

C11 - 6.1 - Simplifying Rationals Notes

Simplify.

$$\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

$$\frac{2}{4} = \frac{\overset{1}{\cancel{2}}}{\cancel{2} \times 2} = \frac{1}{2}$$

$$\frac{6x^2}{2x} = \frac{6 \times x \times \cancel{x}}{2 \times \cancel{x}} = 3x$$

$$\frac{2x + 4}{x + 2} = \frac{2(\cancel{x+2})}{\cancel{x+2}} = 2 \quad \text{Factor, Simplify.}$$

$$\frac{x^2 + 5x + 6}{x + 3} = \frac{(x + 2)(\cancel{x+3})}{x + \cancel{3}} = (x + 2)$$

$$\frac{x + 3}{x^2 - 9} = \frac{\cancel{x+3}}{(\cancel{x+3})(x-3)} = \frac{1}{x-3}$$

$$\frac{1}{2-x} = \frac{1}{-(x-2)} = \frac{-1}{x-2}$$

$\begin{array}{l} 2-x \\ -(-2+x) \\ -(x-2) \end{array}$	OR	$\begin{array}{l} 2-x \\ -(x-2) \end{array}$
<i>GCF = -1</i> <i>Rearrange order of terms</i>		

$$\frac{x-4}{4-x} = \frac{x-4}{-(-4+x)} = \frac{x-4}{-(x-4)} = -1$$

$$\frac{x^2 - 3x - 4}{x^2 - 1} = \frac{(x-4)(\cancel{x+1})}{(x-1)(\cancel{x+1})} = \frac{x-4}{x-1}$$

$$\frac{x^2 - 5x + 6}{x + 2} = \frac{(x-2)(x-3)}{x+2} \quad \text{Cannot Simplify}$$

C11 - 6.2 - Restrictions Notes

$$\frac{8}{0} = \text{und}$$

Can't Divide by Zero

Restrictions: Set Denominator $\neq 0$ and solve

$$\frac{1}{x}$$

$x \neq 0$

$$\frac{2}{x+3}$$

$x + 3 \neq 0$

$x \neq -3$

$$\frac{x}{2}$$

No Restrictions

$$\frac{3}{x^2 + 5x + 6}$$

$$x^2 + 5x + 6 \neq 0$$

$$(x + 3)(x + 2) \neq 0$$

$x + 3 \neq 0 \quad x + 2 \neq 0$

$x \neq -3$

$x \neq -2$

$$\frac{3}{2x^2 + x - 1}$$

$$2x^2 + x - 1 \neq 0$$

$$(2x - 1)(x + 1) \neq 0$$

$2x - 1 \neq 0 \quad x + 1 \neq 0$

$x \neq \frac{1}{2}$

$x \neq -1$

$$\frac{5}{x^2 - 4}$$

$$x^2 - 4 \neq 0$$

$$(x + 2)(x - 2) \neq 0$$

$x + 2 \neq 0 \quad x - 2 \neq 0$

$x \neq -2$

$x \neq 2$

$$\frac{2}{x^2 - 2x}$$

$$x^2 - 2x \neq 0$$

$$x(x - 2) \neq 0$$

$x \neq 0$

$x - 2 \neq 0$

$x \neq 2$

$$\frac{1}{x^2 + 1}$$

$$x^2 + 1 \neq 0$$

$$x^2 \neq -1$$

$$\sqrt{x^2} \neq \sqrt{-1}$$

Can't even root a negative

No Restrictions

C11 - 6.3 - Multiplying Dividing Rationals Notes

$$\frac{1}{2} \times \frac{1}{3} = \left(\frac{1}{6}\right)$$

Multiply Tops
Multiply Bottoms

$$\frac{3}{8} \times \frac{4}{9} = \frac{3 \times 4}{8 \times 9} = \frac{\cancel{3} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{3} \times 3} = \left(\frac{1}{6}\right)$$

$$\frac{3}{8} \times \frac{4}{9} = \frac{\cancel{3}^1 \times \cancel{4}^1}{\cancel{8}_2 \times \cancel{9}_3} = \left(\frac{1}{6}\right)$$

$$\frac{a}{2} \div \frac{1}{3} = \frac{a}{2} \times \frac{3}{1} = \left(\frac{3a}{2}\right)$$

Flip and multiply

$$\frac{1}{x+2} \times (x+2) = (1) \quad \begin{matrix} x+2 \neq 0 \\ x \neq -2 \end{matrix}$$

Restrictions

$$\frac{1}{(x+2)(x+3)} \times (x+3) = \frac{1}{x+2}$$

$$\begin{matrix} x+2 \neq 0 & x+3 \neq 0 \\ x \neq -2 & x \neq -3 \end{matrix}$$

$$\frac{x+2}{x+3} \times \frac{2}{x+2} = \frac{2}{x+3}$$

$$\begin{matrix} x+2 \neq 0 & x+3 \neq 0 \\ x \neq -2 & x \neq -3 \end{matrix}$$

$$\frac{2}{x+1} \times (x+1)(x+2) = 2(x+2)$$

$$\frac{2}{x+1} \times (x+1)(x+2) = 2(x+2)$$

Think what cancels and what are you left with

$$\frac{x+1}{x^2-5x+6} \times \frac{x-2}{x^2+5x+4} = \frac{1}{(x-3)(x+4)}$$

Factor

$$\frac{(x+1)(x-2)}{(x-3)(x-2)(x+4)(x+1)} = \frac{1}{(x-3)(x+4)}$$

$$\begin{matrix} x-2 \neq 0 & x+1 \neq 0 & x-3 \neq 0 & x+4 \neq 0 \\ x \neq 2 & x \neq -1 & x \neq 3 & x \neq -4 \end{matrix}$$

$x \neq 2, -1, 3, -4$

$$\frac{x-4}{x+5} \div \frac{x-4}{x-3} = \frac{x-3}{x+5}$$

Flip and multiply

$$\frac{(x-4)(x-3)}{(x+5)(x-4)} = \frac{x-3}{x+5}$$

$$\begin{matrix} x+5 \neq 0 & x-3 \neq 0 & x-4 \neq 0 \\ x \neq -5 & x \neq 3 & x \neq 4 \end{matrix}$$

$x \neq 3, -5, 4$

$$\frac{x-7}{x+4} \div \frac{x^2-2x-15}{x^2-x-20} = \frac{x-7}{x+3}$$

Factor 1st

$$\frac{x-7}{x+4} \div \frac{(x-5)(x+3)}{(x-5)(x+4)} = \frac{x-7}{x+3}$$

$$\frac{(x-7)(x-5)(x+4)}{(x+4)(x-5)(x+3)} = \frac{x-7}{x+3}$$

$$\begin{matrix} x+4 \neq 0 & x-5 \neq 0 & x+3 \neq 0 \\ x \neq -4 & x \neq 5 & x \neq -3 \end{matrix}$$

$x \neq -4, -3, 5$

C11 - 6.4 - LCD Notes

Find LCD

$$\frac{1}{2} + \frac{1}{3} =$$

$$LCD = 6$$

$$\frac{\square}{6} + \frac{\square}{6} =$$

$$\frac{3 \times 1}{3 \times 2} + \frac{1 \times 2}{3 \times 2} =$$

$$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

$$\frac{\square}{2} + \frac{\square}{3} =$$

$$LCD = 2 \times 3$$

$$\frac{\square}{a} + \frac{\square}{b} =$$

$$LCD = ab$$

$$\frac{\square}{2} + \frac{\square}{6} =$$

$$LCD = 6$$

$$\frac{\square}{2} + \frac{\square}{2 \times 3} =$$

$$LCD = 2 \times 3$$

$$\frac{1}{a} + \frac{1}{ab} =$$

$$LCD = ab$$

$$\frac{\square}{a} + \frac{\square}{bc} =$$

$$LCD = abc$$

$$\frac{1}{a^2} + \frac{1}{a} =$$

$$LCD = a^2$$

$$\frac{\square}{ab} + \frac{\square}{cd} =$$

$$LCD = abcd$$

$$\frac{\square}{2} + \frac{\square}{2+1} =$$

$$LCD = 2 \times (2 + 1)$$

$$\frac{\square}{a} + \frac{\square}{a+1} =$$

$$LCD = a(a + 1)$$

$$\frac{\square}{2+4} + \frac{\square}{2+1} =$$

$$LCD = (2 + 4)(2 + 1)$$

$$\frac{\square}{a+1} + \frac{\square}{a+2} =$$

$$LCD = (a + 1)(a + 2)$$

$$\frac{\square}{a} + \frac{\square}{b} = \frac{\square}{c}$$

$$LCD = abc$$

$$\frac{\square}{a} + 5 = \frac{\square}{a+1}$$

$$LCD = a(a + 1)$$

$$\frac{1}{a} + \frac{1}{a+1} = \frac{1}{a+2}$$

$$LCD = a(a + 1)(a + 2)$$

C11 - 6.4 - Adding Subtracting Rationals Notes

$$\frac{1}{2} + \frac{1}{3} =$$

$$\frac{3 \times 1}{3 \times 2} + \frac{1 \times 2}{1 \times 2} =$$

$$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

$LCD = 6$

LCD
Do to top, do to bottom
Add/subtract

$$\frac{x}{2} + \frac{1}{2} = \frac{x+1}{2}$$

$LCD = 2$

$$\frac{x}{2} - \frac{1}{6} =$$

$$\frac{3 \times x}{3 \times 2} - \frac{1}{6} =$$

$$\frac{3x}{6} - \frac{1}{6} = \frac{3x-1}{6}$$

$LCD = 6$

$$\frac{3}{2} - \frac{x+2}{2} =$$

$$\frac{3-(x+2)}{2} =$$

$$\frac{3-x-2}{2} = \frac{1-x}{2}$$

$LCD = 2$

Don't forget to distribute the negative

Factoring out a negative

$$\frac{1}{x-2} + \frac{1}{2-x}$$

$$\frac{1}{x-2} + \frac{-1}{-(x-2)}$$

$$\frac{1}{x-2} - \frac{1}{(x-2)}$$

$$\frac{x}{x+2} + \frac{1}{x+2} = \frac{x+1}{x+2}$$

$LCD = x+2$ $x+2 \neq 0$
 $x \neq -2$

$$\frac{1}{x+2} + \frac{1}{(x+2)(x+3)} =$$

$$\frac{x+3}{x+3} \times \frac{1}{x+2} + \frac{1}{(x+2)(x+3)} =$$

$$\frac{1}{(x+2)(x+3)} + \frac{1}{(x+2)(x+3)} =$$

$$\frac{x+3+1}{(x+2)(x+3)} = \frac{x+4}{(x+2)(x+3)}$$

$LCD = (x+2)(x+3)$ $x+2 \neq 0$ $x+3 \neq 0$
 $x \neq -2$ $x \neq -3$

$$\frac{1}{x} + \frac{3}{(x+2)} =$$

$$\frac{x+2}{x+2} \times \frac{1}{x} + \frac{3}{(x+2)} \times \frac{x}{x} =$$

$$\frac{1}{x(x+2)} + \frac{3x}{x(x+2)} =$$

$$\frac{x+2+3x}{x(x+2)} = \frac{5x+2}{x(x+2)}$$

$LCD = x(x+2)$ $x \neq 0$ $x+2 \neq 0$
 $x \neq -2$ $x \neq -2$

$$\frac{x+2}{x^2+5x+6} + \frac{1}{x+3} =$$

$$\frac{x+2}{(x+2)(x+3)} + \frac{1}{x+3} =$$

Simplify 1st

$$\frac{1}{x+3} + \frac{1}{x+3} =$$

$$\frac{1+1}{x+3} = \frac{2}{x+3}$$

$LCD = (x+3)$ $x+2 \neq 0$ $x+3 \neq 0$
 $x \neq -2$ $x \neq -3$

$$\frac{x}{(x-2)(x+2)} - \frac{2}{(x-2)(x+2)} =$$

$$\frac{x-2}{(x-2)(x+2)} = \frac{1}{x+2}$$

$LCD = (x+2)(x-2)$ $x+2 \neq 0$ $x+3 \neq 0$
 $x \neq -2$ $x \neq -3$

Simplify at end

C11 - 6.5 - Rational Equations Notes

Solve for x .

$$\begin{aligned} \frac{x}{2} + \frac{1}{4} &= \frac{3}{4} \\ 2 \times \frac{x}{2} + \frac{1}{4} &= \frac{3}{4} \\ \frac{2x}{2} + \frac{1}{4} &= \frac{3}{4} \\ \frac{2x}{4} + \frac{1}{4} &= \frac{3}{4} \\ \left(\frac{2x}{4} + \frac{1}{4} = \frac{3}{4}\right) \times LCD \\ 2x + 1 &= 3 \\ -1 \quad -1 \\ 2x &= 2 \\ \frac{2x}{2} &= \frac{2}{2} \\ x &= 1 \end{aligned}$$

Get an LCD then Multiply by the LCD

OR!

$$\begin{aligned} \frac{x}{2} + \frac{1}{4} &= \frac{3}{4} \quad \text{Multiply by} \\ \frac{x}{2} + \frac{1}{4} &= \frac{3}{4} \quad \text{the } LCD = 4 \\ \left(\frac{x}{2} + \frac{1}{4} = \frac{3}{4}\right) \times 4 \\ \frac{4x}{2} + \frac{4}{4} &= \frac{12}{4} \\ 2x + 1 &= 3 \\ -1 \quad -1 \\ 2x &= 2 \\ \frac{2x}{2} &= \frac{2}{2} \\ x &= 1 \end{aligned}$$

OR!

$$\begin{aligned} \left(\frac{x}{2} + \frac{1}{4} = \frac{3}{4}\right) \times LCD: 4 \\ 2x + 1 &= 3 \\ 2x &= 2 \\ x &= 1 \end{aligned}$$

Instead of actually multiplying by the LCD we are going to multiply and simplify at the same time.

Or Add Fractions/Cross Multiply

$$\begin{aligned} \frac{2}{x+2} + 3 &= \frac{11}{x+2} \\ \left(\frac{2}{x+2} + 3 = \frac{11}{x+2}\right) \times LCD = (x+2) \\ \frac{2(x+2)}{x+2} + 3(x+2) &= \frac{11(x+2)}{x+2} \\ 2 + 3(x+2) &= 11 \\ 2 + 3x + 6 &= 11 \\ 3x &= 3 \\ x &= 1 \end{aligned}$$

OR!

$x+2 \neq 0$
 $x \neq -2$

$$\begin{aligned} \left(\frac{2}{x+2} + 3 = \frac{11}{x+2}\right) \times LCD = (x+2) \\ 2 + 3(x+2) &= 11 \\ 2 + 3x + 6 &= 11 \\ 3x &= 3 \\ x &= 1 \end{aligned}$$

$$\begin{aligned} \frac{2}{x+2} = \frac{4}{x-3} \\ \left(\frac{2}{x+2} = \frac{4}{x-3}\right) \times LCD = (x+2)(x-3) \\ 2(x-3) &= 4(x+2) \\ 2x - 6 &= 4x + 8 \\ -14 &= 2x \\ x &= -7 \end{aligned}$$

OR!

$x+2 \neq 0$
 $x \neq -2$

$x-3 \neq 0$
 $x \neq 3$

$$\begin{aligned} \frac{2}{x+2} = \frac{4}{x-3} \\ 2(x-3) &= 4(x+2) \quad \text{Cross Multiply} \\ 2x - 6 &= 4x + 8 \\ -14 &= 2x \\ x &= -7 \end{aligned}$$

$$\begin{aligned} \frac{15}{x^2+5x+6} - \frac{2}{x+2} = \frac{1}{x+2} \\ \left(\frac{15}{(x+2)(x+3)} - \frac{2}{x+2} = \frac{1}{x+2}\right) \times LCD = (x+2)(x+3) \\ 15 - 2(x+3) &= 1(x+3) \\ 15 - 2x - 6 &= x + 3 \\ 9 &= 3x \\ x &= 3 \end{aligned}$$

Factor

$x+2 \neq 0$
 $x \neq -2$

$x+3 \neq 0$
 $x \neq -3$

$$\begin{aligned} \frac{1}{x+1} + 2 &= \frac{3}{x+2} \\ \left(\frac{1}{x+1} + 2 = \frac{3}{x+2}\right) \times LCD = (x+1)(x+2) \\ 1(x+2) + 2(x+1)(x+2) &= 3(x+2) \\ x + 2 + 2x^2 + 6x + 4 &= 3x - 6 \\ 2x^2 + 4x + 12 &= 0 \\ \frac{2x^2}{2} + \frac{4x}{2} + \frac{12}{2} &= \frac{0}{2} \\ x^2 + 2x + 6 &= 0 \end{aligned}$$

Quadratic Formula: No Solution $b^2 - 4ac < 0$

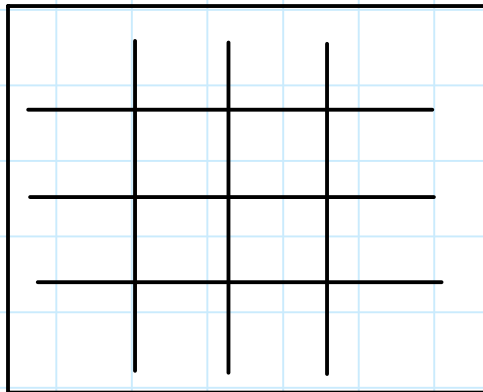
$x+1 \neq 0$
 $x \neq -1$

$x+2 \neq 0$
 $x \neq -2$

C11 - 6.6 - Hoses filling Pool Notes

Two hoses together fill a pool in 2 hours. If only hose A is used, the pool fills in 3 hours. How long would it take to fill the pool if only hose B were used?

	Amount	Time	Rate
Hose A	1 pool	3 hours	$\frac{1 \text{ pool}}{3 \text{ hours}}$
Hose B	1 pool	x hours	$\frac{1 \text{ pool}}{x \text{ hours}}$
Together	1 pool	2 hours	$\frac{1 \text{ pool}}{2 \text{ hours}}$



$$\frac{1}{3} + \frac{1}{x} = \frac{1}{2}$$

$$\left(\frac{1}{3} + \frac{1}{x} = \frac{1}{2}\right) \times 6x$$

$$2x + 6 = 3x$$

$$-2x \quad -2x$$

$$6 = x$$

It will take 6 hours.

Add Rates
Together to
equal the rates
together

$$v = \frac{d}{t} \qquad r = \frac{a}{t}$$

C11 - 6.7 - Sum of Reciprocals Consecutive Integers Notes

The sum of the reciprocals of two consecutive integers is $\frac{5}{6}$. What are the integers?

Let "x" = 1st #
Let x + 1 = 2nd #

$$\frac{1}{x} + \frac{1}{(x+1)} = \frac{5}{6}$$

Restrictions

$$x \neq 0 \quad x \neq -1$$

$$\frac{1}{x} + \frac{1}{(x+1)} = \frac{5}{6}$$
$$\left(\frac{1}{x} + \frac{1}{(x+1)} = \frac{5}{6}\right) \times LCD$$

LCD: $6x(x+1)$

$$6(x+1) + 6x = 5x(x+1)$$

$$6x + 6 + 6x = 5x^2 + 5x$$

$$0 = 5x^2 - 7x - 6$$

$$0 = (5x^2 - 10x) + (3x - 6)$$

$$0 = 5x(x - 2) + 3(x - 2)$$

$$0 = (5x + 3)(x - 2)$$

$$x = 2$$

1st number = 2

2nd number = 3

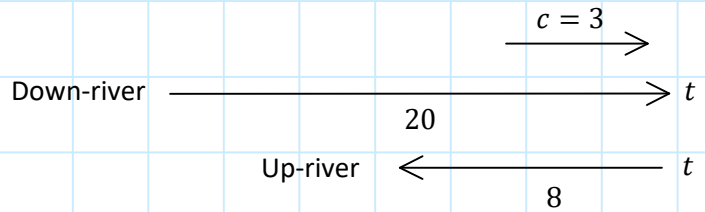
$$\cancel{x = -\frac{3}{5}} \quad x = 2$$

Reject

C11 - 6.8 - Speed Distance Time Notes

Mary paddles down river 20km with a current of 3km/h. It takes her the same time to paddle up river 8km. What is the speed of the boat?

	Speed	Distance	Time
Down-river	$v_b + 3$	20	t
Up-river	$v_b - 3$	8	t



Let $v_b = \text{velocity of boat}$
 $t = \text{time}$

Down river

$$v = \frac{d}{t}$$

$$v_b + 3 = \frac{20}{t}$$

$$v_b = \frac{20}{t} - 3$$

$$v_b = v_b$$

$$\frac{20}{t} - 3 = \frac{8}{t} + 3$$

$$\left(\frac{20}{t} - 3 = \frac{8}{t} + 3\right) \times \text{LCD: } t$$

$$20 - 3t = 8 + 3t$$

$$12 = 6t$$

$$t = 2s$$

Up river

$$v = \frac{d}{t}$$

$$v_b - 3 = \frac{8}{t}$$

$$v_b = \frac{8}{t} + 3$$

$$v_b = \frac{8}{t} + 3$$

$$v_b = \frac{8}{2} + 3$$

$$v_b = 7 \frac{\text{km}}{\text{hr}}$$

$$v = \frac{d}{t}$$

Isolation

Substitution

Solve

Substitution

LCD = t

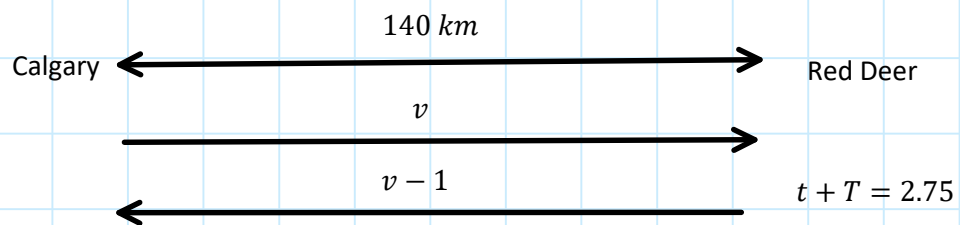
Solve

Mike travels one km per hour faster and completes 4 km 1 minute faster than Sue? How fast are they travelling?

let $v = \text{speed}$

let $t = \text{time } C \rightarrow R$

let $T = \text{time } R \rightarrow C$



$$v = \frac{d}{t}$$

$$v = \frac{140}{t}$$

$$v = \frac{d}{t}$$

$$v - 1 = \frac{140}{T}$$

$$T = 2.75 - t$$

$$\frac{140}{t} - 1 = \frac{140}{2.75 - t}$$

$$v = \frac{140}{1.30764}$$

$$v = 107.06$$

$$t = 1.30764$$

LCD
 Quadform

$$T = 1.44236$$