C11 - 7.0 - Absolutes Exp/Eq + Case - Case Notes

$|x| = -6$ Impossible.

Simplify : $|2| = 2$ $|2| = 2$ $|-3| = 3$ $|2-4| = 2$ $|-2| = 2$ $|3| - |-5| = 3 - 5 = -2$ $-|3| = -3$ $-|-5| = -(5) = -5$

Solve Algebraically. Do whatever is inside the absolute value, then make it positive.

$|x| = 4$

"+" case: $+(x) = 4$, $x = 4$ — Distribute a positive into the absolute value

"−" case: $-(x) = 4$, $x = -4$ — Distribute a negative into the absolute value

$\|x\| = 4$	$\|x\| = 4$	Check Your answer. LHS = RHS
$\|4\| = 4$	$\|-4\| = 4$	
$4 = 4$	$4 = 4$	

$2|x-2| = 6$

"+" case: $+2(x-2) = 6$, $2x - 4 = 6$, $2x = 10$, $x = 5$

"−" case: $-2(x-2) = 6$, $-2x + 4 = 6$, $-2x = 2$, $x = -1$

$\frac{2(x-2)}{2} = \frac{6}{2}$, $x - 2 = 6$, …

If already negative combine

$|x^2 - 1| = x - 1$

"+" case: $+(x^2 - 1) = x - 1$, $x^2 - x = 0$, $x(x-1) = 0$, $x = 0$, $x - 1 = 0$, $x = 1$

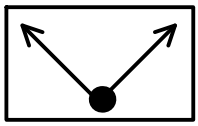
"−" case: $-(x^2 - 1) = x - 1$, $-x^2 + 1 = x - 1$, $x^2 + x - 2 = 0$, $(x+2)(x-1) = 0$, $x - 1 = 0$, $x = 1$, $x + 2 = 0$, $x = -2$

$|x^2 - 1| = x - 1$, $|0^2 - 1| = 0 - 1$, $|-1| = -1$

$|x^2 - 1| = x - 1$, $|1^2 - 1| = 1 - 1$, $|0| = 0$

$|x^2 - 1| = x - 1$, $|(-2)^2 - 1| = -2 - 1$, $|4 - 1| = -2 - 1$, $|3| = -3$

C11 - 7.0 - Absolutes Linear Piecewise Notes



Graphing Absolute Values :

$y = |x + 2|$ "+" case: "−" case:

$y_1 = +(x + 2)$ $y_2 = -(x + 2)$

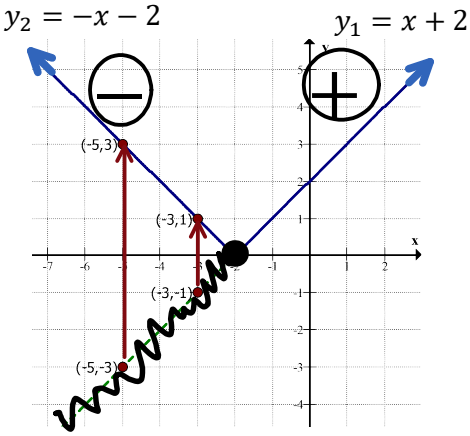
$y_1 = x + 2$ $y_2 = -x - 2$

$y = x + 2 \longrightarrow y = |x + 2|$

x	y
-5	-3
-3	-1
-2	0
-1	1
0	2

x	y
-5	+3
-3	+1
-2	+0
-1	+1
0	+2

$D: x \in \mathbb{R}$ $R: y \geq 0$



Vertex: $(-2,0)$

Piecewise function: $y = \begin{cases} x + 2, if & x \geq -2 \\ -x - 2, if & x < -2 \end{cases}$

Notice: The domain of the negative case is not equal to.

$y = \begin{cases} \text{"+" case,} & \text{Domain of "+" case} \\ \text{"−" case,} & \text{Domain of "−" case} \end{cases}$

Domain of positive case: $x + 2 \geq 0$, $-2 \quad -2$, $x \geq -2$ Set what is inside the absolute value greater than or equal to zero.

Domain of negative case: $x + 2 < 0$, $-2 \quad -2$, $x < -2$ Set what is inside the absolute value less than zero.

Solve algebraically :

$|x + 2| = 4$ "+" case: "−" case:

$+(x + 2) = 4$ $-(x + 2) = 4$

$x + 2 = 4$ $-x - 2 = 4$

$x = 2$ $-x = 6$

$x = -6$

Check your answer.

$|x + 2| = 4$ $|-6 + 2| = 4$

$|2 + 2| = 4$ $|-4| = 4$

$|4| = 4$ $|-4| = 4$

Solve graphically :

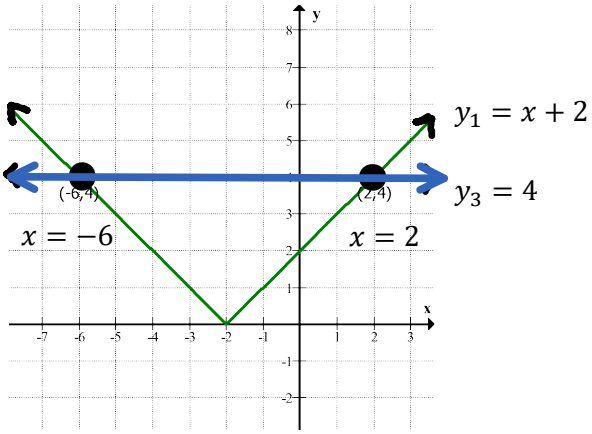
$|x + 2| = 4$ $y_1 = |x + 2|$ y=Left hand side (LHS) $y_3 = 4$ y=Right hand side (RHS)

"+" case: "−" case:

$y_1 = +(x + 2)$ $y_2 = -(x + 2)$

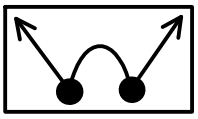
$y_1 = x + 2$ $y_2 = -x - 2$

$y_2 = -x - 2$



$x = 2 \qquad x = -6$

C11 - 7.0 - Absolutes Quadratic Notes



Graphing Absolute Values :

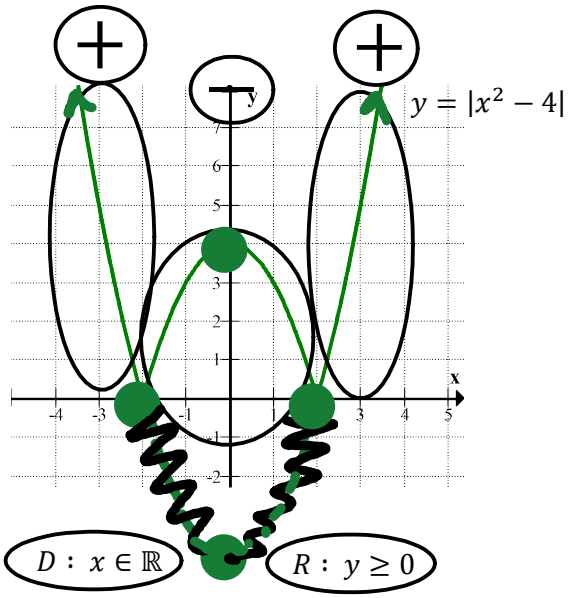
$y = |x^2 - 4|$ "+" case: "−" case:

$y_1 = +(x^2 - 4)$ $y_2 = -(x^2 - 4)$

$y_1 = x^2 - 4$ $y_2 = -x^2 + 4$

$y_1 = x^2 - 4$ $y_2 = -x^2 + 4$

$$y = \begin{cases} x^2 - 4, if & x \le -2, x \ge 2 \\ -x^2 + 4, if & -2 < x < 2 \end{cases}$$



Solve algebraically :

$|x^2 - 4| = x + 2$ "+" case: "−" case:

$+(x^2 - 4) = x + 2$ $-(x^2 - 4) = x + 2$

$x^2 - 4 = x + 2$ $-x^2 + 4 = x + 2$

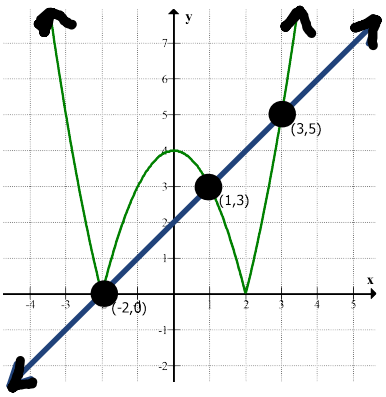
$x^2 - x - 6 = 0$ $0 = x^2 + x - 2$

$(x - 3)(x + 2) = 0$ $0 = (x + 2)(x - 1)$

$x = 3, -2$ $x = -2, 1$

Solve Graphically :

$y = |x^2 - 4|$



$y = x + 2$

$x = 3, -2, 1$

C11 - 7.0 - Reciprocals Notes

Pick a y value, What's one divided by that y value.
Put a point on the graph. X value is same as it was.

Restrictions : See Rationals

$\frac{1}{x-2}$ $x-2=0$ $x=2$ *Set denominator = 0, and solve.*

$\frac{1}{(x+2)(2x-1)}$ Factor $x+2=0$ $x=-2$ $2x-1=0$ $x=\frac{1}{2}$

$y=x+4$ Line : $y=mx+b$

$y=\frac{1}{x+4}$ Reciprocal line

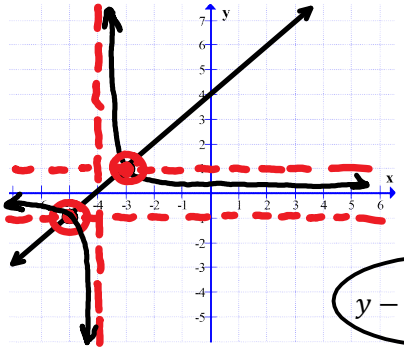
Solve algebraically: set denominator = 0, 1, -1.

Vertical asymptote (VA):
Denominator = 0
$x+4=0$
$x=-4$
$D: x\neq -4$

Invariant points (IP):
Denominator = ± 1
$x+4=1$ $x=-3$ $(-3,1)$
$x+4=-1$ $x=-5$ $(-5,-1)$

Graph :

$y=\frac{1}{x+4}$ $y=x+4$

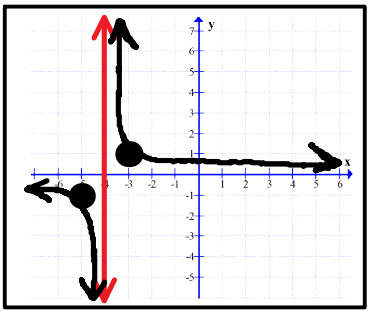


1. Graph original
2. Graph VA: Dotted line $(x-int)$
3. Graph IP's $(x,\pm 1)$
4. Graph Reciprocal
5. y-int

$HA: y=0$
$R: y\neq 0$
$y-int: (0,\frac{1}{4})$

Close to the vertical asymptote, through the point, close the x-axis/vertical asymptote*

$y=\frac{1}{x+4}$



$VA: x=4$
$D: x\neq -4$

Notice:
The invariant points are the intersection of the original and the lines $y=1, y=-1$.
The vertical asymptote(s) of the reciprocal is the x –intercept of the original

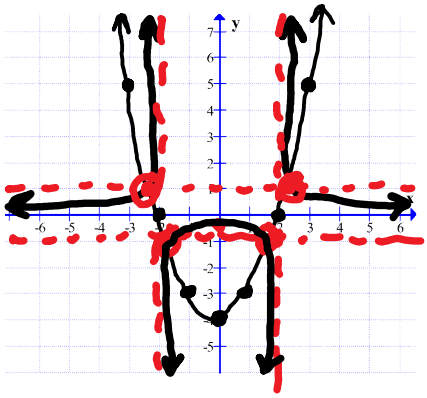
$y=x^2-4$ Parabola

$y=\frac{1}{x^2-4}$ Reciprocal Parabola

Vertical asymptote (VA):
Denominator = 0
$x^2-4=0$
$(x+2)(x-2)=0$
$x=2,-2$

Invariant points (IP):
Denominator = ± 1
$x^2-4=1$
$x^2=5$
$x=\sqrt{5},-\sqrt{5}$
$(\sqrt{5},1)$ $(-\sqrt{5},1)$

$x^2-4=-1$
$x^2=3$
$x=\sqrt{3},-\sqrt{3}$
$(\sqrt{3},-1)$ $(-\sqrt{3},-1)$



1. Graph original
2. Graph VA's: Dotted lines
3. Graph IP's
4. Graph reciprocal
5. y-int

$D: x\neq \pm 2$ $y\neq 0$ $y-int: (0,-\frac{1}{4})$