C11 - 1.1 - Arithmetic Means Notes

Write the first terms 5 of the sequence

$t_1 = 2, d = 3$

$+3$

$\underline{\ 2\ }, \underline{\quad}, \underline{\quad}, \underline{\quad}, \underline{\quad}$

$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5$

$n = 1 \quad n = 2 \quad n = 3 \quad n = 4 \quad n = 5$

2,5,8,11,14

$2 + 3 = 5$
$5 + 3 = 8$
...

$t_1 =$ $1st\ term\ (aka$: "a or u_1")
d $= common\ difference$
$t_n = term\ n, every\ term$
$n = Term\ \#, or\ \#\ of\ terms$

$t_2 = 2, t_5 = -4$ *Logic*

$-d \quad +d \quad +d \quad +d$

$\underline{\quad}, \underline{\ 2\ }, \underline{\quad}, \underline{\quad}, \underline{-4}$

$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5$

$$2 + 3d = -4$$
$$-2 \qquad -2$$
$$3d = -6$$
$$\frac{3d}{3} = -\frac{6}{3}$$
$$d = -2$$

$4 - 1 = 3$

$\underline{\ 4\ }, \underline{\ 2\ }, \underline{\ 0\ }, \underline{-2}. \underline{-4}$

$4, 2, 0, -2, -4$

OR $t_n = t_1 + (n-1)d$

$t_2 = 2, t_5 = -4$ Systems of Equations

$$t_n = t_1 + (n-1)d$$
$$t_2 = t_1 + (2-1)d$$
$$2 = t_1 + d$$
$$t_1 = 2 - d$$

$$t_n = t_1 + (n-1)d$$
$$t_5 = t_1 + (5-1)d$$
$$-4 = t_1 + 4d$$
$$-4 = (2-d) + 4d$$
$$-4 = 2 + 3d$$
$$d = -2$$

$$t_1 = 2 - (-2)$$
$$t_1 = 4$$

$2 - 2 = 0$
$0 - 2 = -2$
...
$2 + 2 = 4$

$t_7 = 26, t_{95} = 378$ *Logic*

$$26 + 88d = 378$$
$$-26 \qquad -26$$
$$88d = 352$$
$$\frac{88d}{88} = \frac{352}{88}$$
$$d = 4$$

$95 - 7 = 88$

OR

$$t_n = t_1 + (n-1)d$$
$$t_7 = t_1 + (7-1)(4)$$
$$26 = t_1 + 24$$
$$t_1 = 2$$

$$t_n = t_1 + (n-1)d$$
$$t_2 = 2 + (2-1)(4)$$
$$t_2 = 6$$

$\underline{\ 2\ }, \underline{\ 6\ }, \underline{\ 10\ }, \underline{\ 14\ }. \underline{\ 18\ }$

2,6,10,14,18

$26 - 4 = 22$
$22 - 4 = 18$
$18 - 4 = 14$
$14 - 4 = 10$
...

C11 - 1.1 - Arithmetic Sequences Notes

2,5,8 ...

$d = ?$ $\quad t_n = ?$ $\quad t_{10} = ?$ $\quad t_n = 53, n = ?$

+3 +3

$\underline{2}$, $\underline{5}$, $\underline{8}$, $\underline{?}$ ... $\underline{?}$ ..... $\underline{53}$

t_1 $\quad t_2$ $\quad t_3$ $\quad t_4$ $\quad t_{10}$ $\quad t_n$

$n = 1$ $\quad n = 2$ $\quad n = 3$ $\quad n = 4$ $\quad n = 10$ $\quad n = ?$

_____, _____, _____ ... _____ ... _____

$t_1 = 2$

$d = t_n - t_{n-1}$
$d = 8 - 5$
$d = 3$

$d = t_n - t_{n-1}$
$d = 5 - 2$
$d = 3$

Difference

$d = t_n - t_{n-1}$

A term subtracted by the term before it

$t_{n-1} = \textit{term before } t_n$

Arithmetic: d must always be the same

Find the General term $t_n = ?$

$t_n = t_1 + (n-1)d$
$t_n = 2 + (n-1)3$
$t_n = 2 + 3n - 3$
$t_n = 3n - 1$

General term formula

$t_n = t_1 + (n-1)d$

The first term plus 'n − 1' differences

What is the tenth term t_{10}?

$t_n = 3n - 1$
$t_{10} = 3(10) - 1$
$t_{10} = 29$

Check your answer:
2,5,8,11,14,17,20,23,26,29 ✓

Or, Start from beginning

$t_n = t_1 + (n-1)d$
$t_{10} = 2 + (10-1)3$
$t_{10} = 2 + 27$
$t_{10} = 29$

Remember: You could have also added the common difference repeatedly

53 is what term, $t_n = 53, n = ?$

$t_n = 3n - 1$
$53 = 3n - 1$
$+1 \qquad +1$
$54 = 3n$
$\frac{54}{3} = \frac{3n}{3}$
$n = 18$

Check your answer:
2,5,8,11,14,17,20,23,26,29,32,35,28,41,44,47,50,53 ✓

C11 - 1.2 - Arithmetic Series Notes

2,5,8 ... $s_{12} = ?$

$s_n = sum\ of\ n\ terms$

+3	+3			
2 ,	5 ,	8 ,	?	... ?
t_1	t_2	t_3	t_4	t_{12}
$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 12$

$t_1 = 2$

$d = t_n - t_{n-1}$
$d = 8 - 5$
$d = 3$

$d = t_n - t_{n-1}$
$d = 5 - 2$
$d = 3$

What is the sum of the first twelve terms s_{12}? $s_{12} = ?, n = 12$.

$s_n = \frac{n}{2}(2t_1 + (n-1)d)$

$s_{12} = \frac{12}{2}(2(2) + (12-1)3)$

$s_{12} = 6(4 + (11)3)$

$s_{12} = 6(4 + 33)$

$s_{12} = 6(37)$

$s_{12} = 222$

$$s_n = \frac{n}{2}(2t_1 + (n-1)d)$$

Sum of "n" terms formula: if t_n is not known.

Check your answer:
$2 + 5 + 8 + 11 + 14 + 17 + 20 + 23 + 26 + 29 + 32 + 35 = 222$ ✓

OR

$s_n = \frac{n}{2}(t_1 + t_n)$

$s_{12} = \frac{12}{2}(2 + t_{12})$

$s_{12} = 6(2 + 35)$

$s_{12} = 222$

$t_n = 3n - 1$
$t_{12} = 3(12) - 1$
$t_{12} = 35$

$$s_n = \frac{n}{2}(t_1 + t_n)$$

Sum of "n" terms formula: if t_n is known.

C11 - 1.3 - Geometric Means Notes

Write the first terms 5 of the sequence

$t_1 = 2, r = 3$

$\times 3$

2 , ____ , ____ , ____ , ____

t_1 t_2 t_3 t_4 t_5

$\times 3$

2 , 6 , 18 , 54 , 162

2,6,18,54,162

t_1= 1st term (aka: "a or u_1")
r = common ratio
t_n= term n, every term
n = Term #, or # of terms

$t_2 = 4, t_4 = 16$

$\div r$ $\times r$ $\times r$ $\times r$

____ , 4 , ____ , 16 , ____

t_1 t_2 t_3 t_4 t_5

$4r^2 = 16$

$r^2 = 4$

$\sqrt{r^2} = \sqrt{4}$

$r = \pm 2$

$4 - 2 = 2$

$\div +2$ $\times +2$ $\times +2$ $\times +2$

2 , 4 , 8 , 16 , 32

$\div +2$ $\times +2$ $\times +2$ $\times +2$

-2 , 4 , -8 , 16 , -32

2,4,8,16,32

$-2, 4, -8, 16, -32$

$t_2 = 9, t_5 = 243$

$9r^3 = 243$

$r^3 = 27$

$\sqrt{r^3} = \sqrt{27}$

$r = 3$

$5 - 2 = 3$

3, 9, 27, 81, 243

$t_1 = 2, t_5 = 162$

$2r^4 = 162$

$r^4 = 81$

$r = \pm 3$

$5 - 1 = 4$

2, 6, 18, 54, 162

2, −6, 18, −54, 162

C11 - 1.3 - Geometric Sequences Notes

3,6,12 …

$r = ?$ $t_n = ?$ $t_5 = ?$ $t_n = 768, n = ?$

$\times 2$ $\times 2$

$\underline{3}$,	$\underline{6}$,	$\underline{12}$,	$\underline{?}$	… $\underline{?}$	….. $\underline{768}$
t_1	t_2	t_3	t_4	t_{10}	t_n
$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = ?$

______, ______, _______ … _______ … ______

$t_1 = 3$

$r = \frac{t_n}{t_{n-1}}$

$r = \frac{6}{3}$

$r = 2$

$r = \frac{t_n}{t_{n-1}}$

$r = \frac{12}{6}$

$r = 2$

Ratio

$r = \frac{t_n}{t_{n-1}}$

A term divided by the term before it

$t_{n-1} = term\ before\ t_n$

Geometric: r must always be the same

Find the General term $t_n = ?$

$t_n = t_1 r^{n-1}$

$t_n = 3(2)^{n-1}$

General term formula

$t_n = t_1 r^{n-1}$

The first term
times 'r − 1' differences

What is the fifth term t_5? $t_5 = ?, n = 5$.

$t_n = 3(2)^{n-1}$

$t_5 = 3(2)^{n-1}$

$t_5 = 3(2)^{5-1}$

$t_5 = 3(2)^4$

$t_5 = 48$

Check your answer: 3,6,12,24,48 ✓

Or, Start from beginning

$t_n = t_1 r^{n-1}$

$t_5 = 3(2)^{5-1}$

$t_5 = 48$

Remember: You could have also multiplied by the common ratio repeatedly

The number 768 is what term? $t_n = 768, n = ?$

$t_n = 3(2)^{n-1}$

$768 = 3(2)^{n-1}$ *divide both sides by* 3

$256 = 2^{n-1}$

$2^8 = 2^{n-1}$ *Change of base*: $256 = 2^8$

$8 = n - 1$ *Same Base, exponents are equal*

$n = 9$

Check your answer: 3,6,12,24,48,96,192,384,768 ✓

3,6,12 ...

$s_8 = ?$ $s_\infty = ?$

$s_n = sum\ of\ n\ terms$

$\times 2$ $\times 2$

$\underline{3}$, $\underline{6}$, $\underline{12}$, $\underline{?}$ ... $\underline{384}$

t_1 t_2 t_3 t_4 t_{10}

$n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 8$

$t_1 = 3$

$$r = \frac{t_n}{t_{n-1}} \qquad r = \frac{t_n}{t_{n-1}}$$

$$r = \frac{6}{3} \qquad r = \frac{12}{6}$$

$$r = 2 \qquad r = 2$$

What is the sum of the first eight terms $\boldsymbol{s_8}$? $\boldsymbol{s_8} = ?, \boldsymbol{n = 8}$.

$$s_n = \frac{t_1(1 - r^n)}{1 - r}$$

$$s_8 = \frac{3(1 - 2^8)}{1 - 2}$$

$$s_8 = 765$$

$$s_n = \frac{t_1(1 - r^n)}{1 - r}$$

Sum of "n" terms formula (if number of terms is known)

Check your answer: $3 + 6 + 12 + 24 + 48 + 96 + 192 + 384 = 765$ ✓

OR

$$s_n = \frac{t_1 - rt_n}{1 - r}$$

$$s_8 = \frac{3 - 2(t_8)}{1 - 2}$$

$$s_8 = \frac{3 - 2(384)}{1 - 2}$$

$$s_8 = 756$$

$$t_n = 3(2)^{n-1}$$

$$t_8 = 3(2)^{8-1}$$

$$t_8 = 3(2)^7$$

$$t_8 = 3(128)$$

$$t_8 = 384$$

$$s_n = \frac{t_1 - rt_n}{1 - r}$$

Sum of "n" terms formula (if last term t_n is known)

What is the sum of an infinite number of terms?

$r = 2$ $r > 1, \therefore no\ sum$

Check your answer: $3 + 6 + 12 + 24 + 48 + 96 + 192 + 384 + 768 + 1536 + 3072 + \cdots = \infty$ ✓

C11 - 1.5 - Infinite Geometric Sequences Notes

8,4,2 ...

$s_\infty = ?$

What is the sum of the infinite sequence?

$\times\frac{1}{2}$ $\quad$ $\times\frac{1}{2}$

$\underset{t_1}{8}\ ,\ \underset{t_2}{4}\ ,\ \underset{t_3}{2}\ ,\ \underset{t_4}{1}\ ,\ \underset{t_5}{\frac{1}{2}}\ ,\ \underset{t_6}{\frac{1}{4}}\ , \ldots$

$t_1 = 8$

$r = \frac{t_n}{t_{n-1}}$ $\quad$ $r = \frac{t_n}{t_{n-1}}$

$r = \frac{4}{8}$ $\quad$ $r = \frac{2}{4}$

$r = \frac{1}{2}$ $\quad$ $r = \frac{1}{2}$

$-1 < r < 1$

$-1 < \frac{1}{2} < 1$

$\therefore$ Convergent, has sum

$$s_\infty = \frac{t_1}{1-r}$$

$$s_\infty = \frac{8}{1-\frac{1}{2}}$$

$$s_\infty = \frac{8}{\frac{1}{2}}$$

$$s_\infty = 16$$

$$s_\infty = \frac{t_1}{1-r}$$

Sum of "n" terms formula (infinite number of terms)

$Check\ your\ answer: 8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 15.9375 \approx 16$

8,16,32 ...

$s_\infty = ?$

What is the sum of the infinite sequence?

$\times 2$ $\quad$ $\times 2$

$\underset{t_1}{8}\ ,\ \underset{t_2}{16}\ ,\ \underset{t_3}{32}\ ,\ \underset{t_4}{64}\ ,\ \underset{t_5}{128}\ ,\ \underset{t_6}{256}\ , \ldots$

$t_1 = 8$

$r = \frac{t_n}{t_{n-1}}$ $\quad$ $r = \frac{t_n}{t_{n-1}}$

$r = \frac{16}{8}$ $\quad$ $r = \frac{32}{16}$

$r = 2$ $\quad$ $r = 2$

$r > 1$

$\therefore$ Divergent

$\therefore$ No sum

$Check\ your\ answer: 8 + 16 + 64 + 128 + 256 + 512 + 1024 + 2048 + \cdots = \infty$

C11 - 1.6 - Sigma Notation - Notes

Find the sum of the terms

Arithmetic

$\sum_{k=1}^{4} 2k = ?$ ← d

$\underline{2}$, $\underline{4}$, $\underline{6}$, $\underline{8}$,

k=1 k=2 k=3 k=4

$2k = 2(1) = 2$ $2k = 2(2) = 4$ $2k = 2(3) = 6$ $2k = 2(4) = 8$

$s_4 = 2 + 4 + 6 + 8 = 20$

Steps
Put in $k =$ bottom number the equation
Put in $k + 1$ (bottom # plus 1)
Repeat until $k =$ top number

k	$2k$
1	2
2	4
3	6
4	8

Arithmetic

$\sum_{k=1}^{100} 2k = ?$

$s_n = \frac{n}{2}(2t_1 + (n-1)d)$

$s_{100} = \frac{100}{2}(2(2) + (100-1)2)$

$s_{100} = 10100$

$s_n = \frac{n}{2}(2t_1 + (n-1)d)$

$\# \ of \ terms = n - k + 1$
= Top # minus Bottom # $+ 1$

$d = 4 - 2$
$d = 2$

$d = 6 - 4$
$d = 2$

$n = 100 - 1 + 1$
$n = 100$

Geometric

$\sum_{k=2}^{6} 8(\frac{1}{2})^{k-1} = ?$

Always r

Not Always t1

$\underline{4}$, $\underline{2}$, $\underline{1}$, $\underline{\frac{1}{2}}$, $\underline{\frac{1}{4}}$,

k=2 k=3 k=4 k=5 k=6

$3(\frac{1}{2})^{k-1} = 8\left(\frac{1}{2}\right)^{2-1} = 4$

$3(\frac{1}{2})^{k-1} = 8\left(\frac{1}{2}\right)^{3-1} = 2$

...

$3(\frac{1}{2})^{k-1} = 8\left(\frac{1}{2}\right)^{6-1} = \frac{1}{4}$

$4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} = 7.75$

$n = 6 - 2 + 1$
$n = 5$

$s_n = \frac{t_1(1 - r^n)}{1 - r}$

$s_n = \frac{t_1(1 - r^n)}{1 - r}$

$s_5 = \frac{4\left(1 - \left(\frac{1}{2}\right)^5\right)}{1 - \left(\frac{1}{2}\right)}$

$s_5 = 7.75$

Infinite Geometric

$\sum_{k=2}^{\infty} 3(2)^{k-1} = ?$

$\underline{4}$, $\underline{2}$, $\underline{1}$, $\underline{\frac{1}{2}}$, $\underline{\frac{1}{4}}$ ...

$r = \frac{2}{4}$
$r = \frac{1}{2}$

$r = \frac{1}{2}$

$-1 < r < 1$
$-1 < \frac{1}{2} < 1$
$\therefore$ Convergent, has sum

$s_\infty = \frac{t_1}{1 - r}$

$s_\infty = \frac{4}{1 - \left(\frac{1}{2}\right)}$

$s_\infty = \frac{4}{\frac{1}{2}}$

$s_\infty = 4 \times \frac{2}{1}$

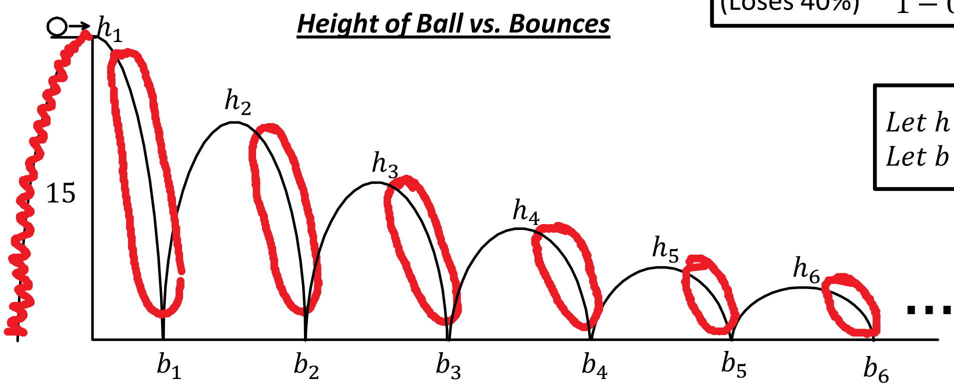
$s_\infty = 8$

C11 - 1.8 - Bouncing Ball Notes (up 60%)

A ball rolls off a building 15 m tall. After each bounce, it rises to 60% of the previous height.

(Loses 40%) $1 - 0.4 = 0.6$

Height of Ball vs. Bounces



Let h = Fall height
Let b = Bounce Number

$r = 60\%$
$r = 0.6$

$\times 0.6 \quad \times 0.6 \quad \times 0.6 \quad \times 0.6 \quad \times 0.6$

$\underline{15}_{t_1}, \underline{9}_{t_2}, \underline{5.4}_{t_3}, \underline{3.24}_{t_4}, \underline{1.94}_{t_5}, \underline{1.17}_{t_6}$

Terms are Fall Heights!

$r = \frac{9}{15}$ $\quad r = \frac{5.4}{9}$

$r = .6$ $\quad r = .6$

How high does the ball bounce after the 1st, 2nd bounce?

Height After 1st Bounce

$15 \times 0.6 = 9\ m$

Height After 2nd Bounce

$9 \times 0.6 = 5.4\ m$

$1 \rightarrow 2!$

$2 \rightarrow 3!$

After $1st = t_2$

After $2nd = t_3$

How high does the ball bounce after the nth bounce? (Find the general formula)

$t_n = t_1 r^{n-1}$

$t_n = t_1 r^{n-1}$

$t_n = 15(0.6)^{n-1}$

How high does the ball bounce after the 4th bounce. t_5

$t_n = t_1(r)^{n-1}$

$t_5 = 15(0.6)^{5-1}$

$t_5 = 15(0.6)^4$

$t_5 = 1.94\ m$

$4 \rightarrow 5!$

After $4th$ bounce $= t_5$

How high does the ball bounce after the 10th bounce. t_{11}

$t_n = t_1 r^{n-1}$

$t_{11} = 15(0.6)^{11-1}$

$t_{11} = 15(0.6)^{10}$

$t_{11} = 0.09\ m$

$10 \rightarrow 11!$

After $10th$ bounce $= t_{11}$

What is the total vertical distance the ball has travelled when it hits the ground for the 5th bounce? $s_5 = ?$

$s_n = \frac{t_1(1 - r^n)}{1 - r}$

$s_5 = \frac{15(1 - (.6)^5)}{1 - .6}$

$s_5 = \frac{15(0.87)}{.4}$

$s_5 = 34.6\ m$

$34.6 \times 2 - 15 = 54.2\ m$

Count it

15
$+9 \times 2$
$+5.4 \times 2$
$+3.24 \times 2$
$+1.94 \times 2$
54.2

If it bounces forever, what is the total vertical distance travelled? $s_\infty = ?$

$s_\infty = \frac{t_1}{1 - r}$

$h_\infty = \frac{h_1}{1 - r}$

$h_\infty = \frac{15}{1 - 0.6}$

$h_\infty = \frac{15}{0.4}$

$h_\infty = 37.5\ m$

$r = 0.6 \quad r < 1$

$37.5 \times 2 - 15 = 60m$

Double it to account for rise heights and subtract the initial height (double counted)