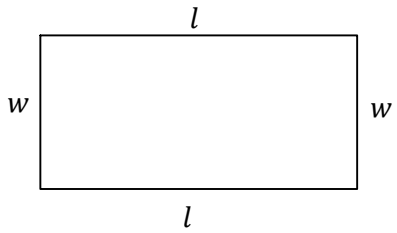
C11 - 4.6 - Rectangular Garden

A rectangular garden has an Area of 36 and a Perimeter of 30. What are the lengths and widths?

Let $w = width$
Let $l = length$



Let statements:

$P = 2l + 2w$

$A = l \times w$

Equation 1, equation 2.

$P = 2l + 2w$
$30 = 2l + 2w$
$\frac{30}{2} = \frac{2l}{2} + \frac{2w}{2}$
$15 = l + w$
$-w \quad -w$
$15 - w = l$
$l = 15 - w$

Equation #1
Isolate a variable

$A = l \times w$
$36 = l \times w$
$36 = (15 - w) \times w$
$36 = 15w - w^2$
$+w^2 \quad +w^2$
$36 + w^2 = 15w$
$-15w \quad -15w$
$w^2 - 15w + 36 = 0$
$(w - 12)(w - 3) = 0$

Equation #2
Substitute the isolated variable

Factor

$w - 12 = 0$
$w = 12$

$w - 3 = 0$
$w = 3$

Solve

$l = 15 - w$
$l = 15 - (12)$
$l = 3$

Substitute w into the other equation.

$Length = 12$
$Width = 3$

List the length and width

OR

$l = 15 - w$
$l = 15 - (3)$
$l = 12$

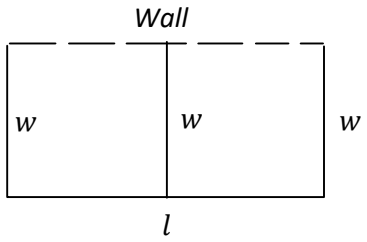
$Length = 3$
$Width = 12$

List the length and width

C11 - 4.6 - Fence Split in Two

A rectangular fence that is split in half is against a wall. The total fencing length is 39, and it has a total area of 66. What are the dimensions of the fence?

Let $w = width$
Let $l = length$



Let statements:

$P = l + 3w$ $\quad$ $A = l \times w$

Equation 1, equation 2.

$P = l + 3w$
$39 = l + 3w$
$-3w \quad -3w$
$39 - 3w = l$
$l = 39 - 3w$

Equation #1
Isolate a variable

$A = l \times w$
$66 = (39 - 3w) \times w$
$66 = 39w - 3w^2$
$+3w^2 \quad +3w^2$
$66 + 3w^2 = 39w$
$-39w \quad -39w$
$3w^2 - 39w + 66 = 0$
$3(w^2 - 13w + 22) = 0$
$3(w - 2)(w - 11) = 0$

Equation #2
Substitute the isolated variable

Factor

$w - 2 = 0$ $\quad$ $w - 11 = 0$
$w = 2$ $\quad$ $w = 11$

Solve

$l = 39 - 3w$
$l = 39 - 3(2)$
$l = 39 - 6$
$l = 33$

Substitute w into the other equation.

$Width = 2$
$Length = 33$

List the length and width

or

$l = 39 - 3w$
$l = 39 - 3(11)$
$l = 39 - 33$
$l = 6$

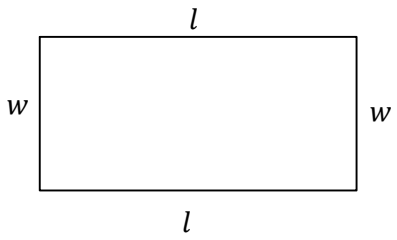
$Width = 11$
$Length = 6$

List the length and width

C11 - 4.6 - Rectangular Garden Quad

A rectangular garden has an area of 61 and a perimeter of 40. What are the lengths and widths?

Let $w = width$
Let $l = length$



Let statements:

$P = 2l + 2w$ $\quad$ $A = l \times w$

Equation 1, equation 2.

$$P = 2l + 2w$$
$$40 = 2l + 2w$$
$$\frac{40}{2} = \frac{2l}{2} + \frac{2w}{2}$$
$$20 = l + w$$
$$-w \quad -w$$
$$20 - w = l$$
$$l = 20 - w$$

Equation #1
Isolate a variable

$$A = l \times w$$
$$91 = l \times w$$
$$61 = (20 - w) \times w$$
$$61 = 20w - w^2$$
$$+w^2 \quad +w^2$$
$$61 + w^2 = 20w$$
$$-20w \quad -20w$$
$$w^2 - 20w + 61 = 0$$

Equation #2
Substitute the isolated variable

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$w = \frac{-(-20) \pm \sqrt{20^2 - 4(1)(61)}}{2(1)}$$

Quadratic Formula

$$w = \frac{20 - \sqrt{156}}{2(1)} \qquad w = \frac{20 + \sqrt{156}}{2(1)}$$
$$w = 3.755 \qquad w = 16.245$$

Solve

$$l = 20 - w$$
$$l = 20 - (16.245)$$
$$l = 3.755$$

Substitute w into the other equation.

$Length = 16.245$
$Width = 3.755$

List the length and width

OR

$$l = 15 - w$$
$$l = 15 - (3.755)$$
$$l = 16.245$$

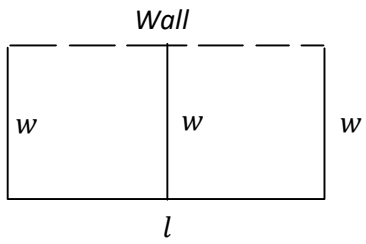
List the length and width

$Length = 3.755$
$Width = 16.245$

C11 - 4.6 - Fence Split in Two Quad

A rectangular fence that is split in half is against a wall. The total fencing length is 61, and it has a total area of 58. What are the dimensions of the fence?

Let $w = width$
Let $l = length$



Let statements:

$P = l + 3w$ $\quad$ $A = l \times w$

Equation 1, equation 2.

Equation #1
Isolate a variable

$$P = l + 3w$$
$$61 = l + 3w$$
$$-3w \quad -3w$$
$$61 - 3w = l$$
$$l = 61 - 3w$$

Equation #2
Substitute the isolated variable

$$A = l \times w$$
$$58 = (61 - 3w) \times w$$
$$58 = 61w - 3w^2$$
$$+3w^2 \qquad +3w^2$$
$$58 + 3w^2 = 61w$$
$$-61w \qquad -61w$$
$$3w^2 - 61w + 58 = 0$$

Quadratic Formula

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$w = \frac{-(-61) \pm \sqrt{61^2 - 4(3)(58)}}{2(3)}$$

$$w = \frac{61 + \sqrt{3025}}{6} \qquad w = \frac{61 - \sqrt{3025}}{6}$$
$$w = 19.\overline{3} \qquad w = 1$$
$$w = \frac{58}{3}$$

Solve

Substitute w into the other equation.

$$l = 61 - 3w$$
$$l = 61 - 3\left(\frac{58}{3}\right)$$
$$l = 61 - 58$$
$$l = 3$$

List the length and width

$$Width = \frac{58}{3}$$
$$Length = 3$$

or

$$l = 61 - 3w$$
$$l = 61 - 3(1)$$
$$l = 61 - 3$$
$$l = 58$$

List the length and width

$$Width = 58$$
$$Length = 1$$