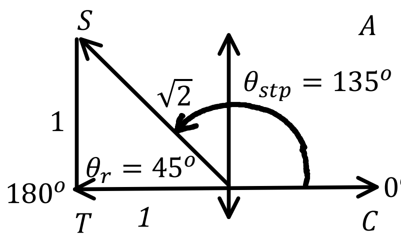
C11 - 2.0 - Trig Rad ASTC/Unit ○ Spec Δ @$^{-1}$(+) Alg $\theta_@$



$180 - 135 = 45$

$cos135^o = -\frac{1}{2} = -0.707$

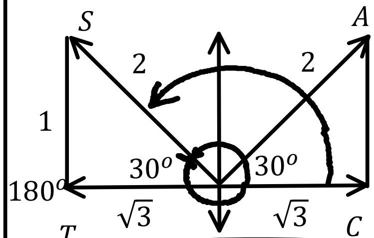
$\theta_{cot} = 135^o \pm 360n$
$\theta_{gen} = 135^o + 360n, n \in I$

$\theta_{cot} = 495^o, -225^o$

$\theta_{pri} = 135^o$

Domain

$sin\theta = \frac{1}{2}$; $-270^o \le \theta < 360^o$



$\theta_{stp} = -210^o, 30^o, 150^o$

Check :

$\sin -210^o = \sin 30^o = \sin 150^o = \frac{1}{2}$

$\theta_{gen} = 30^o + 360^o n, n \in I$
$\theta_{gen} = 150^o + 360^o n, n \in I$

$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

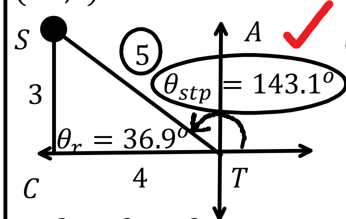
Algebra

$2sin\theta - 1 = 0$
$2sin\theta = 1$
$sin\theta = \frac{1}{2}$

SOH CAH TOA is a magical fairyland to help grade tens learn trigonometry, it is only something that works but not the actual definition.

$5sin\theta + 3 = 0$
$5sin\theta = -3$
$sin\theta = -\frac{3}{5}$

SOH-CAH-TOA
CHO-SHA-CAO



$sin\theta = +\frac{3}{5}$

$cos\theta = -\frac{4}{5}$

$tan\theta = -\frac{3}{4}$

$a^2 + b^2 = c^2$

$c = 5$

$\theta_r = \tan^{-1}\left(+\frac{3}{4}\right)$

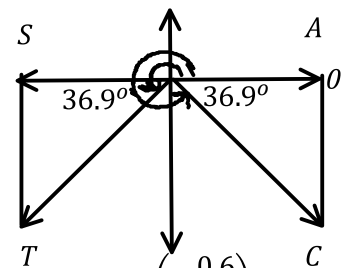
$180 - 36.9 = 143.1$

$\theta_r = 36.9^o$

$sin143.1 = 0.6 = \frac{3}{5}$

θ_r : Only inverse positives.

$sin\theta = -\frac{0.6}{1}$; $0 \le \theta < 360$



$\theta_r = \sin^{-1}\left(+\frac{0.6}{1}\right)$

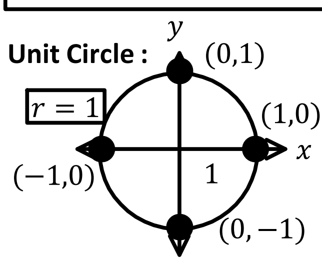
$\theta_r = 36.9^o$ $180^o + 36.9^o$
$360^o - 36.9^o$

$\theta_{stp} = 216.9^o, 323.1^o$

$sin216.9 = -0.6$
$sin323.1 = -0.6$

$\theta_{gen} = 216.9^o + 360^o n, n \in I$
$\theta_{gen} = 323.1 +^o 360^o, n \in I$

Unit Circle :



$sin90^o = 1$

$sin\theta = y$

$\tan 270^o = und$

$tan\theta = \frac{y}{x}$

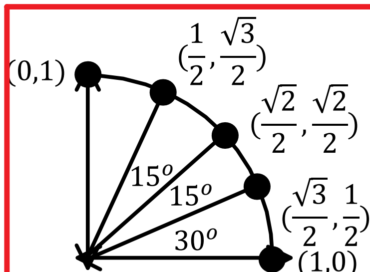
$cos\theta = 0$; $0 < \theta < 360$

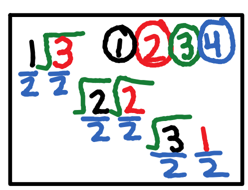
$cos\theta = x$ $\cos 90^o = 0$

$\theta = 90^o, 270^o$ $\cos 270^o = 0$

$\theta_{gen} = 90^o + \pi n, n \in I$

$cos\theta - 2 = -2$
$cos\theta = 0$





1,2,3
1,2,3
Root the tops
All over 2.

SOH-CAH-TOA is a magical fairy land to teach grade 10's trig*.

C11 - 2.0 - Trig NPV's $x^2 + y^2 = r^2$ (x,y) $p(\theta)$ a,A,w

NPV's : Denominator $\neq 0$

$\frac{1}{\tan\theta}$ → $\frac{1}{\frac{\sin\theta}{\cos\theta}}$

$\cos\theta \neq 0$ … $\theta \neq 90^o, 270^o$ … $\theta \neq 90^o + 180^o n, n \in \mathrm{I}$ … $p^* = 270 - 90 = 180$

$\sin\theta \neq 0$ … $\theta \neq 0, 180^o$ … $\theta \neq 180^o n, n \in \mathrm{I}$ … $p^* = 180 - 0 = 180$

$\frac{1}{\cos\theta + 1}$

$\cos\theta + 1 \neq 0$

$\cos\theta \neq -1$

…

$\frac{1}{\sin\theta - \frac{1}{2}}$

$\sin\theta - \frac{1}{2} \neq 0$

$\sin\theta \neq \frac{1}{2}$

…

$\frac{1}{\cos^2 x - 1}$

$\cos^2 x - 1 \neq 0$

$\cos^2 x \neq 1$

$\cos x \neq \pm 1$

…

$\frac{1}{\sin^2 x + 1}$

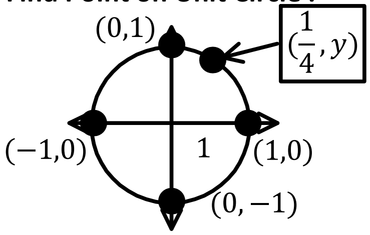
$\sin^2 x + 1 \neq 0$

$\sin^2 x \neq -1$

$\sin x \neq \sqrt{-1}$

No Restrictions

Find Point on Unit Circle :



$x^2 + y^2 = 1$

$\left(\frac{1}{4}\right)^2 + y^2 = 1$

$\frac{1}{16} + y^2 = \frac{16}{16}$

$y^2 = \frac{15}{16}$

$y = \pm\frac{\sqrt{15}}{4}$

Is the Point on the Unit Circle :

$(-\frac{3}{4}, \frac{1}{4})$

$x^2 + y^2 = 1$

$\left(-\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 \neq 1$

$\frac{9}{16} + \frac{1}{16} \neq 1$

$\frac{10}{16} \neq 1$

Not on Unit Circle

$(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$

$x^2 + y^2 = 1$

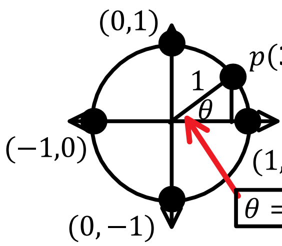
$\left(-\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 = 1$

$\frac{3}{4} + \frac{1}{4} = 1$

$1 = 1$ ✓

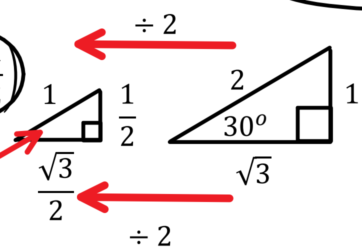
On Unit Circle

Solve the Point on the Unit Circle :

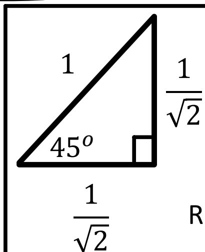


$p(30^o) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\theta = 30^o$



Similar Triangles



$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

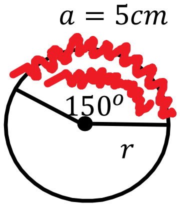
Rationalize!

Arc Length/Sector Area :

Find the Sector Area and Radius of the circle if arc-length subtended by θ.

$let\ a = arc - length$

$a = 5cm$



$\frac{\theta}{360^o} = \frac{arc}{2\pi r}$

$r = \frac{arc(360^o)}{2\pi r\theta}$

$\frac{150}{360} = \frac{5}{2\pi r}$

$r = 1.91cm$

$\frac{150}{360} = \frac{5}{12}$

Logic Check

$C = 2\pi r$

$C = 2\pi(1.91)$

$C = 12cm$

$A = \frac{ar}{2}$

$A = \frac{5 \times 1.91}{2}$

$A = 4.78cm^2$ ✓

$A = \pi r^2$

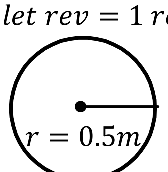
$A = \pi(1.91)^2$

$A = 11.46cm^2$

Find the angular velocity of a wheel travelling 25 $\frac{m}{s}$ if the radius 0.5 m. Find the arc in 0.1 s.

$let\ w = angular\ velocity$

$let\ rev = 1\ revolution$



$C = 2\pi r$

$C = 2\pi(0.5)$

$C = 3.14\ m$

Number of Turns :

$25m$ ⟷ $1s$

$3.14\ m$ ⟷ $\frac{25}{3.14} = 7.96\ Revs$

$w = \frac{\theta}{t}$

$w = \frac{7.96\ revs}{1\ s}$

$w = \frac{7.96\ (360)}{1\ s}$ ✓

$w = \frac{2865^o}{s}$

Length of tire that touches the road.

$1\ Rev = 360^o$

$\frac{\theta}{360^o} = \frac{arc}{2\pi r}$

$arc = \frac{2\pi r\theta}{360^o}$

$arc = \frac{2\pi(0.5)(2865^o)}{360^o}$

$arc = 25m$

If you turn about 8 times with a circumference of about 3m you will be about 25m.

C11 - 2.0 - Trig Alg Fact $let\ m =$ Period Graph/Calc

Algebra :

$sin\theta + sin\theta - 1 = 0$

$2sin\theta = 1$

$sin\theta = \frac{1}{2}$

…

$\cos^2\theta = 1$

$cos\theta = \pm 1$

$cos\theta = 1 \quad cos\theta = -1$

…

$\sin^2\theta = \frac{1}{2} \quad ; \quad 0 \le \theta < 2\pi$

$sin\theta = \pm\frac{1}{\sqrt{2}}$

…

4 triangles!

$\theta = 45^o\ \theta = 135^o$

$\theta = 225^o\ \theta = 315^o$

$\theta_{gen} = 45^o + 90^o n, n \in \mathrm{I}$

$2\sin^2\theta + sin\theta - 1 = 0$ Factoring

$2m^2 + m - 1 = 0$ $let\ m = sin\theta$

$(2m-1)(m+1) = 0$

$2m - 1 = 0 \quad m + 1 = 0$

$m = \frac{1}{2} \quad m = -1$

$sin\theta = \frac{1}{2} \quad sin\theta = -1$

… …

$\frac{cosx}{cosx+1} = -\frac{1}{3}$

$\frac{m}{m+1} = -\frac{1}{3}$ $let\ m = cosx$

$3m = -m - 1$

$m = -\frac{1}{4}$

$cosx = -\frac{1}{4}$

$2sin\theta cos\theta + cos\theta = 0$

$cos\theta(2sin\theta + 1) = 0$

$cos\theta = 0 \quad 2sin\theta + 1 = 0$

…

$sin\theta = \frac{1}{2}$

…

$\tan^2\theta + tan\theta = 3$ $let\ m = tan\theta$

$m^2 + m - 3 = 0$

…

Quadform!

$m = 1.3 \quad m = -2.3$

$tan\theta = 1.3 \quad tan\theta = -2.3$

$\theta = \tan^{-1}(+1.3) \quad \theta = \tan^{-1}(+2.3)$

$\theta_r = 0.915 \quad \theta_r = 1.161$

… …

Period Change : $y = sinbx$ The usual number of answers in the domain times b*.

$sin2\theta = \frac{1}{2} \quad ; \quad 0 \le \theta < 360^o$

$sinm = \frac{1}{2}$ $let\ m = 2\theta$

…

$m = 30^o \quad m = 150^o$

$2\theta = 30^o \quad 2\theta = 150^o$

$\theta = 15^o \quad \theta = 75^o$

Period

$p = \frac{360}{b}$

$p = \frac{360}{2}$

$p = 180$

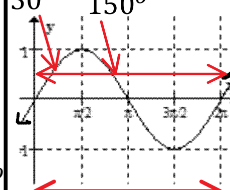
$\theta = 195 + 180$

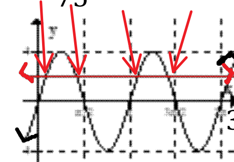
$\theta = 375 > 360^o$

$Calc\ y_1 = y_2$

$y_1 = LHS$

$y_2 = RHS$





$HC = \frac{1}{2}$

$\theta = \theta + p \quad \theta = \theta + p$

$\theta = 15 + 180 \quad \theta = 75 + 180$

$\theta = 195^o \quad \theta = 255^o$

Add/Subtract period until outside of the domain.

$\theta_{gen} = 15^o + 180^0 n, n \in \mathrm{I}$

$\theta_{gen} = 75^o + 180^0 n, n \in \mathrm{I}$

$195 - 15 = 180$

$255 - 75 = 180$

$\cos\frac{1}{2}\theta = 0 \quad ; \quad 0 \le \theta < 360^o$

$cosm = 0$ $let\ m = \frac{1}{2}\theta$

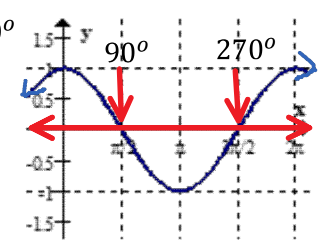
$cos\theta = y$

…

$m = 90^o \quad m = 270$

$\frac{1}{2}\theta = 90^o \quad \frac{1}{2}\theta = 270$

$\theta = 180^o \quad \theta = 540^o$



$p^* = 720^o$

$\theta_{gen} = 180^o + 720^o, n \in \mathrm{I}$

$y = asin(b(x-c)) + d$ $p = 72^o$

$\sin(5(x-60)) = \frac{1}{2} \quad ; \quad 0 \le x < 360^o$

$sinm = \frac{1}{2}$ $let\ m = 5(x-60)$

…

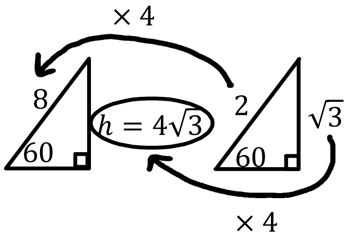
$x = 18^o, 66^o, 90^o, 138^o \ldots 354^o, 378^o$

$\theta_{gen} = 18^o + 72n, n \in \mathrm{I}$

$\theta_{gen} = 66^o + 72n, n \in \mathrm{I}$

C11 - 2.0 - Trig Geometry/Cart Plane

Solve for h.



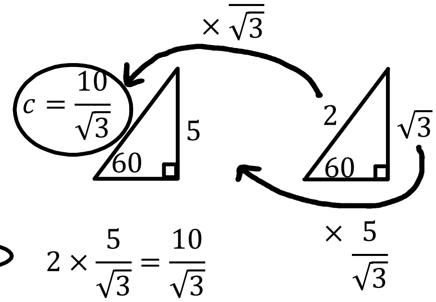
$sin\theta = \frac{opp}{hyp}$

$sin60 = \frac{h}{8}$

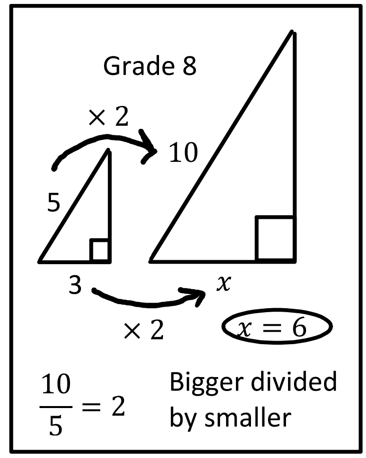
$8 \times \frac{\sqrt{3}}{2} = \frac{h}{8} \times 8$

$h = 4\sqrt{3}$

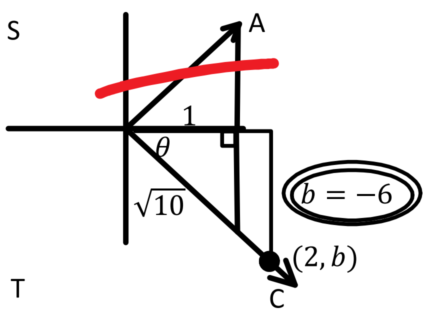
Solve for c.

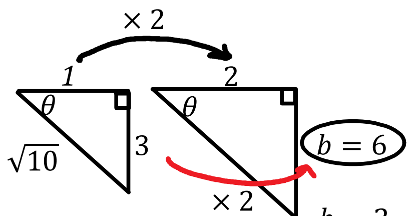


$2 \times \frac{5}{\sqrt{3}} = \frac{10}{\sqrt{3}}$



$cos\theta = \frac{1}{\sqrt{10}}$ $tan\theta < 0$ Find b ; $(2, b)$





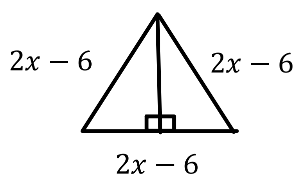
$a^2 + b^2 = c^2$

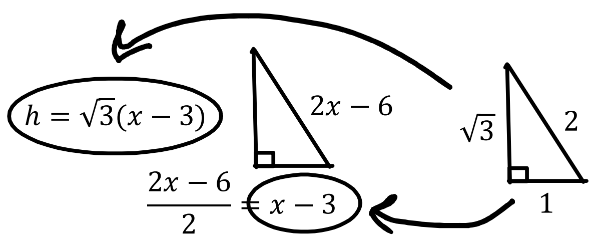
$b = 3$

$\frac{b}{3} = \frac{2}{1}$

$b = 6$

Find Area (Hard)





$\frac{2x-6}{2} = x - 3$

$A = \frac{bh}{2}$

$A = \frac{(2x-6)\sqrt{3}(x-3)}{2}$

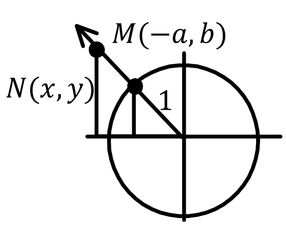
$A = \sqrt{3}(x-3)^2$

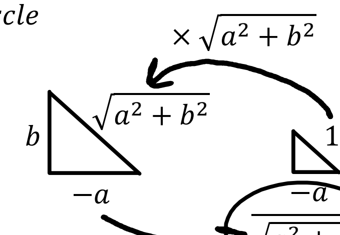
~~$(2x-6)^2 - (x-3)^2 = b^2$~~

~~$4x^2 - 24x + 36 - x^2 + 6x - 9 = b^2$~~

~~$3x^2 - 18x + 25 = b^2$~~

Find N(x, y) on unit circle





$\frac{-a}{\sqrt{a^2+b^2}}$

$\div \sqrt{a^2+b^2}$

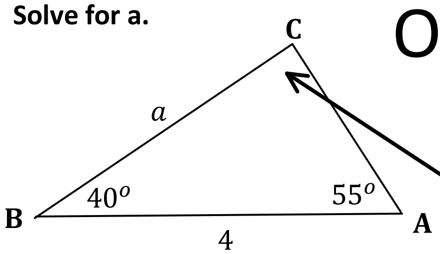
$x = \frac{-a}{\sqrt{a^2+b^2}}$

Squaring negatives gives positive.

$y = \frac{b}{\sqrt{a^2+b^2}}$

C11 - 2.0 - Trig Sine Opp Pair/Cosine SAS/SSS Law

Solve for a.



Opposite Pair!

Put what your looking for on top! Or Algebra!

180° in a triangle:

$C = 180^o - 40^o - 55^o$

$= 85^o$

$$\frac{a}{sinA} = \frac{c}{sinC}$$

$$\frac{a}{sin55^o} = \frac{4}{sin85^o}$$

$$\frac{a}{sin55^o} = \frac{4sin55^o}{sin85^o}$$

Multiply both sides by $sin55^o$

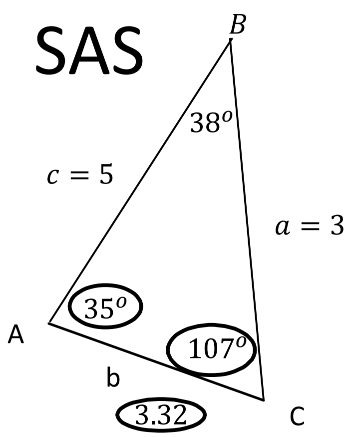
$a = 3.289$

$$\frac{sinA}{a} = \frac{sinB}{b}$$

To Find an Angle

Solve the triangle *Find the angle opposite of the smaller side 1st.

SAS



Cosine Law:

$$c^2 = a^2 + b^2 - 2abCosC$$

$$b^2 = a^2 + c^2 - 2ac \cdot cosB$$

$$b^2 = 3^2 + 5^2 - 2(3)(5) \cdot \cos(38^o)$$

$$b^2 = 9 + 25 - 30\cos(38^o)$$

$$b^2 = 34 - 23.64$$

$$b^2 = 10.36$$

$$\sqrt{b^2} = \sqrt{10.36}$$

$b = 3.22$

Plug into Calculator Square both sides

Sine Law:

$$\frac{sinA}{a} = \frac{sinB}{b}$$

$$\frac{sinA}{3} = \frac{sin38^o}{3.22}$$

$$\frac{sinA}{3} = 0.19$$

$$3 \times \frac{sinA}{3} = 0.19 \times 3$$

$$sinA = 0.57$$

$A = 35^o$

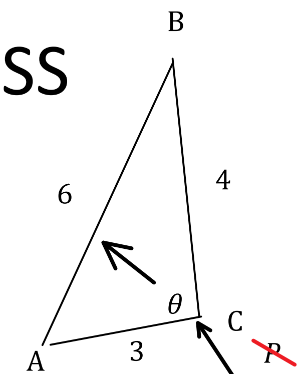
$C = 180^o - 38^o - 35^o$

$= 107^o$

OR

Plug parts into Calculator. Times both sides.

SSS



$$c^2 = b^2 + a^2 - 2abCosC$$

$$6^2 = 3^2 + 4^2 - 2(4)(3)cosC$$

Substitute values in

$$36 = 9 + 16 - 24cosC$$

Calculate the squares, multiply

$$36 = 25 - 24cosC$$

Add

$$-25 \quad -25$$

$$11 = -24cosC$$

Subtract from both sides

$$\frac{11}{-24} = \frac{-24cosC}{-24}$$

Divide both sides

$$-\frac{11}{24} = cosC$$

$$cosC = -\frac{11}{24}$$

$$C = \cos^{-1}\left(-\frac{11}{24}\right)$$

$C = 117.3^o$

Inverse cos

$$C = \cos^{-1}\left(\frac{(a^2 + b^2 - c^2)}{(2ab)}\right)$$

$$c^2 = b^2 + a^2 - 2abcosC$$

$$b^2 = c^2 + a^2 - 2cacosB$$

$$a^2 = b^2 + c^2 - 2cbcosA$$

SAS SAS : Find the smallest angle first (or 180 minus*)

SSS SSS : Find the largest angle first (or 180 minus*)

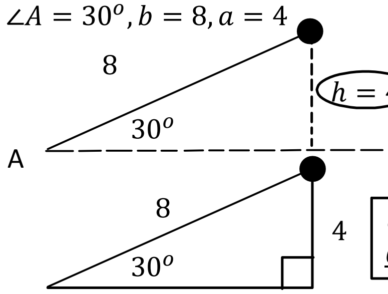
C11 - 2.0 - Trig ASS Sine 0/1/2 Tri

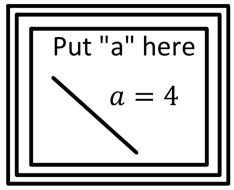
θ_{stp}, Terminal Arm

How many triangles?

Remember: Always find the height first.

$\angle A = 30^o, b = 8, a = 4$





$a = h$
One triangle

1

$\sin\theta = \frac{O}{H}$

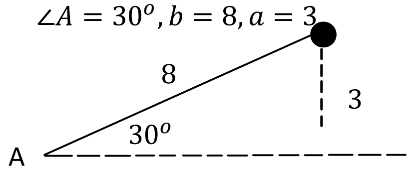
$\sin 30^o = \frac{h}{8}$

$8\sin 30^o = h$

$4 = h$

$h = 4$

$\angle A = 30^o, b = 8, a = 3$

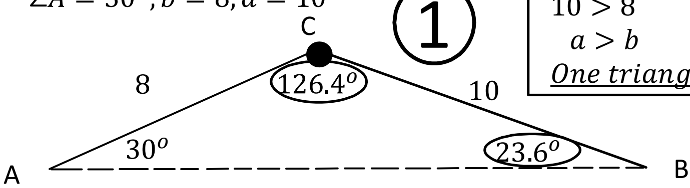


$3 < 4$
$a < H$
no triangle

0

No triangle, can't solve.

$\angle A = 30^o, b = 8, a = 10$



1

$10 > 8$
$a > b$
One triangle

$\frac{sinB}{b} = \frac{sinA}{a}$

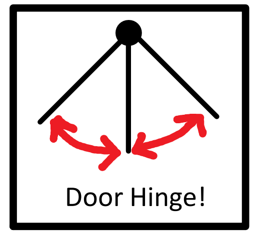
$\frac{sinB}{8} = \frac{\sin 30^O}{10}$

$sinB = \frac{8\sin 30^O}{10}$

$sinB = \frac{8\sin 30^O}{10}$

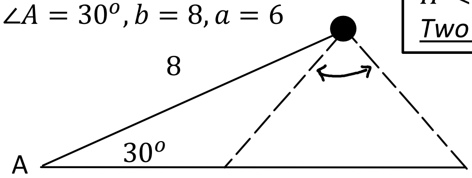
$B = \sin^{-1} 0.4$

$B = 23.6^o$



How many triangles?

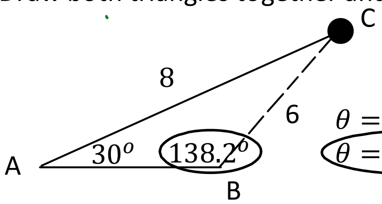
$\angle A = 30^o, b = 8, a = 6$



$4 < 6 < 8$
$H < a < B$
Two triangles

2

Draw both triangles together and separately.



$\theta = 180^o - 41.8^o$

$\theta = 138.2^o$

$\frac{\sin 30^o}{6} = \frac{\sin B}{8}$

$0.08\bar{3} = \frac{sinB}{8}$

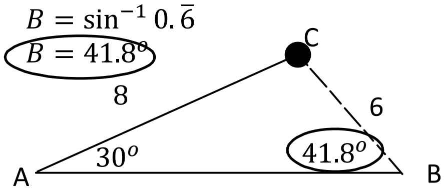
$8 \times 0.08\bar{3} = \frac{sinB}{8} \times 8$

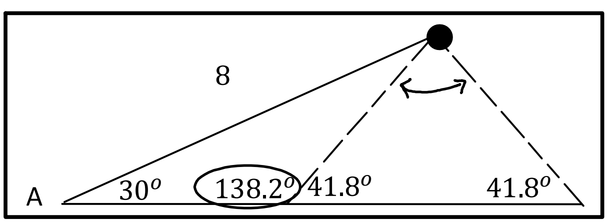
$0.\bar{6} = sinB$

$sinB = 0.\bar{6}$

$B = \sin^{-1} 0.\bar{6}$

$B = 41.8^o$



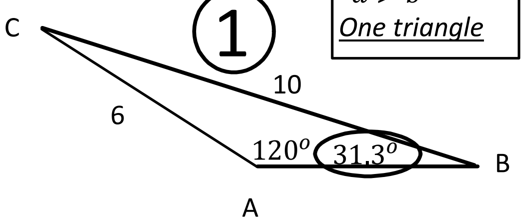


Notice: Both triangles have an angle of 30^o, a side going up of 8, and a side opposite to 30^o of 6 & The isosceles triangle.

$\angle A = 120^o, b = 6, a = 10$

$10 > 6$
$a > b$
One triangle

1



$\frac{sinB}{b} = \frac{sinA}{a}$

$\frac{sinB}{6} = \frac{\sin 120^o}{10}$

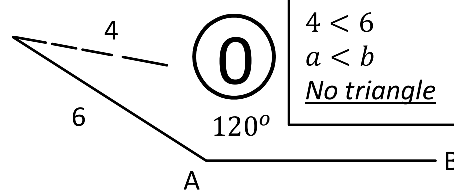
$SinB = \frac{6sin120^o}{10}$

$sinB = 0.52$

$B = \sin^{-1} 0.52$

$B = 31.3^o$

$\angle A = 120^o, b = 6, a = 4$



0

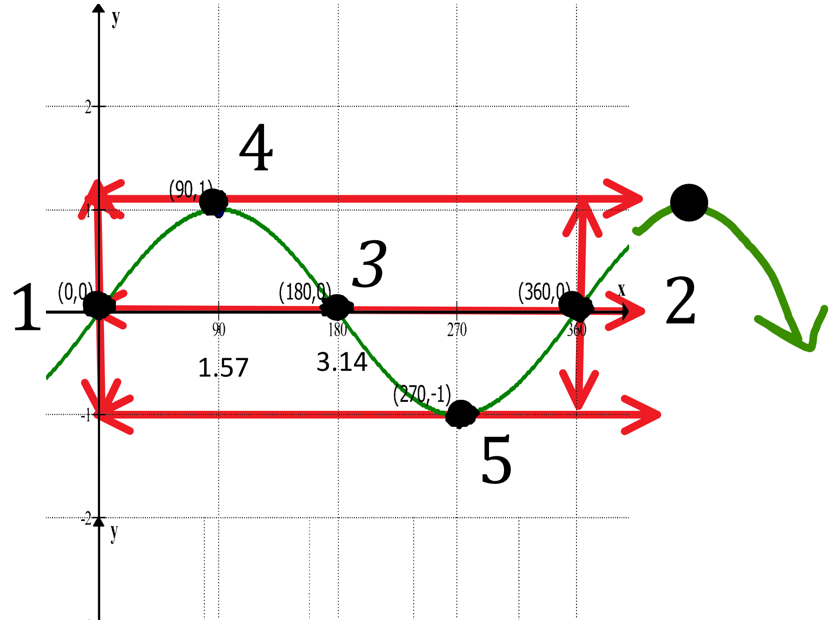
$4 < 6$
$a < b$
No triangle

No triangle. Can't solve.

C11 - 2.0 - Trig Graph Sin/Cos/Tan TOV Notes

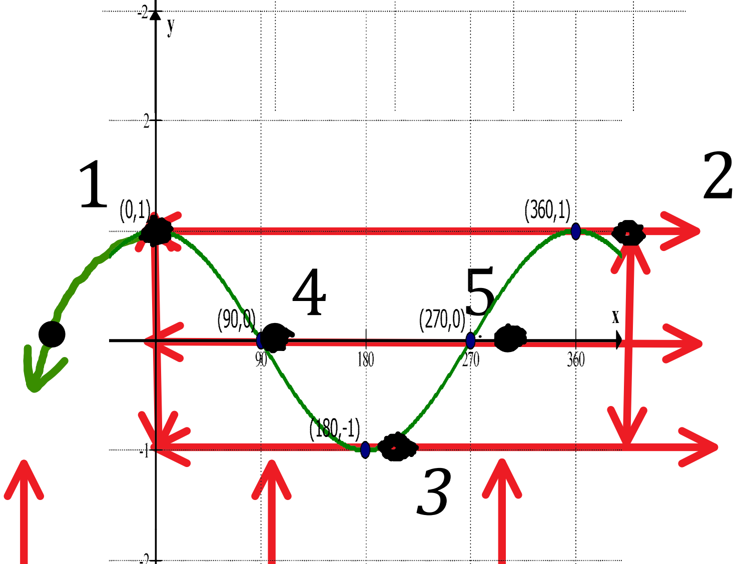
$y = \sin x$

x	y
0º	0
90º	1
180º	0
270º	-1
360º	0



$y = \cos x$

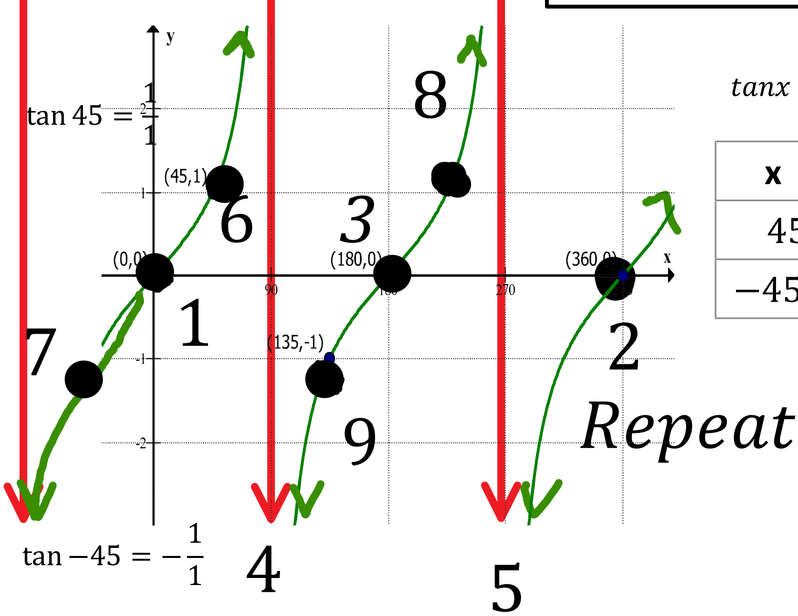
x	y
0º	1
90º	0
180º	-1
270º	0
360º	1



Tan is Zero when sin is zero
Tan is UND when cos is zero

$y = \tan x$

x	y
-45^o	-1
0º	0
45º	1
90º	und
135º	-1
180º	0



$$tanx = \frac{sinx}{cosx}$$

x	y
45	1
-45	-1

Special
Triangles
ASTC