C11 - 5.1 - Adding and Subtracting Radicals Notes

Square Roots

$\sqrt[2]{7} + \sqrt[2]{7} = 2\sqrt[2]{7}$

$5.29 = 5.29$ ✓

Like Radicals: Add or subtract coefficients.

$\boxed{x + x = 2x}$

Like Radicals: Same radicand, same index

$1\sqrt[2]{3} + 1\sqrt[2]{3} = 2\sqrt[2]{3}$

$3.46 = 3.46$ ✓

Index $\rightarrow$ $\sqrt[2]{3}$ $\leftarrow$ Radicand

$2\sqrt[2]{3} + 5\sqrt[2]{3} = 7\sqrt[2]{3}$

$12.12 = 12.12$ ✓

Calcuator

$\sqrt[2]{3} + \sqrt[2]{2} = 1.71 + 1.41 = 3.15$

$\sqrt[2]{3} + \sqrt[2]{2} = \sqrt[2]{3} + \sqrt[2]{2}$

Cannot add/subtract unlike radicals.

Can only add/subtract like radicals.

$4\sqrt[2]{3} - 7\sqrt[2]{2} = -3\sqrt[2]{2}$

$-4.24 = -4.24$ ✓

Simplify Roots

$\sqrt[2]{12} + \sqrt[2]{27} + \sqrt[2]{18} + 5$

$2\sqrt[2]{3} + 3\sqrt[2]{3} + 3\sqrt[2]{2} + 5$

$5\sqrt[2]{3} + 3\sqrt[2]{2} + 5$

$17.9 = 17.9$ ✓

$\sqrt[2]{12} = \sqrt[2]{2 \times 2 \times 3}$

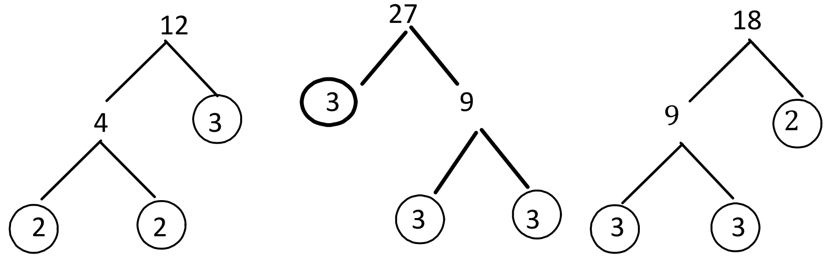
$= 2\sqrt[2]{3}$

$\sqrt[2]{27} = \sqrt[3]{3 \times 3 \times 3}$

$= 2\sqrt[3]{3}$

$\sqrt[2]{18} = \sqrt[2]{3 \times 3 \times 2}$

$= 3\sqrt[2]{2}$



Cube Roots

$\sqrt[3]{7} + \sqrt[3]{7} = 2\sqrt[3]{7}$

$3.83 = 3.83$ ✓

$\sqrt[3]{5} + \sqrt[3]{5} = 2\sqrt[3]{5}$

$3.42 = 3.42$ ✓

$-2\sqrt[3]{5} - 6\sqrt[3]{5} = -8\sqrt[3]{5}$

$-13.68 = -13.68$ ✓

$\sqrt[3]{3} + 1 = \sqrt[3]{3} + 1$

Can only add or subtract like radicals.

C11 - 5.2 - Multiplying and Dividing Radicals Notes

$\sqrt[2]{3} \times \sqrt[2]{3} = \sqrt[2]{3 \times 3}$
$= \sqrt[2]{9}$
$= 3$

$7 \times \sqrt{5} = 7\sqrt{5}$
$\sqrt{5} \times 7 = 7\sqrt{5}$
$13.23 = 13.23$

$\sqrt[2]{5} \times \sqrt[2]{3} = \sqrt[2]{5 \times 3}$
$= \sqrt[2]{15}$ $\quad 3.87 = 3.87$

$3\sqrt[2]{7} \times 2\sqrt[2]{3} = 3 \times 2\sqrt[2]{7 \times 3}$
$= 6\sqrt[2]{21}$

$27.50 = 27.50$

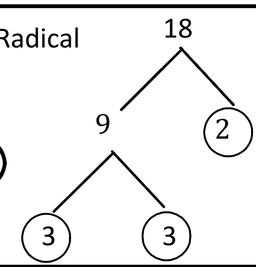
Multiply Coefficients
Multiply Radicands

$2 \times 5\sqrt{3} = 10\sqrt{3}$ $\quad 17.32 = 17.32$
$2\sqrt{5} \times \sqrt{3} = 2\sqrt{15}$ $\quad 7.75 = 7.75$

$5\sqrt[2]{6} \times 7\sqrt[2]{3} = 5 \times 7\sqrt[2]{6 \times 3}$
$= 35\sqrt[2]{18}$
$= 35 \times 3\sqrt[2]{2}$
$= 105\sqrt[2]{2}$ $\quad 148.49 = 148.49$

Simplify Radical

$\sqrt[2]{18} = \sqrt[2]{3 \times 3 \times 2}$
$= 3\sqrt[2]{2}$



$\sqrt[2]{5} \times \sqrt[3]{5} = \sqrt[2]{5} \times \sqrt[3]{5} = 5^{\frac{1}{2}} \times 5^{\frac{1}{3}} = 5^{\frac{5}{6}}$

Can only multiply/divide like indexes.
Cannot multiply/divide unlike indexes.
Change Form, Add Exponents

$3.82 = 3.82$

Distribute

$3(5 + \sqrt{2})$
$15 + 3\sqrt{2}$

$19.24 = 19.24$

$(5 + \sqrt{7})\sqrt{7}$
$5\sqrt{7} + 7$

$20.23 = 20.23$

FOIL

$(2 - \sqrt[2]{3}) \times (1 + \sqrt[2]{5})$
$2 + 2\sqrt{5} - 1\sqrt{3} - \sqrt{15}$

$0.867 = 0.867$

$(2 + \sqrt{3})^2$
$(2 + \sqrt{3})(2 + \sqrt{3})$
...

$\frac{\sqrt[2]{6}}{\sqrt[2]{3}} = \sqrt[2]{\frac{6}{3}}$
$= \sqrt[2]{2}$ $\quad 1.41 = 1.41$

$\frac{10\sqrt[2]{6}}{2\sqrt[2]{3}} = \frac{10}{2}\sqrt[2]{\frac{6}{3}}$
$= 5\sqrt[2]{2}$ $\quad 7.07 = 7.07$

$\frac{\sqrt{24}}{\sqrt{8}} = \frac{2\sqrt{6}}{2\sqrt{2}} = \frac{\sqrt{6}}{\sqrt{2}} = \sqrt{\frac{6}{3}} = \sqrt{2}$

$\sqrt{24} = 2\sqrt{6}$
$\sqrt{8} = 2\sqrt{2}$

OR

$\frac{\sqrt{24}}{\sqrt{8}} = \sqrt{\frac{24}{8}} = \sqrt{3}$ $\quad$ Simplify 1st

C11 - 5.3 - Rationalizing the Denominator Notes

$$\frac{5}{\sqrt[2]{3}} = \frac{5 \times \boxed{\sqrt[2]{3}}}{\sqrt[2]{3} \times \boxed{\sqrt[2]{3}}}$$

Multiply the top and bottom by the root in the denominator.
Only the Root!

$$= \frac{5\sqrt[2]{3}}{\sqrt[2]{3 \times 3}}$$
$$= \frac{5\sqrt[2]{3}}{\sqrt[2]{9}}$$
$$= \frac{5\sqrt[2]{3}}{3}$$

$$\frac{5}{\sqrt[2]{3}} = 2.89 = \frac{5\sqrt[2]{3}}{3}$$

$$\boxed{\sqrt[2]{3^1} = 3^{\frac{1}{2}} \qquad x^{\frac{m}{n}} = \sqrt[n]{x^m}}$$

$$\sqrt{3} \times \sqrt{3} = 3 \qquad 3^{\frac{1}{2}} \times 3^{\frac{1}{2}} = 3^1 \qquad \frac{1}{2} + \frac{1}{2} = 1$$

Add Exponents

$$\frac{5}{2 - \sqrt[2]{6}} = \frac{5 \times \boxed{(2 + \sqrt[2]{6})}}{(2 - \sqrt[2]{6}) \times \boxed{(2 + \sqrt[2]{6})}}$$
$$= \frac{10 + 5\sqrt[2]{6}}{-2}$$

Distribution
Foil

$$\frac{5}{2 - \sqrt[2]{6}} = -11.12 = \frac{10 + 5\sqrt[2]{6}}{-2}$$

Multiply the top/bottom by **Conjugate** of denominator.

$$(2 - \sqrt[2]{6}) \times (2 + \sqrt[2]{6})$$
$$4 + 2\sqrt{6} - 2\sqrt{6} - \sqrt{36}$$
$$4 + \cancel{2\sqrt{6} - 2\sqrt{6}} - \sqrt{36}$$
$$4 - \sqrt{36}$$
$$-2$$

$$(a+b)(a-b) =$$
$$a^2 \cancel{- ab + ab} - b^2 =$$
$$a^2 - b^2$$

$$\boxed{F\cancel{O}\cancel{I}L}$$

$$\frac{4}{\sqrt[2]{5} + \sqrt[2]{3}} = \frac{4 \times (\sqrt[2]{5} - \sqrt[2]{3})}{(\sqrt[2]{5} + \sqrt[2]{3}) \times (\sqrt[2]{5} - \sqrt[2]{3})}$$
$$= \frac{4\sqrt[2]{5} - 4\sqrt[2]{3}}{5 - 3}$$
$$= \frac{4\sqrt[2]{5} - 4\sqrt[2]{3}}{2} \boxed{\frac{\div 2}{\div 2}}$$
$$= 2\sqrt{5} - 2\sqrt{3}$$

Conjugate

Simplify, by dividing the top and bottom by 2.

$$\frac{4}{\sqrt[2]{5} + \sqrt[2]{3}} = 1.01 = 2\sqrt{5} - 2\sqrt{3}$$

$$\frac{5}{\sqrt[3]{3}} = \frac{5 \times \boxed{\sqrt[3]{3}} \times \boxed{\sqrt[3]{3}}}{\sqrt[3]{3} \times \boxed{\sqrt[3]{3}} \times \boxed{\sqrt[3]{3}}}$$
$$= \frac{5\sqrt[3]{9}}{3}$$

$$\frac{5}{\sqrt[3]{3}} = 3.47 = \frac{5\sqrt[3]{9}}{3}$$

Multiply the top and bottom by the cube root of the denominator twice. (Or three times for a fourth root etc.)

$$\boxed{\sqrt[3]{3} = 3^{\frac{1}{3}} \qquad x^{\frac{m}{n}} = \sqrt[n]{x^m}}$$

$$\boxed{\sqrt[3]{3} \times \sqrt[3]{3} \times \sqrt[3]{3} = 3 \qquad 3^{\frac{1}{3}} \times 3^{\frac{1}{3}} \times 3^{\frac{1}{3}} \times = 3^1 \qquad \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1}$$

C11 - 5.4 - Solving Radical Equations/Restrictions Notes

$\sqrt{x+2}=4$
$\left(\sqrt{x+2}\right)^2=(4)^2$
$x+2=16$
$x=14$

Square Both sides (Brackets)

$\sqrt{x+2}=4$
$\sqrt{14+2}=4$
$\sqrt{16}=4$
$4=4$

Check Answer:

LHS=RHS ✓

$x+2\geq 0$
$-2\quad -2$
$x\geq -2$

Restrictions: Set underneath root ≥ 0 and solve.

$\sqrt{x+2}+1=4$
$-1\quad -1$
$\sqrt{x+2}=3$
$\left(\sqrt{x+2}\right)^2=(3)^2$
$x+2=9$
$-2\quad -2$
$x=7$ ✓

Isolate Root

$\sqrt{x+2}+1=4$
$\sqrt{7+2}+1=4$
$\sqrt{9}+1=4$
$3+1=4$
$4=4$

$x+2\geq 0$
$x\geq -2$

$\sqrt{x+3}=\sqrt{2x+5}$
$\left(\sqrt{x+3}\right)^2=\left(\sqrt{2x+5}\right)^2$
$x+3=2x+5$
$-x\quad -x$
$3=x+5$
$-5\quad -5$
$x=-2$ ✓

$\sqrt{x+3}=\sqrt{2x+5}$
$\sqrt{-2+3}=\sqrt{2(-2)+5}$
$\sqrt{1}=\sqrt{1}$

$x+3\geq 0$
$x\geq -3$

$2x+5\geq 0$
$x\geq -\frac{5}{2}$

$\sqrt{x+3}-x-1=0$
$\sqrt{x+3}=x+1$
$\left(\sqrt{x+3}\right)^2=(x+1)^2$
$x+3=(x+1)(x+1)$
$x+3=x^2+2x+1$
$0=x^2+x-2$
$0=(x+2)(x-1)$

$x+2=0$
$\cancel{x=-2}$ ✗

$x-1=0$
$x=1$ ✓

$\sqrt{x+3}=x+1$
$\sqrt{-2+3}=-2+1$
$1\neq -1$

$\sqrt{x+3}=x+1$
$\sqrt{1+3}=1+1$
$2=2$

$x+3\geq 0$
$x\geq -3$

Square Both Sides First

$2\sqrt{x+3}=6$
$\left(2\sqrt{x+3}\right)^2=(6)^2$
$4(x+3)=36$
$\frac{4(x+3)}{4}=\frac{36}{4}$
$x+3=9$
$-3\quad -3$
$x=6$ ✓

Divide First

$2\sqrt{x+3}=6$
$\frac{2\sqrt{x+3}}{2}=\frac{6}{2}$
$\sqrt{x+3}=3$
$\left(\sqrt{x+3}\right)^2=(3)^2$
$x+3=9$
$-3\quad -3$
$x=6$ ✓

$\sqrt{x}=-5$
No Solution ✗

$\sqrt{x+99}=-5$
No Solution

A Square/Even Root Can't Equal a Negative

$\sqrt{x+1}=\sqrt{x}+1$
$\left(\sqrt{x+1}\right)^2=(\sqrt{x}+1)^2$
$x+1=(\sqrt{x}+1)(\sqrt{x}+1)$
$x+1=x+\sqrt{x}+\sqrt{x}+1$
$0=2\sqrt{x}$
$(0)^2=(2\sqrt{x})^2$
$0=4x$
$x=0$ ✓

$\cancel{x+1\geq 0}$
$\cancel{x\geq -1}$

$x\geq 0$

More Restrictive

$\sqrt{x+1}=\sqrt{x}+1$
$\sqrt{0+1}=\sqrt{0}+1$
$1=1$

$\sqrt{x-5}-\sqrt{x-8}=1$
$\sqrt{x-5}=\sqrt{x-8}+1$
$\left(\sqrt{x-5}\right)^2=\left(\sqrt{x-8}+1\right)^2$
$x-5=\left(\sqrt{x-8}+1\right)\left(\sqrt{x-8}+1\right)$
$x-5=x-8+2\sqrt{x-8}+1$
$1=\sqrt{x-8}$
$(1)^2=\left(\sqrt{x-8}\right)^2$
$1=x-8$
$x=9$ ✓

$\sqrt{x-5}-\sqrt{x-8}=1$
$\sqrt{9-5}-\sqrt{9-8}=1$
$\sqrt{4}-\sqrt{1}=1$
$2-1=1$

$x-8\geq 0$
$x\geq 8$

$\cancel{x-5\geq 0}$
$\cancel{x\geq 5}$

$(2x+3)^2=(x+7)^2$
$\sqrt{(2x+3)^2}=\sqrt{(x+7)^2}$
$2x+3=x+7$
$x=4$ ✓

Square Root Both Sides

$(2x+3)^2=(x+7)^2$
$(2(4)+3)^2=((4)+7)^2$
$121=121$