C11 - 3.9 - Set Up Maximize Candy Sales Notes

A student sells candy to all of his friends. Each candy costs 6 dollars, and he has 10 friends who buy the candy each day. Every time he increases the cost by 1 dollar, 1 of his friends decides not to buy the candy. Set up how this question will look.

$Let\ p = price$
$Let\ q = quantity$
$Let\ r = revenue$
$Let\ x = \#\ of\ price\ increases$

$Revenue = price \times quantity$
If $p = 6, \quad q = 10 \quad r = 6 \times 10$
$r = 60$

$p = 6 + 1x \longrightarrow$ Raising the price by 1 dollar x times.

$q = 10 - 1x \longrightarrow$ Each x times he raises the price, 1 less friend will buy the candy.

$r = p \times q$
$r = (6 + 1x) \times (10 - 1x)$

Price

x	p
-2	4
-1	5
0	6
1	7
2	8

Starting Price and Quantity (zero price increase)

Quantity

x	q
-2	12
-1	11
0	10
1	9
2	8

C11 - 3.9 - Maximize Candy Sales Notes

A student sells candy to all of his friends. Each candy costs 6 dollars, and he has 10 friends who buy the candy each day. Every time he increases the cost by 1 dollar, 1 of his friends decides not to buy the candy. What is the price that will maximize revenue?

$Let\ p = price$
$Let\ q = quantity$
$Let\ r = revenue$
$Let\ x = \#\ of\ price\ increases$

$Revenue = price \times quantity$
If $p = 6, \quad q = 10$
$r = 60$

$r = p \times q$
$r = 6 \times 10$
$r = \$60$

$p = 6 + 1x$ → If he decides to raise the price by 1 dollar x times.

$q = 10 - 1x$ → One less friend will buy the candy each time he increases the price.

$r = p \times q$
$r = (6 + x)(10 - x)$
$r = 60 - 6x + 10x - x^2$
$r = 60 + 4x - x^2$
$r = -x^2 + 4x + 60$
$r = -(x^2 - 4x) + 60$ $\times (-1)$
$r = -(x^2 - 4x + 4 - 4) + 60$
$r = -(x^2 - 4x + 4) + 60 + 4$
$r = -(x - 2)^2 + 64$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(-\frac{4}{2}\right)^2 = (-2)^2 = 4$

$y = \max\ revenue = \$64$

Vertex: $(2, 64)$

$x = 2\ price\ increases$

$p = 6 + 1x$
$p = 6 + 1(2)$
$p = 6 + 2$
$p = 8$

$price = 8$

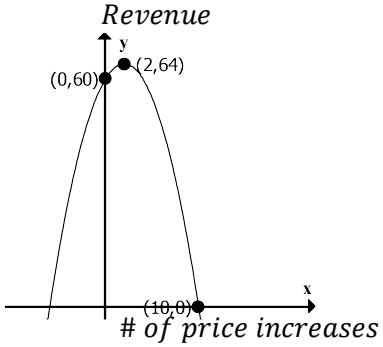
$q = 10 - 1x$
$q = 10 - 1(2)$
$q = 10 - 2$
$q = 8$

$quantity = 8$

Check with Table of Values

Price	Quantity	(x)	Revenue (y)
6	10	0	60
7	9	1	63
8	8	2	64
9	7	3	63
10	6	4	60
11	5	5	55

1st increase
2nd increase
Max revenue



C11 - 3.9 - Maximize Car Sales Notes

A car salesman sells a car for $4000, with 20 people buying the car. For every $200 he takes off the price, 2 more people buy a car. What is the price that will maximize revenue?

$Let\ p = price$
$Let\ q = quantity$
$Let\ r = revenue$
$\boxed{Let\ x = \#\ of\ price\ decreases}$

$Revenue = price \times quantity$
$If\ p = \$4000, \quad q = 20$
$r = \$80{,}000$

If they sell 20 cars at $4000, revenue is $80,000.

$p = 4000 - 200x$ → If he decides to decrease the price by $200 x times.

$q = 20 + 2x$ → Two more people will buy the car each time he decreases the price.

$r = p \times q$
$r = (4000 - 200x)(20 + 2x)$
$r = 80000 + 8000x - 4000x - 400x^2$
$r = -400x^2 + 4000x + 80000$
$r = -400(x^2 - 10x) + 80000$
$r = -400(x^2 - 10x + 25 - 25) + 80000$
$r = -400(x^2 - 10x - 25) + 80000 + 10000$
$r = -400(x - 5)^2 + 90000$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(-\frac{10}{2}\right)^2 = (-5)^2 = 25$

Vertex: $(5, 90000)$

$x = 5\ price\ decreases$

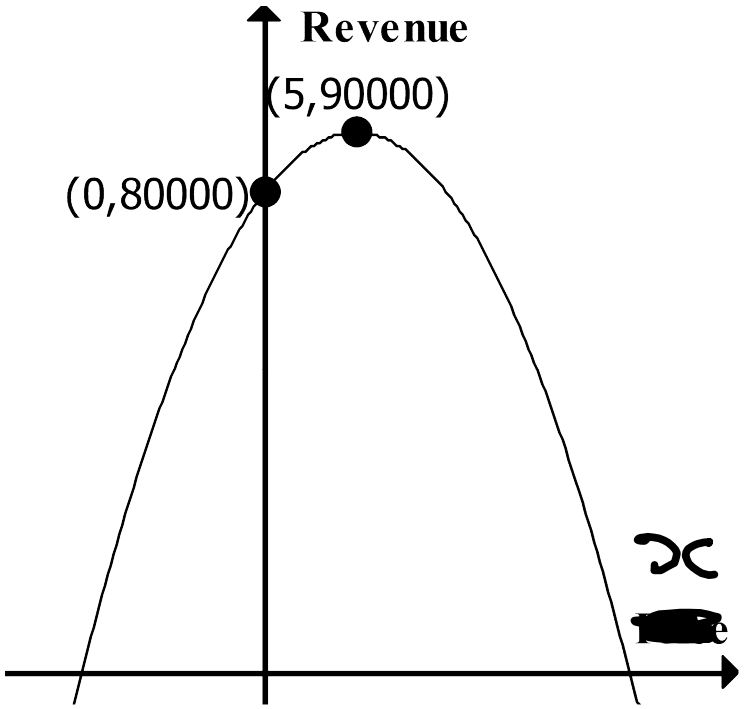
$y = \max\ revenue = \$90000$

$p = 4000 - 200x$
$p = 4000 - 200(5)$
$p = 4000 - 1000$
$p = 3000$

$\boxed{price = \$3000}$

$q = 20 + 2x$
$q = 20 + 2(5)$
$q = 20 + 10$
$q = 30$

$\boxed{quantity = 30\ people}$



C11 - 3.9 - Maximize Car Sales Notes (No Price Increases)

A car salesman sells a car for $2000, with 20 people buying the car. For every $200 he takes off the price, 2 more people buy a car. What is the price that will maximize revenue?

$Let\ p = price$
$Let\ q = quantity$
$Let\ r = revenue$
$\boxed{Let\ x = \#\ of\ price\ decreases}$

$Revenue = price \times quantity$
$If\ p = \$2000, \quad q = 20$
$r = \$40{,}000$
If they sell 20 cars at $8000, revenue is $40,000.

$p = 2000$
$p = 2000 - 200x$ → If he decides to decrease the price by $200 x times.

$q = 20$
$q = 20 + 2x$ → Two more people will buy the car each time he decreases the price.

$r = p \times q$
$r = (2000 - 200x)(20 + 2x)$
$r = 40000 + 4000x - 4000x - 400x^2$
$r = -400x^2 + 40000$
$r = -400(x + 0)^2 + 40000$

Vertex: $(0, 40000)$
$x = 0\ price\ decreases$
$y = \max\ revenue = \$30000$

$p = 2000 - 200x$
$p = 2000 - 200(0)$
$p = 2000$

$\boxed{price = \$2000}$

$q = 20 + 2x$
$q = 20 + 2(0)$
$q = 20 - 0$
$q = 20$

$\boxed{quantity = 20\ people}$