C11 - 5.4 - Solving Radical Equations/Restrictions Notes

$\sqrt{x+2}=4$
$(\sqrt{x+2})^2=(4)^2$
$x+2=16$
$x=14$

Square Both sides (Brackets)

$\sqrt{x+2}=4$
$\sqrt{14+2}=4$
$\sqrt{16}=4$
$4=4$

Check Answer:

LHS=RHS ✓

$x+2\geq 0$
$-2 \quad -2$
$x\geq -2$

Restrictions: Set underneath root ≥ 0 and solve.

$\sqrt{x+2}+1=4$
$-1 \quad -1$
$\sqrt{x+2}=3$
$(\sqrt{x+2})^2=(3)^2$
$x+2=9$
$-2 \quad -2$
$x=7$ ✓

Isolate Root

$\sqrt{x+2}+1=4$
$\sqrt{7+2}+1=4$
$\sqrt{9}+1=4$
$3+1=4$
$4=4$

$x+2\geq 0$
$x\geq -2$

$\sqrt{x+3}=\sqrt{2x+5}$
$(\sqrt{x+3})^2=(\sqrt{2x+5})^2$
$x+3=2x+5$
$-x \quad -x$
$3=x+5$
$-5 \quad -5$
$x=-2$ ✓

$\sqrt{x+3}=\sqrt{2x+5}$
$\sqrt{-2+3}=\sqrt{2(-2)+5}$
$\sqrt{1}=\sqrt{1}$

$x+3\geq 0$
$x\geq -3$

$2x+5\geq 0$
$x\geq -\frac{5}{2}$

$\sqrt{x+3}-x-1=0$
$\sqrt{x+3}=x+1$
$(\sqrt{x+3})^2=(x+1)^2$
$x+3=(x+1)(x+1)$
$x+3=x^2+2x+1$
$0=x^2+x-2$
$0=(x+2)(x-1)$

$x+2=0$
$x=-2$ ✗

$x-1=0$
$x=1$ ✓

$\sqrt{x+3}=x+1$
$\sqrt{-2+3}=-2+1$
$1\neq -1$

$\sqrt{x+3}=x+1$
$\sqrt{1+3}=1+1$
$2=2$

$x+3\geq 0$
$x\geq -3$

Square Both Sides First

$2\sqrt{x+3}=6$
$(2\sqrt{x+3})^2=(6)^2$
$4(x+3)=36$
$\frac{4(x+3)}{4}=\frac{36}{4}$
$x+3=9$
$-3 \quad -3$
$x=6$ ✓

Divide First

$2\sqrt{x+3}=6$
$\frac{2\sqrt{x+3}}{2}=\frac{6}{2}$
$\sqrt{x+3}=3$
$(\sqrt{x+3})^2=(3)^2$
$x+3=9$
$-3 \quad -3$
$x=6$ ✓

$\sqrt{x}=-5$
No Solution ✗

$\sqrt{x+99}=-5$
No Solution

A Square/Even Root Can't Equal a Negative

$\sqrt{x+1}=\sqrt{x}+1$
$(\sqrt{x+1})^2=(\sqrt{x}+1)^2$
$x+1=(\sqrt{x}+1)(\sqrt{x}+1)$
$x+1=x+\sqrt{x}+\sqrt{x}+1$
$0=2\sqrt{x}$
$(0)^2=(2\sqrt{x})^2$
$0=4x$
$x=0$ ✓

$x+1\geq 0$
$x\geq -1$

$x\geq 0$

More Restrictive

$\sqrt{x+1}=\sqrt{x}+1$
$\sqrt{0+1}=\sqrt{0}+1$
$1=1$

$\sqrt{x-5}-\sqrt{x-8}=1$
$\sqrt{x-5}=\sqrt{x-8}+1$
$(\sqrt{x-5})^2=(\sqrt{x-8}+1)^2$
$x-5=(\sqrt{x-8}+1)(\sqrt{x-8}+1)$
$x-5=x-8+2\sqrt{x-8}+1$
$1=\sqrt{x-8}$
$(1)^2=(\sqrt{x-8})^2$
$1=x-8$
$x=9$ ✓

$\sqrt{x-5}-\sqrt{x-8}=1$
$\sqrt{9-5}-\sqrt{9-8}=1$
$\sqrt{4}-\sqrt{1}=1$
$2-1=1$

$x-8\geq 0$
$x\geq 8$

$x-5\geq 0$
$x\geq 5$

$(2x+3)^2=(x+7)^2$
$\sqrt{(2x+3)^2}=\sqrt{(x+7)^2}$
$2x+3=x+7$
$x=4$ ✓

Square Root Both Sides

$(2x+3)^2=(x+7)^2$
$(2(4)+3)^2=((4)+7)^2$
$121=121$