C11 - 9.1 - Linear Inequalities In Two Variables Notes

Graph the following Inequality

$y > x - 2$ Graph: $y = x - 2$

$y = mx + b$

$<, >$ (Open Dots, Dotted line)

Test Point

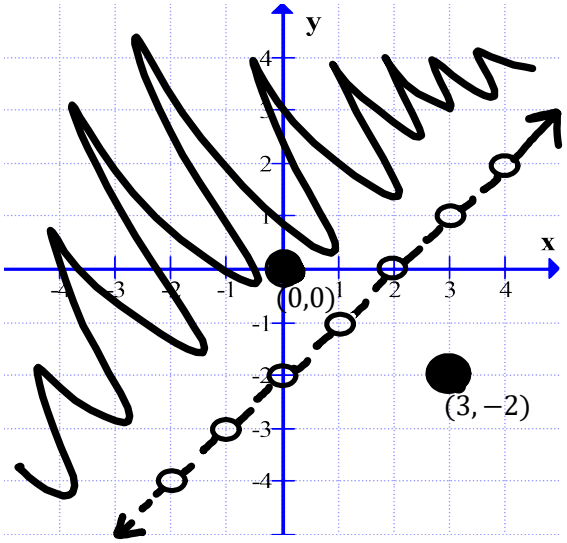
(x, y)
$(0,0)$

Choose a Point on either side of the Line

Zero-Zero Test*

$y > x - 2$
$0 > 0 - 2$
$0 > -2$ ✓ Substitute for x and y.

Correct: Shade the $(0,0)$ side of the line.



$y > x - 2$

Find Equation

Test Point		Equation	
y	$x - 2$	$y = mx + b$	(x, y)
0	$0 - 2$	"Space"	$(0,0)$
0	-2		
$0 > -2$ ✓		Make a correct Statement	
$y > x - 2$		y $x - 2$	

Test Point (x, y) $(3, -2)$

OR

$y > x - 2$
$-2 > 3 - 2$
$-1 > 1$ ✗

Incorrect: Shade the Not $(3, -2)$ side of the line.

Isolate for y or TOV $y = mx + b$

$x - y \geq 2$
$-y \geq -x + 2$
$y \leq x - 2$

OR

$x - y \geq 2$
$x - 2 \geq y$
$y \leq x - 2$

Add y
Subtract 2
Mirror

(Both Sides)

Subtract x
Divide* by -1
Change Sign!

Graph the following Inequality

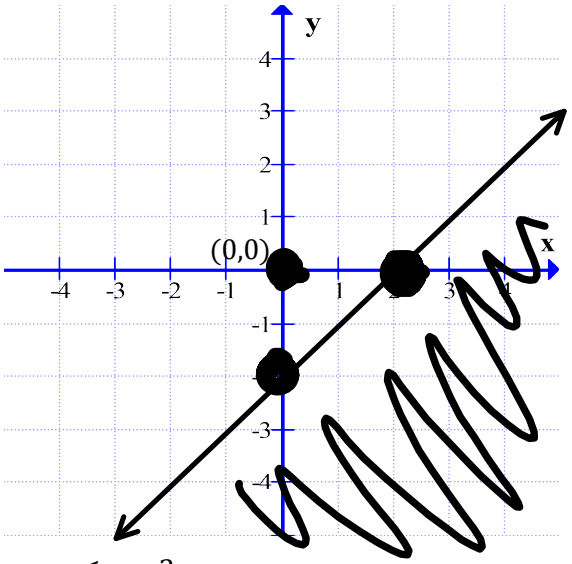
$y \leq x - 2$ Graph $y = x - 2$

$\leq, \geq$ (Closed Dots, Solid Line)

Test Point $(0,0)$

$y \leq x - 2$
$0 \leq 0 - 2$
$0 \leq -2$ ✗

Incorrect: Shade "Not" the $(0,0)$ side of the line.



$y \leq x - 2$

Find Equation

Test Point		Equation	
y	$x - 2$	$y = mx + b$	(x, y)
0	$0 - 2$	"Space"	$(0,0)$
0	-2		
$0 \leq -2$ ✗		Make a Incorrect Statement	
$y \leq x - 2$		y $x - 2$	

Notice: the (0,0) test only works if (0,0) is not on the line. If (0,0) is on the line we must choose a distinct point that is not on the line like (5,0) or (0,2).

OR "Shade" above/below than "the line"

Replace the word y *with "shade"*
Greater than = above/Less than = below
Replace the equation with "the line"

C11 - 9.2 - Linear/Quadratic Inequalities In One Variable Notes

Solve

$x - 2 \leq 0$

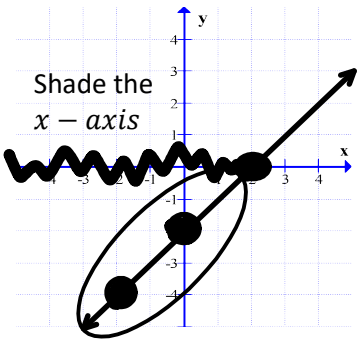
$x - 2 \leq 0$

$+2 \quad +2$ Solve

$x \leq 2$

Graphing

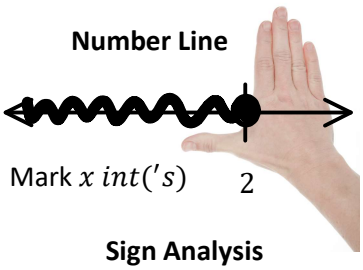
$y\ values \leq 0$
$The\ Thing \leq 0$



$y = x - 2$

What are the x values when $y \leq 0$. Circle them!

Number Line



Sign Analysis

Pick a value

$x \leq 2$ $\quad$ $x \geq 2$

$x = 0$ Substitute $x = 4$

$x - 2 \leq 0$
$0 - 2 \leq 0$
$-2 \leq 0$ ✓

$x - 2 \leq 0$
$4 - 2 \leq 0$
$2 \leq 0$ ✗

Correct: Shade that section

Incorrect: Shade Not that section

$x \leq 2$

$-x^2 + 5x - 4 < 0$

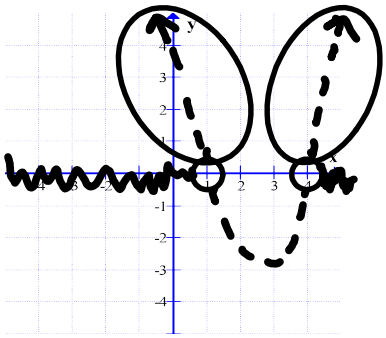
$-x^2 + 5x - 4 < 0$
$-(x^2 - 5x + 4) < 0$
$\frac{-(x^2 - 5x + 4)}{-1} > \frac{0}{-1}$ $\div -1^*$
$x^2 - 5x + 4 > 0$
$(x - 4)(x - 1) > 0$ Factor

$x - intercept's$

$x - 4 = 0$ $\quad$ $x - 1 = 0$
$x = 4$ $\quad$ $x = 1$

Graphing

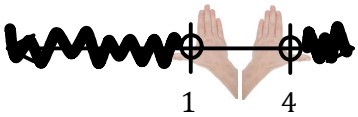
$y\ values > 0$
$The\ Thing > 0$



$y = (x - 4)(x - 1)$

What are the x values when $y > 0$. Circle them!

Number Line



Sign Analysis

Pick a value

$x < 1$ $\quad$ $1 < x < 4$ $\quad$ $x > 4$

$x = 0$ $\quad$ $x = 2$ $\quad$ $x = 5$

Substitute

$(x - 4)(x - 1) > 0$
$(0 - 4)(0 - 1) > 0$
$(-4)(-1) > 0$
$4 > 0$ ✓

$(1)(4) > 0$
$4 > 0$ ✓

$(-2)(1) > 0$
$-2 > 0$ ✗

$x < 1 \quad x > 4$

$x^2 - 4 \leq 0$

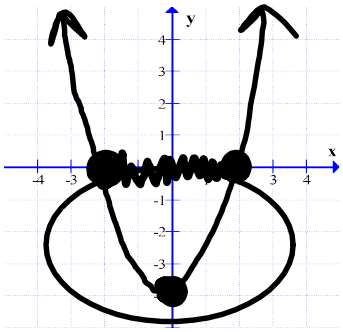
$x^2 - 4 \leq 0$
$(x + 2)(x - 2) \leq 0$

$x + 2 = 0$ $\quad$ $x - 2 = 0$
$x = -2$ $\quad$ $x = 2$

$x - intercept's$

Graphing

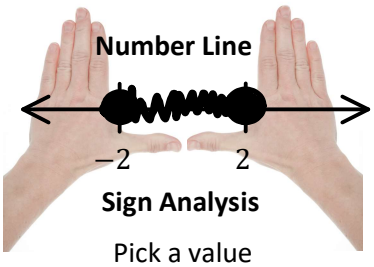
$y\ values \leq 0$
$The\ Thing \leq 0$



$y = x^2 - 4$

What are the x values when $y \geq 0$. Circle them!

Number Line



Sign Analysis

Pick a value

$x \leq -2$ $\quad$ $-2 \leq x \leq 2$ $\quad$ $x \geq 2$

$x = -3$ $\quad$ $x = 0$ $\quad$ $x = 3$

$x^2 - 4 \leq 0$
$(-3)^2 - 4 \leq 0$
$5 \leq 0$ ✗

$x^2 - 4 \leq 0$
$(0)^2 - 4 \leq 0$
$5 \leq 0$ ✓

$x^2 - 4 \leq 0$
$(3)^2 - 4 \leq 0$
$5 \leq 0$ ✗

$-2 \leq x \leq 2$

The answer is only the Domain. The number line and graph is only to help. There is no y involved.

C11 - 9.3 - Quadratic Inequalities in Two Variables Notes

Graph the following inequalities

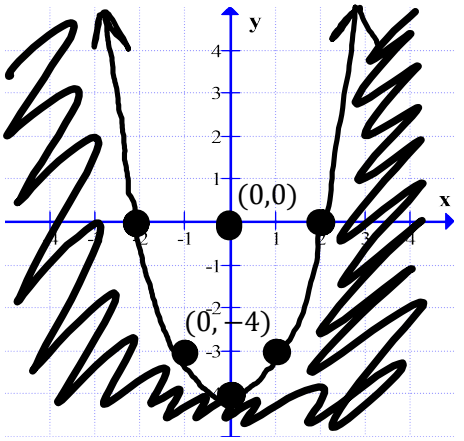
$y \le x^2 - 4$

Graph: $y = x^2 - 4$

TOV

x	y
-2	0
-1	-3
0	-4
1	-3
2	0

(Closed dots, Solid Line) $y = x^2 - 4$



Test Point $(0,0)$

$y \le x^2 - 4$
$0 \le (0)^2 - 4$
$0 \le -4$ ✗

Substitute for x and y.

Incorrect: Shade the "NOT" (0,0) side of the line.

Find Equation

$y = a(x-p)^2 + q$
$y = a(x-0)^2 - 4$
$-3 = a(1-0)^2 - 4$
$-3 = 1a - 4$
$1 = a$
$y = 1(x-0)^2 - 4$
$y = x^2 - 4$

Vertex Form

(x, y)
$(0, -4)$ Vertex

(x, y)
$(1, -3)$ Point

Test Point

y	$x^2 - 4$
0	$0^2 - 4$
$0 \le -4$	✗

$y \le x^2 - 4$

"Space" (x, y) $(0,0)$

Make a Incorrect Statement

$y > x^2 - 2x - 3$

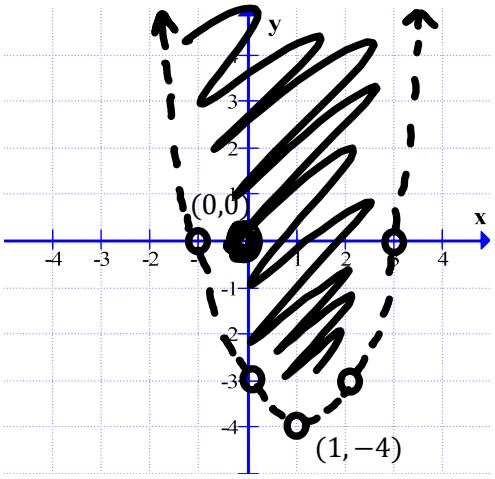
(Open dots, Dotted line) $y = x^2 - 2x - 3$

Graph: $y = x^2 - 2x - 3$

$y = x^2 - 2x - 3$ Complete the square $\left(\frac{b}{2}\right)^2$
$y = (x^2 - 2x) - 3$
$y = (x^2 - 2x + 1 - 1) - 3$
$y = (x-1)^2 - 4$ $(1, -4)$ Vertex

$y = x^2 - 2x - 3$
$y = (x+1)(x-3)$

$x = -1 \quad x = 3$ $x - intercepts$



Test Point $(0,0)$

$y > x^2 - 4$
$0 > 0 - 4$
$0 > -4$ ✓

Substitute for x and y.

Correct: Shade the $(0,0)$ side of the line.

Find Equation

$y = a(x-p)^2 + q$

$y = a(x-1)^2 - 4$
$-3 = a(2-1)^2 - 4$
$-3 = 1a - 4$
$1 = a$
$y = 1(x-1)^2 - 4$

$y = (x-1)^2 - 4$

Vertex Form

(x, y)
$(1, -4)$ Vertex

(x, y)
$(2, -3)$ Point

Test Point

y	$(x-1)^2 - 4$
0	$(0-1)^2 - 4$
$0 \le -3$	✓

$y \le (x-1)^2 - 4$

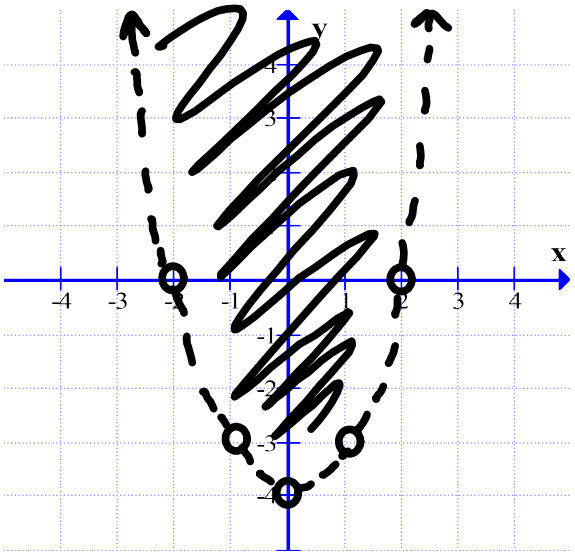
"Space" (x, y) $(0,0)$

Make a Correct Statement

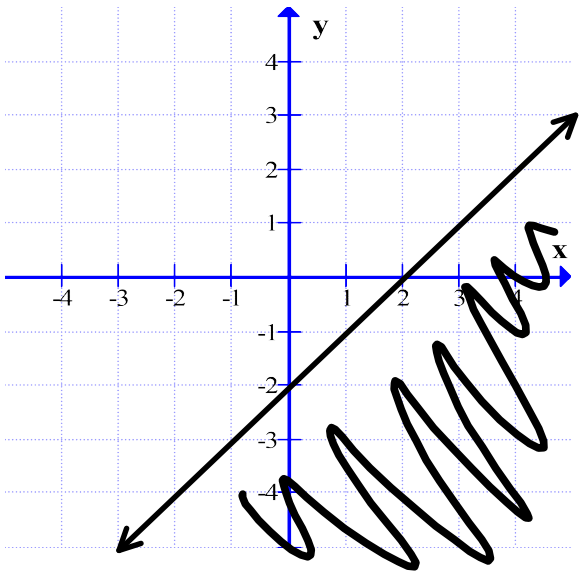
C11 - 9.3 - Inequalities Systems Notes

Solve the following system by graphing:

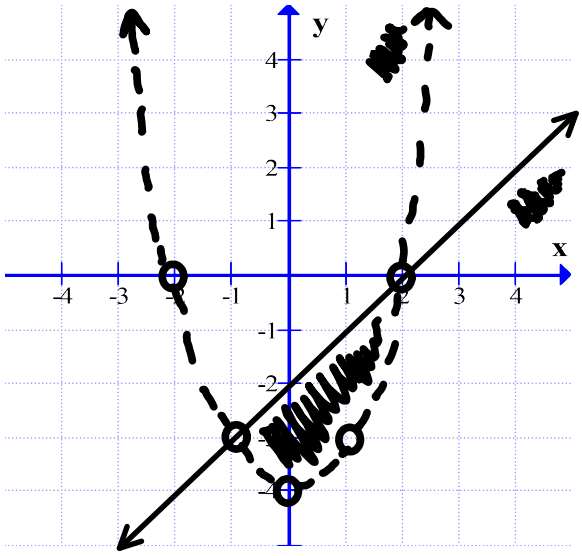
$y > x^2 - 4$



$y \leq x - 2$



$y > x^2 - 4$
$y \leq x - 2$



Notice: we have graphed each equation and shaded only the region which satisfies both inequalities.

C11 - 9.4 - Burgers and Fries Notes

$let\ b = \#\ burgers$
$let\ f = \#\ fries$

$burgers = \$3$
$fries = \$2$

$\$12\ to\ spend$

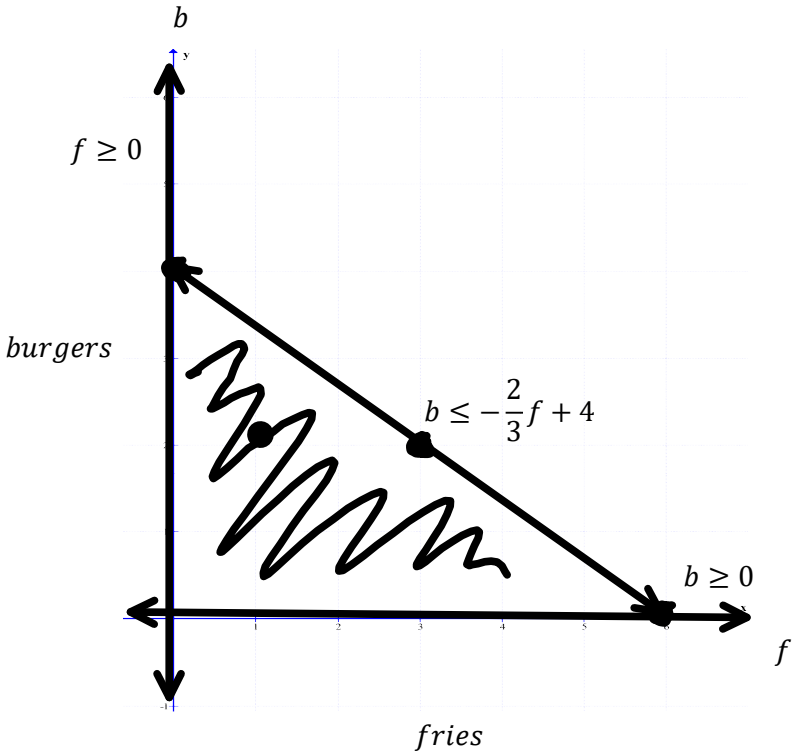
$3b + 2f \leq 12$

$1\ burger = 3 \times 1 = 3$
$3\ burger = 3 \times 2 = 6$
$b\ burger = 3 \times b = 3b$

$3b + 2f \leq 12$
$3b \leq -2f + 12$
$b \leq -\frac{2}{3}f + 4$

$y = mx + b$

f	b
0	4
6	0



(f, b)
$(0,4) = \$12$
$(0,3) = \$9$
$(0,2) = \$6$
$(0,1) = \$3$
$(0,0) = \$0$
$(1,3) = \$11$
$(1,2) = \$8$
$(1,1) = \$5$
$(1,0) = \$2$
$(2,2) = 10$
$(2,1) = \$7$
$(2,0) = \$4$
$(3,2) = \$12$
$(3,1) = \$9$
$(3,0) = \$6$
$(4,1) = \$11$
$(4,0) = \$8$
$(5,0) = \$10$
$(6,0) = \$12$

Test Point: (1,1)

$b \geq 0$
$1 \geq 0$ ✓

$f \geq 0$
$1 \geq 0$ ✓

$b \leq -\frac{2}{3}f + 4$
$1 \leq -\frac{2}{3}(1) + 4$
$1 \leq \frac{10}{3}$ ✓

Restrictions

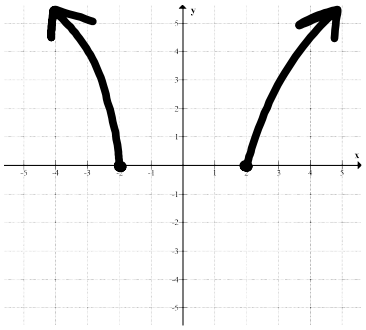
$0 \leq b \leq 4\ \ b \in W$

$0 \leq f \leq 6\ f \in W$

$W: Whole\ Numbers$

C11 - 9.5 - Inequalities Quadratic Restrictions Notes

$y = \sqrt{x^2 - 4}$



$x^2 - 4 \geq 0$

$x^2 \geq 4$

$\sqrt{x^2} \geq \sqrt{4}$

$|x| \geq 2$

$\pm x \geq 2$

$x \geq 2 \qquad -x \geq 2$

$x \leq -2$

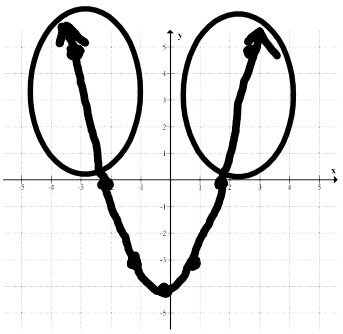
$x \geq +2 \qquad x \leq -2$

x	y
-3	$\sqrt{5}$
-2	0
2	0
3	$\sqrt{5}$

Range

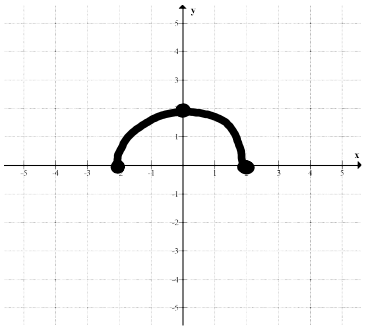
$y \geq 0$

$y = x^2 - 4$



$x \leq -2 \qquad x \geq +2$

$y = \sqrt{4 - x^2}$



$4 - x^2 \geq 0$

$x^2 \leq 4$

$x \leq 2 \qquad x \geq -2$

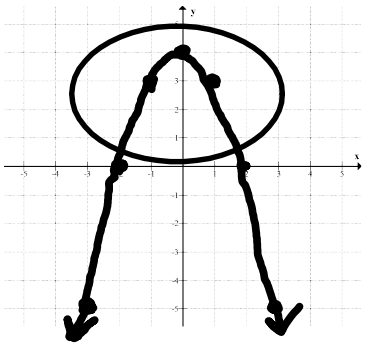
$-2 \leq x \leq 2$

x	y
-2	0
0	2
2	0

Range

$0 \leq y \leq 2$

$y = 4 - x^2$



$-2 \leq x \leq 2$