$y = a(x - \#)(x - \#)$ Find Eq

C11 - 4.0 - Quadratics

$x - int$: *Set y equal to zero*, $(y = 0)$

$(a)(b) = 0$

$a = 0 \qquad b = 0$

Factor to Solve for x − intercepts: $y = 0$

$y = x^2 - 4x + 3$
$y = (x - 1)(x - 3)$
$0 = (x - 1)(x - 3)$

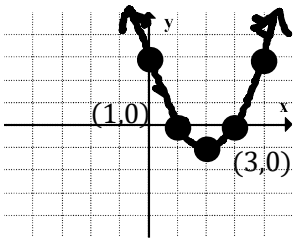
$\_\_-1\_\_ \times \_\_-3\_\_ = 3$
$\_\_-1\_\_ + \_\_-3\_\_ = -4$

Factor.
Set y equal to zero.

$x - 1 = 0 \qquad x - 3 = 0$
$+1 \quad +1 \qquad +3 \quad +3$
$x = +1 \qquad x = +3$

Set the brackets equal to zero seperately. Solve.

$(1,0) \quad x - int: \quad (3,0)$



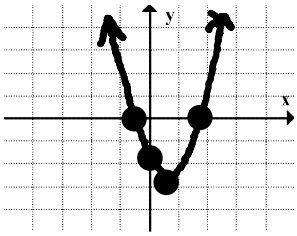
x	y
0	3
1	0
2	-1
3	0
4	3

1) Get $= 0$, $a = 1^*$
2) Factor
3) Set factors/brackets = 0 separately
4) Solve

$y = 2x^2 - 3x - 2$
$y = 2x^2 - 4x + 1x - 2$
$y = (2x^2 - 4x)(+1x - 2)$
$y = 2x(x - 2) + 1(x - 2)$
$y = (x - 2)(2x + 1)$
$0 = (x - 2)(2x + 1)$

$\_\_-4\_\_ \times \_\_1\_\_ = -4$
$\_\_-4\_\_ + \_\_1\_\_ = -3$

Factor
Decompose
Group
GCF
GCF



Algebra

Get = Zero!

$x^2 - 2x = 2x - 3$
$-2x \quad -2x$
$x^2 - 4x = -3$
$+3 \quad +3$
$x^2 - 4x + 3 = 0$
...

$(x - 1)(x - 3) = 2x^2 - 8x + 6$
$x^2 - 4x + 3 = 2x^2 - 8x + 6$
$0 = x^2 - 4x + 3$
...

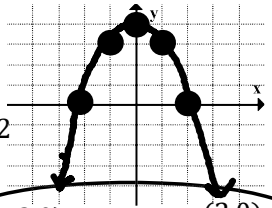
$x - 2 = 0 \qquad 2x + 1 = 0$
$+2 \quad +2 \qquad -1 \quad -1$
$x = 2 \qquad \frac{2x}{2} = -\frac{1}{2}$
$x = -\frac{1}{2}$

$(-\frac{1}{2}, 0) \quad x - int: \quad (2,0)$

$y = -x^2 + 4$
$0 = -x^2 + 4$
$0 = -(x^2 - 4)$
$0 = -(x + 2)(x - 2)$

GCF: -1
Differences of Squares

$x + 2 = 0 \qquad x - 2 = 0$
$-2 \quad -2 \qquad +2 \quad +2$
$x = -2 \qquad x = 2$

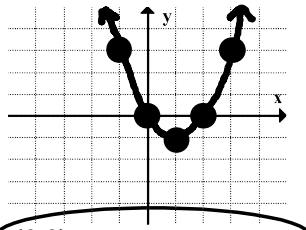


$(-2,0) \quad x - int: \quad (2,0)$

$y = -x^2 + 2x$
$0 = -x^2 + 2x$
$0 = -x(x - 2)$

GCF: $-x$

$x = 0 \qquad x - 2 = 0$
$+2 \quad +2$
$x = 2$



$(0,0) \quad x - int: \quad (2,0)$

Find Standard Form :

$x\ int: (2,0), (6,0)$ Get $= 0$

$x = 2 \qquad x = 6$
$-2 \quad -2 \qquad -6 \quad -6$
$x - 2 = 0 \qquad x - 6 = 0$

$0 = (x - 2)(x - 6)$
$y = x^2 - 8x + 12$

$x\ int: \left(\frac{1}{2}, 0\right), (4,0)$

$x = \frac{1}{2}$
$2 \times x = \frac{1}{2} \times 2$
$2x = 1$
$-1 \quad -1$
$2x - 1 = 0$

$y = \left(x - \frac{1}{2}\right)(x - 4)$
$y = x^2 - \frac{9}{2}x + 2$

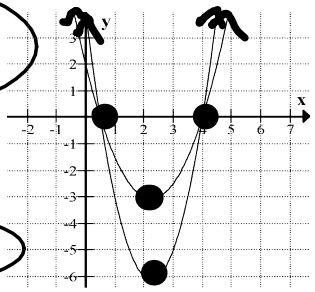
$x = 4$
$-4 \quad -4$
$x - 4 = 0$

OR

Algebra

$y = (2x - 1)(x - 4)$
$y = 2x^2 - 9x + 4$

No Fractions



Notice: Two different graphs in standard form have same x-intercepts.

$x - int = 2 \text{ and } -2$
$a = 2$
$y = 2(x - 2)(x + 2)$
$y = 2x^2 - 8$

$x - int: (-4,0)$
$x = -4$
$x + 4 = 0 \qquad y = (x + 4)^2$

$x - int = -1 \text{ and } 3$
$(2, -6)$

$y = a(x + 1)(x - 3)$
$-6 = a(2 + 1)(2 - 3)$
$-6 = -3a$
$a = 2$
$y = 2(x + 1)(x - 3)$

C11 - 4.0 - Quadratics Square Root Method

$x^2 - 4 = 0$
$+4 \quad +4$
$x^2 = 4$
$\sqrt{x^2} = \pm\sqrt{4}$
$x = \pm 2$
$x = 2$ $x = -2$

OR

$x^2 - 4 = 0$
$(x+2)(x-2) = 0$
$x + 2 = 0 \quad x - 2 = 0$
$x = -2$ $x = 2$

$4x^2 - 9 = 0$
$\frac{4x^2}{4} = \frac{9}{4}$
$x^2 = \frac{9}{4}$
$\sqrt{x^2} = \pm\sqrt{\frac{9}{4}} = \pm\frac{\sqrt{9}}{\sqrt{4}}$
$x = \pm\frac{3}{2}$

$x^2 + 16 = 0$
$-16 \quad -16$
$x^2 = -16$
$\sqrt{x^2} = \pm\sqrt{-16}$
DNE

Can't square root a negative.

$(x+2)^2 + 2 = 0$
$-2 \quad -2$
$(x+2)^2 = -2$
$\sqrt{(x+2)^2} = \pm\sqrt{-2}$
DNE

$(x-2)^2 - 1 = 0$
$+1 \quad +1$
$(x-2)^2 = 1$
$\sqrt{(x-2)^2} = \pm\sqrt{1}$
$x - 2 = \pm 1$
$x - 2 = +1 \quad x - 2 = -1$
$x = 3$ $x = 1$

OR

$(x-2)^2 - 1 = 0$
$(x-2)(x-2) - 1 = 0$
$x^2 - 4x + 4 - 1 = 0$
$x^2 - 4x + 3 = 0$
$(x-1)(x-3) = 0$
$x - 1 = 0 \quad x - 3 = 0$
$x = 1$ $x = 3$

Complete Square Alternative Method

$(x^2 - 4x) + 3 = 0$
$(x^2 - 4x + 4) - 4 + 3 = 0$
$(x-2)^2 - 1 = 0$
...

$x^2 - 4x = -3$
$x^2 - 4x + 4 = -3 + 4$
$(x-2)^2 = 1$
...

$2x^2 + 4x = 1$
$2(x^2 + 2x) = 1$
$2(x^2 + 2x + 1) = 1 + 2$
...

$2(x+1)^2 - 8 = 0$
$+8 \quad +8$
$2(x+1)^2 = 8$
$\frac{2(x+1)^2}{2} = \frac{8}{2}$
$(x+1)^2 = 4$
$\sqrt{(x+1)^2} = \pm\sqrt{4}$
$x + 1 = \pm 2$
$x + 1 = 2 \quad x + 1 = -2$
$x = 1$ $x = -3$

$(x-2)^2 - 7 = 0$
$+7 \quad +7$
$(x-2)^2 = 7$
$\sqrt{(x-2)^2} = \pm\sqrt{7}$
$x - 2 = \pm\sqrt{7} \quad x = \pm\sqrt{7} + 2$
$x = \sqrt{7} + 2$ $x = -\sqrt{7} + 2$

Exact Value

$x = 4.65$ $x = -0.65$

Decimal

$2(x-2)^2 - 7 = 0$
$2(x-2)^2 = 7$
$\sqrt{(x-2)^2} = \pm\sqrt{\frac{7}{2}}$
$x - 2 = \pm\sqrt{\frac{7}{2}}$
$x = \pm\sqrt{\frac{7}{2}} + 2$
$x = \pm\frac{\sqrt{7}}{\sqrt{2}} + \frac{2\sqrt{2}}{\sqrt{2}}$
$x = \frac{\pm\sqrt{7} + 2\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
$x = \frac{\pm\sqrt{14} + 4}{2}$

Rationalize

$\frac{2}{3}\left(x + \frac{1}{2}\right)^2 - 8 = 0$
$\frac{2}{3}\left(x + \frac{1}{2}\right)^2 = 8$
$\left(x + \frac{1}{2}\right)^2 = 12$
$\sqrt{\left(x + \frac{1}{2}\right)^2} = \pm\sqrt{12}$
$x + \frac{1}{2} = \pm\sqrt{12}$
$x = \pm\sqrt{12} - \frac{1}{2}$
$x = \frac{\pm 2\sqrt{12}}{2} - \frac{1}{2}$
$x = \frac{\pm 2\sqrt{12} - 1}{2}$
$x = \frac{\pm 4\sqrt{3} - 1}{2}$

Algebra

Get equal to zero, Divide/Multiply* both sides by "a" → $1^* x^2$

$2x^2 - 8x + 6 = 0$
$\frac{2x^2}{2} - \frac{8x}{2} + \frac{6}{2} = \frac{0}{2}$
$x^2 - 4x + 3 = 0$
...

$-x^2 + 5x - 3 = 0$
$\frac{-x^2}{-1} + \frac{5x}{-1} - \frac{3}{-1} = \frac{0}{-1}$
$x^2 - 5x + 3 = 0$
...

$\frac{1}{2}x^2 - 3x + \frac{5}{4} = 0$
$\left(\frac{1}{2}x^2 - 3x + \frac{5}{4} = 0\right) \times 2$
$x^2 - 6x + \frac{5}{2} = 0$
...

$\frac{3}{2}x^2 - x + 2 = 0$
$\left(\frac{3}{2}x^2 - x + 2 = 0\right) \times \frac{2}{3}$
$x^2 - \frac{2}{3}x + \frac{4}{3} = 0$
...

C11 - 4.0 - Quadform Notes

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant: $b^2 - 4ac$

$x = \dfrac{-b \pm \sqrt{DISCRIMINANT}}{2a}$

Solve Quadratic Equation :

Case 1: $b^2 - 4ac > 0$ *Inside the root is positive*

Two x-intercepts
Two Real Roots
Two Solutions

$b^2 - 4ac > 0$
Discriminant > 0
2 Real Roots.

$1 \quad -4 \quad 3$
$1x^2 - 4x + 3 = 0$
$a = 1$
$b = -4$
$c = 3$

Substitute With Brackets

Get $= 0$

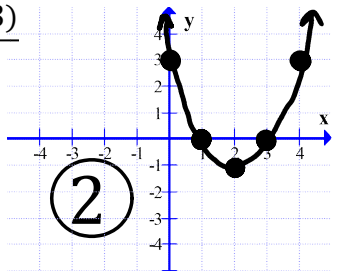
$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$

$x = \dfrac{+4 \pm \sqrt{4}}{2}$ ← Type in Calculator

$x = \dfrac{4 \pm 2}{2}$ (+)

$x = \dfrac{4+2}{2}$ $x = 3$ $\quad$ $x = \dfrac{4-2}{2}$ $x = 1$

2 Rational Roots.



If we add and subtract a positive number we get two answers.

$2x^2 - 6x - 7 = 0$

$x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-7)}}{2(2)}$

$x = \dfrac{6 \pm \sqrt{92}}{4}$

$\sqrt{92} = \sqrt{2 \times 2 \times 23}$
$\sqrt{92} = 2\sqrt{23} = 9.59$

$x = \dfrac{6 \pm 2\sqrt{23}}{4}$

Divide top and bottom by 2

$x = \dfrac{3 \pm \sqrt{23}}{2}$

2 Irrational Roots.

$x = \dfrac{3 + \sqrt{23}}{2}$ Exact Value. $x = \dfrac{3 - \sqrt{23}}{2}$

$x = 3.90$ Decimal. $x = -0.90$

Case 2: $b^2 - 4ac < 0$ *Inside the root is negative*

$\dfrac{3x^2}{3} + \dfrac{12x}{3} + \dfrac{15}{3} = \dfrac{0}{3}$

Divide top and bottom by 3

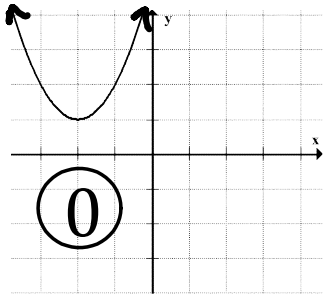
$x^2 + 4x + 5 = 0$

$x = \dfrac{-(4) \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$ (−)

$x = \dfrac{-4 \pm \sqrt{-4}}{2}$ ✕

Cant Square Root Negative

No Solution



$b^2 - 4ac < 0$
Discriminant < 0
No Real Roots.

Zero x-intercepts
No Real Roots/Solutions
Imaginary Roots

Case 3: $b^2 - 4ac = 0$ *Inside the root is zero* $Perfect\ Square$

$x^2 + 4x + 4 = 0$

$x = \dfrac{-(4) \pm \sqrt{(4)^2 - 4(1)(4)}}{2(1)}$

$x = \dfrac{-4 \pm \sqrt{0}}{2}$

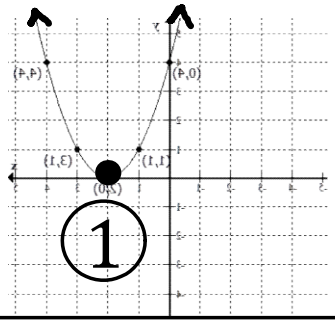
$x = \dfrac{-4 \pm 0}{2}$

$x = -2$

If we add and subtract zero we get one answer

1 Rational Root

0



Perfect Square
$b^2 - 4ac = 0$
Discriminant $= 0$
One Root.

One x-intercepts
Two equal/real roots
One Solution

Find k if :

$x^2 + 10x + k$

$b^2 - 4ac$
$(10)^2 - 4(1)(k)$
$100 - 4k$

2 solutions

$100 - 4k > 0$
$-4k > -100$
$\dfrac{-4k}{-4} < \dfrac{-100}{-4}$
$k < 25$

1 solution

$100 - 4k = 0$
$-4k = -100$
$\dfrac{-4k}{-4} = \dfrac{-100}{-4}$
$k = 25$

0 solutions

$100 - 4k < 0$
$-4k < -100$
$\dfrac{-4k}{-4} < \dfrac{-100}{-4}$
$k > 25$

Sketch the graphs!

$kx^2 + 8x + k$

$b^2 - 4ac$
$(8)^2 - 4(k)(k)$
$64 - 4k^2$

Inequalities Ch9

C11 - 4.0 - Quadratics #'s WPs Notes

Find a # where the sum of itself and its square is six.

$Let\ x = 1st\ \#$

$x + x^2 = 6$

$-6 \quad -6$

$x^2 + x - 6 = 0$

$(x-2)(x+3) = 0$

$x - 2 = 0$ $\quad$ $x + 3 = 0$

$x = 2$ $\quad$ $x = -3$

$2 + 2^2 = 6$ $\quad$ $-3 + (-3)^2 = 6$

$6 = 6$ $\quad$ $-3 + 9 = 6$

Find two consecutive integers whose product is 56.

$Let\ x = 1st\ \#$

$Let\ x + 1 = 2nd\ \#$

$x(x+1) = 56$

$x^2 + x - 56 = 0$

$(x+8)(x-7) = 0$

$x = -8, x = 7$

$7,8$ $\quad$ $-7, -8$

Find two consecutive odd integers whose product is 15.

$Let\ x = 1st\ \#$

$Let\ x + 2 = 2nd\ \#$

$x(x+2) = 15$

$x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$x = -5, x = 3$

$3,5$ $\quad$ $-3, -5$

Two #'s sum to 9, product 20.

$let\ a = 1st\ \#$

$let\ b = 2nd\ \#$

$a + b = 9$

$a = 9 - b$

$a = 9 - b$

$a = 9 - 5$

$a = 4$

$a = 9 - b$

$a = 9 - 4$

$a = 5$

$ab = 20$

$(9-b) \times b = 20$

$9b - b^2 = 20$

$-\frac{b^2}{-1} + \frac{9b}{-1} - \frac{20}{-1} = 0$

$b^2 - 9b + 20 = 0$

$(b-4)(b-5) = 0$

$b - 5 = 0$ $\quad$ $b - 4 = 0$

$b = 5$ $\quad$ $b = 4$

$1st\ \# = 4$

$2nd\ \# = 5$

Check

$5 + 4 = 0 \quad 5 \times 4 = 20$ ✓

C11 - 4.0 - Quadratics Rectangles WPs Notes

A rectangular garden has an Area of $36m^2$ and a Perimeter of 30m. What are the lengths and widths?

Let $w = width$
Let $l = length$

$A = 36$
$P = 30$

l

w

$P = 2l + 2w$
$30 = 2l + 2w$
$\frac{30}{2} = \frac{2l}{2} + \frac{2w}{2}$
$15 = l + w$
$-w \quad -w$
$15 - w = l$
$l = (15 - w)$

$A = l \times w$
$36 = (l) \times w$
$36 = (15 - w) \times w$
$36 = 15w - w^2$
$+w^2 \quad +w^2$
$36 + w^2 = 15w$
$-15w \quad -15w$
$w^2 - 15w + 36 = 0$
$(w - 12)(w - 3) = 0$

$w - 12 = 0$ $\quad$ $w - 3 = 0$

$w = 12$ $\quad$ $w = 3$

$l = 15 - (3)$
$l = 12$

OR

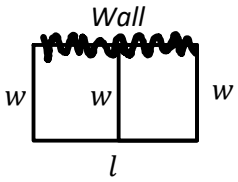
$l = 15 - (12)$
$l = 3$

$Length = 12m$
$Width = 3m$

$Length = 3m$
$Width = 12m$

A rectangular fence that is split in half against a wall. The total fencing length is 39in, and it has a total area of $66in^2$. What are the dimensions of the fence?

Let $w = width$
Let $l = length$



$P = l + 3w$
$39 = l + 3w$
$-3w \quad -3w$
$39 - 3w = l$
$l = (39 - 3w)$

$A = 66$
$P = 39$

$A = (l) \times w$
$66 = (39 - 3w) \times w$
$66 = 39w - 3w^2$
$+3w^2 \quad +3w^2$
$66 + 3w^2 = 39w$
$-39w \quad -39w$
$3w^2 - 39w + 66 = 0$
$3(w^2 - 13w + 22) = 0$
$3(w - 2)(w - 11) = 0$

$w - 2 = 0$
$w = 2$

$w - 11 = 0$
$w = 11$

$l = 39 - 3(2)$
$l = 39 - 6$
$l = 33$

$l = 39 - 3(11)$
$l = 39 - 33$
$l = 6$

OR

$Width = 2in$
$Length = 33in$

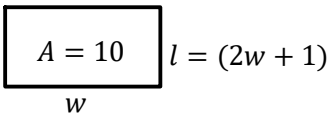
$Width = 11in$
$Length = 6in$

C11 - 4.0 - Quadratics Misc WPs Notes

A rectangle's area is 10 m^2 with a length one meter longer than twice it's width. Find dimensions.

$let\ w = width$

$let\ 2w + 1 = length$



$A = lw$

$10 = w(2w + 1)$

$10 = 2w^2 + w$

$0 = 2w^2 + w - 10$

$0 = (2w + 5)(w - 2)$

$2w + 5 = 0$ $\quad w - 2 = 0$

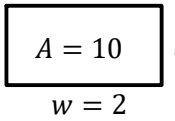
$w = -\frac{5}{2}$ $\quad w = 2m$

$l = 2(2) + 1$

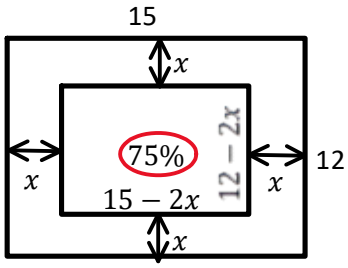
$l = 5m$

No negative lengths.

Check Answer ✓



A rectangular back yard with dimensions 12x15m has a pool with an area of 75% of the total yard with a wrap around deck of the same width. Find the width.



$A = lw$

$A = 12 \times 15$ $\quad 180 \times 0.75 = 135$

$A = 180$

$A = lw$

$135 = (12 - 2x)(15 - 2x)$

...

$0 = 4x^2 - 54x + 45$

...

$x = 0.89m$ $\quad x = 12.6$

$l = 12 - 2x$

$l = 12 - 2(0.89)$

$l = 10.22$

$w = 15 - 2x$

$w = 15 - 2(0.89)$

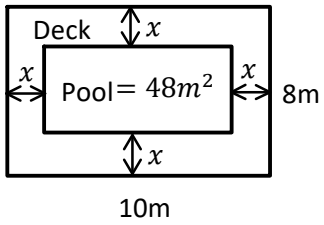
$w = 13.22$

$A = lw$

$A = 10.22(13.22)$

$A = 135$ ✓

A rectangular back yard 10x8m has a pool with an area of $48m^2$ with a wrap around deck of the same width. Find the width of the deck.



$let\ x = deck\ width$

$A = lw$

$48 = (10 - 2x)(8 - 2x)$

$48 = 80 - 36x + 4x^2$

$\frac{0}{4} = \frac{4x^2}{4} - \frac{36x}{4} + \frac{32}{4}$

$0 = x^2 - 9x + 8$

$0 = (x - 8)(x - 1)$

$x - 8 = 0$ $\quad x - 1 = 0$

$x = 8$ $\quad x = 1m$

Can't cut 8 off twice

$l = 10 - 2x$ $\quad w = 10 - 2x$

$l = 10 - 2(1)$ $\quad w = 8 - 2(1)$

$l = 8$ $\quad w = 6$

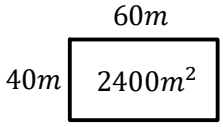
Check $\quad A = lw$

$A = 6 \times 8$

$A = 48$ ✓

Double the area of a 60x40m rectangle by extending dimensions by same amount. Find the extension amount.

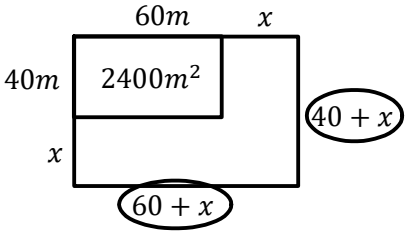
$let\ x = length\ increase$



$A = lw$

$A = 2400 \times 2$

$A = 4800$



$A = lw$

$4800 = (60 + x)(40 + x)$

$4800 = 2400 + 100x + x^2$

$0 = x^2 + 100x - 2400$

$0 = (x - 20)(x + 120)$

$x - 20 = 0$ $\quad x + 120 = 0$

$x = 20m$ $\quad x = -120$

Reject

$l = 60 + 20$ $\quad w = 40 + 20$

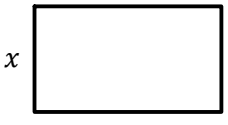
$l = 80$ $\quad w = 60$

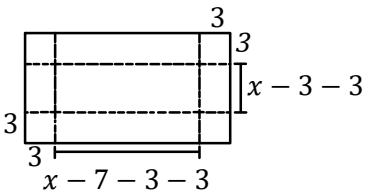
Check $\quad A = lw$

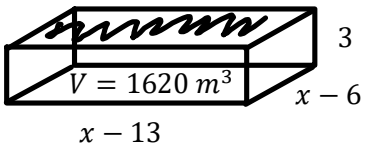
$A = 80 \times 60$

$A = 4800$ ✓

An open top box is made by cutting squares of 3cm from each corner from rectangle piece of cardboard with a width 7 longer than its length, then folding up the sides. Find the length of the square that must be cut from each corner so the box has a volume of 1620 cm^3.







$V = lwh$

$1620 = (x - 6)(x - 13)3$

$540 = x^2 - 19x + 78$

$0 = x^2 - 19x - 462$

$0 = (x - 33)(x + 14)$

$x = 33$ $\quad x = -14$

$V = lwh$

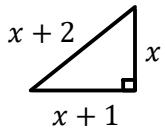
$1620 = (27)(20)(3)$

$1620 = 1620$ ✓

C11 - 4.0 - Quadratics Misc WPs Notes

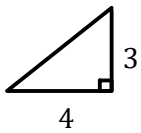
Find the Area of a right angle triangle has consecutive lengths.

$let\ x = a$
$let\ x + 1 = b$
$let\ x + 2 = c$

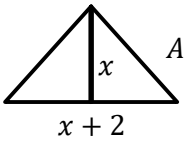


$$a^2 + b^2 = c^2$$
$$(x)^2 + (x+1)^2 = (x+2)^2$$
...
$$x^2 - 2x - 3 = 0$$
...
$x = 3$ ~~$x = -1$~~

$$A = \frac{bh}{2}$$
$$A = \frac{4(3)}{2}$$
$A = 6$



A triangle has a base 2 longer than its height with an area of $40m^2$. Find the base and the height.



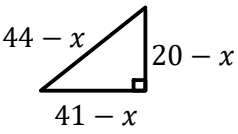
$$A = \frac{bh}{2}$$
$$40 = \frac{x(x+2)}{2}$$
$$80 = x^2 + 2x$$
$$0 = x^2 + 2x - 80$$
$$0 = (x+10)(x-8)$$
~~$x = -10$~~ $x = 8$

$$A = \frac{(10)(80)}{2}$$
$A = 40m^2$ ✓

3 lengths of lengths 20, 41 and 44 have the same length removed to from form a right angle triangle. Find the removed amount.

$let\ x = length\ removed$

$20 - x$
$41 - x$
$44 - x$



$$(20-x)^2 + (41-x)^2 = (44-x)^2$$
...
$$x^2 - 34x + 145 = 0$$
...
$x = 5$ ~~$x = 29$~~

$20 - 5 = 15$
$41 - 5 = 36$
$44 - 5 = 39$

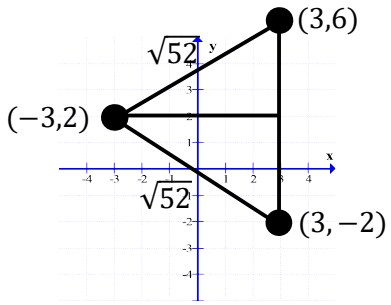
$a^2 + b^2 = c^2$
$15^2 + 36^2 = 39^2$ ✓

Given the distance between the following points, find m.

$d = \sqrt{52}$ $(-3,2)$ $(3,m)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\sqrt{52} = \sqrt{(3 - -3)^2 + (m-2)^2}$$
$$\sqrt{52} = \sqrt{36 + m^2 - 4m + 4}$$
$$\sqrt{52} = \sqrt{m^2 - 4m + 40}$$
$$52 = m^2 - 4m + 40$$
$$0 = m^2 - 4m - 12$$
$$0 = (m-6)(m+2)$$

$m - 6 = 0$ $m + 2 = 0$
$m = 6$ $m = -2$



$$6^2 + 4^2 = \sqrt{52}^2$$