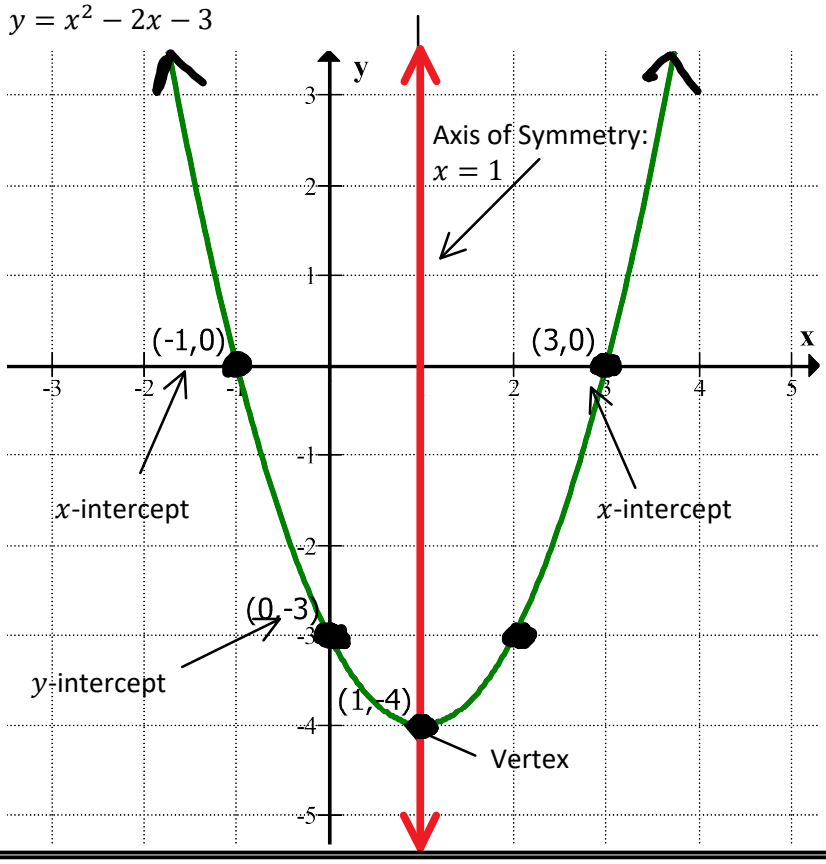
C11 - 3.0 - Key Points of Quadratic Functions Notes $a = 1$

- Vertex
- Shape
- AOS
- x-intercepts
- y-intercept
- Direction of Opening
- Max/Min
- Domain
- Range

$y = x^2 - 2x - 3$



TOV

x	y
-2	5
-1	0
0	-3
1	-4
2	-3
3	0
4	5

Vertex:

$x \in \mathbb{R}$
$Vertex\ (1, -4)$
$AOS\ x = 1$
Opens Up
$Minimum\ y = -4$
$y \geq -4$

	Vertex Form	Standard Form	Factored Form
Equation	$y = (x-1)^2 - 4$	$y = x^2 - 2x - 3$	$y = (x+1)(x-3)$
Info	Vertex: $(1, -4)$	y-intercept: $(0, -3)$	x-intercepts: $(-1,0), (3,0)$

Standard to Vertex

Standard Form: $y = x^2 - 2x - 3$
↓ Complete the square. $y = (x^2 - 2x) - 3$
$y = (x^2 - 2x + 1 - 1) - 3$
Vertex Form: $y = (x-1)^2 - 4$

Standard to Factored

$y = x^2 - 2x - 3$

$y = (x+1)(x-3)$

Standard Form: ↓ Factor. Factored Form:

Vertex to Standard

Vertex Form: $y = (x-1)^2 - 4$
↓ FOIL $y = (x^2 - 2x + 1) - 4$
$y = x^2 - 2x + 1 - 4$
Standard Form: $y = x^2 - 2x - 3$

Factored to Standard

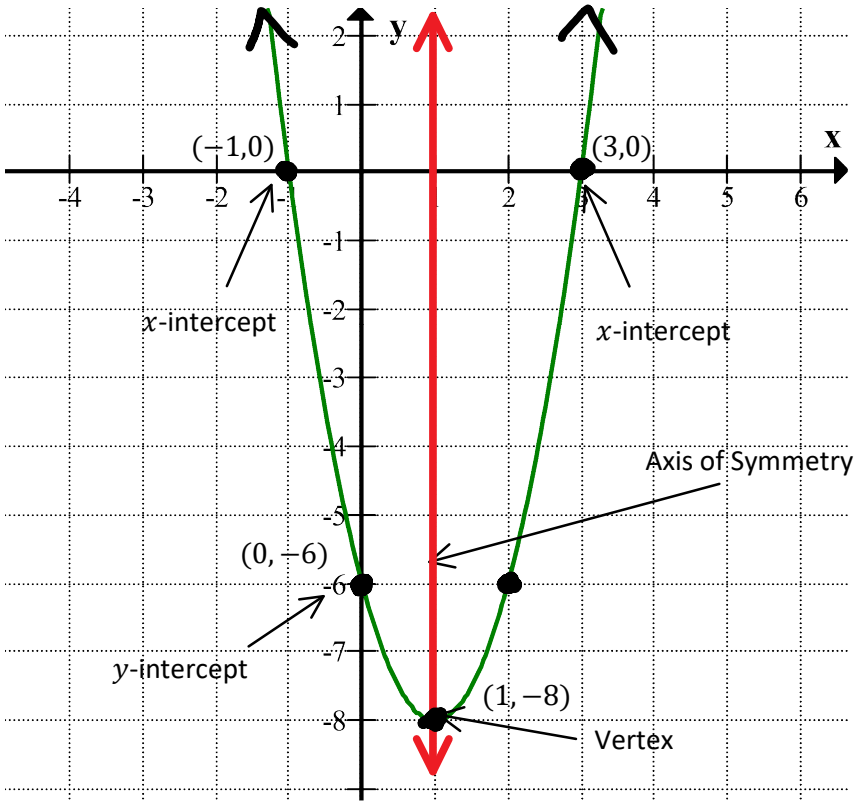
$y = (x+1)(x-3)$
$y = (x^2 - 3x + x - 3)$
$y = x^2 - 2x - 3$

Factored Form: ↓ FOIL Standard Form:

C11 - 3.0 - Key Points of Quadratic Functions Notes $a \neq 1$

- Vertex
- Shape
- AOS
- x-intercepts
- y-intercept
- Direction of Opening
- Max/Min
- Domain
- Range

$y = 2x^2 - 4x - 6$



TOV

x	y
-2	10
-1	0
0	-6
1	-8
2	-6
3	0
4	10

Vertex: $x = 1$

$x \in \mathbb{R}$
$Vertex\ (1, -8)$
$AOS\ x = 1$
Opens Up
$Minimum\ y = -8$
$y \geq -8$

	Vertex Form	Standard Form	Factored Form
Equation	$y = 2(x-1)^2 - 8$	$y = 2x^2 - 4x - 6$	$y = 2(x+1)(x-3)$
Info	Vertex: $(1, -8)$	y-intercept: $(0, -6)$	x-intercepts: $(0,-1)$, $(0,3)$

Standard to Vertex

Standard Form: $y = 2x^2 - 4x - 6$
↓ Complete the square.
$y = 2(x^2 - 2x) - 6$
$y = 2(x^2 - 2x + 1 - 1) - 6$
Vertex Form: $y = 2(x-1)^2 - 8$

Standard to Factored

$y = 2x^2 - 4x - 6$
$y = 2(x^2 - 2x - 3)$
$y = 2(x-3)(x+1)$
Standard Form:
↓ Factor.
Factored Form:

Vertex to Standard

Vertex Form: $y = 2(x-1)^2 - 8$
↓ FOIL
$y = 2(x^2 - 2x + 1) - 8$
$y = 2x^2 - 4x + 2 - 8$
Standard Form: $y = 2x^2 - 4x - 6$

Factored to Standard

$y = 2(x+1)(x-3)$
$y = 2(x^2 - 3x + 1x - 3)$
$y = 2(x^2 - 2x - 3)$
$y = 2x^2 - 4x - 6$
Factored Form:
↓ FOIL
Standard Form:

C11 - 3.0 - Graphing Quadratics Review

Complete the Square ← → FOIL

FOIL ← → Factor

	Vertex Form	General Form	Factored Form (CH.4)
General Equation:	$y = a(x-p)^2 + q$	$y = ax^2 + bx + c$	$y = a(x-z)(x-r)$
Vertex (V):	(p, q) $*\ p\ is\ opposite$	$\left(-\frac{b}{2a}, y\right)$	$\left(\frac{z+r}{2}, y\right)$
Axis of Symmetry (AOS):	$x = p$	$x = -\frac{b}{2a}$	$x = \frac{z+r}{2}$
Domain*:	$x \in R$	$x \in R$	$x \in R$
X intercepts:	$set\ y = 0,$ $and\ solve$	$set\ y = 0,$ $and\ solve$	$Set\ brackets\ separately$ $equal\ to\ zero, and\ solve$ $(x-z) = 0 \mid (x-r) = 0$ $x = z \mid \quad x = r$
Y intercepts:	$set\ x = 0,$ $and\ solve$	$y - int\colon (0, c)$	$set\ x = 0, and\ solve$

Remember: The "p" value is the opposite of the value inside the brackets with x.

" a "			*Range*	*Max/Min Value*
$a > 0$: *Minimum* $(a\ is\ positive)$	*Opens Up*		$y \geq min = q$	$y = min = q$
$a < 0$: *Maximum* $(a\ is\ negative)$	*Opens down*		$y \leq max = q$	$y = max = q$
$a > 1$ OR $a < -1$	*Vertical Expansion*	$a = 2$		
$-1 < a < 1$	*Vertical Compression*	$a = 0.5$		

Find Vertex Form

Vertex: $(-1, -4)$ ***Point***: $(-2, -3)$

$\boxed{y = a(x-p)^2 + q}$

$y = a(x+1)^2 - 4 \quad (-1, -4)$

Opposite, Same

$-3 = a(-2+1)^2 - 4 \quad (-2, -3)$

$-3 = a(-1)^2 - 4$

$-3 = 1a - 4$

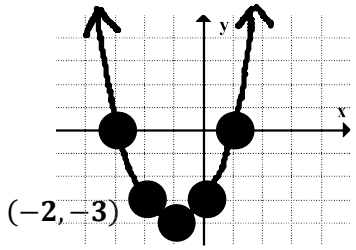
$+4 \qquad +4$

$1 = 1a$

$\frac{1}{1} = \frac{1a}{1}$

$1 = a$

$\boxed{a = 1}$



$(-1, -4)$

$\boxed{y = 1(x+1)^2 - 4}$

Find Standard Form (CH4)

$y - int : (0, -1)$

$x\ int : (-1, 0), \left(\frac{1}{2}, 0\right)$

$x = -1 \qquad x = \frac{1}{2}$

$x + 1 = 0 \qquad 2x = 1$

$2x - 1 = 0$

$y = a(x-\#)(x-\#)$

$y = a(2x-1)(x+1)$

$-1 = a(2(0)-1)(0+1)$

$-1 = -1a$

$a = 1 \qquad \boxed{(0, -1)}$

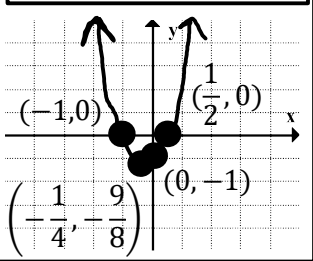
$\boxed{y = 1(2x-1)(x+1)}$

Average $x - int\ values$

$p = \frac{-1+\frac{1}{2}}{2} = -\frac{1}{4}$

$Vertex : \left(-\frac{1}{4}, -\frac{9}{8}\right)$

x	y
$-\frac{1}{4}$	$-\frac{9}{8}$



C11 - 3.0 - Completing the Square Notes

Check by FOIL!

Standard form $\rightarrow$ *Vertex form* $\quad Vertex = (p, q)$

$y = ax^2 + bx + c \rightarrow y = a(x-p)^2 + q$

$y = x^2 + 6x + c$

$y = x^2 + 6x + 9$

$y = (x+3)(x+3)$

$y = (x+3)^2$ ✓

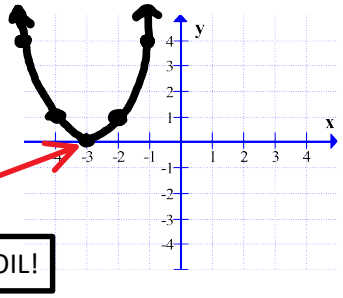
$\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = (3)^2 = 9$

"b" divided by 2 all squared:

Factor

Vertex form: Vertex = $(-3, 0)$

Check by FOIL!



$a = 1$

$y = x^2 - 4x + 3$

Group x terms

$y = (x^2 - 4x) + 3$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

"b" divided by 2 all squared:

$y = (x^2 - 4x + 4 - 4) + 3$

Add and subtract inside brackets

$y = (x^2 - 4x + 4) - 4 + 3$

Remove number not contributing to the perfect square $(-ve)$

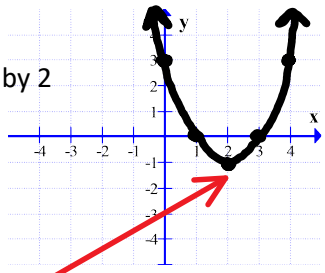
$y = (x-2)(x-2) - 1$

Factor brackets, simplify outside

$y = (x-2)^2 - 1$ ✓

Vertex form: Vertex = $(2, -1)$

Check by FOIL!



$a \neq 1$

$y = 2x^2 - 8x + 3$

Group x terms

Check by FOIL!

$y = (2x^2 - 8x) + 3$

Factor out coefficient of x^2

$y = 2(x^2 - 4x) + 3$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

New "x" coefficient divided by 2 all squared:

x2

$y = 2(x^2 - 4x + 4 - 4) + 3$

Add and subtract inside brackets

$y = 2(x^2 - 4x + 4) - 8 + 3$

Remove number not contributing to perfect square $(-ve)$

Don't forget to multiply by "a"

$y = 2(x-2)(x-2) - 5$

Factor brackets, simplify outside

$y = 2(x-2)^2 - 5$ ✓

Vertex form: Vertex = $(2, -5)$

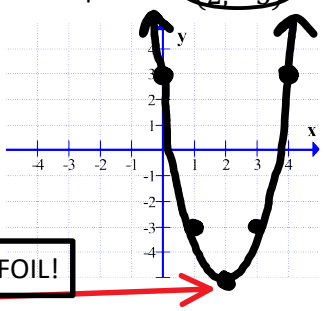
Check by FOIL!

OR $\left(\frac{-b}{2a}, y\right)$

$\left(\frac{-(-8)}{2(2)}, y\right)$ | x | y |
|---|---|
| 2 | -5 |

$(2, y)$

$(2, -5)$



$y = \left(\frac{1}{2}x^2 + \frac{1}{4}x\right) + 2$

Remember: $\frac{b^*}{2}$ is the number that goes inside the brackets with x. $vertex$: $\left(\frac{-b}{2a}, y\right)$

$y = \frac{1}{2}\left(x^2 + \frac{1}{2}x\right) + 2$

Remove GCF

$\frac{1}{4} \div \frac{1}{2} = \frac{1}{4} \times \frac{2}{1} = \left(\frac{1}{2}\right)$

Divide Fractions

Check by FOIL!

$y = \frac{1}{2}\left(x^2 + \frac{1}{2}x + \frac{1}{16} - \frac{1}{16}\right) + 2$

$\left(\frac{b}{2}\right)^2 = \left(\frac{\frac{1}{2}}{\frac{2}{1}}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$\frac{1}{2} \div \frac{2}{1} = \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{4}\right)$

$y = \frac{1}{2}\left(x^2 + \frac{1}{2}x + \frac{1}{16}\right) - \frac{1}{32} + 2$

$-\frac{1}{16} \times \frac{1}{2} = \left(-\frac{1}{32}\right)$

Multiply Fractions

$y = \frac{1}{2}\left(x^2 + \frac{1}{2}x + \frac{1}{16}\right) + \frac{63}{32}$

$-\frac{1}{32} + 2 = -\frac{1}{32} + \frac{2}{1} \times \frac{32}{32} = -\frac{1}{32} + \frac{64}{32} = \left(\frac{63}{32}\right)$

Add/Subtract Fractions

$y = \frac{1}{2}\left(x + \frac{1}{4}\right)^2 + \frac{63}{32}$

Vertex From: Vertex : $\left(-\frac{1}{4}, \frac{63}{32}\right)$