C11 - 9.0 - Inequalities 1 Variable Notes

Solve

$x - 2 \leq 0$

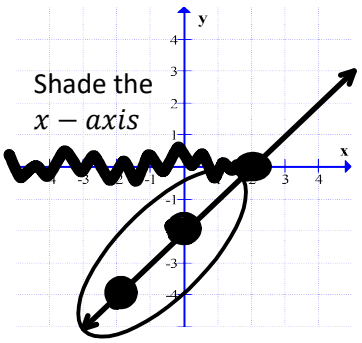
$x - 2 \leq 0$

$+2 \quad +2$ Solve

$x \leq 2$

Graphing

$y\ values \leq 0$
$The\ Thing \leq 0$

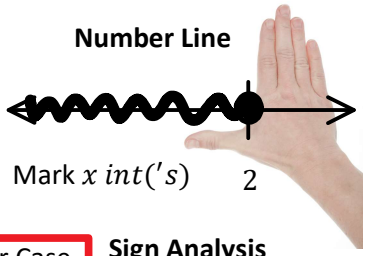


$y = x - 2$

What are the x values when $y \leq 0$. Circle them!

$x \leq 2$

Number Line



Or Case Analysis

Sign Analysis

Pick a value

$x \leq 2$ $\quad$ $x \geq 2$

$x = 0$ Substitute $x = 4$

$x - 2 \leq 0$
$0 - 2 \leq 0$
$-2 \leq 0$ ✓

$x - 2 \leq 0$
$4 - 2 \leq 0$
$2 \leq 0$ ✗

Correct: Shade that section

Incorrect: that section Shade Not

$x \leq 2$

$-x^2 + 5x - 4 < 0$

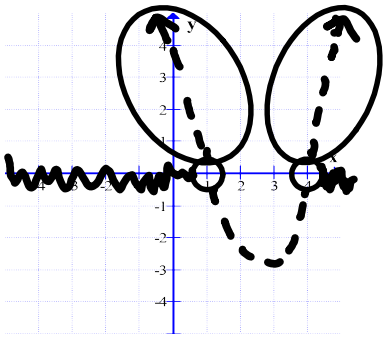
$-x^2 + 5x - 4 < 0$
$-(x^2 - 5x + 4) < 0$
$\frac{-(x^2 - 5x + 4)}{-1} > \frac{0}{-1}$ $\div -1^*$
$x^2 - 5x + 4 > 0$
$(x - 4)(x - 1) > 0$ Factor

$x - intercept's$

$x - 4 = 0$ $\quad$ $x - 1 = 0$
$x = 4$ $\quad$ $x = 1$

Graphing

$y\ values > 0$
$The\ Thing > 0$

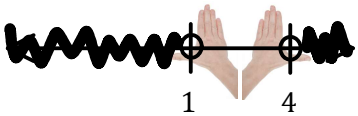


$y = (x - 4)(x - 1)$

What are the x values when $y > 0$. Circle them!

$x < 1$ $\quad$ $x > 4$

Number Line



Sign Analysis

Pick a value

$x < 1$ $\quad$ $1 < x < 4$ $\quad$ $x > 4$

$x = 0$ $\quad$ $x = 2$ $\quad$ $x = 5$

Substitute

$(x - 4)(x - 1) > 0$
$(0 - 4)(0 - 1) > 0$
$(-4)(-1) > 0$
$4 > 0$ ✓

$(1)(4) > 0$
$4 > 0$ ✓

$(-2)(1) > 0$
$-2 > 0$ ✗

$x < 1$ $\quad$ $x > 4$

$0 \geq x^2 - 4 \leq 0$

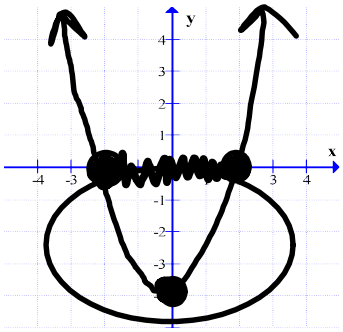
$x^2 - 4 \leq 0$
$(x + 2)(x - 2) \leq 0$

$x + 2 = 0$ $\quad$ $x - 2 = 0$
$x = -2$ $\quad$ $x = 2$

$x - intercept's$

Graphing

$y\ values \leq 0$
$The\ Thing \leq 0$

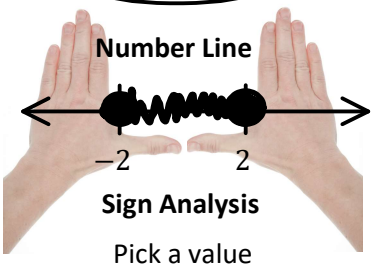


$y = x^2 - 4$

What are the x values when $y \geq 0$. Circle them!

$-2 \leq x \leq 2$

Number Line



Sign Analysis

Pick a value

$x \leq -2$ $\quad$ $-2 \leq x \leq 2$ $\quad$ $x \geq 2$

$x = -3$ $\quad$ $x = 0$ $\quad$ $x = 3$

$x^2 - 4 \leq 0$
$(-3)^2 - 4 \leq 0$
$5 \leq 0$ ✗

$x^2 - 4 \leq 0$
$(0)^2 - 4 \leq 0$
$5 \leq 0$ ✓

$x^2 - 4 \leq 0$
$(3)^2 - 4 \leq 0$
$5 \leq 0$ ✗

$-2 \leq x \leq 2$

The answer is only the Domain. The number line and graph is only to help. There is no y involved.

C11 - 9.0 - Inequalities 2 Variables Notes

Graph the following Inequality

$y > x - 2$ Graph: $y = x - 2$

$y = mx + b$

<,> (Open Dots, Dotted line)

Test Point

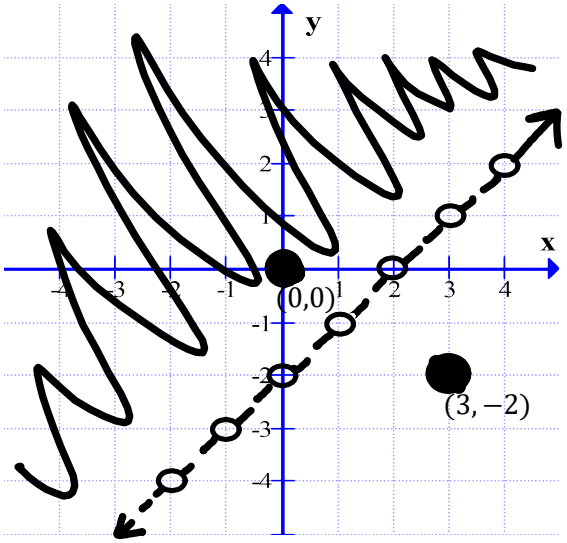
(x, y)
$(0,0)$

Choose a Point on either side of the Line

Zero-Zero Test*

$y > x - 2$
$0 > 0 - 2$
$0 > -2$ ✓ Substitute for x and y.

Correct: Shade the $(0,0)$ side of the line.



$y > x - 2$

Find Equation

Test Point		Equation		
y	$x - 2$	y	$mx + b$	(x, y)
0	$0 - 2$	**"Space"**		$(0,0)$
0	-2			
$0 > -2$ ✓		Make a correct Statement		
$y > x - 2$		y	$x - 2$	

Test Point (x, y) $(3, -2)$ $y > x - 2$ $-2 > 3 - 2$ $-1 > 1$ ✗

OR

Incorrect: Shade the Not $(3, -2)$ side of the line.

Isolate for y or TOV $y = mx + b$

$x - y \geq 2$
$-y \geq -x + 2$
$y \leq x - 2$

OR

$x - y \geq 2$
$x - 2 \geq y$
$y \leq x - 2$

Subtract x
Divide* by -1
Change Sign!

(Both Sides)

Add y
Subtract 2
Mirror

Graph the following Inequality

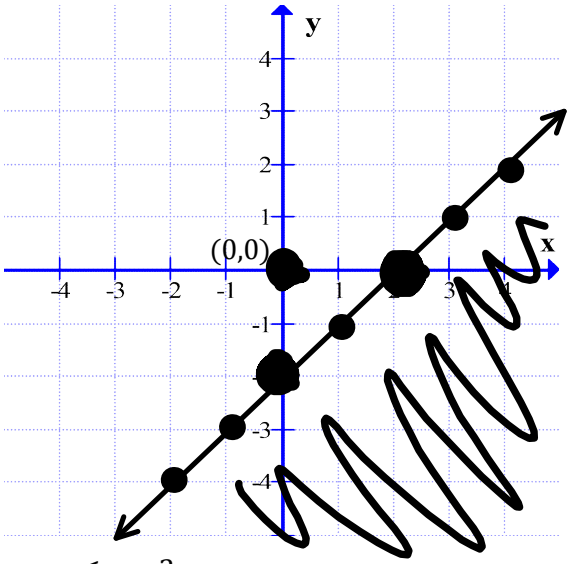
$y \leq x - 2$ Graph $y = x - 2$

≤,≥ (Closed Dots, Solid Line)

Test Point

$(0,0)$

$y \leq x - 2$
$0 \leq 0 - 2$
$0 \leq -2$ ✗ Incorrect: Shade "Not" the $(0,0)$ side of the line.



$y \leq x - 2$

Find Equation

Test Point		Equation		
y	$x - 2$	y	$mx + b$	(x, y)
0	$0 - 2$	**"Space"**		$(0,0)$
0	-2			
$0 \leq -2$ ✗		Make a Incorrect Statement		
$y \leq x - 2$		y	$x - 2$	

Notice: the (0,0) test only works if (0,0) is not on the line. If (0,0) is on the line we must choose a distinct point that is not on the line like* (5,0) or (0,2).

OR "Shade" above/below than "the line"

Replace the word y *with "shade"*
Greater than = above/Less than = below
Replace the equation with "the line"

C11 - 9.0 - Inequalities 2 Variables Notes

Graph the following inequalities

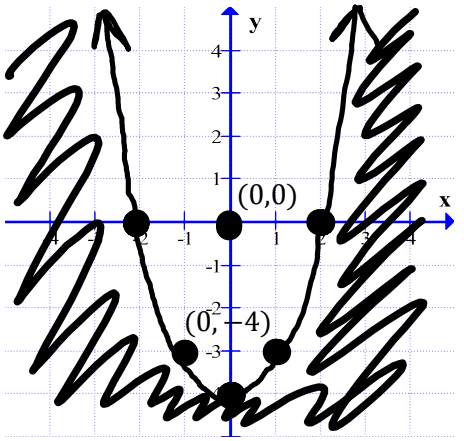
$y \leq x^2 - 4$

Graph: $y = x^2 - 4$

TOV

x	y
-2	0
-1	-3
0	-4
1	-3
2	0

(Closed dots, Solid Line) $y = x^2 - 4$



Test Point $(0,0)$

$y \leq x^2 - 4$
$0 \leq (0)^2 - 4$
$0 \leq -4$ ✗

Substitute for x and y.

Incorrect: Shade the "NOT" (0,0) side of the line.

Find Equation

$y = a(x-p)^2 + q$
$y = a(x-0)^2 - 4$
$-3 = a(1-0)^2 - 4$
$-3 = 1a - 4$
$1 = a$
$y = 1(x-0)^2 - 4$
$y = x^2 - 4$

Test Point

$y \quad x^2 - 4$
$0 \quad 0^2 - 4$
$0 \leq -4$ ✗

$y \leq x^2 - 4$

"Space" (x,y) $(0,0)$

Make a Incorrect Statement

$y > x^2 - 2x - 3$

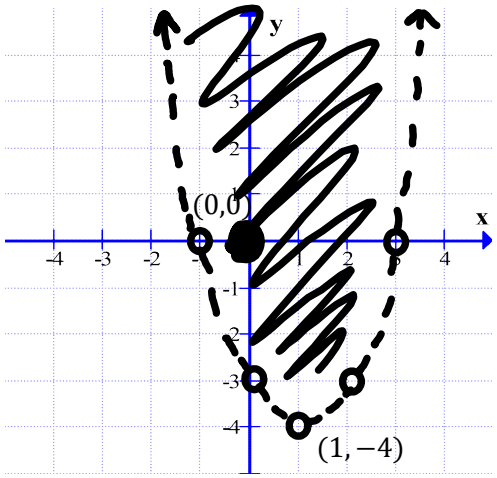
Graph: $y = x^2 - 2x - 3$

(Open dots, Dotted line) $y = x^2 - 2x - 3$

$y = x^2 - 2x - 3$ Complete the square $\left(\frac{b}{2}\right)^2$
$y = (x^2 - 2x) - 3$
$y = (x^2 - 2x + 1 - 1) - 3$
$y = (x-1)^2 - 4$ $(1,-4)$ Vertex

$y = x^2 - 2x - 3$
$y = (x+1)(x-3)$

$x = -1 \quad x = 3$ $x - intercepts$



Test Point $(0,0)$

$y > x^2 - 4$
$0 > 0 - 4$
$0 > -4$ ✓

Substitute for x and y.

Correct: Shade the $(0,0)$ side of the line.

Find Equation

$y = a(x-p)^2 + q$

$y = a(x-1)^2 - 4$
$-3 = a(2-1)^2 - 4$
$-3 = 1a - 4$
$1 = a$
$y = 1(x-1)^2 - 4$

$y = (x-1)^2 - 4$

Vertex Form

(x,y)
$(1,-4)$ Vertex

(x,y)
$(2,-3)$ Point

Test Point

$y \quad (x-1)^2 - 4$
$0 \quad (0-1)^2 - 4$
$0 \leq -3$ ✓

$y \leq (x-1)^2 - 4$

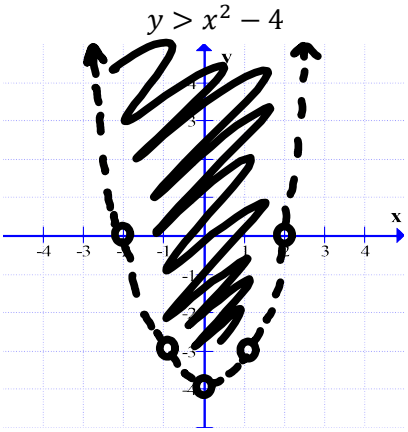
"Space" (x,y) $(0,0)$

Make a Correct Statement

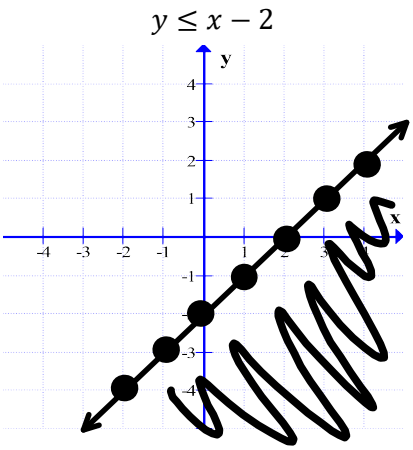
C11 - 9.0 - Systems/Burgers and Fries/Optimization

Solve by graphing :

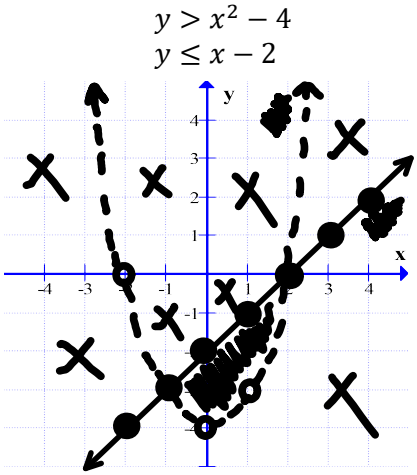
$y > x^2 - 4$



$y \leq x - 2$



$y > x^2 - 4$
$y \leq x - 2$



Test Patch!

Notice: we have graphed each equation and shaded only the region which satisfies both inequalities.

$let\ b = \#\ burgers$ $burgers = \$3$ $\$12\ to\ spend$
$let\ f = \#\ fries$ $fries = \$2$

$1\ burger = 3 \times 1 = 3$
$3\ burger = 3 \times 2 = 6$
$b\ burger = 3 \times b = 3b$

f	b
0	4
6	0

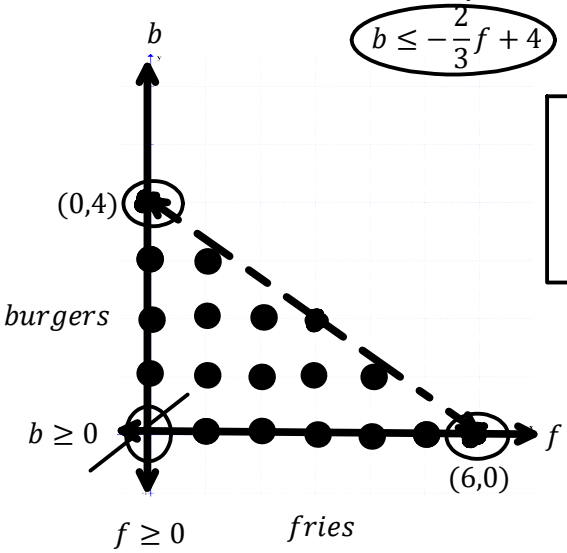
$3b + 2f \leq 12$

$ax + by = c$

$3b + 2f \leq 12$
$3b \leq -2f + 12$
$b \leq -\frac{2}{3}f + 4$

$y = mx + b$

(f, b)	
$(0,4) = \$12$	$(2,2) = 10$
$(0,3) = \$9$	$(2,1) = \$7$
$(0,2) = \$6$	$(2,0) = \$4$
$(0,1) = \$3$	$(3,2) = \$12$
$(0,0) = \$0$	$(3,1) = \$9$
$(1,3) = \$11$	$(3,0) = \$6$
$(1,2) = \$8$	$(4,1) = \$11$
$(1,1) = \$5$	$(5,0) = \$10$
$(1,0) = \$2$	$(6,0) = \$12$



Restrictions*
$0 \leq b \leq 4, b \in W$
$0 \leq f \leq 6, f \in W$
$W: Whole\ Numbers$

Test Point: $(1,1)$

$b \geq 0$ ✓ $1 \geq 0$ $\quad f \geq 0$ ✓ $1 \geq 0$ $\quad b \leq -\frac{2}{3}f + 4$

$1 \leq -\frac{2}{3}(1) + 4$

$1 \leq \frac{10}{3}$ ✓

If burgers are 500 calories and fries are 300 calories and the goal is to maximize calories find the optimization function and the three combinations that will maximize calories. (Foundations)

$let\ C = Total\ calories$ $C = 500b + 300f$ Test Corners (f, b)

$(0,4)$ $(6,0)$ ~~$(0,0)$~~

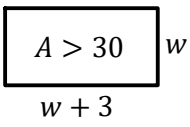
$C = 500(0) + 300(4)$
$C = 1200\ calories$

$C = 500(6) + 300(0)$
$C = 3000\ calories$

Eat 6 Burgers to maximize calories

C11 - 9.0 - Inequalities WP Notes

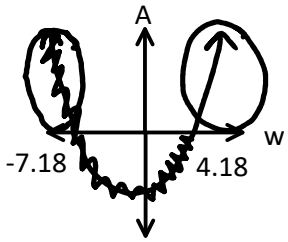
Find the dimensions of a rectangle with a length 3 cm longer than its width and an area greater than 30 cm^2.



$A = lw$
$A = w(w + 3)$

$A > 30$
$w(w + 3) > 30$
$w^2 + 3w - 30 > 0$
...
$w = 4.18, -7.18$

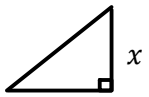
$w > 4.18cm$



$A = lw$
$A = 4.19(7.19)$
$A = 30.1 > 30$ ✓

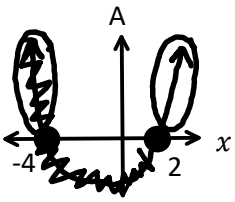
Can't have a negative area or length

Find the legs of a right angle triangle where one leg 2 m longer than the other and an area greater than 4 m^2.

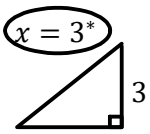


$A = \frac{bh}{2}$
$A = \frac{(x+2)x}{2}$

$A > 4$
$\frac{(x+2)x}{2} > 4$
$2 \times \frac{(x+2)x}{2} > 4 \times 2$
$x^2 + 2x > 8$
$x^2 + 2x - 8 > 0$
$(x + 4)(x - 2) > 0$



~~$x < -4$~~ $x > 2$



$A = \frac{5 \times 3}{2}$
$A = 7.5 > 4$

$x = 1^*$

$A = \frac{1 \times 3}{2}$
$A = 1.5 < 4$

Check a wrong answer is wrong!

A person has $16,000 to invest in stocks and bonds, with at least $2000 in stocks, and at least three times that amount in bonds. Graph.

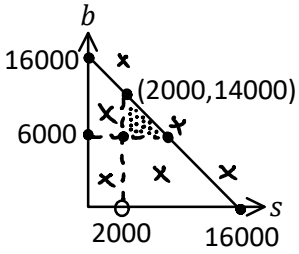
let $b =$ *amount invested in bonds*
let $s =$ *" " " stocks*

$b + s \leq 16000$

$s > 2000$

$3b > s$

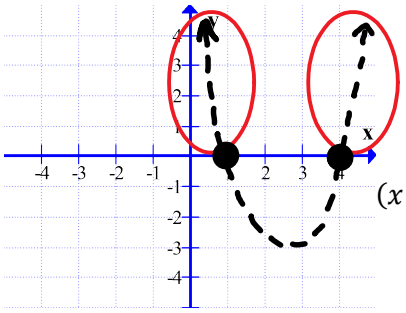
$b > 6000$



s	b
0	16000
16000	0

$b \leq -s + 16000$

C11 - 9.0 - Case Analysis/Scenarios



$x^2-5x > 4$

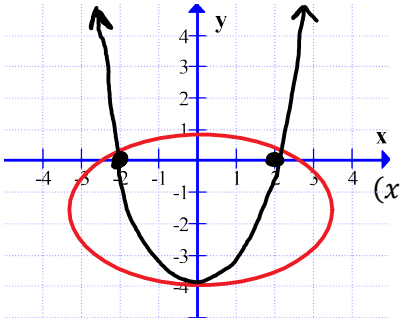
$x^2 - 5x - 4 > 0$

$(x-4)(x-1) > 0$

Case Analysis

$(+)(+) > 0$ OR $(-)(-) > 0$

$x-4>0$ $x>4$ | $x-1>0$ $x>1$ | $x-1<0$ $x<1$ | $x-4<0$ $x<4$



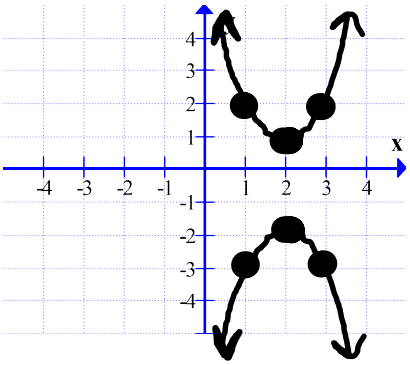
$(x+2)(x-2) \le 0$

Case Analysis

$(+)(-) \le 0$ OR $(-)(+) \le 0$

$x+2 \ge 0$ $x \ge -2$ | $x-2 \le 0$ $x \le 2$ | $x+2 \le 0$ $x \le -2$ | $x-2 \ge 0$ $x \ge 2$

$-2 \le x \le 2$ | *No Sol*

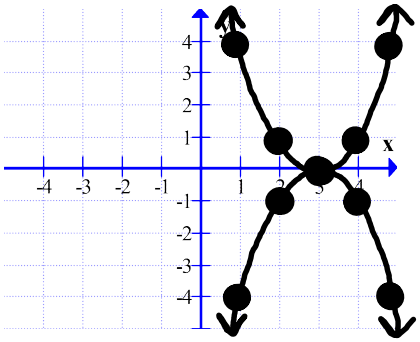


$(x-2)^2 + 1 \ge 0$ $x \in \mathbb{R}$

$(x-2)^2 + 1 \le 0$ $\neq$

$-(x-2)^2 - 1 > 0$ $\neq$

$-(x-2)^2 - 1 < 0$ $x \in \mathbb{R}$

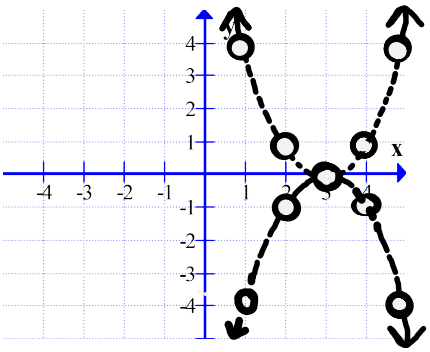


$(x-3)^2 \ge 0$ $x \in \mathbb{R}$

$(x-3)^2 \le 0$ $x = 3$

$-(x-3)^2 > 0$ $\neq$

$-(x-3)^2 < 0$ $x < 3 \quad x > 3$



$(x-3)^2 > 0$ $x < 3 \quad x > 3$

$(x-3)^2 < 0$ $\neq$

$-(x-3)^2 \ge 0$ $x = 3$

$-(x-3)^2 \le 0$ $x \in \mathbb{R}$