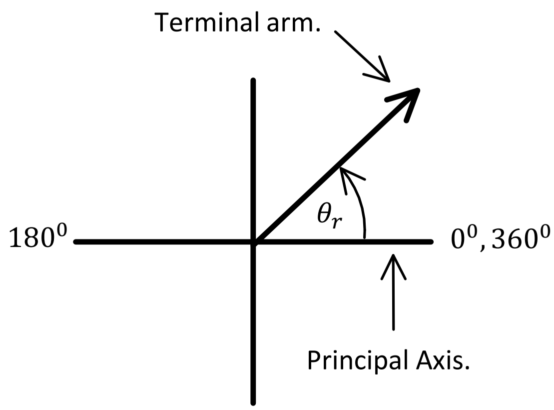
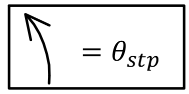
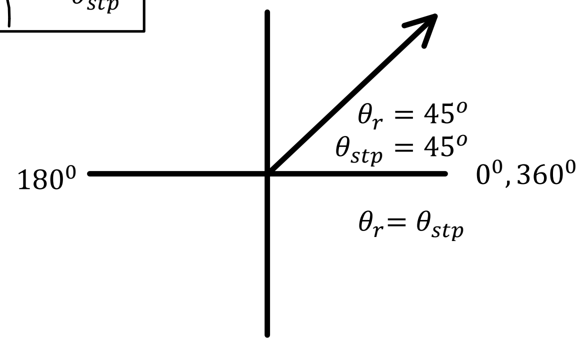
C11 - 2.1 - θ_r, θ_{stp} Notes

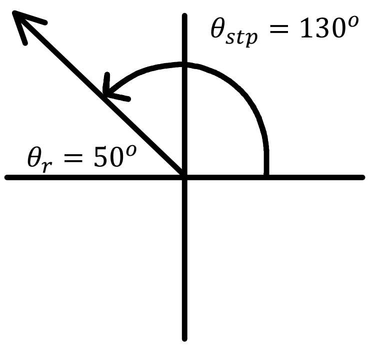
θ_r: the "reference angle"is the angle between the terminal arm and the x-axis ($0^o \le \theta \le 90^o$).

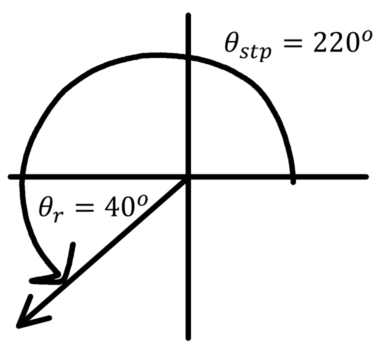
θ_{stp}: the "angle in standard position" from the principal axis (+ x-axis) to the terminal arm.

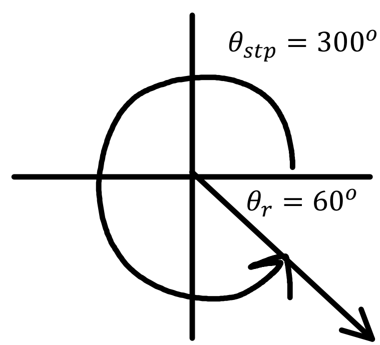












$\theta_{stp} = 180^O - 50^o$
$\theta_{stp} = 130^O$

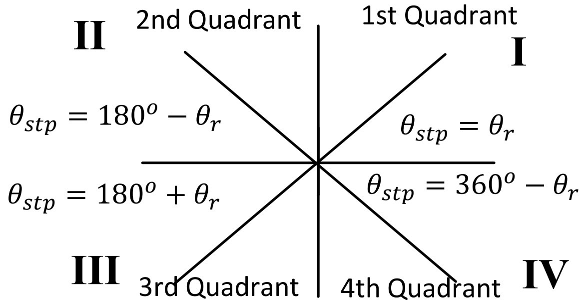
$\theta_r = 180^O - 130^o$
$\theta_r = 50^O$

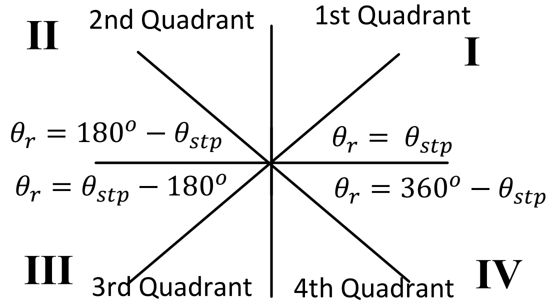
$\theta_{stp} = 180^O + 40^o$
$\theta_{stp} = 220^O$

$\theta_r = 220^o - 180^O$
$\theta_r = 40^O$

$\theta_{stp} = 360^O - 50^o$
$\theta_{stp} = 310^O$

$\theta_r = 360^O - 310^o$
$\theta_r = 50^O$

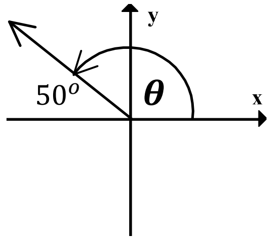




Basic logic will calculate θ_{stp} *and* θ_r much more easily than using these formulas.

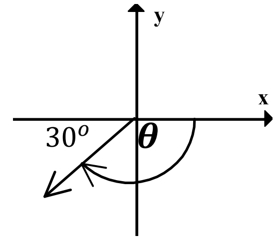
C11 - 2.1 - $\pm\,\theta_{stp}, \theta_{cot}, \theta_{pri}$ Notes

Counter-clockwise rotation is a positive θ_{stp}



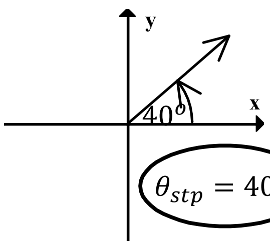
$\theta_{stp} = 180^o - 50^o$
$\theta_{stp} = 130^o$

Clockwise rotation is a negative θ_{stp}

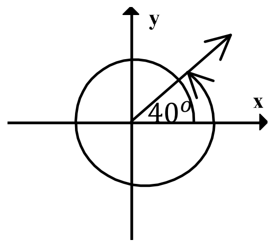


$\theta_{stp} = -(180^o - 30^o)$
$\theta_{stp} = -150^o$

Positive Co-terminal Angles (θ_{cot})



$\theta_{stp} = 40^o$



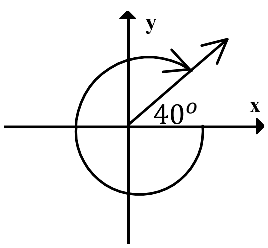
$$\boxed{\theta_{cot} = \theta_{stp} \pm 360^o}$$

$\theta_{cot} = \theta_{stp} \pm 360^o$
$\theta_{cot} = 40^o + 360^o$
$\theta_{cot} = 400^o$

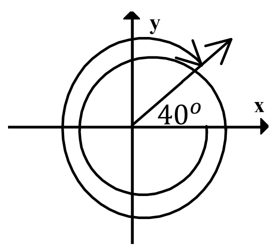
$\theta_{stp} = 40^o, \theta_{stp} = 400^o$

$\theta_{\text{cot}} = 40^o, 400^o, 760^o, 1120^o, 1480^o, \ldots$

Negative Co-terminal Angles (θ_{cot})



$\theta_{cot} = \theta_{stp} \pm 360$
$\theta_{cot} = 40 - 360$
$\theta_{cot} = -320^o$



$\theta_{cot} = \theta_{stp} \pm 360$
$\theta_{cot} = -320 - 360$
$\theta_{cot} = -680^o$

$\theta_{\text{cot}} = 40^o, -320^o, -680^o, -1040^o, -1400^o, \ldots$

$\theta_{principle} = smallest + ve\ \theta_{stp}\ coterminal.$

$\theta_{stp} = 1000^0$

$\theta_{pri} = 1000^0 - 360^0 = 640^0$
$= 640^0 - 360^0 = 280^0$

OR

$\theta_{pri} = 0 \leq \theta_{cot} < 360$

$1000^0 - 2(360^0) = 280^0$

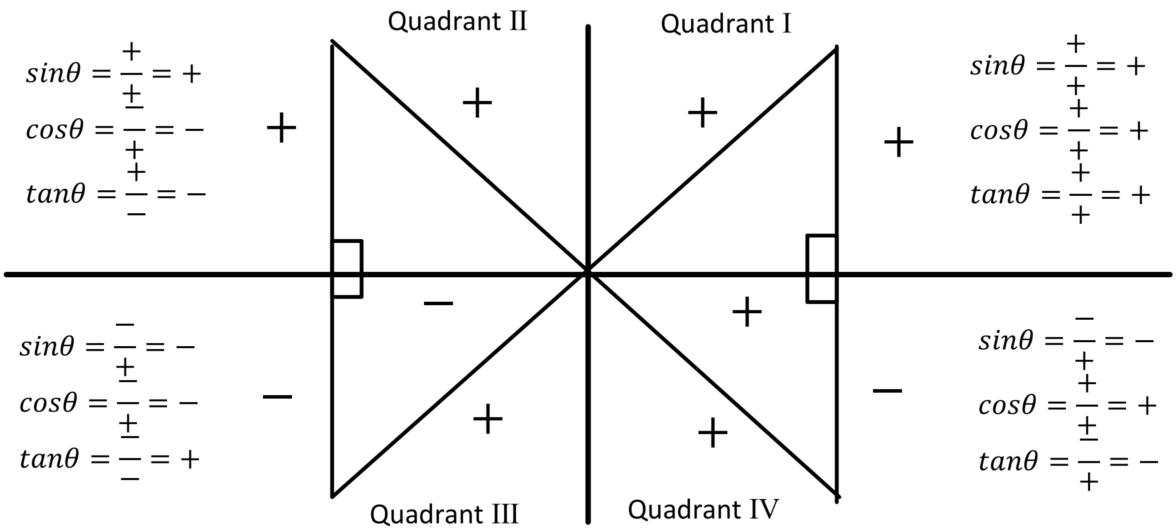
$\dfrac{1000^0}{360^0} = 2.777\ldots$ OR $0.777\ldots \times 360^0 = 280^0$

You may need to add or subtract 360^0 more than once.

$sin\theta = \frac{+}{+} = +$

$cos\theta = \frac{-}{+} = -$

$tan\theta = \frac{+}{-} = -$



$sin\theta = \frac{+}{+} = +$

$cos\theta = \frac{+}{+} = +$

$tan\theta = \frac{+}{+} = +$

$sin\theta = \frac{-}{+} = -$

$cos\theta = \frac{-}{+} = -$

$tan\theta = \frac{-}{-} = +$

$sin\theta = \frac{-}{+} = -$

$cos\theta = \frac{+}{+} = +$

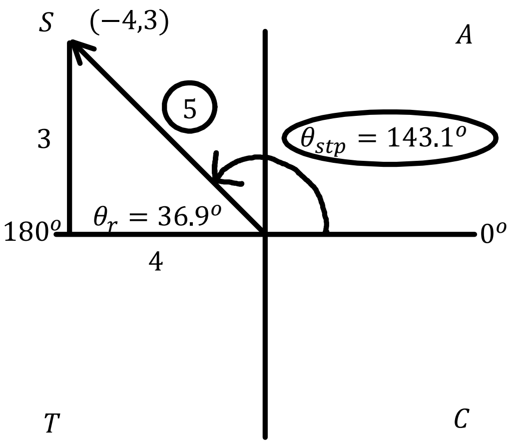
$tan\theta = \frac{-}{+} = -$

$(+)^2 + (-)^2 = +$ $\sqrt{+} = +$	Remember: the hypotenuse is always positive.

S **Students** Only **S**in positive.	**A** **All** **A**ll (sin, cos, tan) positive
Only **T**an positive. **Take** **T**	**Calculus** Only **C**os positive. **C**

C11 - 2.3 - *Trig Ratios* Notes

Find sinx, cox, and tanx for the following point. Find θ_{stp}. SOH CAH TOA



$a^2 + b^2 = c^2$
$3^2 + 4^2 = c^2$
$9 + 16 = c^2$
$25 = c^2$
$\sqrt{25} = \sqrt{c^2}$
$5 = c$

$sin\theta = +\frac{3}{5}$

$cos\theta = -\frac{4}{5}$

$tan\theta = -\frac{3}{4}$

$tan\theta = -\frac{3}{4}$

$\theta = \tan^{-1}(+0.75)$

$\theta = 36.9^o$

$180^o - 36.9^o = 143.1^o$

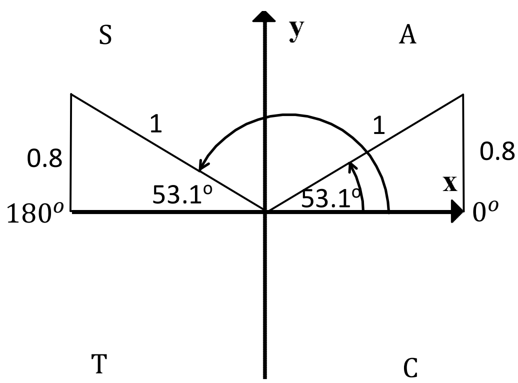
$\theta_{stp} = 143.1$

Check Answer

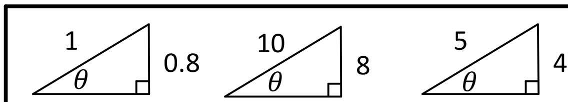
$sin143.1 = +0.6 = +\frac{3}{5}$ ✓

$\theta = \sin^{-1}(-\frac{3}{5})$ $\theta = 36.9$	$\theta = \cos^{-1}(-\frac{4}{5})$ $\theta = 143.1^o$	$\theta = \tan^{-1}(-\frac{3}{4})$ $\theta = -36.9^o$
$\theta = \sin^{-1}(+\frac{3}{5})$ $\theta = 36.9$	$\theta = \cos^{-1}(+\frac{4}{5})$ $\theta = 36.9^o$	$\theta = \tan^{-1}(+\frac{3}{4})$ $\theta = 36.9^o$

$\sin\theta = 0.8$ **Solve for $\theta, 0^o \le \theta < 360^o$ *and general solution***



$sin\theta = \frac{0.8}{1} = \frac{8}{10} = \frac{4}{5}$ *θ is the same!*



Draw two $\Delta's$ where $\sin\theta$ is positive: ASTC Quadrant I, II

Label the triangles according to SOH CAH TOA

Solve for θ_r: $\theta_r = \sin^{-1}(+\frac{O}{H})$

Draw an arrow from the principal axis:
To the first and second terminal arm

$sin\theta = \frac{0.8}{1}$

$\theta_r = \sin^{-1}\left(+\frac{0.8}{1}\right)$

$\theta_r = 53.1^o$

Solve for the arrows θ_{stp} $sin53.1^o = 0.8$ ✓

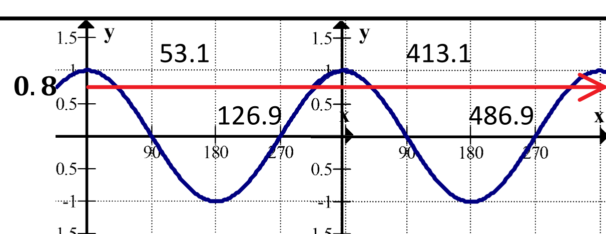
Check your answer: $sin126.9^o = 0.8$ ✓

$\theta_{stp} = 53.1^o$ $\theta_{stp} = 180^o - 53.1^o = 126.9^o$

$\theta_{stp} = 53.1^o, 126.9^o$

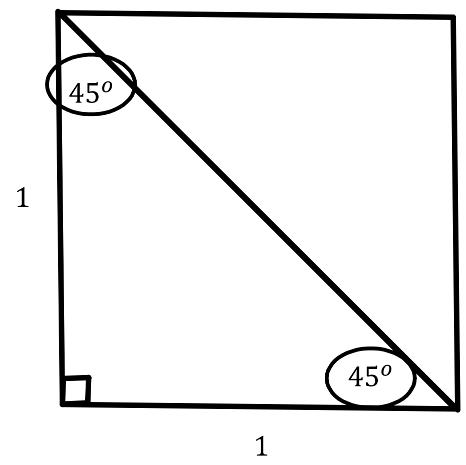
General Solution:
$\theta = \theta_{stp} \pm pn, n \in I$

$\theta = 53.1^o \pm 360^o n, n \in I$
$\theta = 126.9^o \pm 360^o n, n \in I$

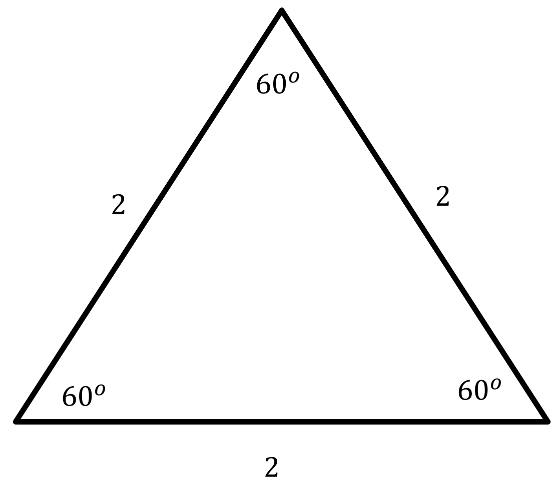


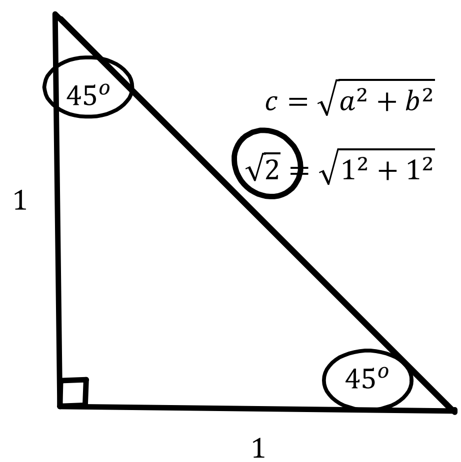
C11 - 2.4 - Special Triangles 30,45,60 sin/cos/tan Notes

Diagonal of a square with sides lengths of 1



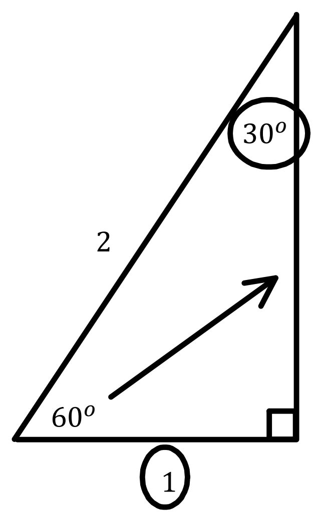
Half an equilateral with sides 2





$$c = \sqrt{a^2 + b^2}$$

$$\sqrt{2} = \sqrt{1^2 + 1^2}$$



$$b = \sqrt{c^2 - a^2}$$

$$\sqrt{3} = \sqrt{2^2 - 1^2}$$

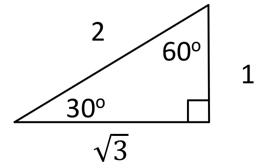
$60 > 30$

$\sqrt{3} > 1$

60 must open up to the root 3. And Vice Versa

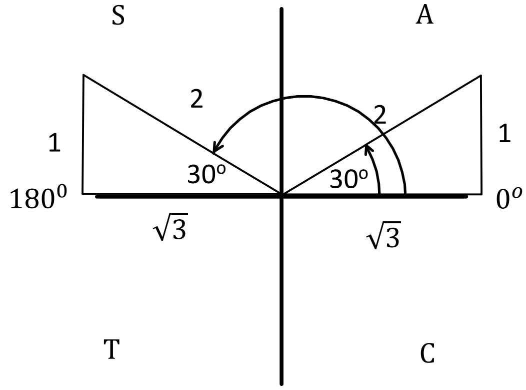
$\sin 45 = \frac{1}{\sqrt{2}}$	$\sin 60 = \frac{\sqrt{3}}{2}$	$\sin 30 = \frac{1}{2}$
$\cos 45 = \frac{1}{\sqrt{2}}$	$\cos 60 = \frac{1}{2}$	$\cos 30 = \frac{\sqrt{3}}{2}$
$\tan 45 = \frac{1}{1}$	$\tan 60 = \frac{\sqrt{3}}{1}$	$\tan 30 = \frac{1}{\sqrt{3}}$

C11 - 2.5 - $sin\theta = \frac{1}{2}$ Notes



$sin\theta = \frac{1}{2}$ **Solve for $\theta, 0^o \leq \theta < 360^o$.**

Between 0 and 360 degrees



Draw Two $\Delta's$ where sin θ is +ve: ASTC Quadrant I, II

Label the $\Delta's$ according to SOH CAH TOA

Label the reference angle according to special Δ's.

Draw an arrow from the principal axis:

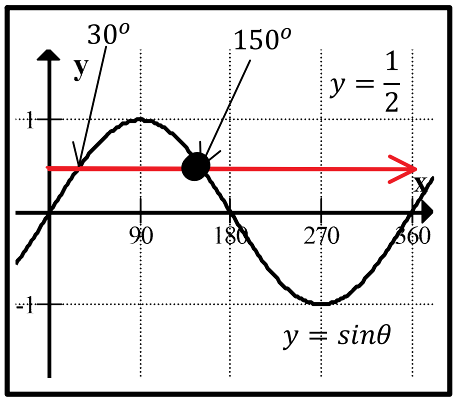
To the first terminal arm and the second terminal arm.

Solve for the arrows θ_{stp}

Check on Calculator

$\theta_{stp} = 30^o$ $\quad \theta_{stp} = 180^o - 30^o = 150^o$

$\theta_{stp} = 30^o, 150^o$



Graphing Calculator

$y = sinx$

$y = \frac{1}{2}$

Zoom 7:

$-360 \leq x \leq 360$

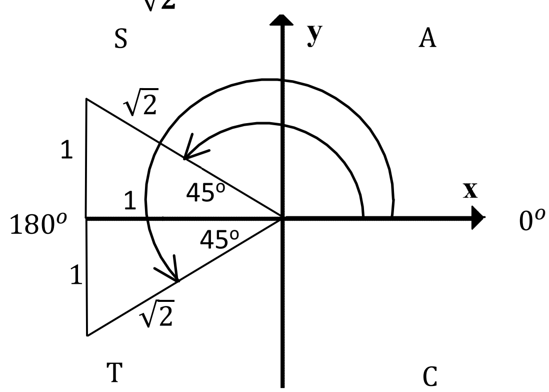
Window = Domain

Find Intersections

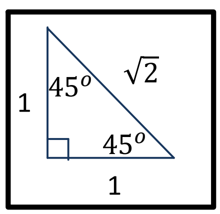
Check your answer: $sin30^o = \frac{1}{2}$ ✓

$\sin\theta = \frac{1}{2}$ $\quad sin150^o = \frac{1}{2}$ ✓

$\boldsymbol{cos\theta = -\frac{1}{\sqrt{2}}}$ **Solve for $\theta, 0^o \leq \theta < 360^o$ *and general solution*.**



Draw two triangles where cos θ is -ve...



$\theta_{stp} = 180^o + 45^o = 225^o$ $\quad \theta_{stp} = 180^o - 45^o = 135^o$

$\boldsymbol{cos\theta = -\frac{1}{\sqrt{2}} = -0.707}$

$\theta_{stp} = 225^o, 135^o$

$\boldsymbol{cos135 = -\frac{1}{\sqrt{2}}}$ ✓ $\quad \boldsymbol{cos225 = -\frac{1}{\sqrt{2}}}$ ✓

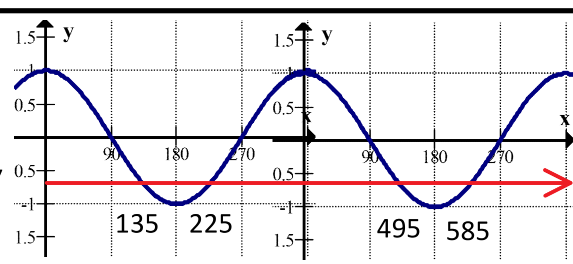
General Solution:

$\theta = \theta_{stp} \pm pn, n \in I$

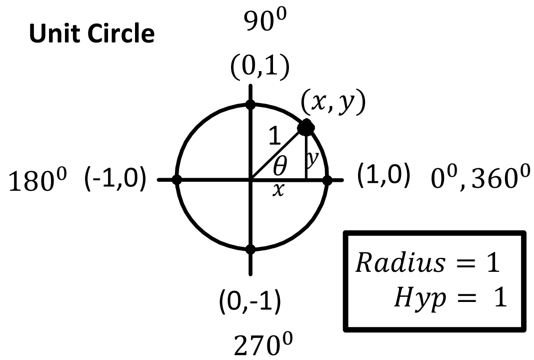
$\theta = 225^o \pm 360^o n, n \in I$

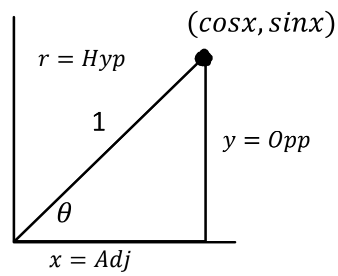
$\theta = 135^o \pm 360^o n, n \in I$

$\boldsymbol{-0.707}$



C11 - 2.6 - Unit Circle sin/cos/tan 90, 180, 270, 360 Notes



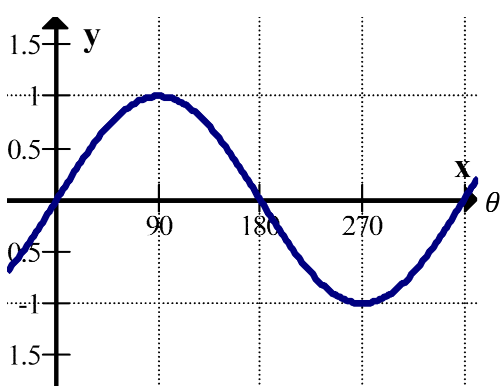


$$(x, y)$$

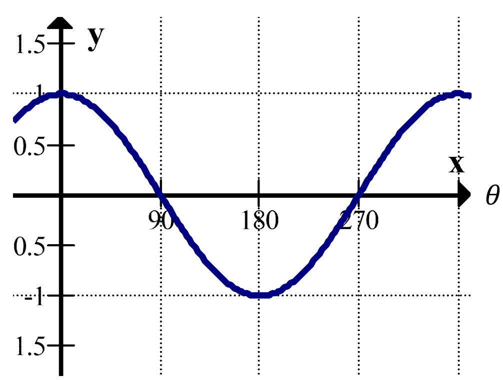
$$(cos\theta, sin\theta)$$

$sin\theta = y$ $\quad$ $cos\theta = x$ $\quad$ $tan\theta = \frac{y}{x}$

$\boldsymbol{sin\theta = \frac{Opp}{Hyp}}$ $sin\theta = \frac{y}{1}$ $sin\theta = y$	$\boldsymbol{cos\theta = \frac{Adj}{Hyp}}$ $cos\theta = \frac{x}{1}$ $cos\theta = x$	$\boldsymbol{tan\theta = \frac{Opp}{Adj}}$ $tan\theta = \frac{y}{x}$
$sin0^o = \frac{0}{1}$ $sin0^o = 0$	$cos0^o = \frac{1}{1}$ $cos0^o = 1$	$tan0^o = \frac{0}{1}$ $tan0^o = 0$
$sin90^o = \frac{1}{1}$ $sin90^o = 1$	$cos90^o = \frac{0}{1}$ $cos90^o = 0$	$tan90^o = \frac{1}{0}$ $tan90^o = UND$
$sin180^o = \frac{0}{1}$ $sin180^o = 0$	$\cos 180^o = -\frac{1}{1}$ $cos180^o = -1$	$tan180^o = \frac{0}{-1}$ $tan180^o = 0$
$sin270^o = \frac{-1}{1}$ $sin270^o = -1$	$cos270^o = \frac{0}{1}$ $cos270^o = 0$	$tan270^o = \frac{-1}{0}$ $tan270^o = UND$
$sin360^o = \frac{0}{1}$ $sin360^o = 0$	$cos360^o = \frac{1}{1}$ $cos360^o = 1$	$tan360^o = \frac{0}{1}$ $tan360^o = 0$



$y = sinx$



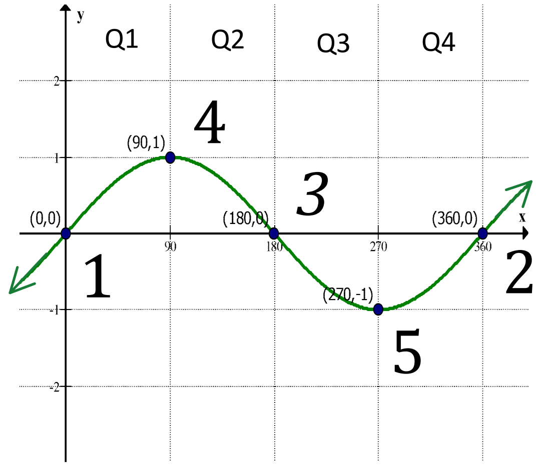
$y = cosx$

C11 - 2.7 - TOV^0 sinx,cosx,tanx Graph TOV Notes

$y = \sin x$

x	y
0°	0
90°	1
180°	0
270°	-1
360°	0

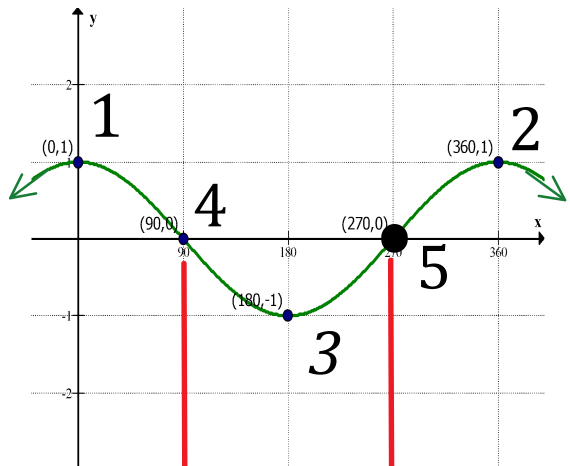
Pt.
(0,0)
(90,1)
(180,0)
(270,-1)
(360,0)



$y = \cos x$

x	y
0°	1
90°	0
180°	-1
270°	0
360°	1

Pt.
(0,1)
(90,0)
(180,-1)
(270,0)
(360,1)

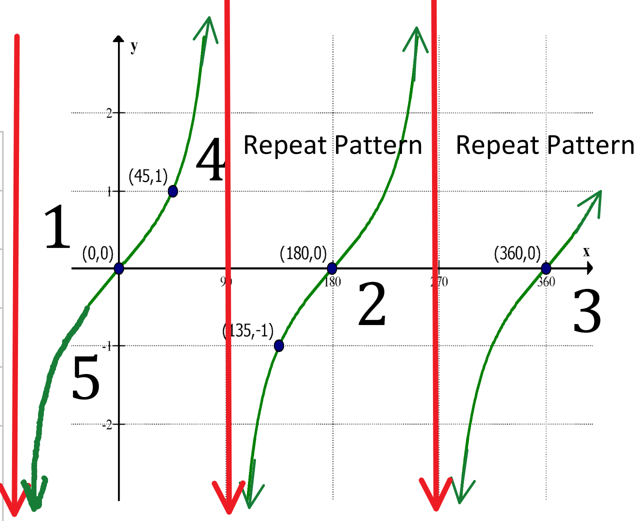


$y = \tan x$

x	y
0°	0
45°	1
90°	und
135°	-1
180°	0

Pt.
(-45,-1)
(0,0)
(45,1)
(90,und)
(135,-1)
(180,0)

ASTC
Special Triangles



Tan is Zero when sin is zero
Tan is UND when cos is zero

$tanx = \frac{sinx}{cosx}$

C11 - 2.8 - $\sin 2\theta$ Notes

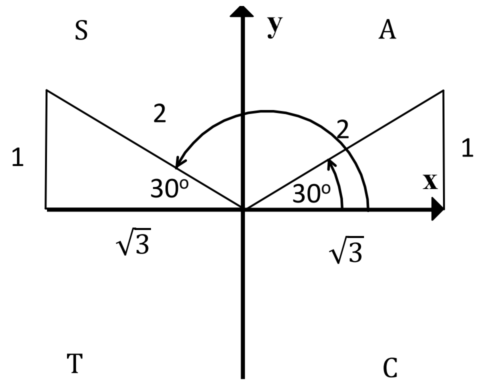
$sin2\theta = \frac{1}{2}$

Solve for θ $0^o \le \theta < 360^o$, *and the general solution.*

$\sin m = \frac{1}{2}$

$Let\ m = 2\theta$

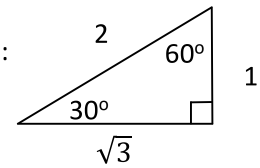
Draw two $\Delta's$ where $\sin m$ is +ve: ASTC Quadrant I, II



Label the triangles according to SOH CAH TOA

Label the reference angle according to special $\Delta's$.

Draw an arrow from the principal axis:
To the first and second terminal arm



Solve for the arrows m_{stp}

Check your answer: $sin2\theta = \frac{1}{2}$

$\sin(2(15)) = \frac{1}{2}$ ✓ $\sin(2(75)) = \frac{1}{2}$ ✓

$sinm = \frac{1}{2}$

$m_{stp} = 30^o$

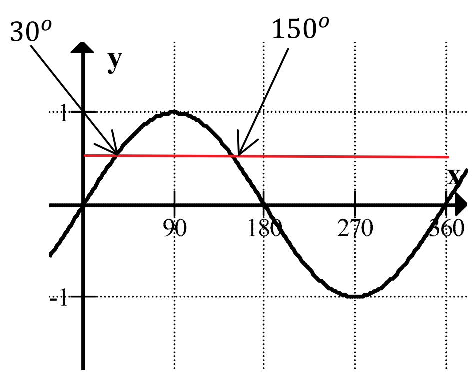
$m = 30^o$
$2\theta = 30^o$
$\frac{2\theta}{2} = \frac{30^o}{2}$
$\theta = 15^o$

$m_{stp} = 180^o - 30^o$
$= 150^o$

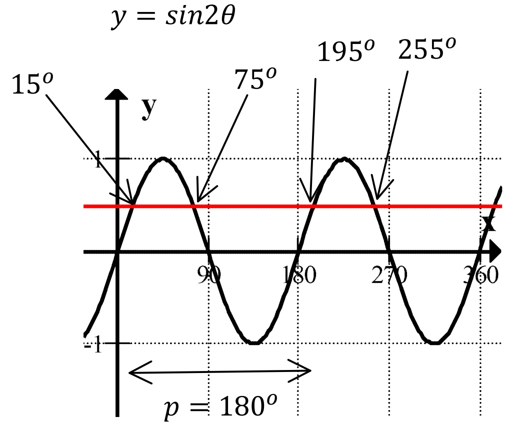
$m = 150^o$
$2\theta = 150^o$
$\frac{2\theta}{2} = \frac{150^o}{2}$
$\theta = 75^o$

Substitute 2θ back in for m.

$y = sin\theta$



$y = sin2\theta$



$HC = \frac{1}{2}$

$p = \frac{360^0}{b}$
$p = \frac{360^o}{2}$
$= 180^o$

$\theta = \theta_{stp} \pm p$
$\theta = 15^o + 180^o$
$\theta = 195^o$

~~$\theta = \theta_{stp} \pm p$~~
~~$\theta = 195^o + 180^o$~~
~~$\theta = 375^o$~~

$\theta = \theta_{stp} \pm p$
$\theta = 75^o + 180^o$
$\theta = 255^o$

$0 \le \theta \le 360^o$
$\theta = 15^o, 75^o, 195^o, 225^o$

$\sin(2(195)) = \frac{1}{2}$ ✓

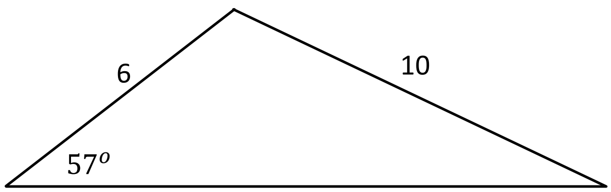
$\sin(2(225)) = \frac{1}{2}$ ✓

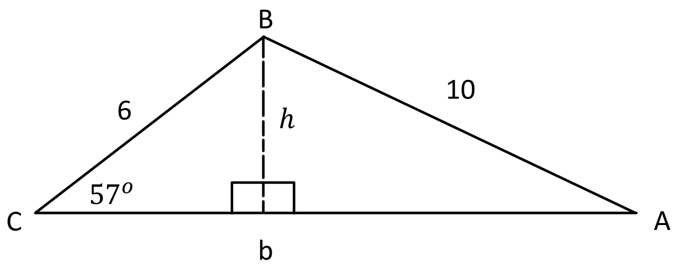
General Solution: $\theta_{gen} = \theta_{stp} \pm pn, n \in I$
$\theta_{gen} = 15^o \pm 180^o\ n, n \in I$

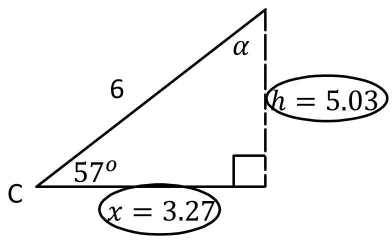
$\theta_{gen} = \theta_{stp} \pm pn, n \in I$
$\theta_{gen} = 75^o \pm 180^o n, n \in I$

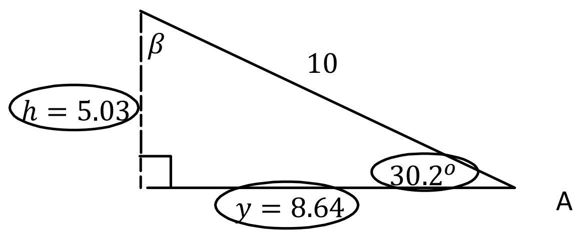
C11 - 2.9 - Solve ASS Triangle Without Sine Law Notes

Solve the triangle with side lengths of 6 m and 10 m, and an angle of 57^o.









$$sin\theta = \frac{O}{H}$$
$$sin57^o = \frac{h}{6}$$
$$6 \times sin57^o = \frac{h}{\cancel{6}} \times \cancel{6}$$
$$6sin57^o = h$$
$$5.03 = h$$
$$\boxed{h = 5.03}$$

$$cos\theta = \frac{A}{H}$$
$$cos57^o = \frac{x}{6}$$
$$6 \times cos57^o = \frac{x}{\cancel{6}} \times \cancel{6}$$
$$6cos57^o = x$$
$$3.27 = x$$
$$\boxed{x = 3.27}$$

$$sin\theta = \frac{O}{H}$$
$$sin\theta = \frac{5.03}{10}$$
$$sin\theta = 0.503$$
$$\theta = \sin^{-1} 0.503$$
$$\boxed{\theta = 30.2^o}$$

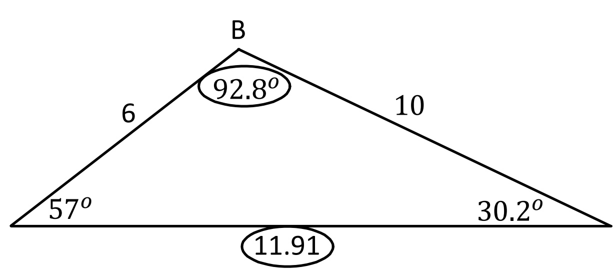
$$cos\theta = \frac{A}{H}$$
$$\cos 30.2^o = \frac{y}{10}$$
$$0.864 = \frac{y}{10}$$
$$10 \times 0.864 = \frac{y}{\cancel{10}} \times \cancel{10}$$
$$8.64 = y$$
$$\boxed{y = 8.64}$$

$$\alpha = 180^o - (57^o + 90^o)$$
$$\alpha = 180^o - 147^o$$
$$\alpha = 33^o$$

$$\beta = 180^o - (30.2^o + 90^o)$$
$$\beta = 180^o - 120.2^o$$
$$\beta = 59.8^o$$

$$B = \alpha + \beta$$
$$= 33^o + 59.8^o$$
$$= \boxed{92.8^o}$$

$$b = x + y$$
$$b = 3.27 + 8.64$$
$$\boxed{b = 11.91}$$

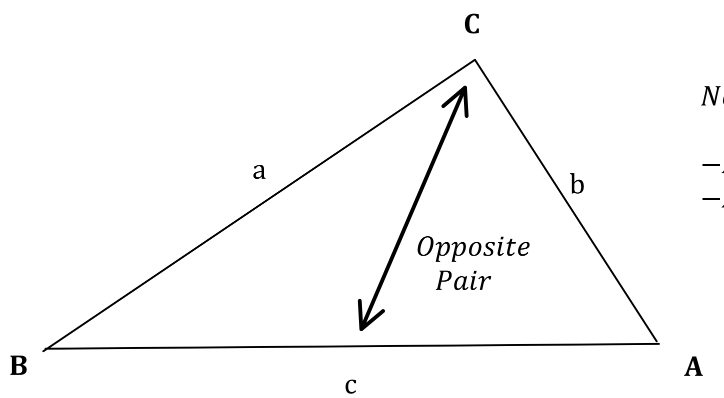


C11 - 2.9 - Sine Law Notes

Sine Law: $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$ **OR** $\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$

$(to\ find\ a\ side)$ $(to\ find\ an\ angle)$

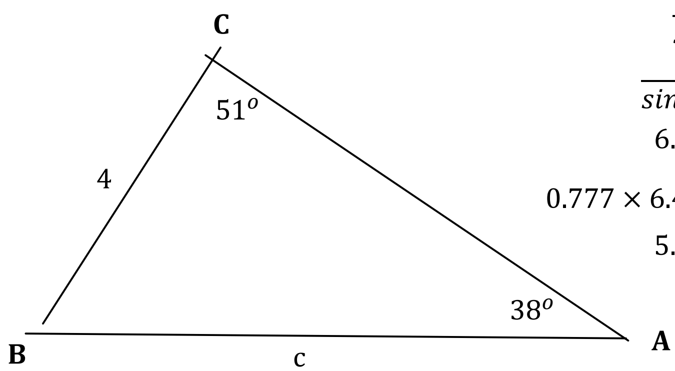
$What\ you\ are\ looking\ for\ goes\ on\ top\ but\ algebra\ allows\ you\ to\ do\ either$



$Notice:\ Use\ the\ Sine\ Law\ if\ you\ have:$

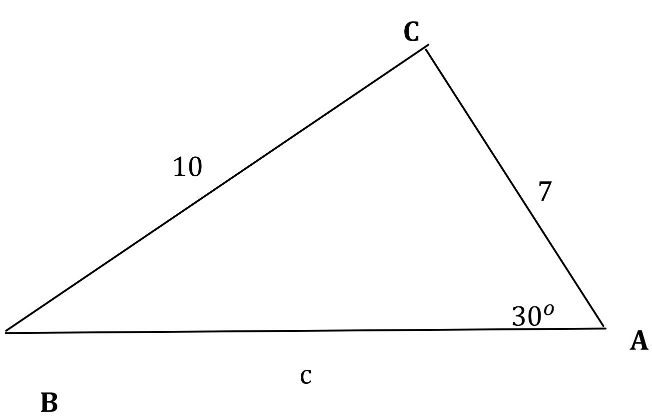
$-An\ opposite\ pair$
$-And\ one\ other\ piece\ of\ information$

Remember: We only sin angles.
$180^o\ in\ a\ triangle$



$$\frac{a}{sinA} = \frac{c}{sinC}$$
$$\frac{4}{sin38^o} = \frac{c}{sin51^o}$$
$$6.497 = \frac{c}{0.777}$$
$$0.777 \times 6.497 = \frac{c}{0.777} \times 0.777$$
$$5.048 = c$$
$$c = 5.048$$

$$\frac{a}{sinA} = \frac{c}{sinC}$$
$$\frac{asinC}{sinA} = c$$

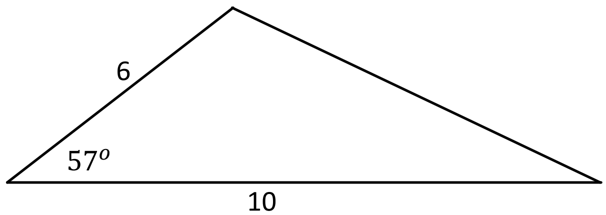


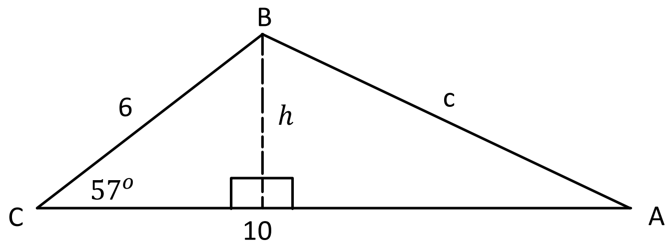
$$\frac{sinA}{a} = \frac{sinB}{b}$$
$$\frac{\sin(30)}{10} = \frac{sinB}{7}$$
$$0.05 = \frac{sinB}{7}$$
$$7 \times .05 = \frac{sinB}{7} \times 7$$
$$0.35 = \text{sinB}$$
$$\text{sinB} = 0.35$$
$$\text{B} = \sin^{-1}(0.35)$$
$$B = 20.5^o$$

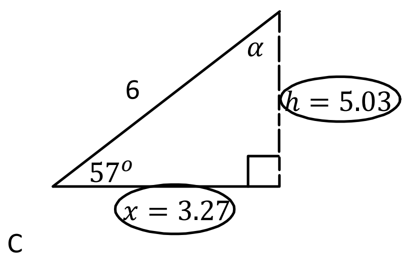
Remember: If you have 2 angles without either opposite side, use 180^o in a triangle.

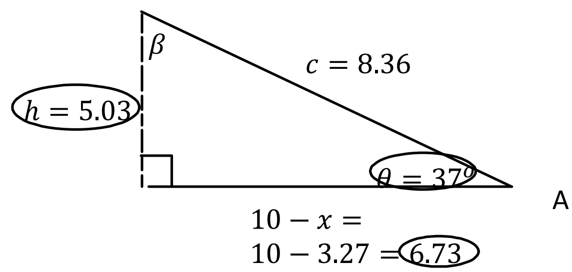
C11 - 2.10 - Solve SAS Triangle Without Cosine Law Notes

Solve the triangle with side lengths of 6 m and 10 m, and an angle between the two given sides of 57^o.









$$sin\theta = \frac{O}{H}$$
$$sin57^o = \frac{h}{6}$$
$$6 \times sin57^o = \frac{h}{6} \times 6$$
$$6sin57^o = h$$
$$5.03 = h$$
$$h = 5.03$$

$$cos\theta = \frac{A}{H}$$
$$cos57^o = \frac{x}{6}$$
$$6 \times cos57^o = \frac{x}{6} \times 6$$
$$6cos57^o = x$$
$$3.27 = x$$
$$x = 3.27$$

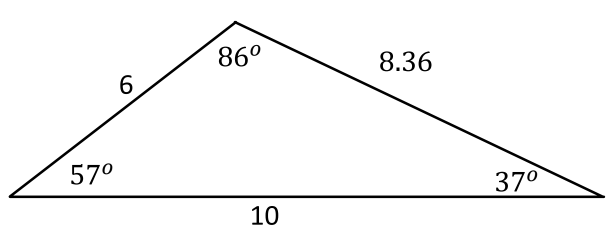
$$tan\theta = \frac{O}{A}$$
$$tan\theta = \frac{5.03}{6.73}$$
$$tan\theta = 0.7474$$
$$\theta = \tan^{-1}(0.7474)$$
$$\theta = 36.77^o$$
$$\theta = 37^o$$

$$sin\theta = \frac{O}{H}$$
$$sin37 = \frac{5.03}{c}$$
$$c \times sin37 = \frac{5.03}{c} \times c$$
$$csin37 = 5.03$$
$$\frac{csin37}{sin37} = \frac{5.03}{sin37}$$
$$c = \frac{5.03}{sin37}$$
$$c = 8.36$$

$$57^o + 90^o + \alpha = 180^o$$
$$147^o + \alpha = 180^o$$
$$-147^o \qquad - 147^o$$
$$\alpha = 33^o$$

$$37^o + 90^o + \beta = 180^o$$
$$127^o + \beta = 180^O$$
$$-127^o \qquad - 127^o$$
$$\beta = 53^o$$

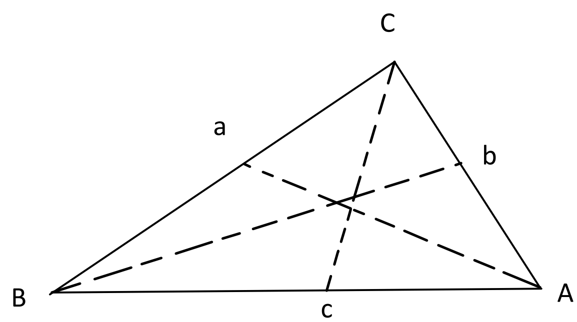
$$B = \alpha + \beta$$
$$= 33^o + 53^o$$
$$= 86^o$$



Remember: Find the smallest angle first, and/or 180 minus

C11 - 2.10 - Cosine Law Notes

Cosine Law



Cosine Law:

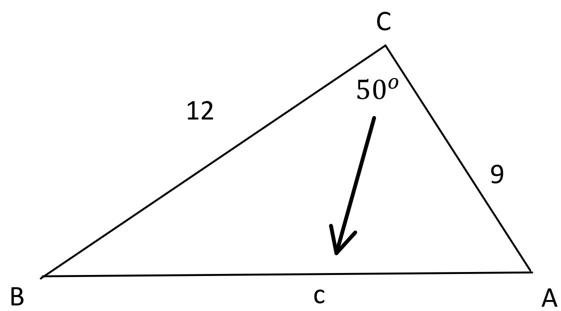
$$c^2 = b^2 + a^2 - 2abcosC$$

Notice: This pattern should occur.

Cosine Law: SSS (hard) and SAS (easy)

Remember: Only one angle in the formula

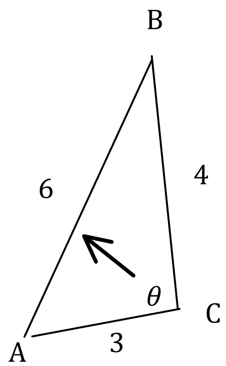
Remember: We only *cos* angles.



$$c^2 = b^2 + a^2 - 2abCosC$$
$$c^2 = 9^2 + 12^2 - 2(12)(9)cos50$$
$$c^2 = 86.2$$
$$\sqrt{c^2} = \sqrt{86.2}$$
$$c = 9.3$$

Plug into calculator

Square root both sides



$$c^2 = b^2 + a^2 - 2abcosC$$

$$6^2 = 3^2 + 4^2 - 2(4)(3)cosC$$ Substitute values in

$$36 = 9 + 16 - 24cosC$$ Calculate the squares, multiply

$$36 = 25 - 24cosC$$ Add

$$36 = 25 - 24cosC$$
$$-25 \quad -25$$ Subtract from both sides

$$11 = -24cosC$$
$$\frac{11}{-24} = \frac{-24cosC}{-24}$$ Divide both sides
$$-\frac{11}{24} = cosC$$
$$cosC = -\frac{11}{24}$$
$$C = \cos^{-1}\left(-\frac{11}{24}\right)$$ Inverse cos
$$C = 117.3^o$$

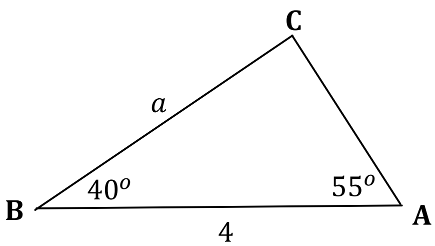
$$C = \cos^{-1}\left(\frac{(a^2+b^2-c^2)}{(2ab)}\right)$$

$$c^2 = b^2 + a^2 - 2abcosC$$
$$b^2 = c^2 + a^2 - 2cacosB$$
$$a^2 = b^2 + c^2 - 2cbcosA$$

See edit below

C11 - 2.11 - Sine/Cosine Law Notes Solve the Triangle

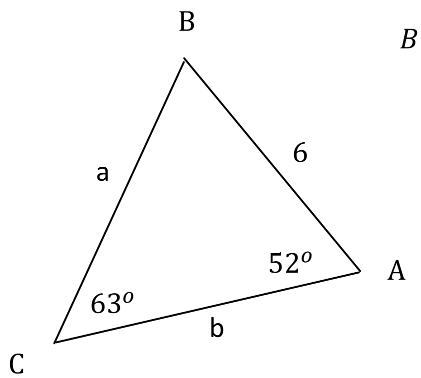
Solve for a.



$C = 180^o - 40^o - 55^o$
$= 85^o$

$$\frac{a}{sinA} = \frac{c}{sinC}$$
$$\frac{a}{sin55^o} = \frac{4}{sin85^o}$$
$$\frac{a}{0.819} = 4.015$$
$$\cancel{0.819 \times} \frac{a}{\cancel{0.819}} = 4.015 \times 0.819$$
$$a = 3.289$$

Solve the triangle.

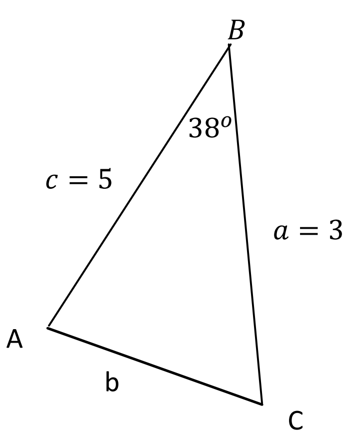


$B = 180^o - 63^o - 52^o$
$= 65^o$

$$\frac{a}{sinA} = \frac{c}{sinC}$$
$$\frac{a}{sin52^o} = \frac{6}{sin63^o}$$
$$\frac{a}{0.788} = 6.734$$
$$\cancel{0.788 \times} \frac{a}{\cancel{0.788}} = 6.734 \times 0.788$$
$$a = 6.734 \times 0.788$$
$$a = 5.306$$

$$\frac{b}{sinB} = \frac{c}{sinC}$$
$$\frac{b}{sin65^o} = \frac{6}{sin63^o}$$
$$\frac{b}{0.906} = 6.734$$
$$\cancel{0.906 \times} \frac{b}{\cancel{0.906}} = 6.734 \times 0.906$$
$$b = 6.101$$

Solve the triangle *Find the angle opposite of the smaller side 1st.



Cosine Law: Switched b and c

$$c^2 = a^2 + b^2 - 2abcosC$$
$$b^2 = a^2 + c^2 - 2ac \cdot cosB$$
$$b^2 = 3^2 + 5^2 - 2(3)(5) \cdot \cos(38^o)$$
$$b^2 = 9 + 25 - 30\cos(38^o)$$
$$b^2 = 34 - 23.64$$
$$b^2 = 10.36$$
$$\sqrt{b^2} = \sqrt{10.36}$$
$$b = 3.22$$

Sine Law:

$$\frac{sinA}{a} = \frac{sinB}{b}$$
$$\frac{sinA}{3} = \frac{sin38^o}{3.22}$$
$$\frac{sinA}{3} = 0.19$$
$$3 \times \frac{sinA}{3} = 0.19 \times 3$$
$$sinA = 0.57$$
$$A = 35^o$$

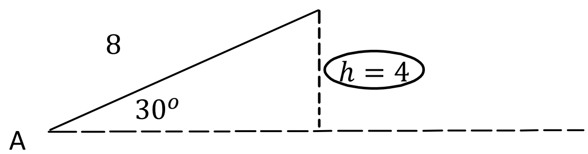
180^o in a triangle:

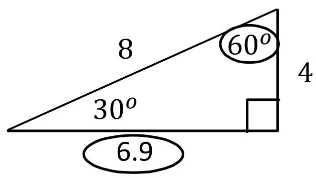
$C = 180^o - 38^o - 35^o$
$= 107^o$

C11 - 2.12 - Ambiguous Case of Sine (ASS) Notes

How many triangles? Solve the triangles. **Remember: Always find the height first.**

$\angle A = 30^o, b = 8, a = 4$





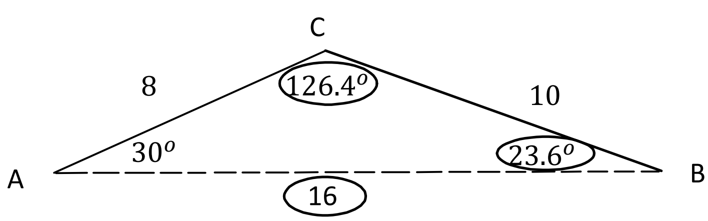
$$\sin\theta = \frac{O}{H}$$
$$\sin 30^o = \frac{h}{8}$$
$$8\sin 30^o = h$$
$$4 = h$$
$$h = 4$$

$$cos\theta = \frac{A}{H}$$
$$cos30^o = \frac{A}{8}$$
$$8\cos 30^o = A$$
$$6.9 = A$$
$$A = 6.9$$

$a = h$
One triangle

$$\theta = 180^o - 30^o - 90^o$$
$$\theta = 60^o$$

$\angle A = 30^o, b = 8, a = 10$



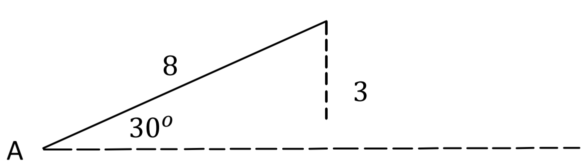
$10 > 8$
$a > b$
One triangle

$$\frac{sinB}{b} = \frac{sinA}{a}$$
$$\frac{sinB}{8} = \frac{\sin 30^O}{10}$$
$$\frac{sinB}{8} = 0.05$$
$$8 \times \frac{sinB}{8} = 0.05 \times 8$$
$$sinB = 0.4$$
$$B = \sin^{-1} 0.4$$
$$B = 23.6^o$$

$$\theta = 180^o - 23.6^o - 30^o$$
$$\theta = 126.4^o$$

$$\frac{c}{sinC} = \frac{a}{sinA}$$
$$\frac{c}{\sin 126.4^o} = \frac{10}{\sin 30^o}$$
$$\frac{c}{0.8} = 20$$
$$0.8 \times \frac{c}{0.8} = 20 \times 0.8$$
$$c = 16$$

$\angle A = 30^o, b = 8, a = 3$



$3 < 4$
$a < H$
no triangle

No triangle, can't solve.

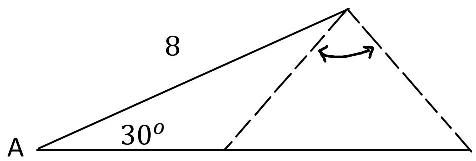
C11 - 2.12 - Ambiguous Case of Sine (ASS) Notes

How many triangles? Solve the triangles.

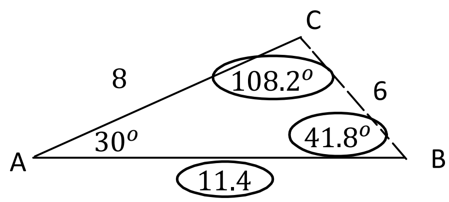
Remember: Always find the height first.

$\angle A = 30^o, b = 8, a = 6$

$4 < 6 < 8$
$H < a < B$
<u>Two triangles</u>



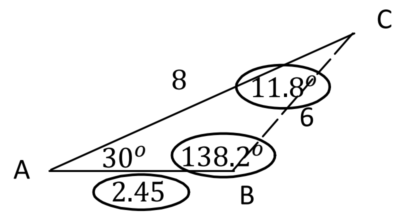
Draw both triangles together and separately.



$$\frac{\sin 30^o}{6} = \frac{\sin B}{8}$$
$$0.08\bar{3} = \frac{sinB}{8}$$
$$8 \times 0.08\bar{3} = \frac{sinB}{8} \times 8$$
$$0.\bar{6} = sinB$$
$$sinB = 0.\bar{6}$$
$$B = \sin^{-1} 0.\bar{6}$$
$$B = 41.8^o$$

$$\theta = 180^o - 30^o - 41.8^o$$
$$\theta = 108.2^o$$

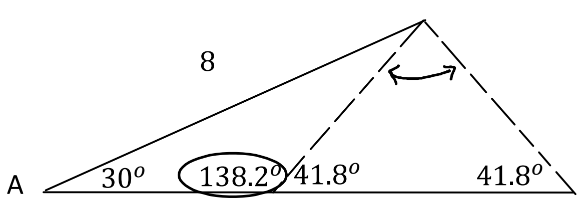
$$\frac{c}{sinC} = \frac{a}{sinA}$$
$$\frac{c}{\sin 108.2^o} = \frac{6}{\sin 30^o}$$
$$\frac{c}{0.95} = 12$$
$$0.95 \times \frac{c}{0.95} = 12 \times 0.95$$
$$c = 11.4$$



$$\theta = 180^o - 41.8^o$$
$$\theta = 138.2^o$$

$$\theta = 180^o - 30^o - 138.2^o$$
$$\theta = 11.8^o$$

$$\frac{c}{sinC} = \frac{a}{sinA}$$
$$\frac{c}{\sin 11.8^o} = \frac{6}{\sin 30^o}$$
$$\frac{c}{0.204} = 12$$
$$0.204 \times \frac{c}{0.204} = 12 \times 0.204$$
$$c = 2.45$$



Notice: Both triangles have an angle of 30^o, a side going up of 8, and a side opposite to 30^o of 6.

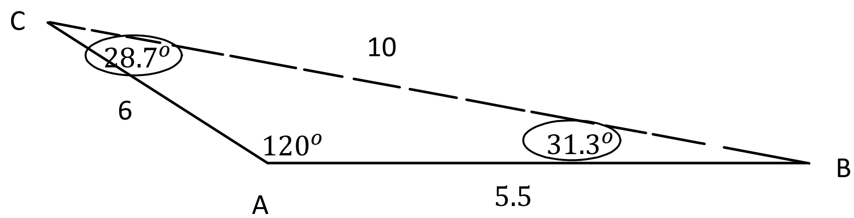
Notice: The isosceles triangle.

C11 - 2.12 - Ambiguous Case of Sine (ASS) Notes

How many triangles? Solve the triangles.

$\angle A = 120^o, b = 6, a = 10$

$10 > 6$
$a > b$
One triangle



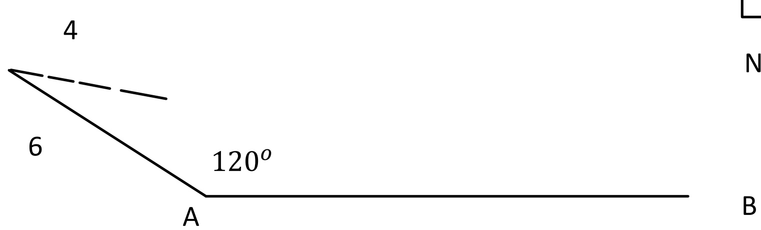
$$\frac{sinB}{b} = \frac{sinA}{a}$$
$$\frac{sinB}{6} = \frac{\sin 120^o}{10}$$
$$\frac{sinB}{6} = 0.0866$$
$$6 \times \frac{sinB}{6} = 0.0866 \times 6$$
$$sinB = 0.52$$
$$B = \sin^{-1} 0.52$$
$$B = 31.3^o$$

$$\theta = 180^o - 31.3^o - 120^o$$
$$\theta = 28.7^o$$

$$\frac{c}{sinC} = \frac{a}{sinA}$$
$$\frac{c}{\sin 28.7^o} = \frac{10}{\sin 120^o}$$
$$\frac{c}{0.48} = 11.55$$
$$0.48 \times \frac{c}{0.48} = 11.55 \times 0.48$$
$$c = 5.5$$

$\angle A = 120^o, b = 6, a = 4$

$4 < 6$
$a < b$
No triangle



No triangle. Can't solve.