C11 - 3.6 - Product of Numbers is a Min Notes

The difference between two numbers is 10. Their product is a minimum.

Let $a = 1st\ \#$
Let $b = 2nd\ \#$

Let statements: get used to using variables other than x and y

① $a - b = 10$

② $a \times b = minimum$
$a \times b = \cancel{minimum}$ y
$y = a \times b$

Equation 1, equation 2.
The minimum or maximum will be y.

$a - b = 10$
$+b \quad +b$
$a = (10 + b)$

Equation #1
Isolate a variable

$y = a \times b$
$y = (10 + b) \times b$
$y = 10b + b^2$
$y = b^2 + 10b$

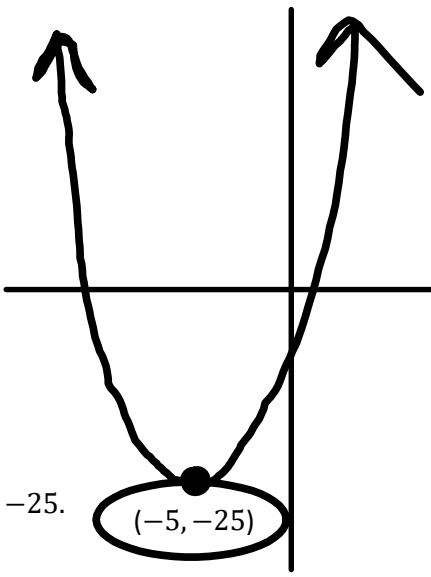
Equation #2
Substitute the isolated variable

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

$y = b^2 + 10b$
$y = (b^2 + 10b + 25 - 25)$
$y = (b^2 + 10b + 25) - 25$
$y = (b + 5)^2 - 25$

Vertex = $(-5, -25)$

b Minimum



$a = 10 + b$
$a = 10 - 5$
$a = 5$

Substitute b into the other equation.

$a = 5$
$b = -5$

List the two numbers and the minimum.

The minimum product is -25.

$(-5, -25)$

(x, y)
(b, min)

C11 - 3.6 - Product of Numbers is a Min Notes

Two numbers differ by 10. The product of the larger number and twice the smaller number is a minimum. What are the numbers?

Let $a = 1st\ \#$
Let $b = 2nd\ \#$

Let statements:

(1) $a - b = 10$

(2) $a \times 2b = minimum$
$a \times 2b = \cancel{minimum}\ \ y$
$y = a \times 2b$

Equation 1, equation 2.
The minimum or maximum will be y.

$a - b = 10$
$a = 10 + b$

Equation #1
Isolate a variable

$y = a \times 2b$
$y = (10 + b) \times 2b$
$y = 20b + 2b^2$
$y = 2b^2 + 20b$

Equation #2
Substitute the isolated variable

$y = 2b^2 + 20b$ ×2
$y = 2(b^2 + 10b + 25 - 25)$
$y = 2(b^2 + 10b + 25) - 50$
$y = 2(b + 5)^2 - 50$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

Vertex = (-5, -50)

b Minimum

$a = 10 + b$
$a = 10 - 5$
$a = 5$

Substitute b into the other equation.

$a = 5$
$b = -5$

List the two numbers and the minimum.

The minimum product is -50.

C11 - 3.6 - Sum of Squares is a Min Notes

Two numbers sum to 8. The sum of their squares is a minimum.

Let $a = 1st\ \#$
Let $b = 2nd\ \#$

Let statements:

① $a + b = 8$

② $a^2 + b^2 = minimum$
$a^2 + b^2 = \cancel{minimum}\ y$
$y = a^2 + b^2$

Equation 1, equation 2.
The minimum or maximum will be y.

$a + b = 8$
$-b \quad -b$
$a = 8 - b$
$a = (8 - b)$

Equation #1
Isolate a variable

$y = a^2 + b^2$
$y = (8 - b)^2 + b^2$
$y = 64 - 16b + b^2 + b^2$
$y = 2b^2 - 16b + 64$

Equation #2
Substitute the isolated variable

$y = 2b^2 - 16b + 64$
$y = 2(b^2 - 8b) + 64$
$y = 2(b^2 - 8b + 16 - 16) + 64$
$y = 2(b^2 - 8b + 16) + 64 - 32$
$y = 2(b - 4)^2 + 32$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{8}{2}\right)^2 = (4)^2 = 16$

Vertex = (4, 32)

b

Minimum

$a = 8 - b$
$a = 8 - (4)$
$a = 4$

Substitute b into the other equation.

$a = 4$
$b = 4$

List the two numbers and the maximum.

The minimum product is 32.

C11 - 3.6 - Product of Numbers is a Max Notes

The sum of two times one number and six times another is sixty. Find the numbers if their product is a maximum.

Let $a = 1st\ \#$
Let $b = 2nd\ \#$

Let statements:

① $2a + 6b = 60$

② $a \times b = maximum$
$a \times b = \cancel{maximum}\ y$
$y = a \times b$

Equation 1, equation 2. The minimum or maximum will be y.

$\frac{2a}{2} + \frac{6b}{2} = \frac{60}{2}$
$a + 3b = 30$
$a = 30 - 3b$

Equation #1
Isolate a variable

$y = a \times b$
$y = (30 - 3b) \times b$
$y = 30b - 3b^2$
$y = -3b^2 + 30b$

Equation #2
Substitute the isolated variable

$y = -3b^2 + 30b$
$y = -3(b^2 - 10b + 25 - 25)$
$y = -3(b^2 - 10b + 25) + 75$
$y = -3(b - 5)^2 + 75$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

Vertex = (5, 75)

b — Maximum

$a = 30 - 3b$
$a = 30 - 3(5)$
$a = 15$

Substitute b into the other equation.

$a = 15$
$b = 5$

List the two numbers and the maximum.

The maximum product is 75