C11 - 3.0 - Completing the Square Notes

Check by FOIL!

Standard form $\rightarrow$ *Vertex form* $\quad Vertex = (p, q)$

$y = ax^2 + bx + c \rightarrow y = a(x - p)^2 + q$

$y = x^2 + 6x + c$

$y = x^2 + 6x + 9$

$y = (x + 3)(x + 3)$

$y = (x + 3)^2$

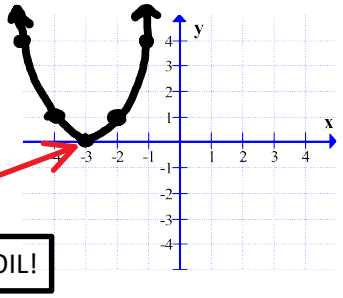
$\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = (3)^2 = 9$

"b" divided by 2 all squared:

Factor

Vertex form: Vertex = $(-3, 0)$

Check by FOIL!



$a = 1$

$y = x^2 - 4x + 3$

$y = (x^2 - 4x) + 3$ *Group x terms*

$y = (x^2 - 4x + 4 - 4) + 3$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

"b" divided by 2 all squared:

Add and subtract inside brackets

$y = (x^2 - 4x + 4) - 4 + 3$

Remove number not contributing to the perfect square $(-ve)$

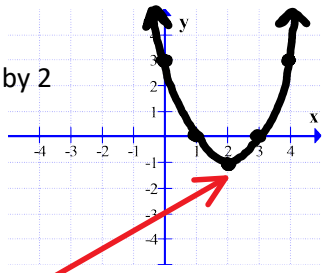
$y = (x - 2)(x - 2) - 1$

Factor brackets, simplify outside

$y = (x - 2)^2 - 1$

Vertex form: Vertex = $(2, -1)$

Check by FOIL!



$a \neq 1$

$y = 2x^2 - 8x + 3$

$y = (2x^2 - 8x) + 3$ *Group x terms*

Check by FOIL!

Factor out coefficient of x^2

$y = 2(x^2 - 4x) + 3$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

New "x" coefficient divided by 2 all squared:

OR $\left(\frac{-b}{2a}, y\right)$

$\left(\frac{-(-8)}{2(2)}, y\right)$ | x: 2 | y: -5

$(2, y)$

$(2, -5)$

x2

$y = 2(x^2 - 4x + 4 - 4) + 3$

Add and subtract inside brackets

$y = 2(x^2 - 4x + 4) - 8 + 3$

Remove number not contributing to perfect square $(-ve)$

Don't forget to multiply by "a"

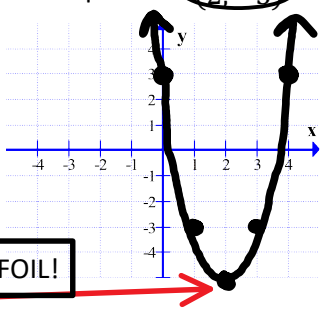
$y = 2(x - 2)(x - 2) - 5$

Factor brackets, simplify outside

$y = 2(x - 2)^2 - 5$

Vertex form: Vertex = $(2, -5)$

Check by FOIL!



$y = \left(\frac{1}{2}x^2 + \frac{1}{4}x\right) + 2$

Remember: $\frac{b^*}{2}$ is the number that goes inside the brackets with x. $vertex$: $\left(\frac{-b}{2a}, y\right)$

$y = \frac{1}{2}\left(x^2 + \frac{1}{2}x\right) + 2$ Remove GCF

$\frac{1}{4} \div \frac{1}{2} = \frac{1}{4} \times \frac{2}{1} = \frac{1}{2}$ Divide Fractions

Check by FOIL!

$y = \frac{1}{2}\left(x^2 + \frac{1}{2}x + \frac{1}{16} - \frac{1}{16}\right) + 2$

$\left(\frac{b}{2}\right)^2 = \left(\frac{\frac{1}{2}}{\frac{2}{1}}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$\frac{1}{2} \div \frac{2}{1} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$y = \frac{1}{2}\left(x^2 + \frac{1}{2}x + \frac{1}{16}\right) - \frac{1}{32} + 2$

$-\frac{1}{16} \times \frac{1}{2} = -\frac{1}{32}$ Multiply Fractions

$y = \frac{1}{2}\left(x^2 + \frac{1}{2}x + \frac{1}{16}\right) + \frac{63}{32}$

$-\frac{1}{32} + 2 = -\frac{1}{32} + \frac{2}{1} \times \frac{32}{32} = -\frac{1}{32} + \frac{64}{32} = \frac{63}{32}$

Add/Subtract Fractions

$y = \frac{1}{2}\left(x + \frac{1}{4}\right)^2 + \frac{63}{32}$

Vertex From: Vertex : $\left(-\frac{1}{4}, \frac{63}{32}\right)$