
-2	-8
-1	-9
0	-8
1	-5

Vertex:



C11 - 4.3 - Solving by Square Root Method Notes

$$x^2 - 4 = 0$$
$$+4 \quad +4$$
$$x^2 = 4$$
$$\sqrt{x^2} = \pm\sqrt{4}$$
$$x = \pm 2$$
$$x = 2 \qquad x = -2$$

$$x^2 - 4 = 0$$
$$(x+2)(x-2) = 0$$
$$x + 2 = 0 \qquad x - 2 = 0$$
$$x = -2 \qquad x = 2$$

$$2(x+1)^2 - 8 = 0$$
$$+8 \quad +8$$
$$2(x+1)^2 = 8$$
$$\frac{2(x+1)^2}{2} = \frac{8}{2}$$
$$(x+1)^2 = 4$$
$$\sqrt{(x+1)^2} = \pm\sqrt{4}$$
$$x + 1 = \pm 2$$
$$x + 1 = 2 \qquad x + 1 = -2$$
$$-1 \quad -1 \qquad -1 \quad -1$$
$$x = 1 \qquad x = -3$$

$$(x-2)^2 - 1 = 0$$
$$+1 \quad +1$$
$$(x-2)^2 = 1$$
$$\sqrt{(x-2)^2} = \pm\sqrt{1}$$
$$x - 2 = \pm 1$$
$$x - 2 = 1 \qquad x - 2 = -1$$
$$x = 3 \qquad x = 1$$

$$(x-2)^2 - 1 = 0$$
$$(x-2)(x-2) - 1 = 0$$
$$x^2 - 4x + 4 - 1 = 0$$
$$x^2 - 4x + 3 = 0$$
$$(x-1)(x-3) = 0$$
$$x - 1 = 0 \qquad x - 3 = 0$$
$$x = 1 \qquad x = 3$$

$$(x-2)^2 - 7 = 0$$
$$+7 \quad +7$$
$$(x-2)^2 = 7$$
$$\sqrt{(x-2)^2} = \pm\sqrt{7}$$
$$x - 2 = \pm\sqrt{7}$$
$$x = \pm\sqrt{7} + 2$$
$$x = \sqrt{7} + 2 \qquad x = -\sqrt{7} + 2$$

$$x^2 + 16 = 0$$
$$-16 \quad -16$$
$$x^2 = -16$$
$$\sqrt{x^2} = \pm\sqrt{-16}$$
$$DNE$$

Can't square root a negative.

$$(x+2)^2 + 2 = 0$$
$$-2 \quad -2$$
$$(x+2)^2 = -2$$
$$\sqrt{(x+2)^2} = \pm\sqrt{-2}$$
$$DNE$$

$$\left(x - \frac{1}{2}\right)^2 - 7 = 0$$
$$\left(x - \frac{1}{2}\right)^2 = 7$$
$$x - \frac{1}{2} = \pm\sqrt{7}$$
$$x = \pm\sqrt{7} + \frac{1}{2}$$
$$x = \pm\sqrt{7} \times \frac{2}{2} + \frac{1}{2}$$
$$x = \frac{\pm 2\sqrt{7}}{2} + \frac{1}{2}$$
$$x = \frac{\pm 2\sqrt{7} + 1}{2}$$

$$2\left(x + \frac{1}{2}\right)^2 - 8 = 0$$
$$2\left(x + \frac{1}{2}\right)^2 = 8$$
$$\left(x + \frac{1}{2}\right)^2 = 4$$
$$\sqrt{\left(x + \frac{1}{2}\right)^2} = \pm\sqrt{4}$$
$$x + \frac{1}{2} = \pm 2$$
$$x = \pm 2 - \frac{1}{2}$$
$$x = 1.5 \qquad x = -2.5$$

$$2(x-2)^2 - 7 = 0$$
$$2(x-2)^2 = 7$$
$$\sqrt{(x-2)^2} = \pm\sqrt{\frac{7}{2}}$$
$$x - 2 = \pm\sqrt{\frac{7}{2}}$$
$$x = \pm\sqrt{\frac{7}{2}} + 2$$
$$x = \pm\frac{\sqrt{7}}{\sqrt{2}} + \frac{2\sqrt{2}}{\sqrt{2}}$$
$$x = \frac{\pm\sqrt{7} + 2\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$x = \frac{\pm\sqrt{14} + 4}{2}$$

$$2\left(x - \frac{2}{3}\right)^2 - 7 = 0$$
$$2\left(x - \frac{2}{3}\right)^2 = 7$$
$$\sqrt{\left(x - \frac{2}{3}\right)^2} = \pm\sqrt{\frac{7}{2}}$$
$$x - \frac{2}{3} = \pm\sqrt{\frac{7}{2}}$$
$$x = \pm\sqrt{\frac{7}{2}} + \frac{2}{3}$$
$$x = \pm\frac{\sqrt{7}}{\sqrt{2}} \times \frac{3}{3} + \frac{2}{3} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$x = \frac{\pm 3\sqrt{7} + 2\sqrt{3}}{3\sqrt{2}}$$
$$x = \frac{\pm 3\sqrt{14} + 2\sqrt{6}}{6}$$

Rationalize

C11 - 4.4 - Quadratic Equation Notes

Solve

1 − 4 3

$1x^2 - 4x + 3 = 0$

$a = 1$
$b = -4$
$c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Equation

Substitute With Brackets

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{+4 \pm \sqrt{4}}{2}$$

$(-4)^2 - 4(1)(3) = 4$

Type underneath Square Root into Calculator

$$x = \frac{4 \pm 2}{2}$$

$$x = \frac{4+2}{2} \qquad x = \frac{4-2}{2}$$

$x = 3 \qquad x = 1$

2 Rational Roots.

2 + 5 1

$2x^2 + 5x + 1 = 0$

$a = 2$
$b = -5$
$c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(+5) \pm \sqrt{(5)^2 - 4(2)(1)}}{2(2)}$$

$$x = \frac{-5 \pm \sqrt{17}}{4}$$

$$x = \frac{-5 + \sqrt{17}}{4} \qquad x = \frac{-5 - \sqrt{17}}{4}$$

Exact Value

$x = -0.21 \qquad x = -2.28$

Decimal

2 Irrational Roots.

$b^2 - 4ac > 0$
Discriminant > 0
2 Real Roots.

2 − 6 − 7

$2x^2 - 6x - 7 = 0$

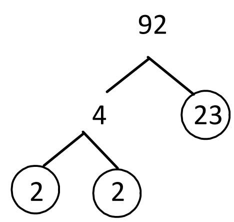
$a = 2$
$b = -6$
$c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-7)}}{2(2)}$$

$$x = \frac{6 \pm \sqrt{92}}{4}$$

$\sqrt{92} = \sqrt{2 \times 2 \times 23}$
$\sqrt{92} = 2\sqrt{23}$



$$x = \frac{6 \pm 2\sqrt{23}}{4}$$

Divide top and bottom by 2 $\quad \frac{6}{2} = 3 \quad \frac{2}{2} = 1 \quad \frac{4}{2} = 2$

$$x = \frac{3 \pm \sqrt{23}}{2}$$

$$x = \frac{3 + \sqrt{23}}{2} \qquad x = \frac{3 - \sqrt{23}}{2}$$

1 6 11

$x^2 + 6x + 11 = 0$

$a = 1$
$b = 6$
$c = 11$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(11)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{-8}}{2}$$

Cant Square Root Negative

$$x = \frac{-6 \pm \sqrt{-8}}{2}$$

$b^2 - 4ac < 0$
Discriminant < 0
No Real Roots.

3 − 6 3

$3x^2 - 6x + 3 = 0$

$a = 3$
$b = -6$
$c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(3)}}{2(3)}$$

$$x = \frac{6 \pm \sqrt{0}}{6}$$

$$x = \frac{6 \pm 0}{6}$$

$x = 1$

$b^2 - 4ac = 0$
Discriminant $= 0$
One Roots.

$3x^2 - 6x + 3 = 0$

$$\frac{3x^2}{3} - \frac{6x}{3} + \frac{3}{3} = \frac{0}{3}$$

$x^2 - 2x + 1 = 0$

1 − 2 1

$a = 1$
$b = -2$
$c = 1$

Simplify 1st!

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{0}}{2}$$

$$x = \frac{2 \pm 0}{2}$$

$x = 1$

C11 - 4.5 - Discriminant Notes

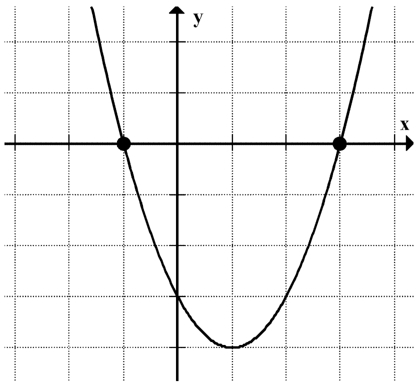
Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{DISCRIMINANT}}{2a}$$

Discriminant: $b^2 - 4ac$

Case 1: $b^2 - 4ac > 0$ *Inside the root is positive*



$x^2 - 2x - 3$

$b^2 - 4ac$
$(-2)^2 - 4(1)(-3)$
$4 + 12$
$+16$

$+$

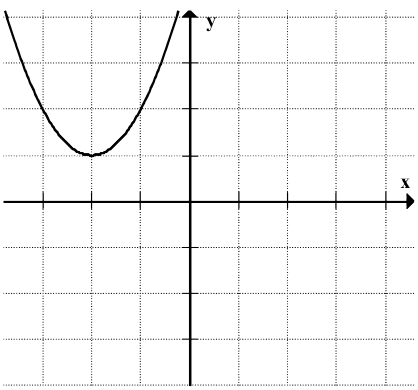
$$x = \frac{2 \pm \sqrt{16}}{2}$$

$x = 3$ $\quad x = -1$

Two x-intercepts
Two Real Roots
Two Solutions

If we add and subtract a positive number we get two answers

Case 2: $b^2 - 4ac < 0$ *Inside the root is negative*



$x^2 + 4x + 5$

$b^2 - 4ac$
$(4)^2 - 4(1)(5)$
$16 - 20$
-4

$-$

$$x = \frac{-4 \pm \sqrt{-4}}{2}$$

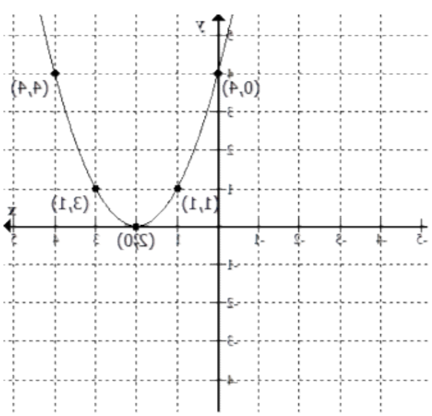
No Solution

Zero x-intercepts
No Real Roots
No Solutions
Imaginary Roots

Cant Square Root Negatives

Case 3: $b^2 - 4ac = 0$ *Inside the root is zero*

$b^2 - 4ac = 0, Perfect\ Square$



$x^2 + 4x + 4$

$b^2 - 4ac$
$(4)^2 - 4(1)(4)$
$16 - 16$
0

0

$$x = \frac{-4 \pm \sqrt{0}}{2}$$

$x = -2$

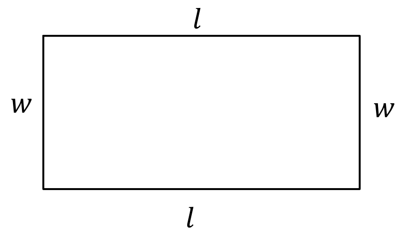
One x-intercepts
Two equal/real roots
One Solution

If we add and subtract zero we get one answer

C11 - 4.6 - Rectangular Garden

A rectangular garden has an Area of 36 and a Perimeter of 30. What are the lengths and widths?

Let $w = width$
Let $l = length$



Let statements:

$$P = 2l + 2w$$

$$A = l \times w$$

Equation 1, equation 2.

$$\begin{aligned} P &= 2l + 2w \\ 30 &= 2l + 2w \\ \frac{30}{2} &= \frac{2l}{2} + \frac{2w}{2} \\ 15 &= l + w \\ -w & \quad\; -w \\ 15 - w &= l \\ l &= 15 - w \end{aligned}$$

Equation #1
Isolate a variable

$$\begin{aligned} A &= l \times w \\ 36 &= l \times w \\ 36 &= (15 - w) \times w \\ 36 &= 15w - w^2 \\ +w^2 & \quad\; +w^2 \\ 36 + w^2 &= 15w \\ -15w & \quad\; -15w \\ w^2 - 15w + 36 &= 0 \\ (w - 12)(w - 3) &= 0 \end{aligned}$$

Equation #2
Substitute the isolated variable

Factor

$$w - 12 = 0 \qquad w - 3 = 0$$
$$w = 12 \qquad w = 3$$

Solve

$$\begin{aligned} l &= 15 - w \\ l &= 15 - (12) \\ l &= 3 \end{aligned}$$

Substitute w into the other equation.

$$Length = 12$$
$$Width = 3$$

List the length and width

OR

$$\begin{aligned} l &= 15 - w \\ l &= 15 - (3) \\ l &= 12 \end{aligned}$$

$$Length = 3$$
$$Width = 12$$

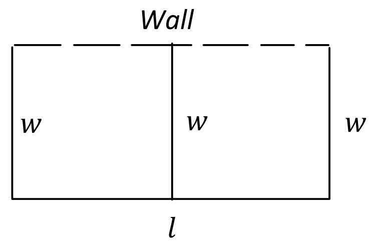
List the length and width

C11 - 4.6 - Fence Split in Two

A rectangular fence that is split in half is against a wall. The total fencing length is 39, and it has a total area of 66. What are the dimensions of the fence?

Let statements:

Let $w = width$
Let $l = length$



Equation 1, equation 2.

$$P = l + 3w$$

$$A = l \times w$$

Equation #1
Isolate a variable

$$\begin{aligned} P &= l + 3w \\ 39 &= l + 3w \\ -3w &\quad -3w \\ 39 - 3w &= l \\ l &= 39 - 3w \end{aligned}$$

Equation #2
Substitute the isolated variable

$$\begin{aligned} A &= l \times w \\ 66 &= (39 - 3w) \times w \\ 66 &= 39w - 3w^2 \\ +3w^2 &\quad + 3w^2 \\ 66 + 3w^2 &= 39w \\ -39w &\quad - 39w \\ 3w^2 - 39w + 66 &= 0 \\ 3(w^2 - 13w + 22) &= 0 \\ 3(w - 2)(w - 11) &= 0 \end{aligned}$$

Factor

Solve

$$\begin{aligned} w - 2 &= 0 \\ w &= 2 \end{aligned} \qquad \begin{aligned} w - 11 &= 0 \\ w &= 11 \end{aligned}$$

Substitute w into the other equation.

$$\begin{aligned} l &= 39 - 3w \\ l &= 39 - 3(2) \\ l &= 39 - 6 \\ l &= 33 \end{aligned}$$

List the length and width

$$Width = 2$$
$$Length = 33$$

or

$$\begin{aligned} l &= 39 - 3w \\ l &= 39 - 3(11) \\ l &= 39 - 33 \\ l &= 6 \end{aligned}$$

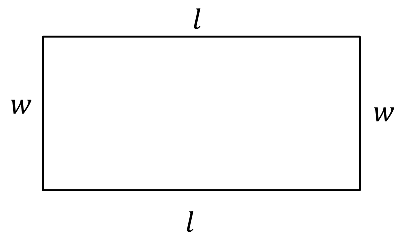
List the length and width

$$Width = 11$$
$$Length = 6$$

C11 - 4.6 - Rectangular Garden Quad

A rectangular garden has an area of 61 and a perimeter of 40. What are the lengths and widths?

Let $w = width$
Let $l = length$



Let statements:

$$P = 2l + 2w \qquad A = l \times w$$

Equation 1, equation 2.

$$\begin{aligned} P &= 2l + 2w \\ 40 &= 2l + 2w \\ \frac{40}{2} &= \frac{2l}{2} + \frac{2w}{2} \\ 20 &= l + w \\ -w & \quad\;\; -w \\ 20 - w &= l \\ l &= 20 - w \end{aligned}$$

Equation #1
Isolate a variable

$$\begin{aligned} A &= l \times w \\ 91 &= l \times w \\ 61 &= (20 - w) \times w \\ 61 &= 20w - w^2 \\ +w^2 & \quad\;\; + w^2 \\ 61 + w^2 &= 20w \\ -20w & \quad\;\; - 20w \\ w^2 - 20w + 61 &= 0 \end{aligned}$$

Equation #2
Substitute the
isolated variable

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$w = \frac{-(-20) \pm \sqrt{20^2 - 4(1)(61)}}{2(1)}$$

Quadratic Formula

$$w = \frac{20 - \sqrt{156}}{2(1)} \qquad w = \frac{20 + \sqrt{156}}{2(1)}$$

$$w = 3.755 \qquad w = 16.245$$

Solve

$$\begin{aligned} l &= 20 - w \\ l &= 20 - (16.245) \\ l &= 3.755 \end{aligned}$$

Substitute w into the
other equation.

$$\begin{aligned} Length &= 16.245 \\ Width &= 3.755 \end{aligned}$$

List the length and width

OR

$$\begin{aligned} l &= 15 - w \\ l &= 15 - (3.755) \\ l &= 16.245 \end{aligned}$$

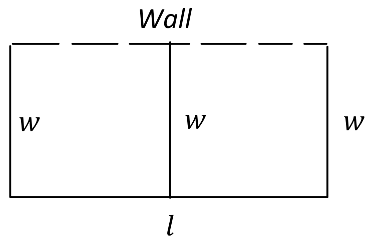
List the length and width

$$\begin{aligned} Length &= 3.755 \\ Width &= 16.245 \end{aligned}$$

C11 - 4.6 - Fence Split in Two Quad

A rectangular fence that is split in half is against a wall. The total fencing length is 61, and it has a total area of 58. What are the dimensions of the fence?

Let $w = width$
Let $l = length$



Let statements:

$$P = l + 3w \qquad A = l \times w$$

Equation 1, equation 2.

$$\begin{aligned} P &= l + 3w \\ 61 &= l + 3w \\ -3w & \quad -3w \\ 61 - 3w &= l \\ l &= 61 - 3w \end{aligned}$$

Equation #1
Isolate a variable

$$\begin{aligned} A &= l \times w \\ 58 &= (61 - 3w) \times w \\ 58 &= 61w - 3w^2 \\ +3w^2 & \quad + 3w^2 \\ 58 + 3w^2 &= 61w \\ -61w & \quad - 61w \\ 3w^2 - 61w + 58 &= 0 \end{aligned}$$

Equation #2
Substitute the isolated variable

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$w = \frac{-(-61) \pm \sqrt{61^2 - 4(3)(58)}}{2(3)}$$

Quadratic Formula

$$w = \frac{61 + \sqrt{3025}}{6} \qquad w = \frac{61 - \sqrt{3025}}{6}$$

$$w = 19.\overline{3} \qquad w = 1$$

$$w = \frac{58}{3}$$

Solve

$$\begin{aligned} l &= 61 - 3w \\ l &= 61 - 3\left(\frac{58}{3}\right) \\ l &= 61 - 58 \\ l &= 3 \end{aligned}$$

Substitute w into the other equation.

$$Width = \frac{58}{3}$$
$$Length = 3$$

List the length and width

or

$$\begin{aligned} l &= 61 - 3w \\ l &= 61 - 3(1) \\ l &= 61 - 3 \\ l &= 58 \end{aligned}$$

$$Width = 58$$
$$Length = 1$$

List the length and width

C11 - 5.1 - Adding and Subtracting Radicals Notes

Square Roots

$\sqrt[2]{7} + \sqrt[2]{7} = 2\sqrt[2]{7}$

$5.29 = 5.29$ ✓

$1\sqrt[2]{3} + 1\sqrt[2]{3} = 2\sqrt[2]{3}$

$3.46 = 3.46$ ✓

$2\sqrt[2]{3} + 5\sqrt[2]{3} = 7\sqrt[2]{3}$

$12.12 = 12.12$ ✓

Like Radicals: Add or subtract coefficients. $\boxed{x + x = 2x}$

Like Radicals: Same radicand, same index

Index → $\sqrt[2]{3}$ ← Radicand

$\sqrt[2]{3} + \sqrt[2]{2} = \sqrt[2]{3} + \sqrt[2]{2}$

Cannot add/subtract unlike radicals.

Can only add/subtract like radicals.

Calcuator

$\sqrt[2]{3} + \sqrt[2]{2} = 1.71 + 1.41 = 3.15$

$4\sqrt[2]{3} - 7\sqrt[2]{2} = -3\sqrt[2]{2}$

$-4.24 = -4.24$ ✓

$\sqrt[2]{12} + \sqrt[2]{27} + \sqrt[2]{18} + 5$

$2\sqrt[2]{3} + 3\sqrt[2]{3} + 3\sqrt[2]{2} + 5$

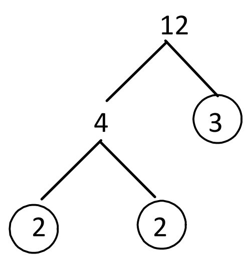
$5\sqrt[2]{3} + 3\sqrt[2]{2} + 5$

$17.9 = 17.9$ ✓

Simplify Roots

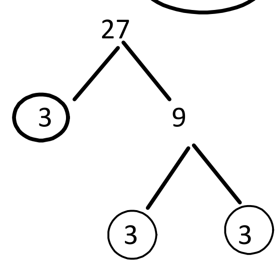
$\sqrt[2]{12} = \sqrt[2]{2 \times 2 \times 3}$

$= 2\sqrt[2]{3}$



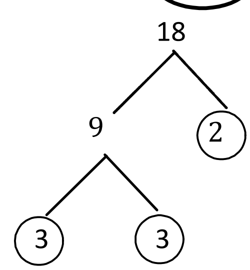
$\sqrt[2]{27} = \sqrt[3]{3 \times 3 \times 3}$

$= 2\sqrt[3]{3}$



$\sqrt[2]{18} = \sqrt[2]{3 \times 3 \times 2}$

$= 3\sqrt[2]{2}$



Cube Roots

$\sqrt[3]{7} + \sqrt[3]{7} = 2\sqrt[3]{7}$

$3.83 = 3.83$ ✓

$\sqrt[3]{5} + \sqrt[3]{5} = 2\sqrt[3]{5}$

$3.42 = 3.42$ ✓

$-2\sqrt[3]{5} - 6\sqrt[3]{5} = -8\sqrt[3]{5}$

$-13.68 = -13.68$ ✓

$\sqrt[3]{3} + 1 = \sqrt[3]{3} + 1$

Can only add or subtract like radicals.

C11 - 5.2 - Multiplying and Dividing Radicals Notes

$\sqrt[2]{3} \times \sqrt[2]{3} = \sqrt[2]{3 \times 3}$
$= \sqrt[2]{9}$
$= 3$

$7 \times \sqrt{5} = 7\sqrt{5}$
$\sqrt{5} \times 7 = 7\sqrt{5}$
$13.23 = 13.23$ ✓

$\sqrt[2]{5} \times \sqrt[2]{3} = \sqrt[2]{5 \times 3}$
$= \sqrt[2]{15}$ $\quad 3.87 = 3.87$ ✓

$3\sqrt[2]{7} \times 2\sqrt[2]{3} = 3 \times 2\sqrt[2]{7 \times 3}$
$= 6\sqrt[2]{21}$

$27.50 = 27.50$ ✓

Multiply Coefficients
Multiply Radicands

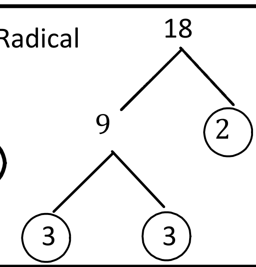
$2 \times 5\sqrt{3} = 10\sqrt{3}$ $\quad 17.32 = 17.32$ ✓

$2\sqrt{5} \times \sqrt{3} = 2\sqrt{15}$ $\quad 7.75 = 7.75$ ✓

$5\sqrt[2]{6} \times 7\sqrt[2]{3} = 5 \times 7\sqrt[2]{6 \times 3}$
$= 35\sqrt[2]{18}$
$= 35 \times 3\sqrt[2]{2}$
$= 105\sqrt[2]{2}$ $\quad 148.49 = 148.49$ ✓

Simplify Radical

$\sqrt[2]{18} = \sqrt[2]{3 \times 3 \times 2}$
$= 3\sqrt[2]{2}$



$\sqrt[2]{5} \times \sqrt[3]{5} = \sqrt[2]{5} \times \sqrt[3]{5} = 5^{\frac{1}{2}} \times 5^{\frac{1}{3}} = 5^{\frac{5}{6}}$

Can only multiply/divide like indexes.
Cannot multiply/divide unlike indexes.
Change Form, Add Exponents

$3.82 = 3.82$ ✓

Distribute

$3(5 + \sqrt{2})$
$15 + 3\sqrt{2}$

$19.24 = 19.24$ ✓

$(5 + \sqrt{7})\sqrt{7}$
$5\sqrt{7} + 7$

$20.23 = 20.23$ ✓

FOIL

$(2 - \sqrt[2]{3}) \times (1 + \sqrt[2]{5})$
$2 + 2\sqrt{5} - 1\sqrt{3} - \sqrt{15}$

$0.867 = 0.867$ ✓

$(2 + \sqrt{3})^2$
$(2 + \sqrt{3})(2 + \sqrt{3})$
...

$\frac{\sqrt[2]{6}}{\sqrt[2]{3}} = \sqrt[2]{\frac{6}{3}}$
$= \sqrt[2]{2}$ $\quad 1.41 = 1.41$ ✓

$\frac{10\sqrt[2]{6}}{2\sqrt[2]{3}} = \frac{10}{2}\sqrt[2]{\frac{6}{3}}$
$= 5\sqrt[2]{2}$ $\quad 7.07 = 7.07$ ✓

$\frac{\sqrt{24}}{\sqrt{8}} = \frac{2\sqrt{6}}{2\sqrt{2}} = \frac{\sqrt{6}}{\sqrt{2}} = \sqrt{\frac{6}{3}} = \sqrt{2}$

$\sqrt{24} = 2\sqrt{6}$
$\sqrt{8} = 2\sqrt{2}$

OR

$\frac{\sqrt{24}}{\sqrt{8}} = \sqrt{\frac{24}{8}} = \sqrt{3}$ $\quad$ Simplify 1st

C11 - 5.3 - Rationalizing the Denominator Notes

$$\frac{5}{\sqrt[2]{3}} = \frac{5 \times \boxed{\sqrt[2]{3}}}{\sqrt[2]{3} \times \boxed{\sqrt[2]{3}}}$$

Multiply the top and bottom by the root in the denominator.
Only the Root!

$$= \frac{5\sqrt[2]{3}}{\sqrt[2]{3 \times 3}}$$

$$= \frac{5\sqrt[2]{3}}{\sqrt[2]{9}}$$

$$= \frac{5\sqrt[2]{3}}{3}$$

$$\frac{5}{\sqrt[2]{3}} = 2.89 = \frac{5\sqrt[2]{3}}{3} \checkmark$$

$$\boxed{\sqrt[2]{3^1} = 3^{\frac{1}{2}} \qquad x^{\frac{m}{n}} = \sqrt[n]{x^m}}$$

$$\sqrt{3} \times \sqrt{3} = 3 \qquad 3^{\frac{1}{2}} \times 3^{\frac{1}{2}} = 3^1 \qquad \frac{1}{2} + \frac{1}{2} = 1$$

Add Exponents

$$\frac{5}{2 - \sqrt[2]{6}} = \frac{5 \times \boxed{(2 + \sqrt[2]{6})}}{(2 - \sqrt[2]{6}) \times \boxed{(2 + \sqrt[2]{6})}}$$

Multiply the top/bottom by **Conjugate** of denominator.

$$(2 - \sqrt[2]{6}) \times (2 + \sqrt[2]{6})$$

$$4 + 2\sqrt{6} - 2\sqrt{6} - \sqrt{36}$$

$$4 + \cancel{2\sqrt{6} - 2\sqrt{6}} - \sqrt{36}$$

$$4 - \sqrt{36}$$

$$-2$$

$$(a + b)(a - b) =$$

$$a^2 \cancel{- ab + ab} - b^2 =$$

$$a^2 - b^2$$

$$\boxed{F\cancel{O}\cancel{I}L}$$

$$= \frac{10 + 5\sqrt[2]{6}}{-2}$$

Distribution
Foil

$$\frac{5}{2 - \sqrt[2]{6}} = -11.12 = \frac{10 + 5\sqrt[2]{6}}{-2} \checkmark$$

$$\frac{4}{\sqrt[2]{5} + \sqrt[2]{3}} = \frac{4 \times (\sqrt[2]{5} - \sqrt[2]{3})}{(\sqrt[2]{5} + \sqrt[2]{3}) \times (\sqrt[2]{5} - \sqrt[2]{3})}$$

Conjugate

$$= \frac{4\sqrt[2]{5} - 4\sqrt[2]{3}}{5 - 3}$$

$$= \frac{4\sqrt[2]{5} - 4\sqrt[2]{3}}{2} \boxed{\frac{\div 2}{\div 2}}$$

$$= 2\sqrt{5} - 2\sqrt{3}$$

Simplify, by dividing the top and bottom by 2.

$$\frac{4}{\sqrt[2]{5} + \sqrt[2]{3}} = 1.01 = 2\sqrt{5} - 2\sqrt{3} \checkmark$$

$$\frac{5}{\sqrt[3]{3}} = \frac{5 \times \sqrt[3]{3} \times \sqrt[3]{3}}{\sqrt[3]{3} \times \sqrt[3]{3} \times \sqrt[3]{3}}$$

Multiply the top and bottom by the cube root of the denominator twice. (Or three times for a fourth root etc.)

$$= \frac{5\sqrt[3]{9}}{3}$$

$$\boxed{\sqrt[3]{3} = 3^{\frac{1}{3}} \qquad x^{\frac{m}{n}} = \sqrt[n]{x^m}}$$

$$\frac{5}{\sqrt[3]{3}} = 3.47 = \frac{5\sqrt[3]{9}}{3} \checkmark$$

$$\sqrt[3]{3} \times \sqrt[3]{3} \times \sqrt[3]{3} = 3 \qquad 3^{\frac{1}{3}} \times 3^{\frac{1}{3}} \times 3^{\frac{1}{3}} \times = 3^1 \qquad \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

C11 - 5.4 - Solving Radical Equations/Restrictions Notes

$$\sqrt{x+2} = 4$$
$$\left(\sqrt{x+2}\right)^2 = (4)^2$$
$$x + 2 = 16$$
$$x = 14$$

Square Both sides (Brackets)

Check Answer:

$$\sqrt{x+2} = 4$$
$$\sqrt{14+2} = 4$$
$$\sqrt{16} = 4$$
$$4 = 4$$

LHS=RHS

$$x + 2 \geq 0$$
$$-2 \quad -2$$
$$x \geq -2$$

Restrictions: Set underneath root ≥ 0 and solve.

Isolate Root

$$\sqrt{x+2} + 1 = 4$$
$$-1 \quad -1$$
$$\sqrt{x+2} = 3$$
$$\left(\sqrt{x+2}\right)^2 = (3)^2$$
$$x + 2 = 9$$
$$-2 \quad -2$$
$$x = 7$$

$$\sqrt{x+2} + 1 = 4$$
$$\sqrt{7+2} + 1 = 4$$
$$\sqrt{9} + 1 = 4$$
$$3 + 1 = 4$$
$$4 = 4$$

$$x + 2 \geq 0$$
$$x \geq -2$$

$$\sqrt{x+3} = \sqrt{2x+5}$$
$$\left(\sqrt{x+3}\right)^2 = \left(\sqrt{2x+5}\right)^2$$
$$x + 3 = 2x + 5$$
$$-x \quad -x$$
$$3 = x + 5$$
$$-5 \quad -5$$
$$x = -2$$

$$\sqrt{x+3} = \sqrt{2x+5}$$
$$\sqrt{-2+3} = \sqrt{2(-2)+5}$$
$$\sqrt{1} = \sqrt{1}$$

$$x + 3 \geq 0 \qquad 2x + 5 \geq 0$$
$$x \geq -3 \qquad x \geq -\frac{5}{2}$$

$$\sqrt{x+3} - x - 1 = 0$$
$$\sqrt{x+3} = x + 1$$
$$\left(\sqrt{x+3}\right)^2 = (x+1)^2$$
$$x + 3 = (x+1)(x+1)$$
$$x + 3 = x^2 + 2x + 1$$
$$0 = x^2 + x - 2$$
$$0 = (x+2)(x-1)$$

$$x + 2 = 0 \qquad x - 1 = 0$$
$$\cancel{x = -2} \qquad x = 1$$

$$\sqrt{x+3} = x + 1$$
$$\sqrt{-2+3} = -2 + 1$$
$$1 \neq -1$$

$$\sqrt{x+3} = x + 1$$
$$\sqrt{1+3} = 1 + 1$$
$$2 = 2$$

$$x + 3 \geq 0$$
$$x \geq -3$$

Square Both Sides First

$$2\sqrt{x+3} = 6$$
$$\left(2\sqrt{x+3}\right)^2 = (6)^2$$
$$4(x+3) = 36$$
$$\frac{4(x+3)}{4} = \frac{36}{4}$$
$$x + 3 = 9$$
$$-3 \quad -3$$
$$x = 6$$

Divide First

$$2\sqrt{x+3} = 6$$
$$\frac{2\sqrt{x+3}}{2} = \frac{6}{2}$$
$$\sqrt{x+3} = 3$$
$$\left(\sqrt{x+3}\right)^2 = (3)^2$$
$$x + 3 = 9$$
$$-3 \quad -3$$
$$x = 6$$

$\sqrt{x} = -5$
No Solution

$\sqrt{x+99} = -5$
No Solution

A Square/Even Root Can't Equal a Negative

$$\sqrt{x+1} = \sqrt{x} + 1$$
$$\left(\sqrt{x+1}\right)^2 = (\sqrt{x} + 1)^2$$
$$x + 1 = (\sqrt{x}+1)(\sqrt{x}+1)$$
$$x + 1 = x + \sqrt{x} + \sqrt{x} + 1$$
$$0 = 2\sqrt{x}$$
$$(0)^2 = (2\sqrt{x})^2$$
$$0 = 4x$$
$$x = 0$$

$$\cancel{x + 1 \geq 0}$$
$$\cancel{x \geq -1}$$
$$x \geq 0$$

More Restrictive

$$\sqrt{x+1} = \sqrt{x} + 1$$
$$\sqrt{0+1} = \sqrt{0} + 1$$
$$1 = 1$$

$$\sqrt{x-5} - \sqrt{x-8} = 1$$
$$\sqrt{x-5} = \sqrt{x-8} + 1$$
$$\left(\sqrt{x-5}\right)^2 = \left(\sqrt{x-8} + 1\right)^2$$
$$x - 5 = \left(\sqrt{x-8} + 1\right)\left(\sqrt{x-8} + 1\right)$$
$$x - 5 = x - 8 + 2\sqrt{x-8} + 1$$
$$1 = \sqrt{x-8}$$
$$(1)^2 = \left(\sqrt{x-8}\right)^2$$
$$1 = x - 8$$
$$x = 9$$

$$\sqrt{x-5} - \sqrt{x-8} = 1$$
$$\sqrt{9-5} - \sqrt{9-8} = 1$$
$$\sqrt{4} - \sqrt{1} = 1$$
$$2 - 1 = 1$$

$$x - 8 \geq 0$$
$$x \geq 8$$

$$\cancel{x - 5 \geq 0}$$
$$\cancel{x \geq 5}$$

$$(2x+3)^2 = (x+7)^2$$
$$\sqrt{(2x+3)^2} = \sqrt{(x+7)^2}$$
$$2x + 3 = x + 7$$
$$x = 4$$

Square Root Both Sides

$$(2x+3)^2 = (x+7)^2$$
$$(2(4)+3)^2 = ((4)+7)^2$$
$$121 = 121$$

C11 - 6.1 - Simplifying Rationals Notes

Simplify.

$$\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

$$\boxed{\frac{2}{4} = \frac{{}^{1}\cancel{2}}{\cancel{2} \times 2} = \frac{1}{2}}$$

$$\frac{6x^2}{2x} = \frac{6 \times x \times \cancel{x}}{2 \times \cancel{x}} = 3x$$

$$\frac{2x+4}{x+2} = \frac{2\cancel{(x+2)}}{\cancel{x+2}} = 2$$

Factor, Simplify.

$$\frac{x^2+5x+6}{x+3} = \frac{(x+2)\cancel{(x+3)}}{\cancel{x+3}} = (x+2)$$

$$\frac{x+3}{x^2-9} = \frac{\cancel{x+3}}{\cancel{(x+3)}(x-3)} = \frac{1}{x-3}$$

$$\frac{1}{2-x} = \frac{1}{-(x-2)} = \frac{-1}{x-2}$$

$2-x$ $-(-2+x)$ $-(x-2)$	$GCF = -1$ Rearrange order of terms	OR	$2-x$ $-(x-2)$

$$\frac{x-4}{4-x} = \frac{x-4}{-(-4+x)} = \frac{x-4}{-(x-4)} = -1$$

$$\frac{x^2-3x-4}{x^2-1} = \frac{(x-4)\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} = \frac{x-4}{x-1}$$

$$\frac{x^2-5x+6}{x+2} = \frac{(x-2)(x-3)}{x+2}$$

Cannot Simplify

C11 - 6.2 - Restrictions Notes

$\frac{8}{0} = und$ Can't Divide by Zero

$\boxed{\text{Restrictions: Set Denominator} \neq 0 \; and \; solve}$

$\frac{1}{x}$ $x \neq 0$

$\frac{2}{x+3}$

$x + 3 \neq 0$

$x \neq -3$

$\frac{x}{2}$ No Restrictions

$\frac{3}{x^2 + 5x + 6}$

$x^2 + 5x + 6 \neq 0$
$(x+3)(x+2) \neq 0$

$x + 3 \neq 0$ $x + 2 \neq 0$

$x \neq -3$ $x \neq -2$

$\frac{3}{2x^2 + x - 1}$

$2x^2 + x - 1 \neq 0$
$(2x-1)(x+1) \neq 0$

$2x - 1 \neq 0$ $x + 1 \neq 0$

$x \neq \frac{1}{2}$ $x \neq -1$

$\frac{5}{x^2 - 4}$

$x^2 - 4 \neq 0$
$(x+2)(x-2) \neq 0$

$x + 2 \neq 0$ $x - 2 \neq 0$

$x \neq -2$ $x \neq 2$

$\frac{2}{x^2 - 2x}$

$x^2 - 2x \neq 0$
$x(x-2) \neq 0$

$x \neq 0$ $x - 2 \neq 0$

$x \neq 2$

$\frac{1}{x^2 + 1}$

$x^2 + 1 \neq 0$
$x^2 \neq -1$
$\sqrt{x^2} \neq \sqrt{-1}$

Can' t even root a negative

No Restrictions

C11 - 6.3 - Multiplying Dividing Rationals Notes

$\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$

Multiply Tops
Multiply Bottoms

$\frac{3}{8}\times\frac{4}{9}=\frac{3\times 4}{8\times 9}=\frac{\cancel{3}\times\cancel{2}\times\cancel{2}}{2\times\cancel{2}\times\cancel{2}\times\cancel{3}\times 3}=\frac{1}{6}$

$\frac{3}{8}\times\frac{4}{9}=\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{8}}}\times\frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{9}}}=\frac{1}{6}$

$\frac{a}{2}\div\frac{1}{3}=$

$\frac{a}{2}\times\frac{3}{1}=\frac{3a}{2}$

Flip and multiply

$\frac{1}{\cancel{x+2}}\times\cancel{(x+2)}=1$ $\quad x+2\neq 0$, $x\neq -2$

Restrictions

$\frac{1}{(x+2)\cancel{(x+3)}}\times\cancel{(x+3)}=\frac{1}{x+2}$

$x+2\neq 0$, $x\neq -2$ $\quad x+3\neq 0$, $x\neq -3$

$\frac{x+2}{x+3}\times\frac{2}{x+2}=$

$\frac{2\cancel{(x+2)}}{(x+3)\cancel{(x+2)}}=\frac{2}{x+3}$

$x+2\neq 0$, $x\neq -2$ $\quad x+3\neq 0$, $x\neq -3$

$\frac{2}{x+1}\times(x+1)(x+2)=$

$x+1\neq 0$, $x\neq -1$

$\frac{2}{\cancel{x+1}}\times\cancel{(x+1)}(x+2)=$

$2(x+2)$

Think what cancels and
what are you left with

$\frac{x+1}{x^2-5x+6}\times\frac{x-2}{x^2+5x+4}=$

$x-2\neq 0$, $x\neq 2$ $\quad x+1\neq 0$, $x\neq -1$ $\quad x-3\neq 0$, $x\neq 3$ $\quad x+4\neq 0$, $x\neq -4$

$\frac{x+1}{(x-3)(x-2)}\times\frac{x-2}{(x+4)(x+1)}=$ Factor

$\frac{\cancel{(x-2)}\cancel{(x+1)}}{(x-3)\cancel{(x-2)}(x+4)\cancel{(x+1)}}=\frac{1}{(x-3)(x+4)}$

$x\neq 2,-1,3,-4$

$\frac{x-4}{x+5}\div\frac{x-4}{x-3}=$

$x+5\neq 0$, $x\neq -5$ $\quad x-3\neq 0$, $x\neq 3$ $\quad x-4\neq 0$, $x\neq 4$

$\frac{x-4}{x+5}\times\frac{x-3}{x-4}=$

Flip and multiply

$\frac{(x-4)(x-3)}{(x+5)(x-4)}=\frac{x-3}{x+5}$

$x\neq 3,-5,4$

$\frac{x-7}{x+4}\div\frac{x^2-2x-15}{x^2-x-20}$

$x+4\neq 0$, $x\neq -4$ $\quad x-5\neq 0$, $x\neq 5$ $\quad x+3\neq 0$, $x\neq -3$

$\frac{x-7}{x+4}\div\frac{(x-5)(x+3)}{(x-5)(x+4)}=$

Factor 1st

$x\neq -4,-3,5$

$\frac{x-7}{x+4}\times\frac{(x-5)(x+4)}{(x-5)(x+3)}=$

$\frac{(x-7)\cancel{(x-5)}\cancel{(x+4)}}{\cancel{(x+4)}\cancel{(x-5)}(x+3)}=\frac{x-7}{x+3}$

C11 - 6.4 - LCD Notes

Find LCD

$\frac{1}{2}+\frac{1}{3}=$ $\qquad LCD = 6 \qquad$ $\frac{\square}{6}+\frac{\square}{6}=$ $\qquad \frac{3\times 1}{3\times 2}+\frac{1\times 2}{3\times 2}= \qquad$ $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$

$\frac{\square}{2}+\frac{\square}{3}=$

$LCD = 2\times 3$

$\frac{\square}{a}+\frac{\square}{b}=$

$LCD = ab$

$\frac{\square}{2}+\frac{\square}{6}=$

$LCD = 6$

$\frac{\square}{2}+\frac{\square}{2\times 3}=$

$LCD = 2\times 3$

$\frac{1}{a}+\frac{1}{ab}=$

$LCD = ab$

$\frac{\square}{a}+\frac{\square}{bc}=$

$LCD = abc$

$\frac{1}{a^2}+\frac{1}{a}=$

$LCD = a^2$

$\frac{\square}{ab}+\frac{\square}{cd}=$

$LCD = abcd$

$\frac{\square}{2}+\frac{\square}{2+1}=$

$LCD = 2\times(2+1)$

$\frac{\square}{a}+\frac{\square}{a+1}=$

$LCD = a(a+1)$

$\frac{\square}{2+4}+\frac{\square}{2+1}$

$LCD = (2+4)(2+1)$

$\frac{\square}{a+1}+\frac{\square}{a+2}$

$LCD = (a+1)(a+2)$

$\frac{\square}{a}+\frac{\square}{b}=\frac{\square}{c}$

$LCD = abc$

$\frac{\square}{a}+5=\frac{\square}{a+1}$

$LCD = a(a+1)$

$\frac{1}{a}+\frac{1}{a+1}=\frac{1}{a+2}$

$LCD = a(a+1)(a+2)$

C11 - 6.4 - Adding Subtracting Rationals Notes

$$\frac{1}{2}+\frac{1}{3}=$$
$$\frac{3\times 1}{3\times 2}+\frac{1\times 2}{3\times 2}=$$
$$\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$$

$LCD\ =\ 6$

LCD
Do to top, do to bottom
Add/subtract

$$\frac{x}{2}+\frac{1}{2}=\frac{x+1}{2}$$

$LCD\ =2$

$$\frac{x}{2}-\frac{1}{6}=$$
$$\frac{3\times x}{3\times 2}-\frac{1}{6}=$$
$$\frac{3x}{6}-\frac{1}{6}=\frac{3x-1}{6}$$

$LCD\ =6$

$$\frac{3}{2}-\frac{x+2}{2}=$$
$$\frac{3-(x+2)}{2}=$$
$$\frac{3-x-2}{2}=\frac{1-x}{2}$$

$LCD\ =2$

Don't forget to distribute the negative

Factoring out a negative

$$\frac{1}{x-2}+\frac{1}{2-x}$$
$$\frac{1}{x-2}+\frac{1}{-(x-2)}$$
$$\frac{1}{x-2}-\frac{1}{(x-2)}$$

$$\frac{x}{x+2}+\frac{1}{x+2}=\frac{x+1}{x+2}$$

$LCD\ =x+2$

$x+2\neq 0$
$x\neq -2$

$$\frac{1}{x+2}+\frac{1}{(x+2)(x+3)}=$$
$$\frac{x+3}{x+3}\times\frac{1}{x+2}+\frac{1}{(x+2)(x+3)}=$$
$$\frac{x+3}{(x+2)(x+3)}+\frac{1}{(x+2)(x+3)}=$$
$$\frac{x+3+1}{(x+2)(x+3)}=\frac{x+4}{(x+2)(x+3)}$$

$LCD=(x+2)(x+3)$

$\boldsymbol{x+2\neq 0}$
$\boldsymbol{x\neq -2}$

$x+3\neq 0$
$x\neq -3$

$$\frac{1}{x}+\frac{3}{(x+2)}=$$
$$\frac{x+2}{x+2}\times\frac{1}{x}+\frac{3}{(x+2)}\times\frac{x}{x}=$$
$$\frac{x+2}{x(x+2)}+\frac{3x}{x(x+2)}=$$
$$\frac{x+2+3x}{x(x+2)}=\frac{5x+2}{x(x+2)}$$

$LCD=x(x+2)$

$x\neq 0$

$\boldsymbol{x+2\neq 0}$
$\boldsymbol{x\neq -2}$

$$\frac{x+2}{x^2+5x+6}+\frac{1}{x+3}=$$
$$\frac{\cancel{x+2}}{\cancel{(x+2)}(x+3)}+\frac{1}{x+3}=$$
$$\frac{1}{x+3}+\frac{1}{x+3}=$$
$$\frac{1+1}{x+3}=\frac{2}{x+3}$$

Simplify 1st

$LCD=(x+3)$

$\boldsymbol{x+2\neq 0}$
$\boldsymbol{x\neq -2}$

$x+3\neq 0$
$x\neq -3$

$$\frac{x}{x^2-4}-\frac{2}{x^2-4}=$$
$$\frac{x}{(x-2)(x+2)}-\frac{2}{(x-2)(x+2)}=$$
$$\frac{\cancel{x-2}}{\cancel{(x-2)}(x+2)}=\frac{1}{x+2}$$

$LCD=(x+2)(x-2)$

Simplify at end

$\boldsymbol{x+2\neq 0}$
$\boldsymbol{x\neq -2}$

$x+3\neq 0$
$x\neq -3$

C11 - 6.5 - Rational Equations Notes

Solve for x.

Get an LCD then Multiply by the LCD

$$\frac{x}{2}+\frac{1}{4}=\frac{3}{4}$$
$$\frac{2\times x}{2\times 2}+\frac{1}{4}=\frac{3}{4}$$
$$\frac{2x}{4}+\frac{1}{4}=\frac{3}{4}$$
$$\left(\frac{2x}{4}+\frac{1}{4}=\frac{3}{4}\right)\times LCD$$
$$2x+1=3$$
$$-1 \quad -1$$
$$2x=2$$
$$\frac{2x}{2}=\frac{2}{2}$$
$$x=1$$

OR!

Multiply by the $LCD=4$

$$\frac{x}{2}+\frac{1}{4}=\frac{3}{4}$$
$$\left(\frac{x}{2}+\frac{1}{4}=\frac{3}{4}\right)\times 4$$
$$\frac{4x}{2}+\frac{4}{4}=\frac{12}{4}$$
$$2x+1=3$$
$$-1 \quad -1$$
$$2x=2$$
$$\frac{2x}{2}=\frac{2}{2}$$
$$x=1$$

OR!

$$\left(\frac{x}{2}+\frac{1}{4}=\frac{3}{4}\right)\times LCD:4$$
$$2x+1=3$$
$$2x=2$$
$$x=1$$

Instead of actually multiplying by the LCD we are going to multiply and simplify at the same time.

Or Add Fractions/Cross Multiply

$$\frac{2}{x+2}+3=\frac{11}{x+2}$$
$$\left(\frac{2}{x+2}+3=\frac{11}{x+2}\right)\times LCD=(x+2)$$
$$\frac{2(x+2)}{x+2}+3(x+2)=\frac{11(x+2)}{x+2}$$
$$2+3(x+2)=11$$
$$2+3x+6=11$$
$$3x=3$$
$$x=1$$

$x+2\neq 0$

$x\neq -2$

OR!

$$\left(\frac{2}{x+2}+3=\frac{11}{x+2}\right)\times LCD=(x+2)$$
$$2+3(x+2)=11$$
$$2+3x+6=11$$
$$3x=3$$
$$x=1$$

$$\frac{2}{x+2}=\frac{4}{x-3}$$
$$\left(\frac{2}{x+2}=\frac{4}{x-3}\right)\times LCD=(x+2)(x-3)$$
$$2(x-3)=4(x+2)$$
$$2x-6=4x+8$$
$$-14=2x$$
$$x=-7$$

$x+2\neq 0$ $x-3\neq 0$

$x\neq -2$ $x\neq 3$

OR!

Cross Multiply

$$\frac{2}{x+2}=\frac{4}{x-3}$$
$$2(x-3)=4(x+2)$$
$$2x-6=4x+8$$
$$-14=2x$$
$$x=-7$$

$$\frac{15}{x^2+5x+6}-\frac{2}{x+2}=\frac{1}{x+2}$$

Factor

$$\left(\frac{15}{(x+2)(x+3)}-\frac{2}{x+2}=\frac{1}{x+2}\right)\times LCD=(x+2)(x+3)$$
$$15-2(x+3)=1(x+3)$$
$$15-2x-6=x+3$$
$$9=3x$$
$$x=3$$

$x+2\neq 0$ $x+3\neq 0$

$x\neq -2$ $x\neq -3$

$$\frac{1}{x+1}+2=\frac{3}{x+2}$$
$$\left(\frac{1}{x+1}+2=\frac{3}{x+2}\right)\times LCD=(x+1)(x+2)$$
$$1(x+2)+2(x+1)(x+2)=3(x+2)$$
$$x+2+2x^2+6x+4=3x-6$$
$$2x^2+4x+12=0$$
$$\frac{2x^2}{2}+\frac{4x}{2}+\frac{12}{2}=\frac{0}{2}$$
$$x^2+2x+6=0$$

$x+1\neq 0$ $x+2\neq 0$

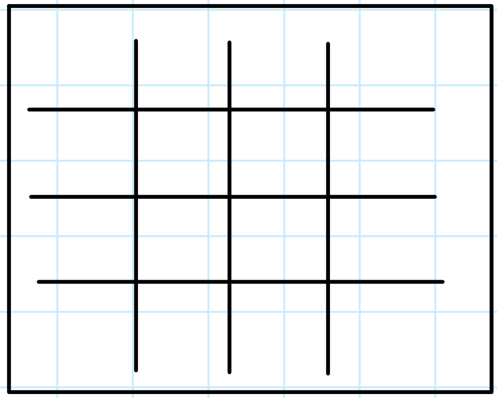
$x\neq -1$ $x\neq -2$

Quadratic Formula: $No\ Solution$ $b^2-4ac<0$

C11 - 6.6 - Hoses filling Pool Notes

Two hoses together fill a pool in 2 hours. If only hose A is used, the pool fills in 3 hours. How long would it take to fill the pool if only hose B were used?

	Amount	Time	Rate
Hose A	1 pool	3 hours	$\frac{1\ pool}{3\ hours}$
Hose B	1 pool	x hours	$\frac{1\ pool}{x\ hours}$
Together	1 pool	2 hours	$\frac{1\ pool}{2\ hours}$



$v = \frac{d}{t}$ $\qquad r = \frac{a}{t}$

$$\frac{1}{3} + \frac{1}{x} = \frac{1}{2}$$

$$\left(\frac{1}{3} + \frac{1}{x} = \frac{1}{2}\right) \times 6x$$

$$2x + 6 = 3x$$

$$-2x \qquad -2x$$

$$6 = x$$

It will take 6 hours.

Add Rates Together to equal the rates together

C11 - 6.7 - Sum of Reciprocals Consecutive Integers Notes

The sum of the reciprocals of two consecutive integers is $\frac{5}{6}$. What are the integers?

$Let\ "x"\ =\ 1st\ \#$
$Let\ x+1 = 2nd\ \#$

$$\frac{1}{x}+\frac{1}{(x+1)}=\frac{5}{6}$$

Restrictions

$x \neq 0$ $x \neq -1$

$$\frac{1}{x}+\frac{1}{(x+1)}=\frac{5}{6}$$

$$\left(\frac{1}{x}+\frac{1}{(x+1)}=\frac{5}{6}\right)\times LCD$$

$LCD: 6x(x+1)$

$$6(x+1)+6x = 5x(x+1)$$
$$6x+6+6x = 5x^2+5x$$
$$0 = 5x^2-7x-6$$
$$0 = (5x^2-10x)+(3x-6)$$
$$0 = 5x(x-2)+3(x-2)$$
$$0 = (5x+3)(x-2)$$

$x = 2$

$\cancel{x = -\frac{3}{5}}$ $x = 2$

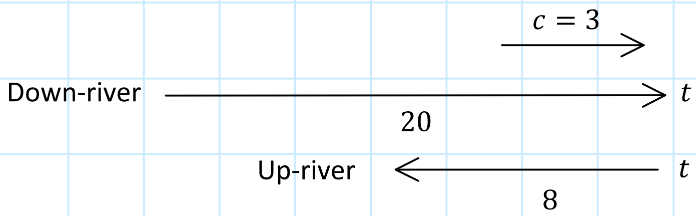
Reject

$1st\ number = 2$
$2nd\ number = 3$

C11 - 6.8 - Speed Distance Time Notes

Mary paddles down river 20km with a current of 3km/h. It takes her the same time to paddle up river 8km. What is the speed of the boat?

	Speed	Distance	Time
Down-river	$v_b + 3$	20	t
Up-river	$v_b - 3$	8	t



$Let\ v_b = velocity\ of\ boat$
$t = time$

Down river

$$v = \frac{d}{t}$$
$$v_b + 3 = \frac{20}{t}$$
$$v_b = \frac{20}{t} - 3$$

Up river

$$v = \frac{d}{t}$$
$$v_b - 3 = \frac{8}{t}$$
$$v_b = \frac{8}{t} + 3$$

$$v = \frac{d}{t}$$

Isolation

Substitution

$$v_b = v_b$$
$$\frac{20}{t} - 3 = \frac{8}{t} + 3$$

Solve

$$\left(\frac{20}{t} - 3 = \frac{8}{t} + 3\right) \times LCD{:}\, t$$
$$20 - 3t = 8 + 3t$$
$$12 = 6t$$
$$t = 2s$$

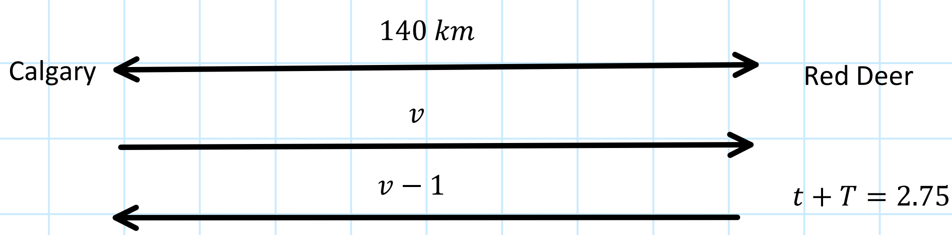
$$v_b = \frac{8}{t} + 3$$
$$v_b = \frac{8}{2} + 3$$
$$v_b = 7\frac{km}{hr}$$

Substitution LCD $= t$

Solve

Mike travels one km per hour faster and completes 4 km 1 minute faster than Sue? How fast are they travelling?

$let\ v = speed$
$let\ t = time\ C \rightarrow R$
$let\ T = time\ R \rightarrow C$



$$v = \frac{d}{t}$$
$$v = \frac{140}{t}$$

$$v = \frac{d}{t}$$
$$v - 1 = \frac{140}{T}$$

$$T = 2.75 - t$$

$$\frac{140}{t} - 1 = \frac{140}{2.75 - t}$$

LCD
Quadform

$$v = \frac{140}{1.30764}$$
$$t = 1.30764$$
$$v = 107.06$$
$$T = 1.44236$$

C11 - 7.1 - Absolute Value: $|x|$ Notes

$|2| =$
$|2| = 2$

$|-3| = 3$

$|2-4| =$
$|-2| = 2$

$|3| - |-5| =$
$3 - 5 = -2$

$-|3| = -3$

$-|-5| =$
$-(5) = -5$

Do whatever is inside the absolute value, then make it positive.

Solve algebraically.

$|x| = 4$

"+" case:

$\cancel{+(x) = 4}$
$x = 4$

Distribute a positive into the absolute value

$|x| = 4$
$|4| = 4$
$4 = 4$ ✓

"−" case:

$-(x) = 4$
$x = -4$

Distribute a negative into the absolute value

$|x| = 4$
$|-4| = 4$
$4 = 4$ ✓

$\boxed{|x| = -6 \quad \text{Impossible.}}$

Check your answer. (Left Hand Side LHS = RHS Right Hand Side)

$|x-2| = 2$

"+" case:

$+(x-2) = 2$
$x - 2 = 2$
$x = 4$

$|x-2| = 2$
$|4-2| = 2$
$|2| = 2$ ✓

"−" case:

$-(x-2) = 2$
$-x + 2 = 2$
$-x = 0$
$x = 0$

$|x-2| = 2$
$|0-2| = 2$
$|-2| = 2$ ✓

$2|x-2| = 6$

"+" case:

$+2(x-2) = 6$
$2x - 4 = 6$
$2x = 10$
$x = 5$

$2|x-2| = 6$
$2|5-2| = 6$
$2|3| = 6$ ✓

"−" case:

$-2(x-2) = 6$
$-2x + 4 = 6$
$-2x = 2$
$x = -1$

$2|x-2| = 6$
$2|-1-2| = 6$
$2|-3| = 6$ ✓

$|x^2 - 1| = x - 1$

"+" case:

$+(x^2-1) = x - 1$
$x^2 - x = 0$
$x(x-1) = 0$

$x = 0$

$x - 1 = 0$
$\cancel{x = 1}$

"−" case:

$-(x^2-1) = x - 1$
$-x^2 + 1 = x - 1$
$x^2 + x - 2 = 0$
$(x+2)(x-1) = 0$

$x - 1 = 0$
$\cancel{x = 1}$

$x + 2 = 0$
$\cancel{x = -2}$

$|x^2-1| = x - 1$
$|1^2 - 1| = 1 - 1$
$|0| = -0$ ✓

$|x^2-1| = x - 1$
$|0^2 - 1| = 0 - 1$
$|-1| = -1$ ✗

$|x^2-1| = x - 1$
$|(-2)^2 - 1| = -2 - 1$
$|4-1| = -2 - 1$
$|3| = -3$ ✗

C11 - 7.1 - Absolute Value Inequalities: $|x|$ Notes

$|x| \geq 2$

"+" case:

$+(x) \geq 2$

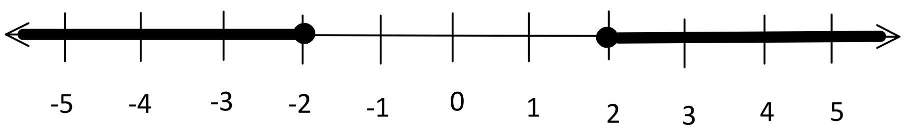
$x \geq 2$

"−" case:

$-(x) \geq 2$

$x \leq -2$

Divide by a negative, change direction of sign.



$\geq, \leq =$ ●

Shade greater than two, and less than negative two.

Check your answer. Test values in shaded region.

$|3| \geq$
$|3| \geq 3$
$3 \geq 2$

$|-3| \geq$
$|-3| \geq$
$3 \geq 2$

$|x-3| < 2$

"+" case:

$+(x-3) < 2$

$x - 3 < 2$

$x < 5$

"−" case:

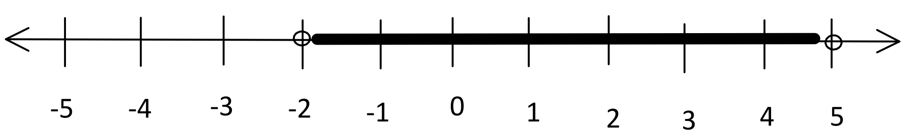
$-(x-3) < 2$

$-x + 3 < 2$

$-x < 2$

$x > -2$

Divide by a negative, change direction of sign.



$>, < =$ ○

Shade less than five, and greater than negative two.

Check your answer. Test values in shaded region.

$|3| \geq$
$|3| \geq 3$
$3 \geq 2$

$|-3| \geq$
$|-3| \geq$
$3 \geq 2$

C11 - 7.2 - $y = |x + c|$ Piecewise Linear Absolute Value Notes

Graphing Absolute Values

$y = |x + 2|$

"+" case:

$y_1 = +(x + 2)$

$y_1 = x + 2$

"−" case:

$y_2 = -(x + 2)$

$y_2 = -x - 2$

Distribute a positive into the absolute value

Distribute a negative into the absolute value

If already negative combine

$y = |x + 2|$

Table of Values

x	y
-5	-3
-3	-1
-2	0
-1	1
0	2

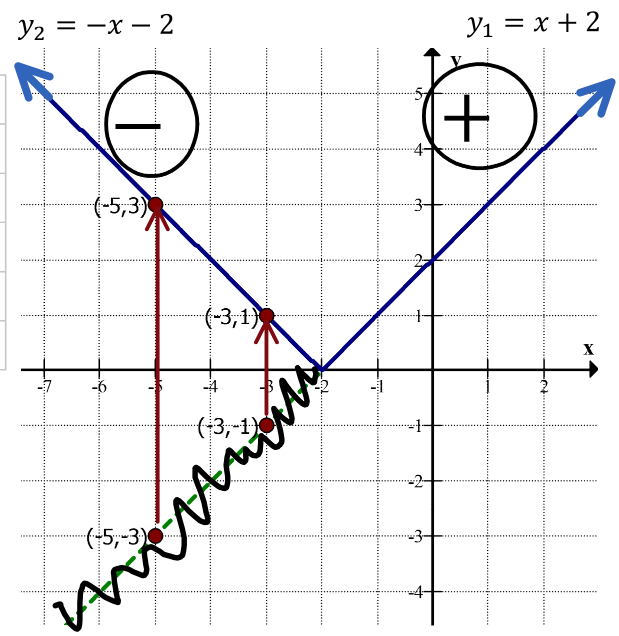
$y = x + 2$

→

x	y
-5	3
-3	1
-2	0
-1	1
0	2

$y = |x + 2|$

Pt.
(-5,2)
(-3,1)
(-2,0)
(-1,1)
(0,2)



Set inside absolute value = 0 and solve

TOV

Vertex: $(-2,0)$

Notice the graph of $y = |x + 2|$ is the graph of $y = x + 2$ and $y = -x - 2$ without any negative y values. Transfer any negative y value to a positive y value.

Piecewise function: $y = \begin{cases} x + 2, if & x \geq -2 \\ -x - 2, if & x < -2 \end{cases}$ $\quad y = \begin{cases} \text{"+" case,} & \text{Domain of "+" case} \\ \text{"−" case,} & \text{Domain of "−" case} \end{cases}$

Notice: The domain of the negative case is not equal to.

Domain of positive case:

$x + 2 \geq 0$

$-2 \quad -2$

$x \geq -2$

Set what is inside the absolute value greater than or equal to zero.

Domain of negative case:

$x + 2 < 0$

$-2 \quad -2$

$x < -2$

Set what is inside the absolute value less than zero.

C11 - 7.3 - $|x| = c$ Equations Absolute Value Notes

Solve algebraically

$|x+2| = 4$

"+" case:

$$+(x+2) = 4$$
$$x+2 = 4$$
$$x = 2$$

"−" case:

$$-(x+2) = 4$$
$$-x-2 = 4$$
$$-x = 6$$
$$x = -6$$

Check your answer.

$$|x+2| = 4$$
$$|2+2| = 4$$
$$|4| = 4 \checkmark$$

$$|-6+2| = 4$$
$$|-4| = 4$$
$$|-4| = 4 \checkmark$$

Solve graphically.

$|x+2| = 4$ Left hand side (LHS) = Right hand side (RHS)

$y = |x+2|$

$y = 4$

y=Left hand side (LHS)

y=Right hand side (RHS)

"+" case:

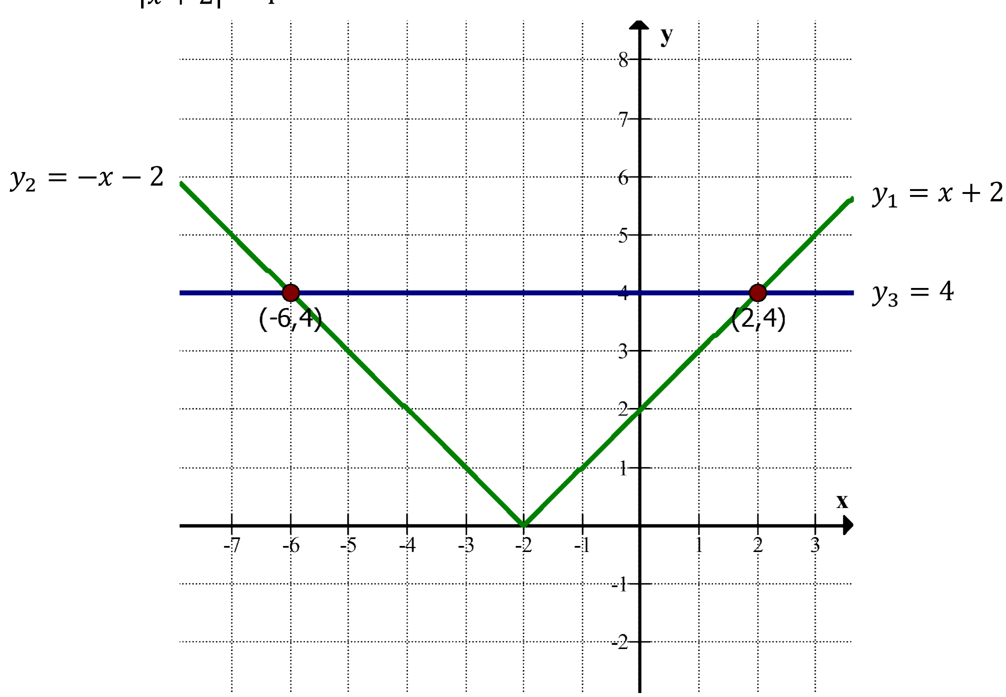
$$y_1 = +(x+2)$$
$$y_1 = x+2$$

"−" case:

$$y_2 = -(x+2)$$
$$y_2 = -x-2$$

$$y_3 = 4$$

$|x+2| = 4$



C11 - 7.4 - Quadratic Absolute Value Notes

$y = |x^2 - 4|$

"+" case:

$y_1 = +(x^2 - 4)$

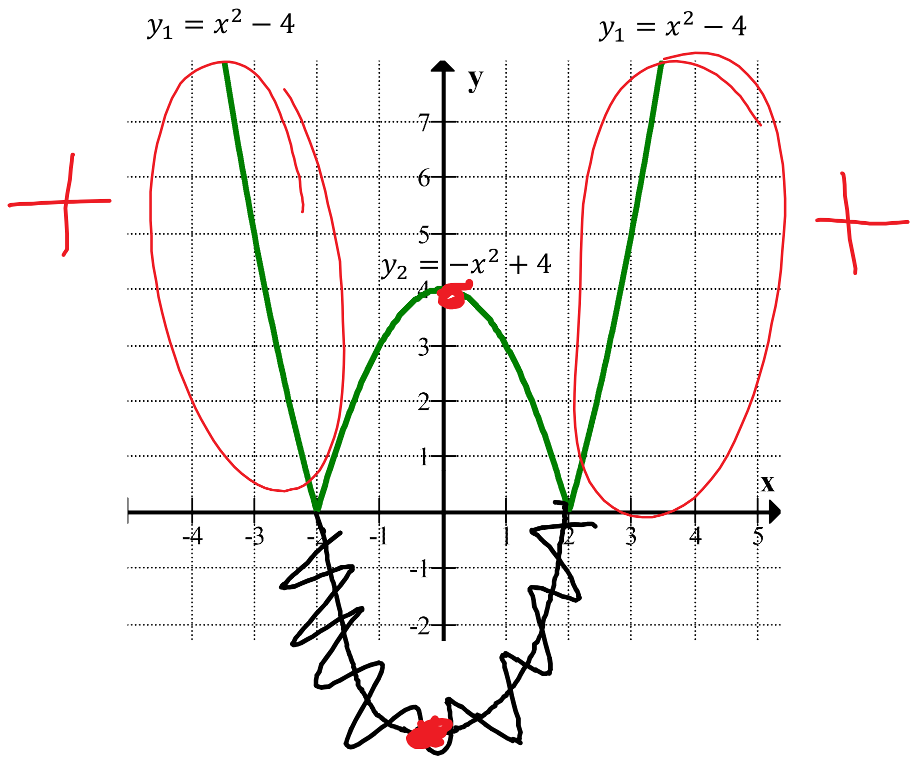
$y_1 = x^2 - 4$

"−" case:

$y_2 = -(x^2 - 4)$

$y_2 = -x^2 + 4$

$y = |x^2 - 4|$



Notice the graph of $y = |x^2 - 4|$ is the graph of $y_1 = x^2 - 4$ less than two and greater than two and is the graph of $y_2 = -x^2 + 4$ less than two and greater than negative two.

Piecewise function:

$$y = \begin{cases} x^2 - 4, & if\ x \geq 2, x \leq -2 \\ -x^2 + 4, & if\ -2 < x < 2 \end{cases}$$

C11 - 7.5 - Quadratic Absolute Value Equations Notes

Solve algebraically.

$|x^2 - 4| = x + 2$

"+" case:

$$+(x^2-4) = x+2$$
$$x^2 - 4 = x + 2$$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x = 3, -2$$

"−" case:

$$-(x^2-4) = x+2$$
$$-x^2 + 4 = x + 2$$
$$0 = x^2 + x - 2$$
$$0 = (x+2)(x-1)$$
$$x = -2, 1$$

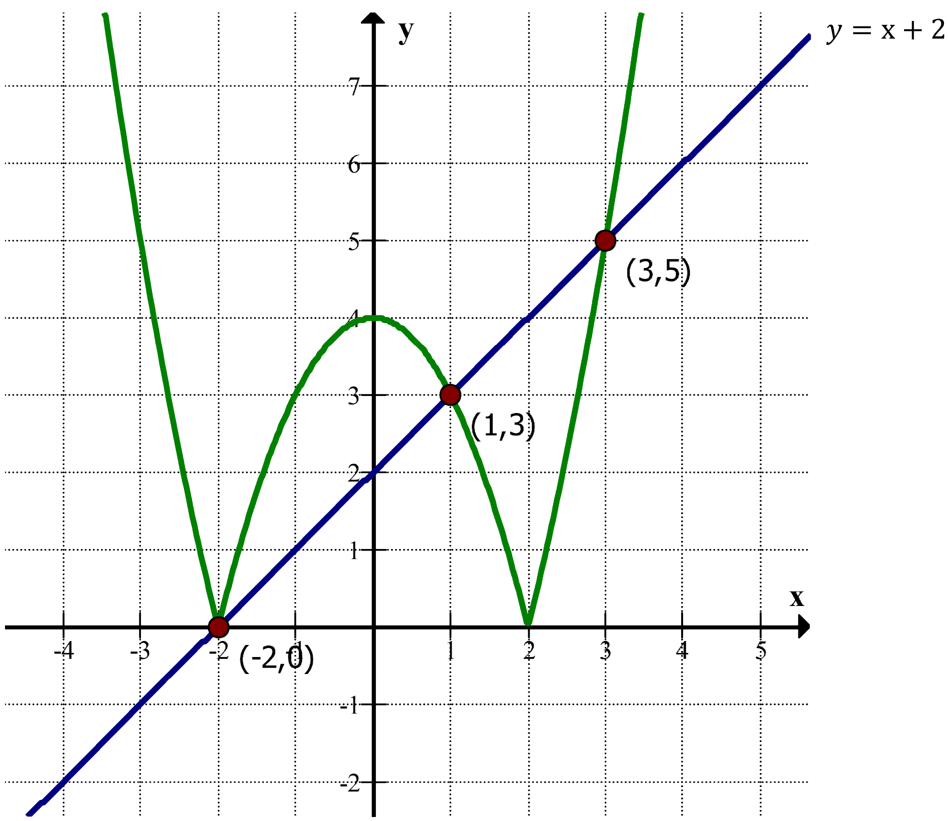
Check Answers!

$x = 3, -2$

$x = -2, 1$

Solve Graphically

$y = |x^2 - 4|$



C11 - 7.6 - Reciprocal Restrictions Notes

Find the restrictions

$$\frac{1}{x-2}$$

$Set\ denominator = 0, and\ solve.$

$x - 2 = 0$

$x = 2$

$$\frac{1}{(x+2)(2x-1)}$$

$Set\ denominator = 0, and\ solve.$

$\boxed{2x^2 + 3x - 2 = (x+2)(2x-1)}$

$x + 2 = 0$

$x = -2$

$2x - 1 = 0$

$x = \frac{1}{2}$

C11 - 7.7 - Linear Reciprocals Notes

$y = x + 4$ Line

$y = \dfrac{1}{x+4}$ Reciprocal line

Pick a y value, What's one divided by that y value. Put a point on the graph. X value is same as it was.

Solve algebraically: set denominator = 0, 1, -1.

Vertical asymptote (VA):
Denominator = 0

$$x + 4 = 0$$
$$x = -4$$

VA: $x = -4$

$D: x \neq -4$

Invariant points (IP):
Denominator = 1

$$x + 4 = 1$$
$$x = -3$$

$(-3, 1)$

Invariant points (IP):
Denominator = -1

$$x + 4 = -1$$
$$x = -5$$

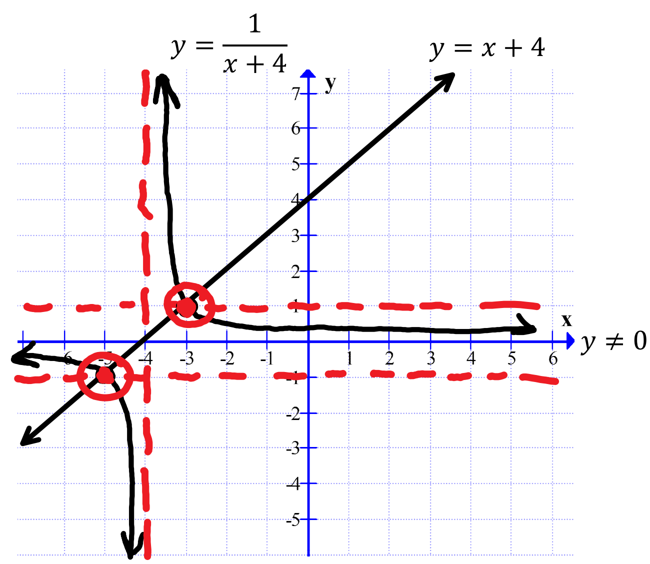
$(-5, -1)$

1. Graph original
2. Graph VA: Dotted line
3. Graph IP's
4. Graph reciprocal

x	y
-5	-1
-4	0
-3	1

x	$\dfrac{1}{x+4}$
-100	$-.01$
-5	-1
-4.1	-10
-4.01	-100
-4	UND
-3.99	100
-3.9	10
-3	1
100	.01

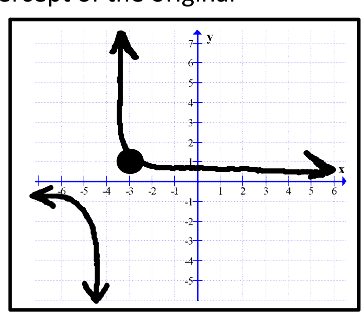
$D: x \neq -4$



Close to the vertical asymptote, through the point, close the x-axis/vertical asymptote

Notice: The invariant points are the intersection of the original and the lines $y = 1, y = -1$

Notice: The vertical asymptote(s) of the reciprocal is the X intercept of the original



C11 - 7.8 - Quadratic Reciprocals Notes

$y = x^2 - 4$ Parabola

$y = \dfrac{1}{x^2 - 4}$ Reciprocal Parabola

Solve algebraically: set denominator = 0, 1, -1.

Vertical asymptote (VA):
Denominator $= 0$

$$x^2 - 4 = 0$$
$$(x+2)(x-2) = 0$$
$$x = 2, -2$$

VA's: $x = 2$
$x = -2$

Invariant points (IP):
Denominator $= 1$

$$x^2 - 4 = 1$$
$$x^2 = 5$$
$$x = \sqrt{5}, -\sqrt{5}$$

$(\sqrt{5}, 1)$
$(-\sqrt{5}, 1)$

Invariant points (IP):
Denominator $= -1$

$$x^2 - 4 = -1$$
$$x^2 = 3$$
$$x = \sqrt{3}, -\sqrt{3}$$

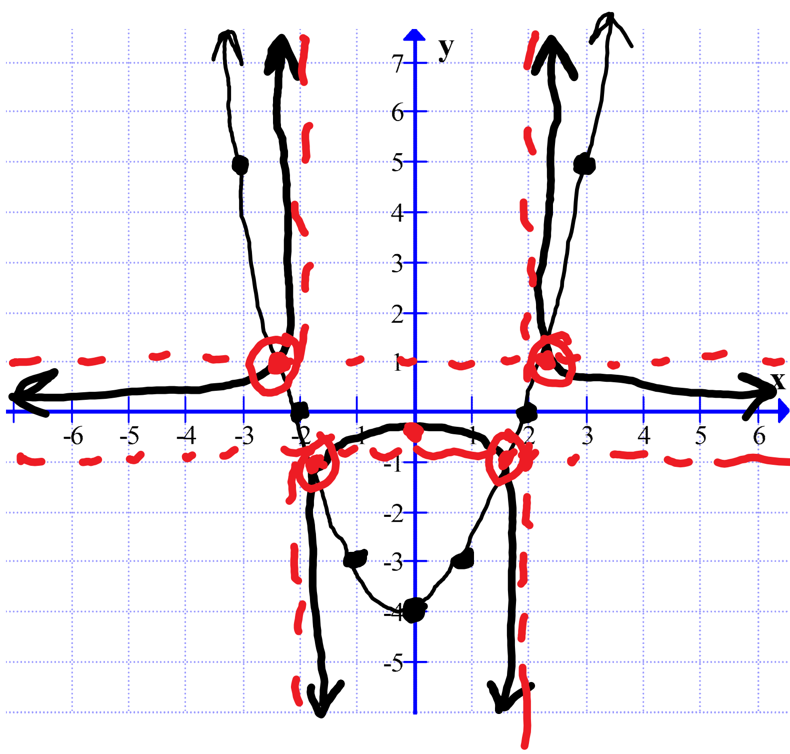
$(\sqrt{3}, -1)$
$(-\sqrt{3}, -1)$

Solve graphically.

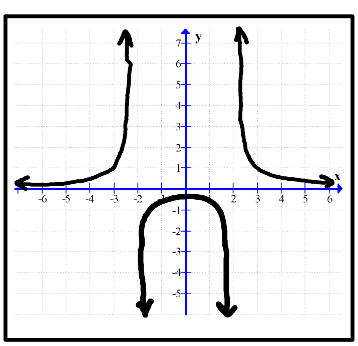
$$y = x^2 - 4$$
$$y = \frac{1}{x^2 - 4}$$

1. Graph original
2. Graph VA's: Dotted lines
3. Graph IP's
4. Graph reciprocal
5. y-int

$D: x \neq \pm 2$



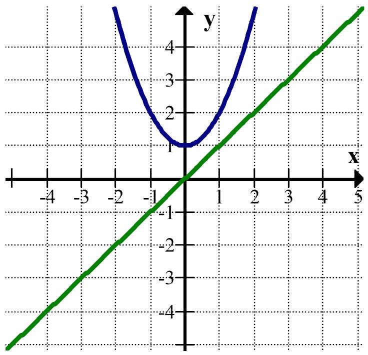
$y \neq 0$

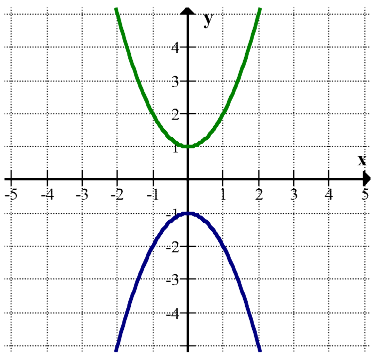


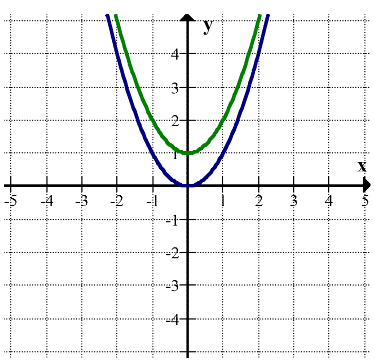
$(0, -4) \longrightarrow (0, -\frac{1}{4})$ $\quad \frac{1}{y} \quad \frac{1}{-4}$

C11 - 8.1 - Number of Intersections/Solutions Notes

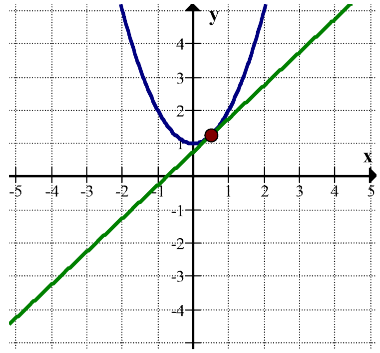
No Solutions

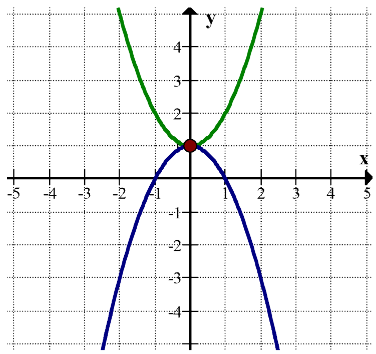


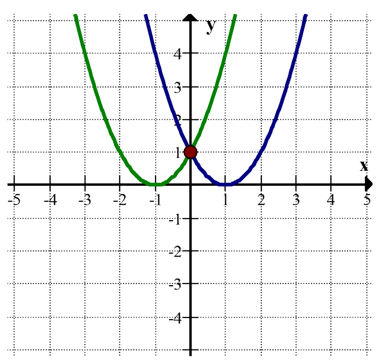




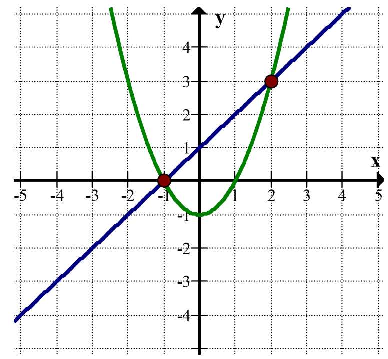
One Solution

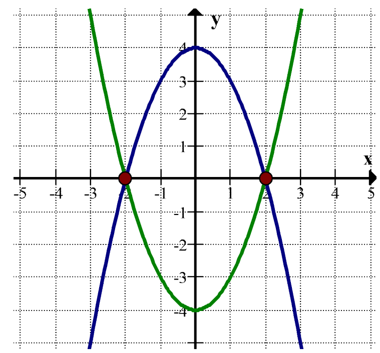


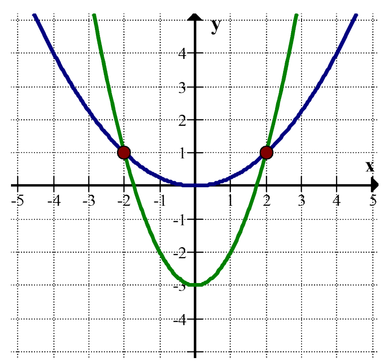




Two Solutions







OR INFINITE SOLUTIONS: Congruent Graphs

C11 - 8.2 - Linear/Quadratic Systems Substitution Notes

Solve by Substitution.

$y = x + 1$ $\quad$ $y = x^2 - 1$ $\qquad$ Equation 1 $\qquad$ Equation 2

$$x + 1 = x^2 - 1$$
$$-1 \qquad\quad -1$$
$$x = x^2 - 2$$
$$-x \qquad -x$$
$$0 = x^2 - x - 2$$

Equation 1 = Equation 2

$$0 = (x + 1)(x - 2)$$

Equation #3

$$x = -1, 2$$

Solve for x

$y = x + 1$
$y = (-1) + 1$
$y = 0$

$y = x + 1$
$y = (2) + 1$
$y = 3$

Solve for y $\qquad$ Solve for y

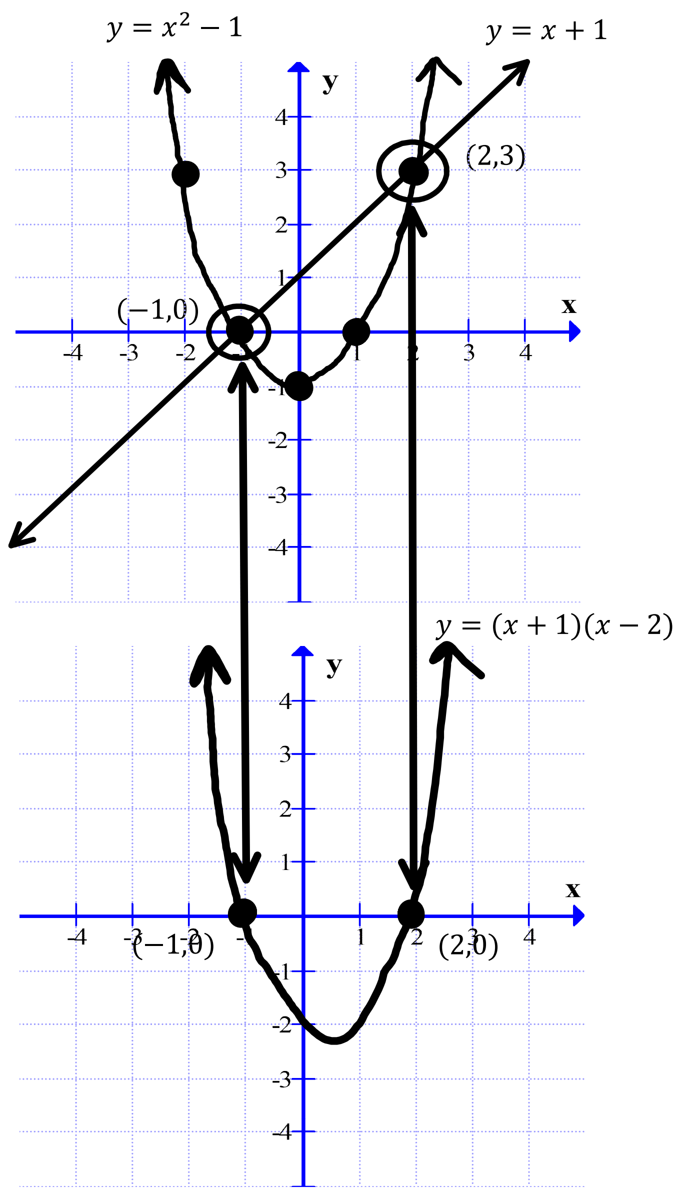
$(-1,0)$ $\qquad$ $(2,3)$

Intersection #1 $\qquad$ Intersection #2

Solve by graphing.

$y = x + 1$ $\quad$ Equation 1
$y = x^2 - 1$ $\quad$ Equation 2

$(-1,0)$ $\qquad$ $(2,3)$



$y = (x + 1)(x - 2)$ $\qquad$ Equation #3

$x + 1 = 0$ $\qquad$ $x - 2 = 0$
$x = -1$ $\qquad$ $x = 2$

Notice the graph of the third equation x-intercepts is the x answer to the question.

C11 - 8.3 - Quadratic Systems $b^2 - 4ac < 0$ Notes

Solve by Substitution.

$y = x^2 - 4x + 5$ $\qquad$ $y = -x^2 + 4x - 6$

$$x^2 - 4x + 5 = -x^2 + 4x - 6$$
$$2x^2 - 8x + 11 = 0$$

Algebra
Cannot Factor

$$2x^2 - 8x + 11 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(11)}}{2(2)}$$
$$x = \frac{8 \pm \sqrt{-24}}{4}$$

No Solution

Discriminant

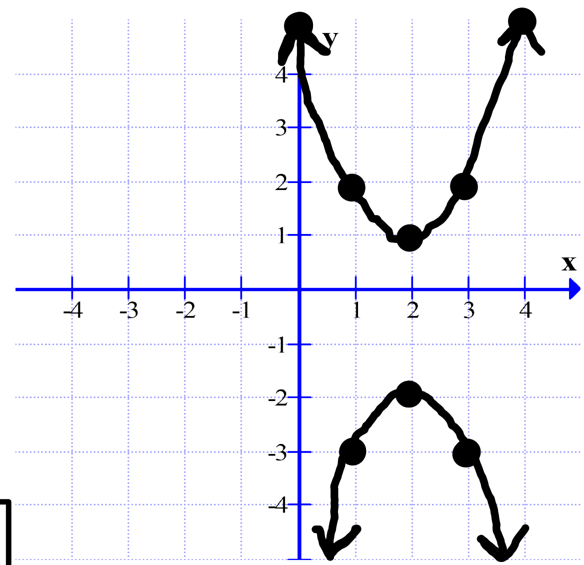
$$\boxed{b^2 - 4AC < 0}$$

$b^2 - 4ac$

$(-8)^2 - 4(2)(11) = -24$

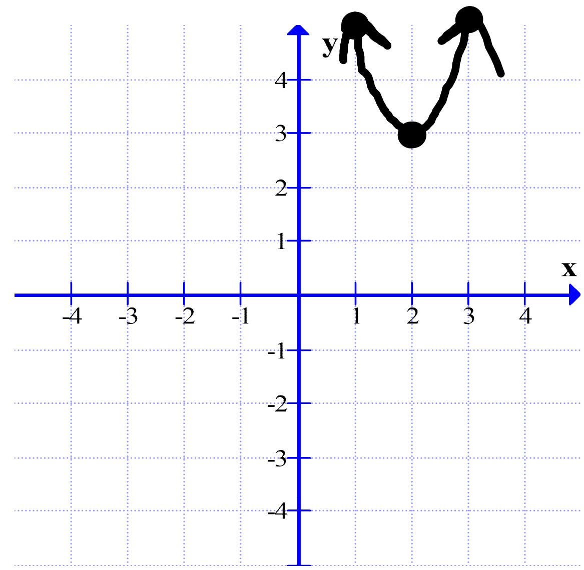
No Solution

$y = x^2 - 4x + 5$



$y = -x^2 + 4x - 6$

$y = 2x^2 - 8x + 11$



C11 - 9.1 - Linear Inequalities In Two Variables Notes

Graph the following Inequality

$y > x - 2$ Graph: $y = x - 2$

$y = mx + b$

$<, >$	○	– – – –	(Open Dots, Dotted line)

Test Point

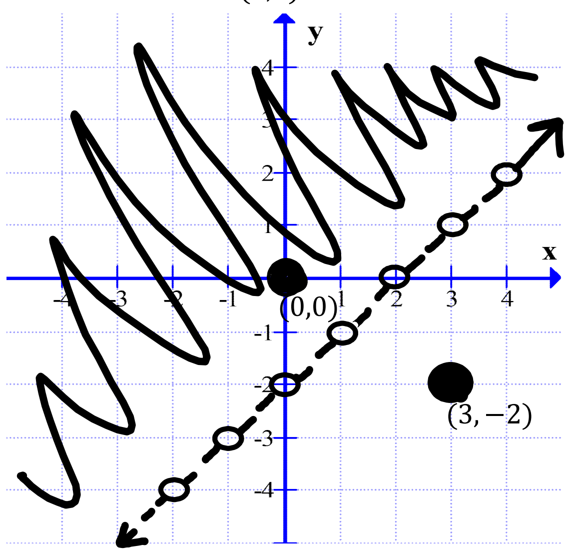
(x, y)
$(0,0)$

Choose a Point on either side of the Line

Zero-Zero Test*

$y > x - 2$
$0 > 0 - 2$
$0 > -2$ ✓ Substitute for x and y.

Correct: Shade the $(0,0)$ side of the line.



$y > x - 2$

Find Equation

Test Point	**Equation**	
$y \quad x - 2$	$y \quad mx + b$	(x, y)
$0 \quad 0 - 2$	**"Space"**	$(0,0)$
$0 \quad -2$		
$0 > -2$ ✓	Make a correct Statement	
$y > x - 2$	$y \quad x - 2$	

Test Point (x, y) $(3, -2)$

$y > x - 2$
$-2 > 3 - 2$
$-1 > 1$ ✗

OR

Incorrect: Shade the Not $(3, -2)$ side of the line.

Isolate for y or TOV $y = mx + b$

$x - y \geq 2$		$x - y \geq 2$
$-y \geq -x + 2$	OR	$x - 2 \geq y$
$y \leq x - 2$	⟷	$y \leq x - 2$
Add y Subtract 2 Mirror	(Both Sides)	Subtract x Divide* by -1 Change Sign!

Graph the following Inequality

$y \leq x - 2$ Graph $y = x - 2$

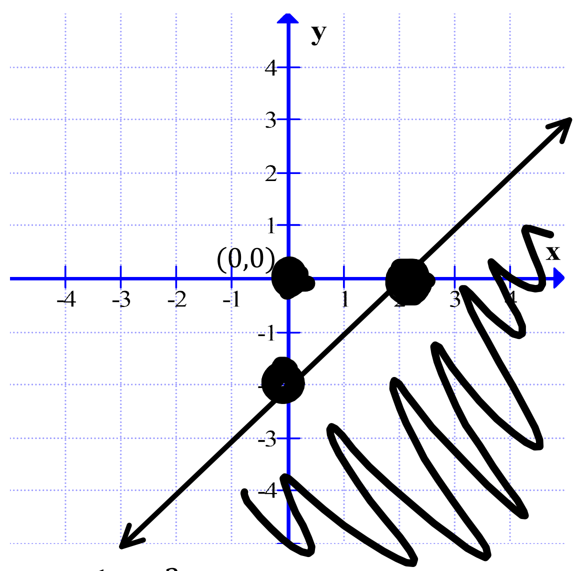
$\leq, \geq$	●	———	(Closed Dots, Solid Line)

Test Point

$(0,0)$

$y \leq x - 2$
$0 \leq 0 - 2$
$0 \leq -2$ ✗

Incorrect: Shade "Not" the $(0,0)$ side of the line.



$y \leq x - 2$

Find Equation

Test Point	**Equation**	
$y \quad x - 2$	$y \quad mx + b$	(x, y)
$0 \quad 0 - 2$	**"Space"**	$(0,0)$
$0 \quad -2$		
$0 \leq -2$ ✗	Make a Incorrect Statement	
$y \leq x - 2$	$y \quad x - 2$	

Notice: the (0,0) test only works if (0,0) is not on the line. If (0,0) is on the line we must choose a distinct point that is not on the line like (5,0) or (0,2).

OR	"Shade" above/below than "the line"	Replace the word y $with$ "$shade$" Greater than = above/Less than = below Replace the equation with "the line"

C11 - 9.2 - Linear/Quadratic Inequalities In One Variable Notes

Solve

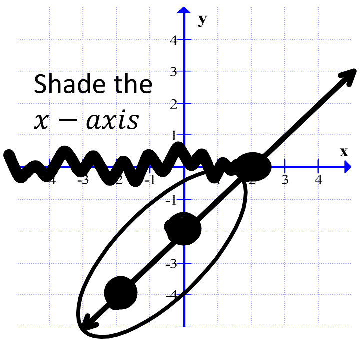
$x - 2 \leq 0$

$x - 2 \leq 0$
$\;\;+2 \quad +2$ Solve

$x \leq 2$

Graphing $\boxed{\begin{matrix} y\ values \leq 0 \\ The\ Thing \leq 0 \end{matrix}}$

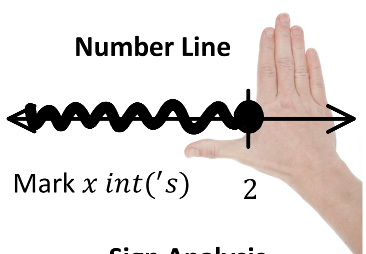
Shade the $x - axis$



$y = x - 2$

What are the x values when $y \leq 0$. Circle them!

Number Line



Mark $x\ int('s)$ 2

Sign Analysis

Pick a value

$x \leq 2$ $\quad x \geq 2$

$x = 0$ Substitute $x = 4$

$x - 2 \leq 0$
$0 - 2 \leq 0$
$-2 \leq 0$

$x - 2 \leq 0$
$4 - 2 \leq 0$
$2 \leq 0$

Correct: Shade that section

Incorrect: Shade Not that section

$x \leq 2$

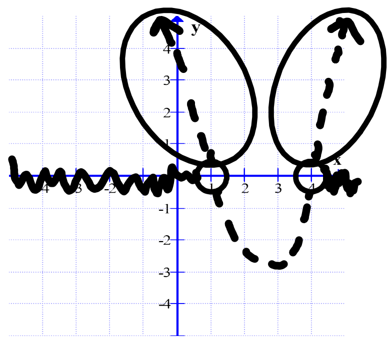
$-x^2 + 5x - 4 < 0$

$-x^2 + 5x - 4 < 0$
$-(x^2 - 5x + 4) < 0$
$-\dfrac{(x^2 - 5x + 4)}{-1} > \dfrac{0}{-1}$ $\quad \div -1^*$
$x^2 - 5x + 4 > 0$
$\boxed{(x-4)(x-1) > 0}$ Factor

$x - intercept's$

$x - 4 = 0 \quad x - 1 = 0$
$x = 4 \quad x = 1$

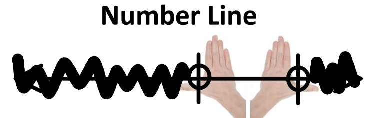
Graphing $\boxed{\begin{matrix} y\ values > 0 \\ The\ Thing > 0 \end{matrix}}$



$y = (x-4)(x-1)$

What are the x values when $y > 0$. Circle them!

Number Line



1 4

Sign Analysis

Pick a value

$x < 1 \quad 1 < x < 4 \quad x > 4$

$x = 0 \quad x = 2 \quad x = 5$

Substitute

$(x-4)(x-1) > 0$
$(0-4)(0-1) > 0$
$(-4)(-1) > 0$
$4 > 0$

$(1)(4) > 0$
$4 > 0$

$(-2)(1) > 0$
$-2 > 0$

$x < 1 \quad x > 4$

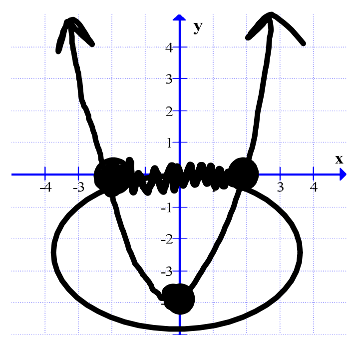
$x^2 - 4 \leq 0$

$x^2 - 4 \leq 0$
$(x+2)(x-2) \leq 0$

$x + 2 = 0 \quad x - 2 = 0$
$x = -2 \quad x = 2$

$x - intercept's$

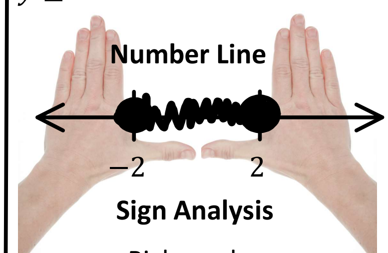
Graphing $\boxed{\begin{matrix} y\ values \leq 0 \\ The\ Thing \leq 0 \end{matrix}}$



$y = x^2 - 4$

What are the x values when $y \geq 0$. Circle them!

Number Line



$-2 \quad 2$

Sign Analysis

Pick a value

$x \leq -2 \quad -2 \leq x \leq 2 \quad x \geq 2$

$x = -3 \quad x = 0 \quad x = 3$

$x^2 - 4 \leq 0$
$(-3)^2 - 4 \leq 0$
$5 \leq 0$

$x^2 - 4 \leq 0$
$(3)^2 - 4 \leq 0$
$5 \leq 0$

$x^2 - 4 \leq 0$
$(0)^2 - 4 \leq 0$
$5 \leq 0$

$-2 \leq x \leq 2$

The answer is only the Domain. The number line and graph is only to help. There is no y involved.

C11 - 9.3 - Quadratic Inequalities in Two Variables Notes

Graph the following inequalities

$y \leq x^2 - 4$

Graph: $y = x^2 - 4$

Test Point $(0,0)$

$y \leq x^2 - 4$
$0 \leq (0)^2 - 4$
$0 \leq -4$

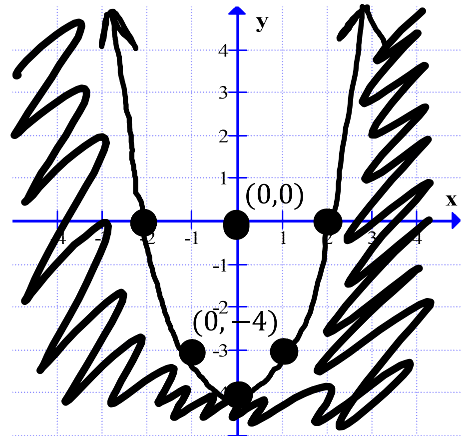
Substitute for x and y.

TOV

x	y
-2	0
-1	-3
0	-4
1	-3
2	0

Incorrect: Shade the "NOT" (0,0) side of the line.

(Closed dots, Solid Line) $y = x^2 - 4$



Find Equation

$y = a(x-p)^2 + q$
$y = a(x-0)^2 - 4$
$-3 = a(1-0)^2 - 4$
$-3 = 1a - 4$
$1 = a$
$y = 1(x-0)^2 - 4$
$y = x^2 - 4$

Vertex Form

(x, y)
$(0, -4)$ Vertex

(x, y)
$(1, -3)$ Point

Test Point

$y \quad x^2 - 4$
$0 \quad 0^2 - 4$
$0 \leq -4$

$y \leq x^2 - 4$

"Space" (x, y) $(0,0)$

Make a Incorrect Statement

$y > x^2 - 2x - 3$

Graph: $y = x^2 - 2x - 3$

$y = x^2 - 2x - 3$
$y = (x^2 - 2x) - 3$
$y = (x^2 - 2x + 1 - 1) - 3$
$y = (x-1)^2 - 4$

Complete the square $\left(\frac{b}{2}\right)^2$

$(1, -4)$ Vertex

$y = x^2 - 2x - 3$
$y = (x+1)(x-3)$

$x = -1$ $x = 3$ $x - intercepts$

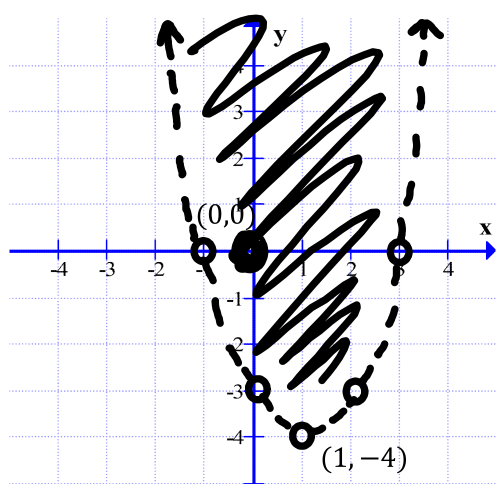
Test Point $(0,0)$

$y > x^2 - 4$
$0 > 0 - 4$
$0 > -4$

Substitute for x and y.

Correct: Shade the (0,0) side of the line.

(Open dots, Dotted line) $y = x^2 - 2x - 3$



Find Equation

$y = a(x-p)^2 + q$

$y = a(x-1)^2 - 4$
$-3 = a(2-1)^2 - 4$
$-3 = 1a - 4$
$1 = a$
$y = 1(x-1)^2 - 4$

$y = (x-1)^2 - 4$

Vertex Form

(x, y)
$(1, -4)$ Vertex

(x, y)
$(2, -3)$ Point

Test Point

$y \quad (x-1)^2 - 4$
$0 \quad (0-1)^2 - 4$
$0 \leq -3$

$y \leq (x-1)^2 - 4$

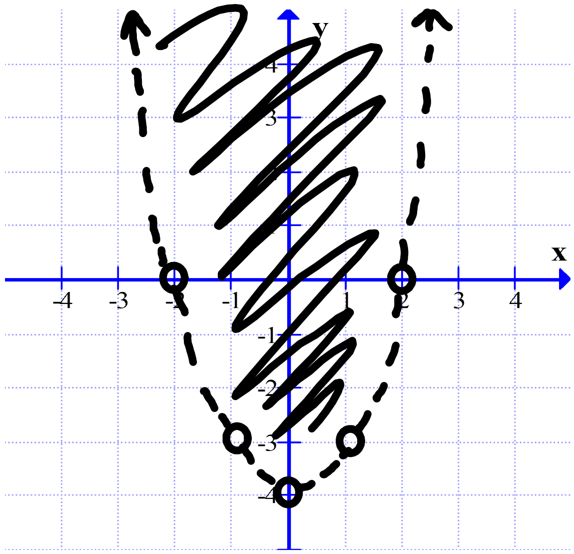
"Space" (x, y) $(0,0)$

Make a Correct Statement

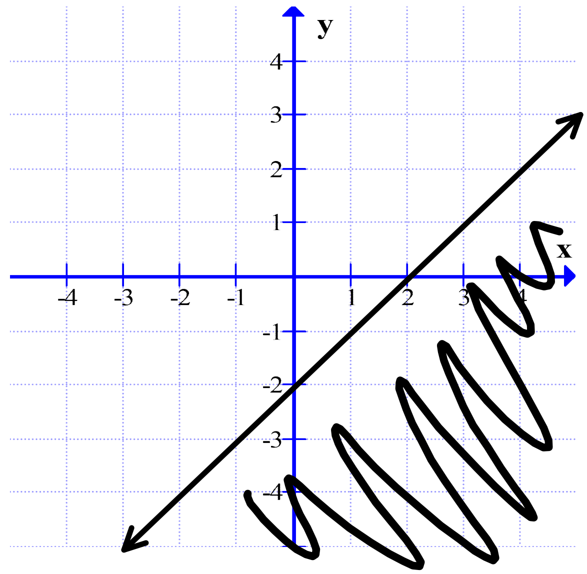
C11 - 9.3 - Inequalities Systems Notes

Solve the following system by graphing:

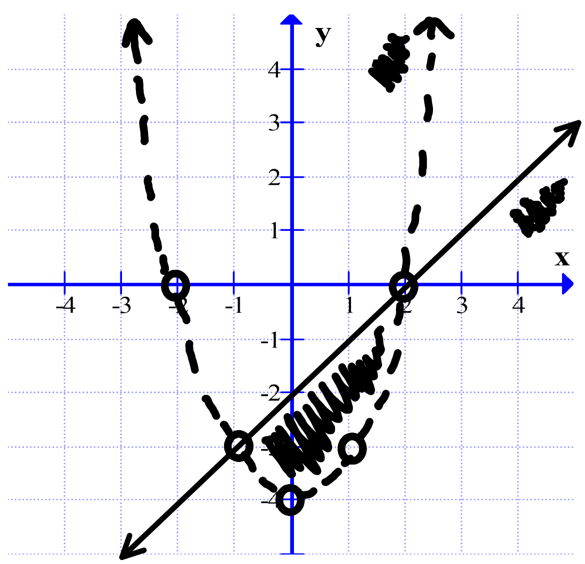
$y > x^2 - 4$



$y \leq x - 2$



$y > x^2 - 4$
$y \leq x - 2$



Notice: we have graphed each equation and shaded only the region which satisfies both inequalities.

C11 - 9.4 - Burgers and Fries Notes

$let\ b = \#\ burgers$
$let\ f = \#\ fries$

$burgers = \$3$
$fries = \$2$

$\$12\ to\ spend$

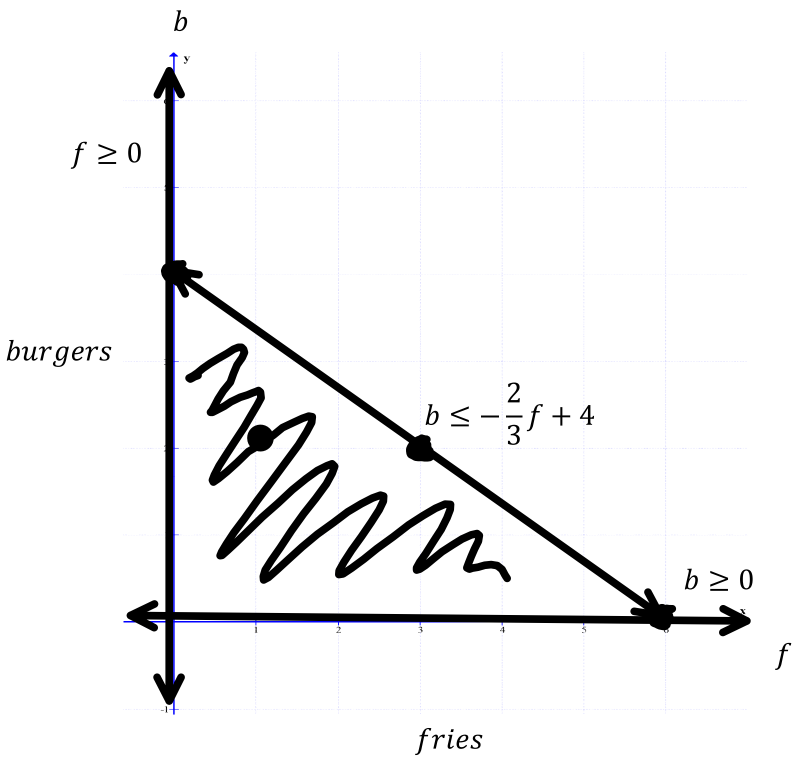
$\boxed{\begin{array}{l} 1\ burger = 3 \times 1 = 3 \\ 3\ burger = 3 \times 2 = 6 \\ b\ burger = 3 \times b = 3b \end{array}}$

$3b + 2f \leq 12$

$$3b + 2f \leq 12$$
$$3b \leq -2f + 12$$
$$b \leq -\frac{2}{3}f + 4$$

$\boxed{y = mx + b}$

f	b
0	4
6	0



(f, b)
$(0,4) = \$12$
$(0,3) = \$9$
$(0,2) = \$6$
$(0,1) = \$3$
$(0,0) = \$0$
$(1,3) = \$11$
$(1,2) = \$8$
$(1,1) = \$5$
$(1,0) = \$2$
$(2,2) = 10$
$(2,1) = \$7$
$(2,0) = \$4$
$(3,2) = \$12$
$(3,1) = \$9$
$(3,0) = \$6$
$(4,1) = \$11$
$(4,0) = \$8$
$(5,0) = \$10$
$(6,0) = \$12$

Test Point: (1,1)

$b \geq 0$
$1 \geq 0$ ✓

$f \geq 0$
$1 \geq 0$ ✓

$$b \leq -\frac{2}{3}f + 4$$
$$1 \leq -\frac{2}{3}(1) + 4$$
$$1 \leq \frac{10}{3}$$ ✓

Restrictions

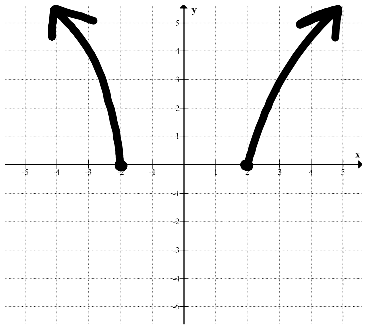
$0 \leq b \leq 4\ \ b \in W$

$0 \leq f \leq 6\ f \in W$

$W: Whole\ Numbers$

C11 - 9.5 - Inequalities Quadratic Restrictions Notes

$y = \sqrt{x^2 - 4}$



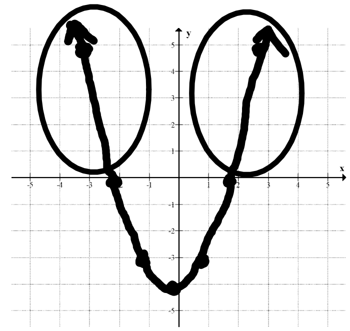
$$x^2 - 4 \geq 0$$
$$x^2 \geq 4$$
$$\sqrt{x^2} \geq \sqrt{4}$$
$$|x| \geq 2$$
$$\pm x \geq 2$$

$x \geq 2$ $\quad -x \geq 2$

$\qquad x \leq -2$

$x \geq +2 \quad x \leq -2$

$y = x^2 - 4$



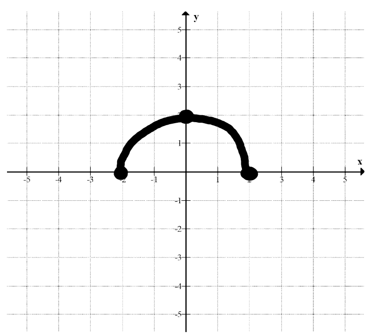
$x \leq -2 \quad x \geq +2$

x	y
-3	$\sqrt{5}$
-2	0
2	0
3	$\sqrt{5}$

Range

$y \geq 0$

$y = \sqrt{4 - x^2}$

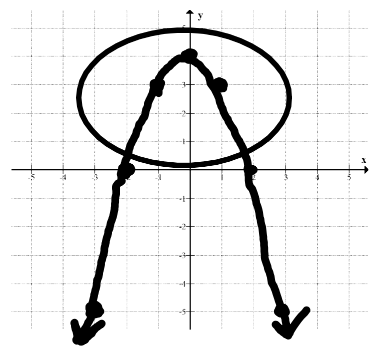


$$4 - x^2 \geq 0$$
$$x^2 \leq 4$$

$x \leq 2 \quad x \geq -2$

$-2 \leq x \leq 2$

$y = 4 - x^2$



$-2 \leq x \leq 2$

x	y
-2	0
0	2
2	0

Range

$0 \leq y \leq 2$

The End