C11 - 6.0 - Rational Simplify NPV's/Restrictions

Simplify : $\dfrac{2}{4} = \dfrac{2 \div 2}{4 \div 2} = \boxed{\dfrac{1}{2}}$ $\boxed{\dfrac{2}{4} = \dfrac{\cancel{1}\cancel{2}}{\cancel{2} \times 2} = \dfrac{\mathbf{1}}{2}}$

$\dfrac{6x^2}{2x} = \dfrac{6 \times x \times \cancel{x}}{2 \times \cancel{x}} = \boxed{3x}$ $\dfrac{2x+4}{x+2} =$ $\dfrac{x^2+5x+6}{x+3} =$ $\dfrac{x+3}{x^2-9} =$

$\dfrac{2(x+2)}{\cancel{x+2}} = \boxed{2}$ $\dfrac{(x+2)(\cancel{x+3})}{\cancel{x+3}} =$ $\dfrac{\cancel{x+3}}{(\cancel{x+3})(x-3)} = \boxed{\dfrac{1}{x-3}}$

$\boxed{(x+2)}$

$\dfrac{1}{2-x} =$ $\boxed{GCF = -1}$ $\dfrac{2-x}{-(-2+x)}$ $\boxed{OR}$ $\dfrac{2-x}{\boxed{-(x-2)}}$ Rearrange order of terms $\dfrac{x-4}{4-x} =$ $\dfrac{1}{x(4-x)} =$

$\dfrac{1}{-(x-2)} = \boxed{\dfrac{-1}{x-2}} = -\dfrac{1}{x-2}$ $\boxed{-(x-2)}$ $\dfrac{x-4}{-(-4+x)} =$ $\boxed{\dfrac{1}{-x(x-4)}}$

$\dfrac{\cancel{x-4}}{-(x-4)} = \boxed{-1}$

$\dfrac{x^2-3x-4}{x^2-1} =$ $\dfrac{x^2-5x+6}{x+2} =$

$\dfrac{(x-4)(\cancel{x+1})}{(x-1)(\cancel{x+1})} = \boxed{\dfrac{x-4}{x-1}}$ $\boxed{\dfrac{(x-2)(x-3)}{x+2}}$ Cannot Simplify $\boxed{\dfrac{x+2}{x^2+4}}$

$\dfrac{2x^2+3x+1}{x+1} = \dfrac{(x+1)(2x+1)}{x+1} = \boxed{2x+1}$

$\boxed{\textbf{Restrictions :}\ \text{Set Denominator} \neq 0\ and\ solve}$ Non-Permissible Value (NPV's)

$\dfrac{8}{0} = \boxed{und}$ $\dfrac{1}{x}$ $\dfrac{2}{x+3}$ $\dfrac{x}{2}$ $\dfrac{3}{x^2+5x+6}$ $x^2+5x+6 \neq 0$
                                                                                                                  $(x+3)(x+2) \neq 0$

$\boxed{\text{Can't Divide by Zero}}$ $\boxed{x \neq 0}$ $x+3 \neq 0$ $x+3 \neq 0$ $x+2 \neq 0$
                                                     $\boxed{x \neq -3}$ $\boxed{\text{No Restrictions}}$ $\boxed{x \neq -3}$ $\boxed{x \neq -2}$

$\dfrac{3x}{2x^2+x-1}$ $2x^2+x-1 \neq 0$ $\dfrac{5}{x^2-4}$ $x^2-4 \neq 0$

                   $(2x-1)(x+1) \neq 0$ $(x+2)(x-2) \neq 0$

              $2x-1 \neq 0$ $x+1 \neq 0$ $x+2 \neq 0$ $x-2 \neq 0$

              $\boxed{x \neq \dfrac{1}{2}}$ $\boxed{x \neq -1}$ $\boxed{x \neq -2}$ $\boxed{x \neq 2}$

$\dfrac{2}{x^2-2x}$ $x^2-2x \neq 0$ $\dfrac{1}{x^3+1}$ $x^3+1 \neq 0$ $\dfrac{1}{x^2+1}$ $x^2+1 \neq 0$

             $x(x-2) \neq 0$ $x^3 \neq -1$ $x^2 \neq -1$

     $\boxed{x \neq 0}$ $x-2 \neq 0$ $\sqrt[3]{x^3} \neq \sqrt[3]{-1}$ $\sqrt{x^2} \neq \sqrt{-1}$

              $\boxed{x \neq 2}$ $\boxed{x \neq -1}$ $\boxed{\text{No Restrictions}}$

                                Can odd root a negative   Can't even root a negative

$\boxed{\begin{array}{l}\text{If the method}\\\text{you were using}\\\text{to find}\\\text{something does}\\\text{not work it}\\\text{means what}\\\text{you were}\\\text{looking for does}\\\text{not exist!}\end{array}}$

$\dfrac{1}{5s^2 t}$ $s^2 \neq 0$ $\boxed{t \neq 0}$ $\dfrac{1}{x+2y}$ $x+2y \neq 0$ $x \neq -2y$

              $\boxed{s = 0}$ $\boxed{x \neq -2y}$ $\boxed{y \neq -\dfrac{x}{2}}$

C11 - 6.0 - Rational Common Mistakes

Expand and Simplify

Every single question you ever do*!

$2(x+3)$ Distribute
$2x+6$ Multiply

Check Answer
$x = 3^*$

$2(x+3)$	$2x+6$
$2((3)+3)$	$2(3)+6$
$\boxed{12}$	$\boxed{12}$

Pick an Arbitrary $x\ value$.
Substitute into question and answer.
Must be equal!

$$\frac{4+3}{4} = \boxed{\frac{7}{4}} \qquad \frac{\cancel{4}+3}{\cancel{4}} \neq \frac{1+3}{1} = \boxed{4}$$

Can do	Can't do
$\dfrac{(\cancel{a})(\cancel{b})(c)}{(\cancel{a})(\cancel{b})} = c$	$\dfrac{(\cancel{a})+(b)}{(\cancel{a})(b)} \neq \dfrac{1+b}{b}$

Four plus 3 is seven. Divided by 4 is seven fourths or one and three quarters. Not anything else. Assume brackets around the top of the fraction rendering bedmas optional*.

$$\frac{4+3}{4} \neq 1+3 \qquad \boxed{let\ x=4}$$

$\dfrac{\frac{4}{4}+\frac{3}{4}}{\frac{4}{4}}$ $\boxed{\begin{array}{c}\div 4\\ \div 4\end{array}}$ Divide top and botom by 4

$\dfrac{1+\frac{3}{4}}{1}$

$\boxed{1+\frac{3}{4}}$ → $1+\frac{3}{4}$

$\dfrac{4\times 1}{4\times 1}+\dfrac{3}{4}$ Add Fractions

$\dfrac{4}{4}+\dfrac{3}{4}$

$\boxed{\dfrac{4+3}{4}}$

You could divide the top and the bottom by four or x. But then you would have to add fractions and get back to where you started.

You cannot randomly cross things off from the top and the bottom without thinking through fraction factoring and exponent theory.

$$\frac{x+3}{x} \neq 1+3$$

$\dfrac{\frac{x}{x}+\frac{3}{x}}{\frac{x}{x}}$ $\boxed{\begin{array}{c}\div x\\ \div x\end{array}}$ Divide top and botom by x

$\dfrac{1+\frac{3}{x}}{1}$

$\boxed{1+\frac{3}{x}}$ → $1+\frac{3}{x}$

$\dfrac{x\times 1}{x\times 1}+\dfrac{3}{x}$ Add Fractions

$\dfrac{x}{x}+\dfrac{3}{x}$

$\boxed{\dfrac{x+3}{x}}$

$\boxed{\begin{array}{c}\dfrac{x+3}{x}\\[2mm] \dfrac{x}{x}+\dfrac{3}{x} \\[2mm] 1+\dfrac{3}{x}\end{array}}$ Separate Fractions

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Separate

$\dfrac{ax+x}{x}$

$\dfrac{\frac{ax}{x}+\frac{x}{x}}{\frac{x}{x}}$ $\boxed{\begin{array}{c}\div x\\ \div x\end{array}}$

$\dfrac{a+1}{1}$

$\boxed{a+1}$

You can divide the top and bottom by x.

Factor

$\dfrac{ax+x}{x}$

$\dfrac{x(a+1)}{x}$

$\boxed{a+1}$

You can separate fractions with a monomial divisor (denominator) because if you were to reverse the process and add fractions you would be back where you started.

Cancel

$\dfrac{a\cancel{x}+\cancel{x}}{\cancel{x}}$

$\dfrac{a+\overset{1}{\cancel{x}}}{\cancel{x}\,1}$

$\boxed{a+1}$

Visually it is nice to factor out an x. Then cancel from the top and the bottom.

Cancel

$\dfrac{ax^2+x}{x^3}$

$\dfrac{ax^{\cancel{2}}+\overset{1}{\cancel{x}}}{x^{\cancel{3}}2}$ $\boxed{\begin{array}{c}\div x\\ \div x\end{array}}$

$\dfrac{ax+1}{x^2}$

$\boxed{\dfrac{ax+1}{x^2}}$

You can cancel from the top and the bottom using exponent rules and fractions and factor theory.

Factor

$\dfrac{ax^2+x}{x^3}$

$\dfrac{x(ax+1)}{x^3}$

$\dfrac{ax+1}{x^2}$

$\boxed{\dfrac{ax+1}{x^2}}$

Separate

$\dfrac{ax^2+x}{x^3}$

$\dfrac{ax^2}{x^3}+\dfrac{x}{x^3}$

$\dfrac{a}{x}+\dfrac{1}{x^2}$

$\boxed{\dfrac{ax+1}{x^2}}$

Don't simplify too far you may have to re add fractions again.

Can't do
$\dfrac{x+3}{x+1} \neq \dfrac{\overset{1}{\cancel{x}}+3}{\underset{1}{\cancel{x}}+1} \neq \dfrac{3}{1}? = \dfrac{1+3}{1+1} = \dfrac{4}{2}?$
$\boxed{let\ x=4}\quad \dfrac{x+3}{x+1} = \dfrac{4+3}{4+1} = \boxed{\dfrac{7}{5}}$

You cannot cancel a binomial (ie. $x+1$)denominator unless the top is factored and one factor is identical to the denominator. See Simplification.

Rat Page 2

Simplify/Cross-Cancel (Not Cross-Multiply)

$$\frac{3}{8} \times \frac{4}{9} = \frac{3 \times 4}{8 \times 9} = \frac{\cancel{3} \times \cancel{2} \times 2}{2 \times \cancel{2} \times 2 \times \cancel{3} \times 3} = \boxed{\frac{1}{6}} \qquad \frac{3}{8} \times \frac{4}{9} = \frac{\cancel{3}^{1}}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{\cancel{9}_{3}} = \boxed{\frac{1}{6}}$$

Multiply & Divide :

$$\frac{a}{2} \div \frac{1}{3} =$$

$$\frac{1}{2} \times \frac{1}{3} = \boxed{\frac{1}{6}} \quad \text{Multiply Tops} \qquad \frac{a}{2} \times \frac{3}{1} = \boxed{\frac{3a}{2}} \quad \text{Flip and}$$
Multiply Bottoms $\qquad\qquad$ multiply

$$\frac{\dfrac{a}{b}}{\dfrac{c}{d}} = \qquad \frac{a}{\left(\dfrac{b}{c}\right)} = \qquad \frac{\left(\dfrac{a}{b}\right)}{c} =$$

$$\frac{a}{b} \div \frac{c}{d} = \qquad a \div \frac{b}{c} = \qquad \frac{a}{b} \div c =$$

$$\frac{a}{b} \times \frac{d}{c} = \boxed{\frac{ad}{bc}} \qquad a \times \frac{c}{b} = \boxed{\frac{ac}{b}} \qquad \frac{a}{b} \times \frac{1}{c} = \boxed{\frac{a}{bc}}$$

Restrictions

$$\frac{1}{\cancel{x+2}} \times \cancel{(x+2)} = \boxed{1} \qquad x + 2 \neq 0$$
$$\boxed{x \neq -2}$$

$$\frac{1}{(x+2)\cancel{(x+3)}} \times \cancel{(x+3)} = \boxed{\frac{1}{x+2}} \qquad \begin{array}{c} x + 2 \neq 0 \\ \boxed{x \neq -2} \\ x + 3 \neq 0 \\ \boxed{x \neq -3} \end{array}$$

Think what cancels and what are you left with

$$\frac{x+2}{x+3} \times \frac{2}{x+2} =$$
$$\frac{2(x+2)}{(x+3)(x+2)} = \boxed{\frac{2}{x+3}}$$

$$\frac{2}{x+1} \times (x+1)(x+2) =$$
$$\frac{2}{\cancel{x+1}} \times \cancel{(x+1)}(x+2) = \boxed{2(x+2)}$$

$$\frac{x+1}{x^2 - 5x + 6} \times \frac{x-2}{x^2 + 5x + 4} = \boxed{\text{Factor}}$$
$$\frac{x+1}{(x-3)(x-2)} \times \frac{x-2}{(x+4)(x+1)} =$$
$$\frac{(x-2)(x+1)}{(x-3)(x-2)(x+4)(x+1)} =$$
$$\boxed{\frac{1}{(x-3)(x+4)}} \quad \boxed{x \neq 2, -1, 3, -4}$$

$$\frac{x-4}{x+5} \div \frac{x-4}{x-3} =$$
$$\frac{x-4}{x+5} \times \frac{x-3}{x-4} = \boxed{\text{Flip and multiply}}$$
$$\frac{(x-4)(x-3)}{(x+5)(x-4)} =$$
$$\boxed{\frac{x-3}{x+5}} \quad \boxed{x \neq 3, -5, 4}$$

$$\frac{x-7}{x+4} \div \frac{x^2 - 2x - 15}{x^2 - x - 20}$$
$$\frac{x-7}{x+4} \div \frac{(x-5)(x+3)}{(x-5)(x+4)} =$$
$$\frac{x-7}{x+4} \times \frac{(x-5)(x+4)}{(x-5)(x+3)} =$$
$$\frac{(x-7)(x-5)(x+4)}{(x+4)(x-5)(x+3)} =$$
$$\boxed{\frac{x-7}{x+3}} \quad \boxed{x \neq -4, -3, 5}$$

Complex Fractions : Multiply the top and bottom by the LCD

$$\frac{3 + \dfrac{2}{x}}{5 - \dfrac{1}{x}} \quad \boxed{\begin{array}{c} \times LCD \\ \times LCD \end{array}}$$
$$\boxed{LCD = x}$$
$$\boxed{\frac{3x+2}{5x-1}}$$

$$\frac{3 + \dfrac{2}{x+1}}{5 - \dfrac{1}{x}} \quad \boxed{\begin{array}{c} \times LCD \\ \times LCD \end{array}}$$
$$\boxed{LCD = x(x+1)}$$
$$\frac{3x(x+1) + 2x}{5x(x+1) - (x+1)}$$
$$\frac{3x^2 + 5x}{5x^2 + 4x - 1}$$
$$\boxed{\frac{x(3x+5)}{(x+1)(5x-1)}}$$

$$5 - \frac{1}{x} \neq 0$$
$$\boxed{x \neq \frac{1}{5}}$$

$$\frac{1 - \dfrac{2}{x+1}}{1 + \dfrac{1}{x-2}} \quad \boxed{\begin{array}{c} \times LCD \\ \times LCD \end{array}}$$
$$\frac{(x+1)(x-2) - 2(x+2)}{(x+1)(x-2) + 1(x+1)}$$
$$\frac{x^2 - 3x - 6}{x^2 - 1}$$
$$\boxed{\frac{x^2 - 3x - 6}{(x+1)(x-1)}}$$

$$\boxed{LCD = (x+1)(x+2)}$$

$$1 + \frac{1}{x-2} \neq 0$$
$$\frac{1}{x-2} \neq -1$$
$$1 \neq -1(x-2)$$
$$1 \neq -x + 2$$
$$\boxed{x \neq 1}$$

OR

$$\frac{3 + \dfrac{2}{x}}{1 - \dfrac{2}{x}} = \frac{\dfrac{3x+2}{x}}{\dfrac{1x-2}{x}} = \frac{3x+2}{x} \times \frac{x}{1x-2} = \boxed{\frac{3x+2}{1x-2}}$$

Or Add Fractions, Top & Bottom, Flip and Multiply

$$\frac{1}{x} \div y$$
$$\frac{1}{x} \times \frac{1}{y} \quad \boxed{x, y \neq 0}$$
$$\boxed{\frac{1}{xy}}$$

$$\frac{1}{x} \div \frac{2}{y} \times \frac{3}{z}$$
$$\frac{1}{x} \times \frac{y}{2} \times \frac{3}{z} \quad \boxed{x, y, z \neq 0}$$
$$\boxed{\frac{3y}{2xz}}$$

$$\frac{\dfrac{1}{x-2} - 3}{\dfrac{2}{x-2} + 4} \quad \boxed{\begin{array}{c} \times LCD \\ \times LCD \end{array}}$$
$$\frac{1 - 3(x-2)}{2 + 4(x-2)} \quad \boxed{LCD = x - 2}$$
$$\frac{7 - 3x}{4x - 6}$$

C11 - 6.0 - Rational LCD Add/Subtract Notes

$$\frac{\square}{a}+\frac{\square}{b}=\frac{\square}{c}$$
$$LCD = abc$$

Find LCD :

$\frac{1}{2}+\frac{1}{3}=$ $\qquad$ $LCD = 6 = (2\times3)$

$\frac{\square}{a}+\frac{\square}{b}=$ $\qquad$ $LCD = (ab)$

$\frac{\square}{2}+\frac{\square}{6}=$ $\qquad$ $LCD = (6)$ $\qquad$ $\frac{\square}{2}+\frac{\square}{2\times3}=$ $\qquad$ $LCD = (2\times3)$

$\frac{1}{a}+\frac{1}{ab}=$ $\qquad$ $LCD = (ab)$

$\frac{\square}{a}+\frac{\square}{bc}=$ $\qquad$ $LCD = (abc)$

$\frac{1}{a^2}+\frac{1}{a}=$ $\qquad$ $LCD = (a^2)$

$\frac{\square}{x}+\frac{\square}{x^2}+\frac{\square}{x^3}=$ $\qquad$ $LCD = (x^3)$

$\frac{\square}{ab}+\frac{\square}{cd}=$ $\qquad$ $LCD = (abcd)$

$\frac{\square}{2}+\frac{\square}{2+1}=$ $\qquad$ $LCD = (2\times(2+1))$

$\frac{\square}{a}+\frac{\square}{a+1}=$ $\qquad$ $LCD = (a(a+1))$

$\frac{\square}{1+1}+\frac{\square}{2+4}=$ $\qquad$ $LCD = ((1+1)(2+4))$

$\frac{\square}{a+1}+\frac{\square}{a+2}=$ $\qquad$ $LCD = ((a+1)(a+2))$

$\frac{\square}{a}+5=\frac{\square}{a+1}$ $\qquad$ $LCD = (a(a+1))$

$\frac{1}{a}+\frac{1}{a+1}=\frac{1}{a+2}$ $\qquad$ $LCD = (a(a+1)(a+2))$

Adding & Subtracting :

$\frac{1}{2}+\frac{1}{3}=$
$\frac{3\times1}{3\times2}+\frac{1\times2}{3\times2}=$
$\frac{3}{6}+\frac{2}{6}=\left(\frac{5}{6}\right)$
$(LCD = 6)$

LCD
Do to top
Do to bottom
Add/subtract

$\frac{x}{2}+\frac{1}{2}=\left(\frac{x+1}{2}\right)$
$LCD = 2$

$\frac{x}{2}-\frac{1}{6}=$
$\frac{3\times x}{3\times2}-\frac{1}{6}=$
$\frac{3x}{6}-\frac{1}{6}=\left(\frac{3x-1}{6}\right)$

$\frac{3}{2}-\frac{x+2}{2}=$
$\frac{3-(x+2)}{2}=$
$\frac{3-x-2}{2}=\left(\frac{1-x}{2}\right)$

Distribute The negative

$\frac{1}{x-2}+\frac{1}{2-x}$
$\frac{1}{x-2}+\frac{1}{-(x-2)}$
$\frac{1}{x-2}-\frac{1}{(x-2)}$
...
$=0$

Factoring out a negative

$\frac{x}{x+2}+\frac{1}{x+2}=\left(\frac{x+1}{x+2}\right)$

Restrictions
$x+2\neq0$
$(x\neq-2)$

$\frac{1}{x+2}+\frac{1}{(x+2)(x+3)}=$
$\frac{x+3}{x+3}\times\frac{1}{x+2}+\frac{1}{(x+2)(x+3)}=$
$\frac{x+3}{(x+2)(x+3)}+\frac{1}{(x+2)(x+3)}=$
$\frac{x+3+1}{(x+2)(x+3)}=$
$\left(\frac{x+4}{(x+2)(x+3)}\right)$

$\frac{1}{x}+\frac{3}{(x+2)}=$
$\frac{x+2}{x+2}\times\frac{1}{x}+\frac{3}{(x+2)}\times\frac{x}{x}=$ $\boxed{LCD = x(x+2)}$
$\frac{x+2}{x(x+2)}+\frac{3x}{x(x+2)}=$
$\frac{x+2+3x}{x(x+2)}=\left(\frac{4x+2}{x(x+2)}\right)$

$\frac{x+2}{x^2+5x+6}+\frac{1}{x+3}=$
$\frac{x+2}{(x+2)(x+3)}+\frac{1}{x+3}=$
$\frac{1}{x+3}+\frac{1}{x+3}=$ Simplify 1st
$\frac{1+1}{x+3}=\left(\frac{2}{x+3}\right)$

$\frac{x}{x^2-4}-\frac{2}{x^2-4}=$
$\frac{x}{(x-2)(x+2)}-\frac{2}{(x-2)(x+2)}=$
$\frac{x-2}{(x-2)(x+2)}=\left(\frac{1}{x+2}\right)$ Simplify at end

C11 - 6.0 - Rational LCD Equations Notes

Solve for x.

$$\frac{x}{2}+\frac{1}{4}=\frac{3}{4}$$

$$\frac{2\times x}{2\times 2}+\frac{1}{4}=\frac{3}{4}$$ Get an LCD then Multiply by the LCD

$$\frac{2x}{4}+\frac{1}{4}=\frac{3}{4}$$

$$\left(\frac{2x}{4}+\frac{1}{4}=\frac{3}{4}\right)\times LCD$$

$$2x+1=3$$
$$-1\quad -1$$
$$2x=2$$
$$\frac{2x}{2}=\frac{2}{2}$$
$$\boxed{x=1}$$

OR

$$\frac{x}{2}+\frac{1}{4}=\frac{3}{4}$$ Multiply by the $LCD=4$

$$\left(\frac{x}{2}+\frac{1}{4}=\frac{3}{4}\right)\times 4$$

$$\frac{4x}{2}+\frac{4}{4}=\frac{12}{4}$$
$$2x+1=3$$
$$-1\quad -1$$
$$2x=2$$
$$\frac{2x}{2}=\frac{2}{2}$$
$$\boxed{x=1}$$

OR

$$\left(\frac{x}{2}+\frac{1}{4}=\frac{3}{4}\right)\times LCD:4$$
$$2x+1=3$$
$$2x=2$$
$$\boxed{x=1}$$

Instead of actually multiplying by the LCD we are going to multiply and simplify at the same time.

Or Add Fractions/Cross Multiply

$$\left(\frac{2}{x+2}+3=\frac{11}{x+2}\right)\times LCD=(x+2)$$
$$2+3(x+2)=11$$
$$2+3x+6=11 \qquad x+2\neq 0$$
$$3x=3 \qquad \boxed{x\neq -2}$$
$$\boxed{x=1}$$

$$\frac{2}{x+2}=\frac{4}{x-3} \qquad x+2\neq 0 \qquad x-3\neq 0$$
$$\boxed{x\neq -2} \qquad \boxed{x\neq 3}$$
$$\left(\frac{2}{x+2}=\frac{4}{x-3}\right)\times LCD=(x+2)(x-3)$$
$$2(x-3)=4(x+2)$$
$$2x-6=4x+8$$
$$-14=2x$$
$$\boxed{x=-7}$$

OR

$$\frac{2}{x+2}=\frac{4}{x-3}$$
$$2(x-3)=4(x+2) \quad \text{Cross Multiply}$$
$$2x-6=4x+8$$
$$-14=2x$$
$$\boxed{x=-7}$$

$$\frac{15}{x^2+5x+6}-\frac{2}{x+2}=\frac{1}{x+2}$$
$$\left(\frac{15}{(x+2)(x+3)}-\frac{2}{x+2}=\frac{1}{x+2}\right)\times LCD=(x+2)(x+3)$$ Factor
$$15-2(x+3)=1(x+3)$$
$$15-2x-6=x+3$$
$$6=3x \qquad x+2\neq 0 \qquad x+3\neq 0$$
$$\boxed{x=2} \qquad \boxed{x\neq -2} \qquad \boxed{x\neq -3}$$

$$\frac{1}{x+1}+2=\frac{3}{x+2}$$
$$\left(\frac{1}{x+1}+2=\frac{3}{x+2}\right)\times LCD=(x+1)(x+2)$$
$$x+1\neq 0 \qquad x+2\neq 0$$
$$\boxed{x\neq -1} \qquad \boxed{x\neq -2}$$
$$1(x+2)+2(x+1)(x+2)=3(x+1)$$
$$x+2+2x^2+6x+4=3x+3$$
$$2x^2+4x+3=0$$

Quadratic Formula: $\boxed{No\ Solution}$ $b^2-4ac<0$

$$x-\frac{1}{x}=1$$
$$\left(x-\frac{1}{x}=1\right)\times x$$
$$x^2-1=x$$
$$x^2-x-1=0$$
$$\ldots$$
$$\boxed{x=1.618,-0.618}$$

$$\frac{4}{x^2}-\frac{3}{x}=1$$
$$\left(\frac{4}{x^2}-\frac{3}{x}=1\right)\times x^2$$
$$4-3x=x^2$$
$$x^2+3x-4=0$$
$$(x-1)(x+4)=0$$
$$\ldots$$
$$\boxed{x=1,-4}$$

C11 - 6.0 - Rational Together/Resistance WPs Notes

Two hoses together fill a pool in 2 hours. If only hose A is used, the pool fills in 3 hours. How long would it take to fill the pool if only hose B were used?

	Amount	Time	Rate
Hose A	1 pool	3 hours	$\dfrac{1\ pool}{3\ hours}$
Hose B	1 pool	x hours	$\dfrac{1\ pool}{x\ hours}$
Together	1 pool	2 hours	$\dfrac{1\ pool}{2\ hours}$

$let\ x = Hose\ B\ time$

$$v = \frac{d}{t} \qquad r = \frac{a}{t}$$

$$\frac{1}{3} + \frac{1}{x} = \frac{1}{2}$$

$$\boxed{LCD}$$

$$\left(\frac{1}{3} + \frac{1}{x} = \frac{1}{2}\right) \times 6x$$

$$2x + 6 = 3x$$

$$-2x \qquad\qquad - 2x$$

$$\boxed{6 = x}$$

It will take Hose b 6 hours.

$$\boxed{\text{Add Rates Together to equal the rates together}}$$

$$\frac{2}{1+2} + 3 = 4 = \frac{11}{1+2}$$

x (tic-tac-toe grid)
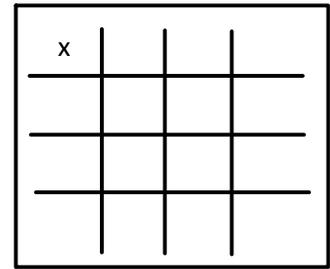
It takes Jerry 9 hours longer than George to clean a pool and together it takes 20 hours. How long does it take George by himself.

	Amount	Time	Rate
Jerry	1 pool	$t+9$	$\dfrac{1}{t+9}$
George	1 pool	t	$\dfrac{1}{t}$
Together	1 pool	20	$\dfrac{1}{20}$

$let\ t = time\ (hours)$

$$\frac{1}{t+9} + \frac{1}{t} = \frac{1}{20}$$

$$\left(\frac{1}{t+9} + \frac{1}{t} = \frac{1}{20}\right) \times 20t(t+9)$$

$$20t + 20(t+9) = t(t+9)$$

$$\ldots$$

$$t^2 - 31t - 180 = 0$$

$$(t-36)(t+5) = 0$$

$$\boxed{t = 36} \quad \cancel{t = -5}$$

It would take George 36 hours to clean the pool.

$$\boxed{r_t = 1.2 \qquad r_2 = r_1 + 1}$$

$$\frac{1}{r_t} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$\frac{1}{1.2} = \frac{1}{r_1} + \frac{1}{r_1+1}$$

$$\left(\frac{5}{6} = \frac{1}{r_1} + \frac{1}{r_1+1}\right) \times 6r_1(r_1+1)$$

$$5r_1(r_1+1) = 6(r_1+1) + 6r_1$$

$$5r_1^2 + 5r_1 = 6r_1 + 6 + 6r_1$$

$$5r_1^2 - 7r_1 - 6 = 0$$

$$\ldots$$

$$(5_r + 3)(r_1 - 2) = 0$$

$$5r_1 + 3 = 0 \qquad r_1 - 2 = 0$$

$$\cancel{r_1 = -\frac{3}{5}} \qquad \boxed{r_1 = 2}$$

Can't have a negative resistance

C11 - 6.0 - Rational Area/Reciprocals WPs Notes

$A = x^2 - 9$ $l = x + 3$

$w = \dfrac{x^2 - 2x - 3}{x + 1}$

$w = \dfrac{x^2 - 2x - 3}{x + 1}$

$w = \dfrac{(x - 3)(x + 1)}{x + 1}$

$w = x - 3$

$A = lw$
$x^2 - 9 = l(x - 3)$
$l = \dfrac{x^2 - 9}{x - 3}$
$l = \dfrac{(x + 3)(x - 3)}{x - 3}$
$l = x + 3$

$A = lw$
$x^2 - 9 = l\left(\dfrac{x^2 - 2x - 3}{x + 1}\right)$

$l = \dfrac{x^2 - 9}{\dfrac{x^2 - 2x - 3}{x + 1}}$

$l = (x^2 - 9) \div \dfrac{x^2 - 2x - 3}{x + 1}$

$l = (x^2 - 9) \times \dfrac{x + 1}{x^2 - 2x - 3}$

$l = (x + 3)(x - 3) \times \dfrac{x + 1}{(x + 1)(x - 3)}$

$l = x + 3$

Find two consecutive integers where the sum of their reciprocals is $\frac{5}{6}$.

$Let\ "x" = 1st\ \#$
$Let\ x + 1 = 2nd\ \#$

$1st\ \# = 2$
$2nd\ \# = 3$

$\dfrac{1}{x} + \dfrac{1}{(x + 1)} = \dfrac{5}{6}$

$\left(\dfrac{1}{x} + \dfrac{1}{(x + 1)} = \dfrac{5}{6}\right) \times 6x(x + 1)$

$6(x + 1) + 6x = 5x(x + 1)$
$6x + 6 + 6x = 5x^2 + 5x$
$5x^2 - 7x - 6 = 0$
$(5x^2 - 10x) + (3x - 6) = 0$
$5x(x - 2) + 3(x - 2) = 0$
$(5x + 3)(x - 2) = 0$

$\dfrac{1}{2} + \dfrac{1}{3} = \dfrac{6}{6}$ $x = -\dfrac{3}{5}$ $x = 2$ $x \neq 0$ $x \neq -1$

Reject

Two numbers sum to 12 and the sum of their reciprocals is $\frac{3}{8}$.

$let\ a = 1st\ \#$
$let\ b = 1st\ \#$

$a + b = 12$
$a = 12 - b$

$a = 12 - 4$
$a = 8$

$a = 12 - 8$
$a = 4$

$\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{3}{8}$

$\left(\dfrac{1}{12 - b} + \dfrac{1}{b} = \dfrac{3}{8}\right) \times 8b(12 - b)$

$8b + 8(12 - b) = 3b(12 - b)$
...
$3b^2 - 36b + 96 = 0$
$b^2 - 12b + 32 = 0$
$(b - 4)(b - 8) = 0$
...
$b = 4, 8$

$1st\ \# = 4$
$2nd\ \# = 8$

$4 + 8 = 12$ $\dfrac{1}{4} + \dfrac{1}{8} = \dfrac{3}{8}$

The difference of a number and twice its reciprocal is -1. **#'s = 1, -2**

$let\ x = \#$

$x - 2 \times \dfrac{1}{x} = -1$

$\left(x - \dfrac{2}{x} = -1\right) \times x$

$x^2 - 2 = -x$
$x^2 + x - 2 = 0$
$(x + 2)(x - 1) = 0$

$x = -2$ $x = 1$

$-2 - 2 \times \dfrac{1}{-2} = -1$
$2 + 1 = -1$

$1 - 2 \times \dfrac{1}{1} = -1$
$1 - 2 = -1$

Tailwind | & | Headwind | OR | No Wind
w d | | w d | | d
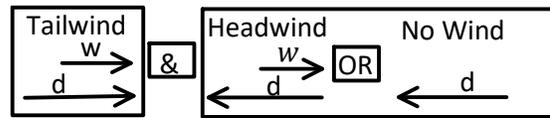
Mary paddles down river 20km with a current of 3km/h. It takes her the same time to paddle up river 8km. What is the speed of the boat?

$Let\ v_b =$ velocity of boat in still water
$t = time$

| Down-river | | Up-river |

$c = 3$ $c = 3$

20   t 8 t
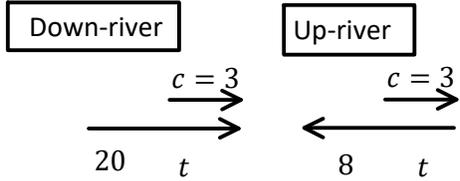
	Speed	Distance	Time
Down	$v_b + 3$	20	t
Up	$v_b - 3$	8	t

$$v = \frac{d}{t}$$
$$v_b + 3 = \frac{20}{t}$$
$$v_b = \left(\frac{20}{t} - 3\right) \rightarrow$$

$$v = \frac{d}{t}$$
$$(v_b) - 3 = \frac{8}{t}$$
$$\left(\frac{20}{t} - 3\right) - 3 = \frac{8}{t}$$
$$\left(\frac{20}{t} - 6 = \frac{8}{t}\right) \times t$$

$$v_b = \frac{20}{t} - 3$$
$$v_b = \frac{20}{(2)} - 3$$
$$v_b = 7\frac{km}{hr}$$

$$20 - 6t = 8$$
$$\frac{-6t}{-6} = \frac{-12}{-6}$$
$$t = 2hr$$

Mike travels 10 km/hr faster and completes 120 km two hours faster than Sue. How fast are they travelling?

$let\ v = Sue's\ speed$
$let\ t = Sue's\ time$

120 km

v t $v + 10$ $t - 2$
Sue   Mike

$$v = \frac{d}{t}$$

$$v = \frac{d}{t}$$
$$v = \left(\frac{120}{t}\right)$$
$$v = \frac{120}{6}$$
$$v = 20\frac{km}{hr}$$

$$v = \frac{d}{t}$$
$$(v) + 10 = \frac{120}{t - 2}$$
$$\left(\frac{120}{t}\right) + 10 = \frac{120}{t - 2}$$
$$120(t - 2) + 10t(t - 2) = 120t$$
$$\dots$$
$$10t^2 - 20t - 240 = 0$$
$$t^2 - 2t - 24 = 0$$
$$\dots$$
$$t = 6 \quad t = 4$$

$$20 + 10 = \frac{120}{6 - 2}$$

Dirk travels there and back 140 km each way. If he travels ten km per hour slower there, what is the average speed there if he travelled for 2 hours and 45 minutes?

$let\ v\left(\frac{km}{hr}\right) = speed$
$let\ T\ (hours) = time\ there$
$let\ t(hours) = time\ back$

There   140 km $v - 10$ T
Back   140 km v t

$$\frac{45}{60} = 0.75$$

$$t + T = 2.75$$
$$T = 2.75 - t$$
$$T = 2.75 - 1.3$$
$$T = 1.45\ h$$

$$\dots$$
$$\dots$$
$$t = 1.3 \quad t = 29.4$$

$$v = \frac{d}{t}$$
$$v - 10 = \frac{140}{T}$$
$$\left(\frac{140}{t} - 10 = \frac{140}{2.75 - t}\right) \times t(2.75 - t)$$
$$140(2.75 - t) - 10t(2.75 - t) = 140t$$
$$10t^2 - 307.5t + 385 = 0$$

$$v = \frac{d}{t}$$
$$v = \frac{140}{t}$$
$$v = \frac{140}{1.3}$$
$$v = 107.7\frac{km}{h}$$