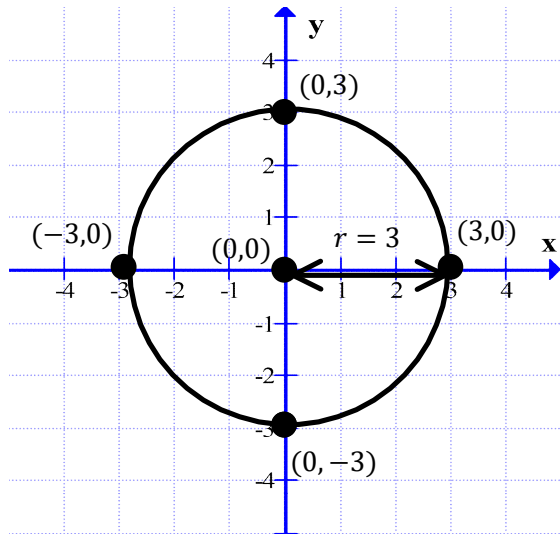


# C12 - 12.0 - Conics Circles/Ellipse Notes

Circle:  $x^2 + y^2 = 9$

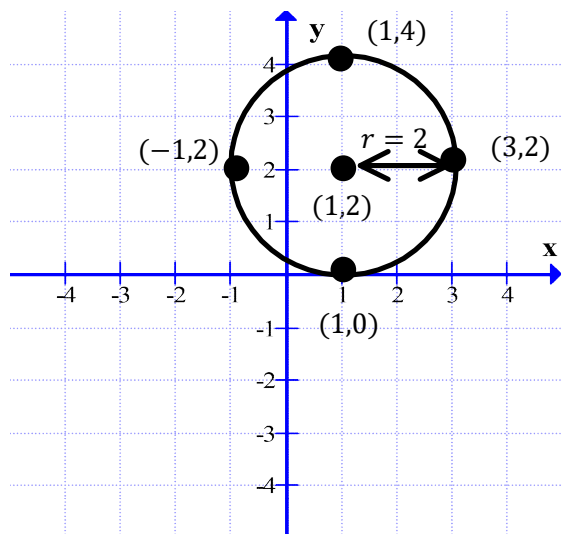


$x^2 + y^2 = r^2$       Radius =  $r$

$x^2 + y^2 = 3^2$                $r = 3$

$(x - 0)^2 + (y - 0)^2 = 3^2$       Center:  $(0,0)$

$(x - 1)^2 + (y - 2)^2 = 4$



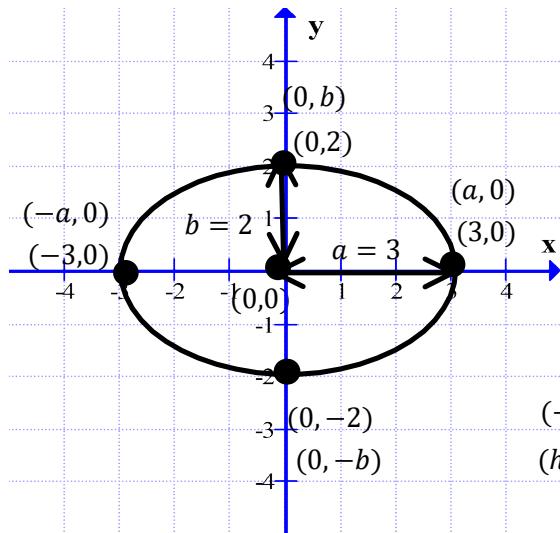
$(x - h)^2 + (y - k)^2 = r^2$       Center:  $(h, k)$

$(x - 1)^2 + (y - 2)^2 = 2^2$                $r = 2$

Center:  $(1,2)$

Ellipse:  $\frac{x^2}{9} + \frac{y^2}{4} = 1$

Get equal to 1.



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$x$  - Radius =  $a$   
 $y$  - Radius =  $b$

$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$

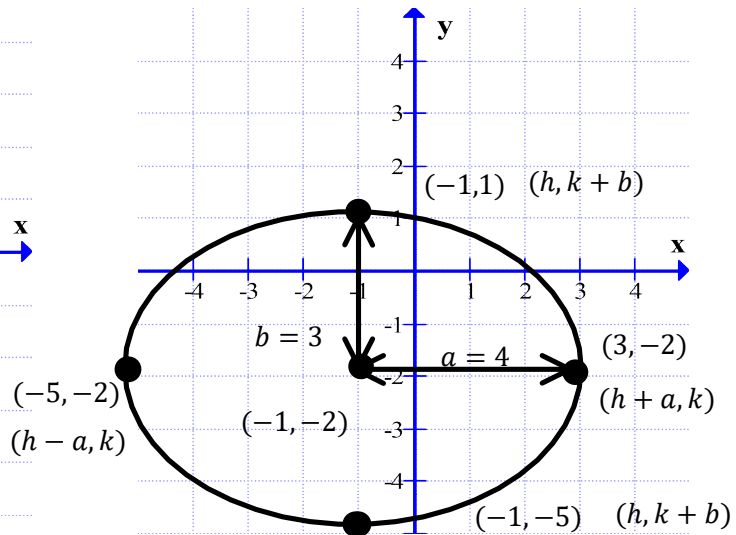
$a = 3$        $b = 2$

Center:  $(0,0)$

$4x^2 + 9y^2 = 36$

;  $\times$  both sides by LCD: 36

$\frac{(x + 1)^2}{16} + \frac{(y + 2)^2}{9} = 1$       Major Axis - Long  
Minor Axis - Short



$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$       Center:  $(h, k)$

$\frac{(x + 1)^2}{4^2} + \frac{(y + 2)^2}{3^2} = 1$        $a = 4$        $b = 3$

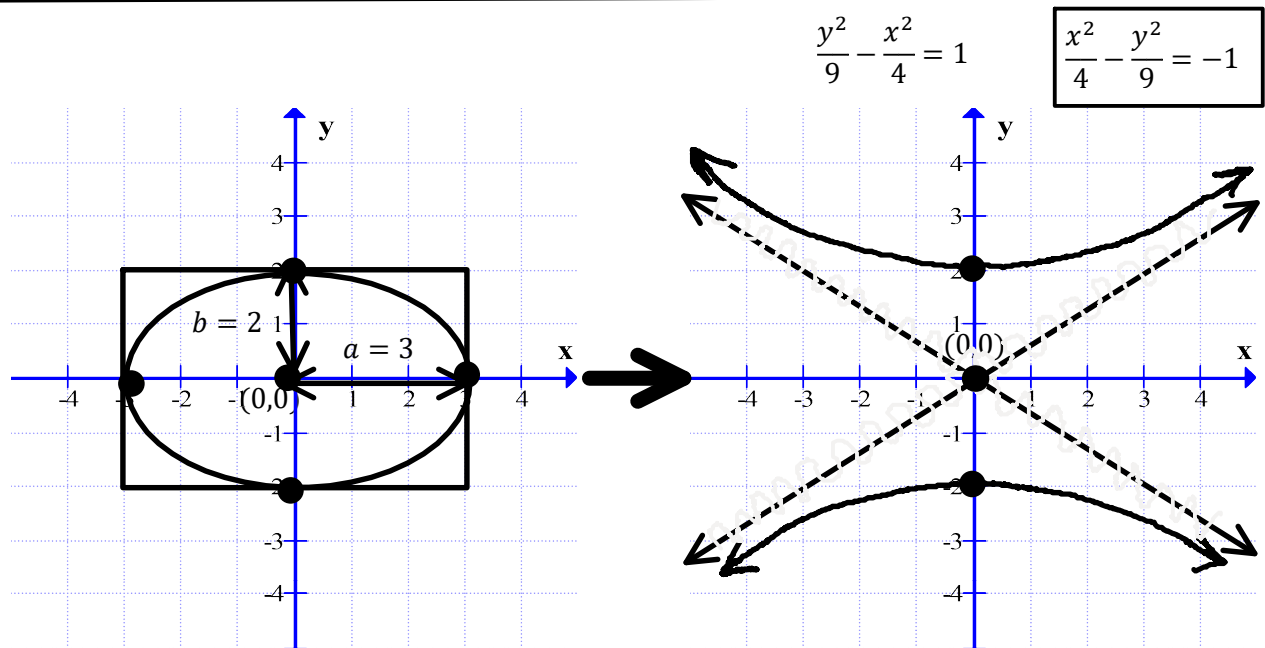
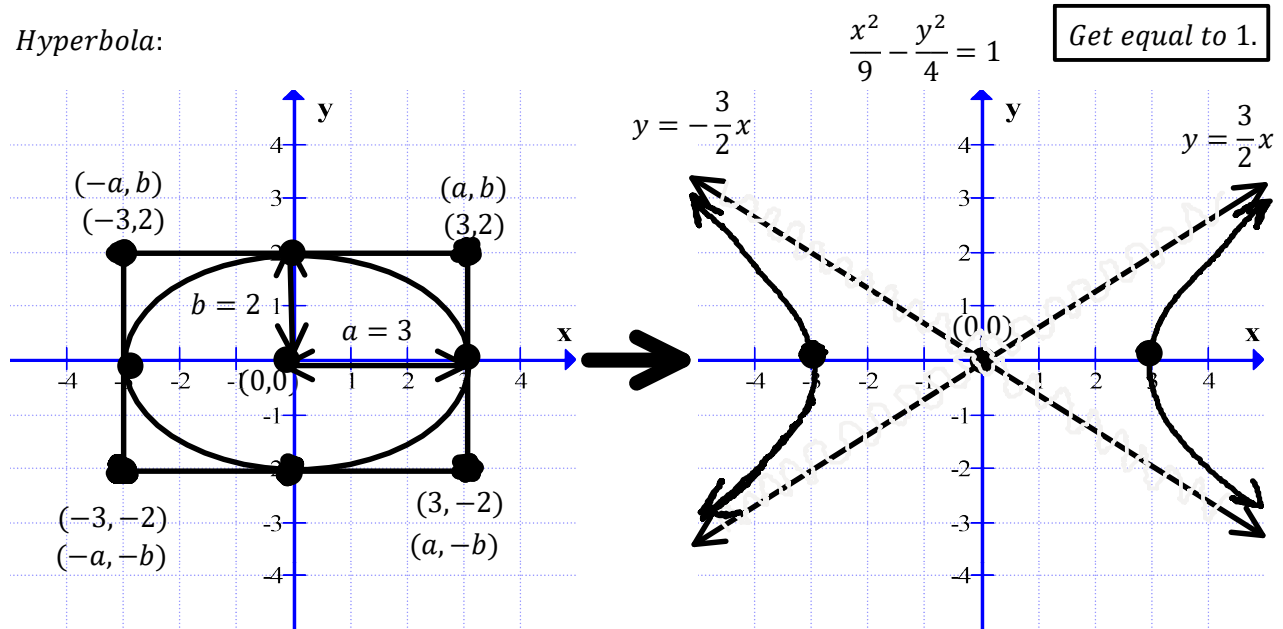
Center:  $(-1, -2)$

$9(x - 1)^2 + 16(y - 1)^2 = 144$   
 $9x^2 - 18x + 16y^2 - 32y - 119 = 0$

LCD  
FOIL  
Algebra

# C12 - 12.0 - Conics Ellipse/Hyperbola Notes

Hyperbola:



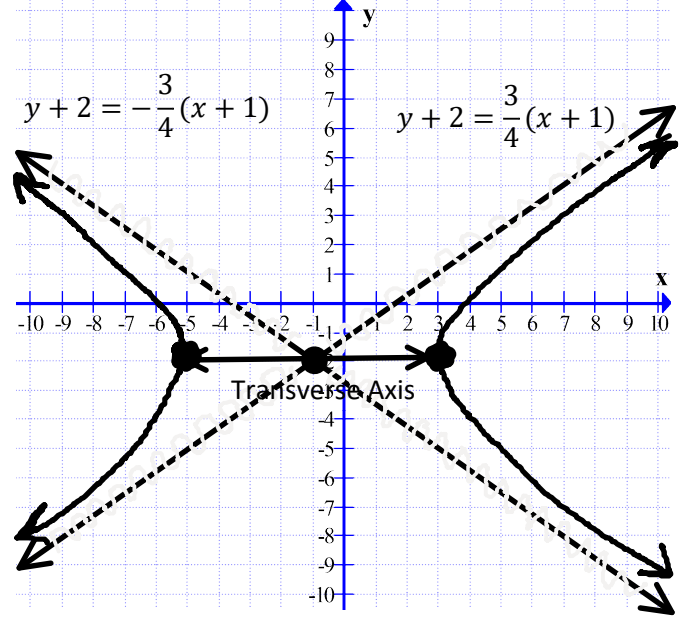
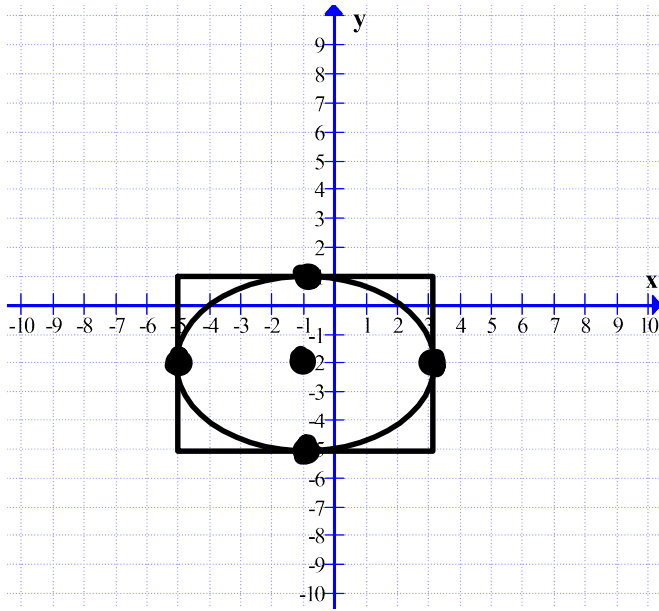
A Hyperbola and an ellipse fuse together at  $(\pm a, 0)$  or  $(0, \pm b)$

Draw a box around the Hyperbola; lines through the vertices of the rectangle are the equations of the asymptotes.

# C12 - 12.0 - Conics Hyperbola/Conics Notes

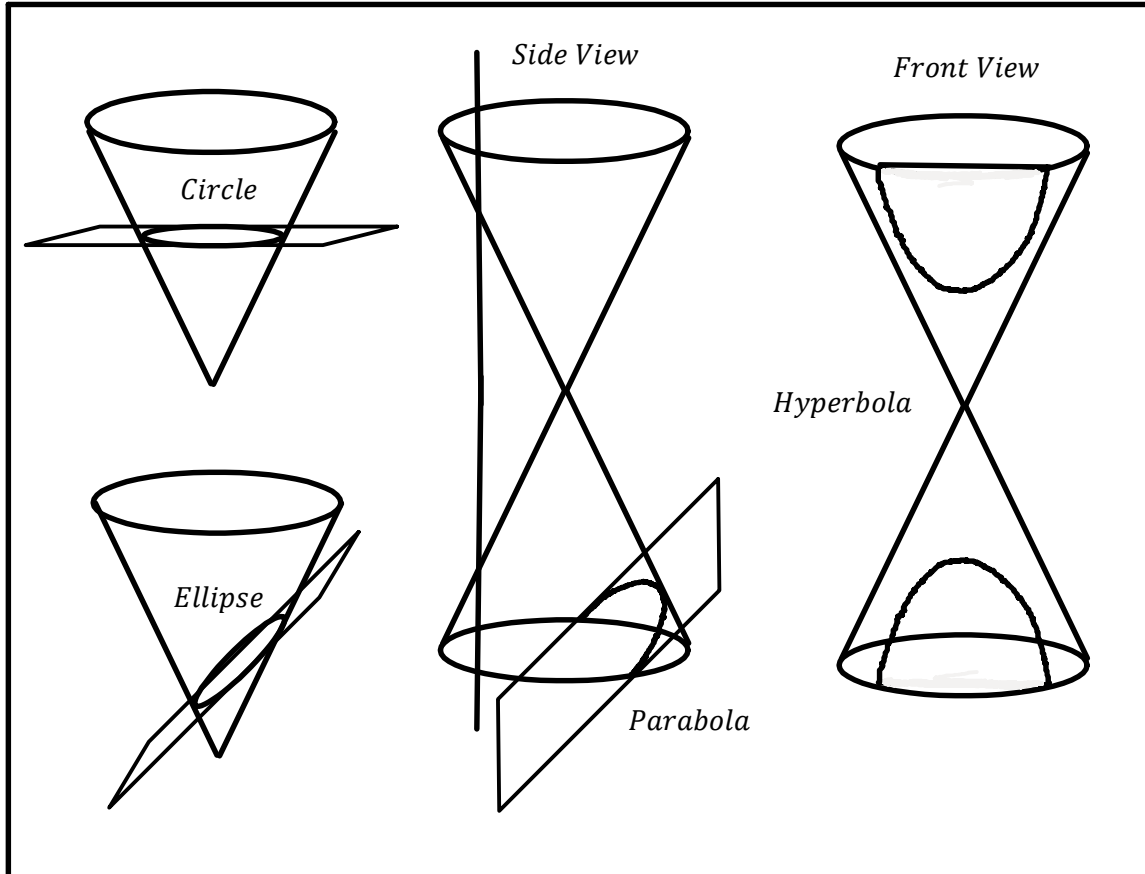
Hyperbola:

$$\frac{(x + 1)^2}{16} - \frac{(y + 2)^2}{9} = 1$$



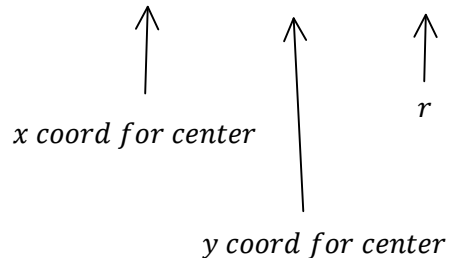
$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1 \quad \text{Center: } (h, k)$$

$$\text{Asymptotes: } y - k = m(x - h) \quad m = \pm \frac{b}{a}$$



# C12 - 12.0 - Conics Completing the Square: x and y

$$\begin{aligned}
 x^2 - 4x + y^2 - 6y &= 0 \\
 (x^2 - 4x) + (y^2 - 6y) &= 0 \\
 (x^2 - 4x + 4) - 4 + (y^2 - 6y + 9) - 9 &= 0 \\
 (x - 2)^2 + (y - 3)^2 - 13 &= 0 \\
 (x - 2)^2 + (y - 3)^2 &= 13 \\
 (x - 2)^2 + (y - 3)^2 &= \sqrt{13}^2
 \end{aligned}$$



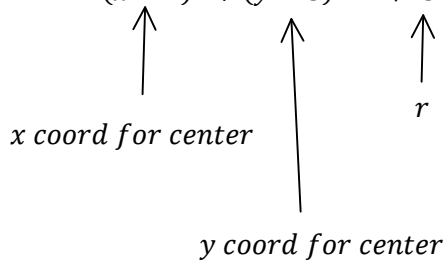
$$0 = Ax^2 + Bxy + Cy^2 + Dx + Ey + F$$

- Circles:  $x^2, y^2$  same sign,  $A = C$
- Ellipse:  $x^2, y^2$  same sign,  $A, C > 0, A \neq C$
- Hyperbola:  $x^2, y^2$  Opposite sign,  $A < 0 < C$  or  $A > 0 > C$
- Parabola:  $x^2$  or  $y^2 = 0$ ,  $A$  or  $C = 0$

$$\begin{aligned}
 3x^2 - 12x - 18y + 3y^2 &= 0 \\
 3(x^2 - 4x - 6y + y^2) &= 0 \\
 \frac{3(x^2 - 4x - 6y + y^2)}{3} &= \frac{0}{3?}
 \end{aligned}$$

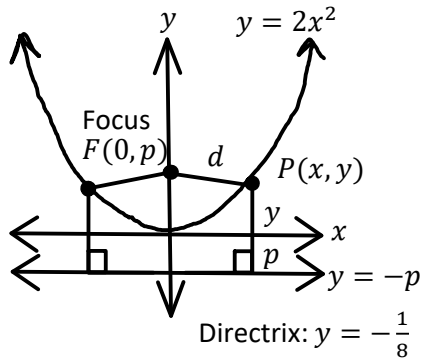
GCF

$$\begin{aligned}
 x^2 - 4x - 6y + y^2 &= 0 \\
 x^2 - 4x + y^2 - 6y &= 0 \\
 (x^2 - 4x) + (y^2 - 6y) &= 0 \\
 (x^2 - 4x + 4) - 4 + (y^2 - 6y + 9) - 9 &= 0 \\
 (x - 2)^2 + (y - 3)^2 - 13 &= 0 \\
 (x - 2)^2 + (y - 3)^2 &= 13 \\
 (x - 2)^2 + (y - 3)^2 &= \sqrt{13}^2
 \end{aligned}$$



# C12 - 12.0 - Conics Notes

Parabola: The distance from the focus to the parabola and straight down to the directrix is equal.  $d = y + p$



$$y = ax^2 \dots$$

$$y = \frac{x^2}{4p}$$

$$a = \frac{1}{4p}$$

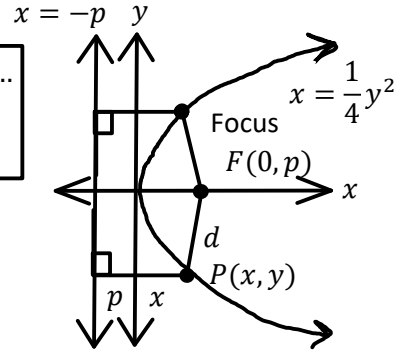
$$x = ay^2 \dots$$

$$x = \frac{y^2}{4p}$$

$$\frac{1}{4p} = 2$$

$$p = \frac{1}{8}$$

Focus:  $(0, \frac{1}{8})$



$$\frac{1}{4p} = \frac{1}{4}$$

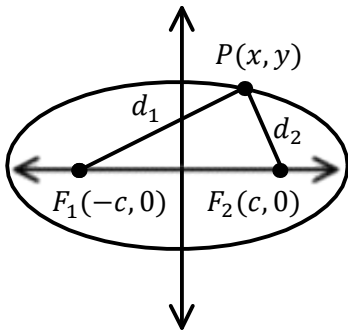
$$p = 1$$

Directrix:  $x = -1$

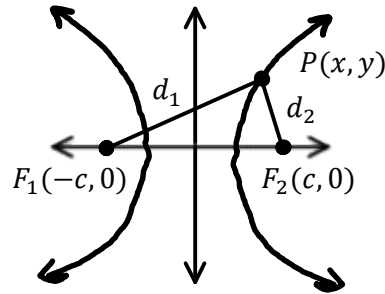
Focus:  $(1, 0)$

Ellipse: The sum distances from the Ellipse to the Foci is constant.  $d_1 + d_2$ ; constant!

Hyperbola: The difference in the distance from the Hyperbola to the Foci is constant.  $d_1 - d_2$ ; constant!



Focus/i: Inside Parabola\*  
Directrix: Outside Parabola\*



$c$ : Distance from Centre on Major/Transverse-Axis

$$c = \sqrt{a^2 - b^2}; a \geq b$$

$$c = \sqrt{b^2 - a^2}; b \geq a$$

Eccentricity:  
 $e = \frac{c}{a} \quad a > b$   
 $e = \frac{c}{b} \quad b > a$

$$c = \sqrt{a^2 + b^2}$$

$$\frac{(x+1)^2}{16} + \frac{(y+2)^2}{9} = 1$$

$$c = \sqrt{16 - 9}$$

$$c = \sqrt{5}$$

Foci:  $(-1 \pm \sqrt{5}, -2)$

Directrix: Line Perpendicular away from Centre on Major/Transverse-Axis  
 $x = \frac{a^2}{c}; a > b$   
 $y = \frac{b^2}{c}; b > a$

$$\frac{(x+1)^2}{16} - \frac{(y+2)^2}{9} = 1$$

$$c = \sqrt{16 + 9}$$

$$c = 5$$

Foci:  $(-1 + 5, -2)$   
 $(4, -2) \text{ \& } (-6, -2)$

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Circles:  $x^2, y^2$  same sign,  $A = C$   
 Ellipse:  $x^2, y^2$  same sign,  $A, C > 0, A \neq C$   
 Hyperbola:  $x^2, y^2$  Opposite sign,  $A < 0 < C$  or  $A > 0 > C$   
 Parabola:  $x^2$  or  $y^2 = 0, A$  or  $C = 0$