

C11 - 1.6 - Sigma Notation - Notes

Find the sum of the terms

Arithmetic

$$\sum_{k=1}^4 2k = ? \quad \frac{2}{k=1}, \quad \frac{4}{k=2}, \quad \frac{6}{k=3}, \quad \frac{8}{k=4}$$

$2k = 2(1) = 2$
 $2k = 2(2) = 4$
 $2k = 2(3) = 6$
 $2k = 2(4) = 8$

$s_4 = 2 + 4 + 6 + 8 = 20$

Steps

Put in $k =$ bottom number the equation
 Put in $k + 1$ (bottom # plus 1)
 Repeat until $k =$ top number

k	$2k$
1	2
2	4
3	6
4	8

Arithmetic

$$\sum_{k=1}^{100} 2k = ?$$

$$s_n = \frac{n}{2}(2t_1 + (n-1)d)$$

$$s_{100} = \frac{100}{2}(2(2) + (100-1)2)$$

$s_{100} = 10100$

$$s_n = \frac{n}{2}(2t_1 + (n-1)d)$$

$$\# \text{ of terms} = n - k + 1$$

$$= \text{Top \# minus Bottom \#} + 1$$

$d = 4 - 2$
 $d = 2$

$d = 6 - 4$
 $d = 2$

$n = 100 - 1 + 1$
 $n = 100$

Geometric

$$\sum_{k=2}^6 8\left(\frac{1}{2}\right)^{k-1} = ? \quad \frac{4}{k=2}, \quad \frac{2}{k=3}, \quad \frac{1}{k=4}, \quad \frac{1}{2}{k=5}, \quad \frac{1}{4}{k=6}$$

$3\left(\frac{1}{2}\right)^{k-1} = 3\left(\frac{1}{2}\right)^{k-1} = \dots = 3\left(\frac{1}{2}\right)^{6-1} = 3\left(\frac{1}{2}\right)^5 = \frac{3}{32}$
 $8\left(\frac{1}{2}\right)^{2-1} = 8\left(\frac{1}{2}\right)^{3-1} = \dots = 8\left(\frac{1}{2}\right)^{6-1} = 8\left(\frac{1}{2}\right)^5 = \frac{8}{32} = \frac{1}{4}$

$4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} = 7.75$

$n = 6 - 2 + 1$
 $n = 5$

$$s_n = \frac{t_1(1-r^n)}{1-r}$$

$$s_5 = \frac{4\left(1 - \left(\frac{1}{2}\right)^5\right)}{1 - \left(\frac{1}{2}\right)}$$

$s_5 = 7.75$

Infinite Geometric

$$\sum_{k=2}^{\infty} 3(2)^{k-1} = ? \quad \frac{4}{k=2}, \quad \frac{2}{k=3}, \quad \frac{1}{k=4}, \quad \frac{1}{2}{k=5}, \quad \frac{1}{4}{k=6}, \quad \dots$$

$r = \frac{2}{4}$
 $r = \frac{1}{2}$
 $r = \frac{1}{2}$

$-1 < r < 1$
 $-1 < \frac{1}{2} < 1$
 \therefore Convergent, has sum

$$s_{\infty} = \frac{t_1}{1-r}$$

$$s_{\infty} = \frac{4}{1 - \left(\frac{1}{2}\right)}$$

$$s_{\infty} = \frac{4}{\frac{1}{2}}$$

$$s_{\infty} = 4 \times \frac{2}{1}$$

$s_{\infty} = 8$