Mathbook PC 12 Notes

Knack Academics

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C12 - 1.1 - HT Translations Theory

$y = x^2$

Let's take the function $f(x) = x^2$

$\boxed{y = f(x) = x^2}$

$(2,4)$

Let's take the point $(2,4)$

4

$\downarrow$

$y = 4$

Remember: The function doesn't change

$\boxed{y = (x+2)^2}$

Now, let's take the function $\boxed{\begin{array}{l} y = x^2 \\ y = (x+2)^2 \end{array}}$

$\boxed{g(x) = (x+2)^2}$

$\uparrow$

$x \to x + 2$ Put $x + 2$ in for x

Let's call it $g(x)$

$(\ ?\ , 4)$

If $y = 4$, What does x have to be?
What plus two all squared equals four?

$$\begin{aligned} y &= (x+2)^2 \\ 4 &= (x+2)^2 \\ \sqrt{4} &= \sqrt{(x+2)^2} \\ 2 &= x + 2 \end{aligned}$$

$x = 0$

$x = 0$

$x = 0$

The $x - value$ was 2
Now the $x - value$ is 0
The $x - value$ minus 2

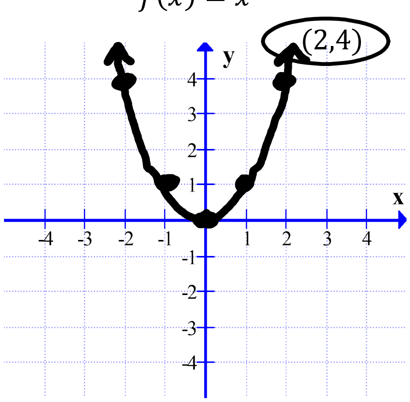
$(0,4)$

$x - 2$ *Left* 2

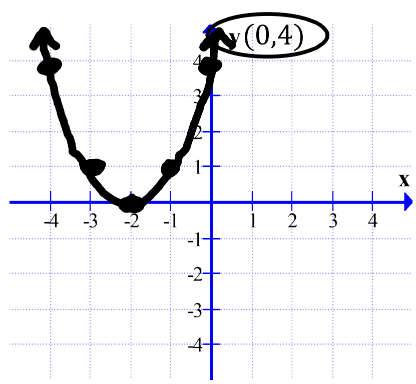
They are all equal to each other

$y = x^2$
$y = f(x)$
$f(x) = x^2$

$y = (x+2)^2$
$g(x) = f(x+2)$
$g(x) = (x+2)^2$



$\boxed{\leftarrow}$



$\boxed{HT = -2}$

Horizontal Translation

General Form

$\boxed{y = f(x - h)}$

$\boxed{y = f(x + 2)}$

Left 2

Horizontal Translations are the **Opposite** of what you see inside the brackets to the **x-value**. Attached to the variable.

C12 - 1.1 - VT Translations Theory

$y = x^2$

Let's take the function $f(x) = x^2$

$\boxed{y = f(x) = x^2}$

$(2,4)$

Let's take the point (2,4)

2

$x = 2$

$\boxed{y = x^2 - 2}$

Now, let's take the function

$$\boxed{\begin{array}{c} y = x^2 \\ y + 2 = x^2 \\ y = x^2 - 2 \end{array}}$$

$\boxed{y \rightarrow y + 2 \quad \text{Put } y + 2 \text{ in for } y}$

$\boxed{m(x) = x^2 - 2}$

Let's call it m(x)

$(2, ?)$

If $x = 2$, What does y equal?
2 squared minus 2 equals 2?

$y = 2$

$$\boxed{\begin{array}{l} y = x^2 - 2 \\ y = (2)^2 - 2 \\ y = 2 \end{array}}$$

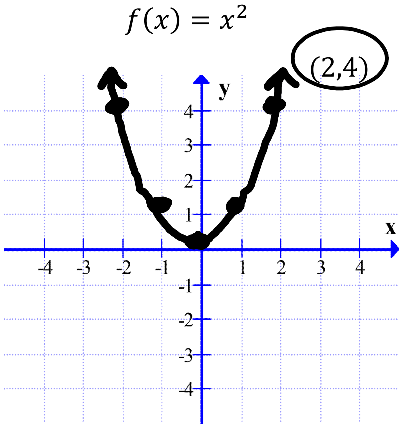
$y = 2$

The y$-value$ was 4
Now the y$-value$ is 2
The $y - value$ minus 2

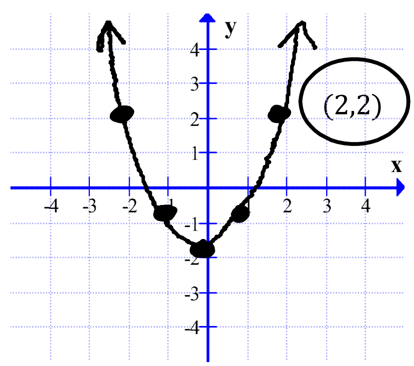
$(2,2)$

$y - 2$ $\quad$ $Down\ 2$

$y = x^2$
$y = f(x)$
$f(x) = x^2$



$y = x^2 - 2$
$m(x) = f(x) - 2$
$m(x) = x^2 - 2$



$\boxed{VT = -2}$

Vertical Translation

General Form

$$\boxed{\begin{array}{l} y - k = f(x) \\ \qquad y = f(x) + k \end{array}}$$

$$\boxed{\begin{array}{l} y + 2 = f(x) \\ \qquad y = f(x) - 2 \end{array}}$$

$Down\ 2$

Vertical Translations are the **Opposite** of what you see on the left hand side to the **y-value**. Attached to the variable.
"k" may be on the left hand side of the equation: $y - k = f(x)$. So add or subtract "k" to both sides.
Do exactly what you see outside of the brackets on the right-hand side to the **y-value**

C12 - 1.2 - HCE Transformations Theory

$y = x^2$

Let's take the function $f(x) = x^2$

$\boxed{y = f(x) = x^2}$

$(2, 4)$

Let's take the point (2,4)

4

$y = 4$

$\downarrow$

$\boxed{y = (2x)^2}$

$\uparrow$

Now, let's take the function $\boxed{\begin{matrix} y = x^2 \\ y = (2x)^2 \end{matrix}}$

$\boxed{m(x) = (2x)^2}$

$\boxed{x \to 2x \qquad \text{Put } 2x \text{ in for } x}$

Let's call it m(x)

$(\,?\,, 4)$

If $y = 4$, What does x have to be?
What times two all squared equals four?

$x = 1$

$$
\boxed{\begin{aligned} y &= (2x)^2 \\ 4 &= (2x)^2 \\ \sqrt{4} &= \sqrt{(2x)^2} \\ 2 &= 2x \\ x &= 1 \end{aligned}}
$$

$x = 1$

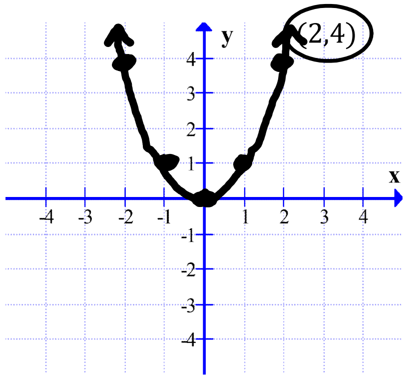
$(1,4)$

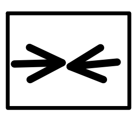
The $x - value$ was 2
Now the $x - value$ is 1
The $x - value$ divided by 2

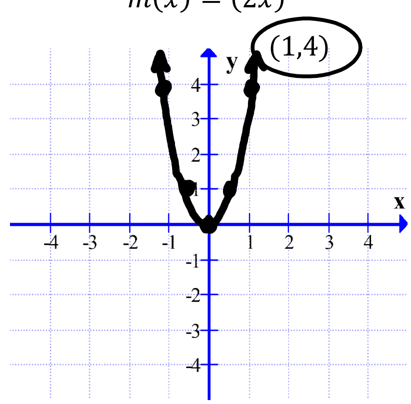
$x \div 2$

$y = x^2$
$y = f(x)$
$f(x) = x^2$

$y = (2x)^2$
$m(x) = f(2x)$
$m(x) = (2x)^2$







$$\boxed{HC = \frac{1}{2}}$$

Horizontal Compression

General Form

$\boxed{y = f(bx)}$

$\boxed{y = f(2x)}$

Horizontal Expansions and Compressions are the **<u>Reciprocal</u>** of what you see inside the brackets to the **<u>x-value</u>**

C12 - 1.2 - VCE Transformations Theory

$y = x^2$

$(1,1)$

1

$\downarrow$

$\boxed{y = 2x^2}$

$\uparrow$

$(1, ?)$

$y = 2$

$(1,2)$

Let's take the function $f(x) = x^2$

$\boxed{y = f(x) = x^2}$

Let's take the point (1,1)

$x = 1$

$$y = x^2$$
$$\frac{1}{2}y = x^2$$
$$y = 2x^2$$

Now, let's take the function

$\boxed{p(x) = 2x^2}$

$y \to \frac{1}{2}y$	Put $\frac{1}{2}y$ in for y

Let's call it p(x)

If $x = 1$, What does y equal?
1 squared times 2 equals 2?

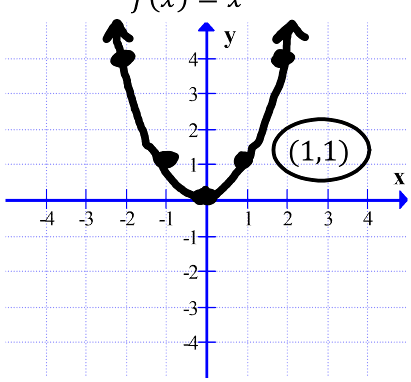
$y = 2$

$$y = 2x^2$$
$$y = 2(1)^2$$
$$y = 2$$

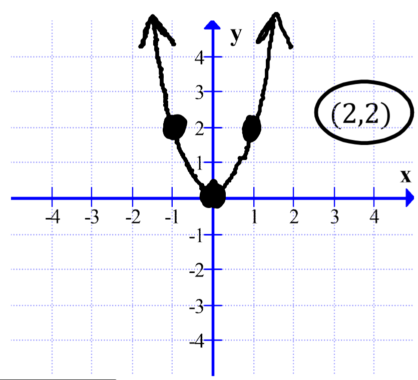
The y$-value$ was 1
Now the y$-value$ is 2
The $y - value$ times 2

$y \times 2$

$$y = x^2$$
$$y = f(x)$$
$$f(x) = x^2$$



$$y = 2x^2$$
$$p(x) = 2f(x)$$
$$p(x) = 2x^2$$



General Form

$\boxed{VE = 2}$

Vertical Expansion

$$\boxed{\begin{aligned} ay &= f(x) \\ y &= af(x) \end{aligned}}$$

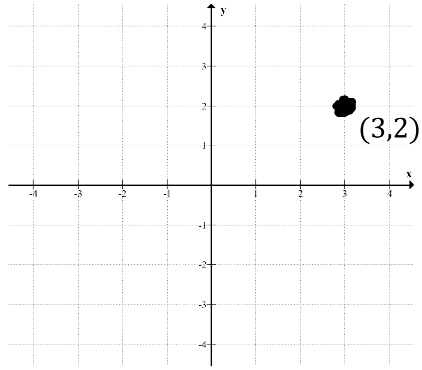
$$\boxed{\begin{aligned} \frac{1}{2}y &= f(x) \\ y &= 2f(x) \end{aligned}}$$

Vertical Expansions and Compressions are the **<u>Reciprocal</u>** of what you see on the left hand side to the **<u>y-value.</u>**
"a" may be on the left side of the equation: $ay = f(x)$. So multiply or divide by "a" to both sides.
Do exactly what you see outside of the brackets on the right-hand side to the **<u>y-value</u>**

C12 - 1.1 - VHT Point Notes

Find new point.

$\boxed{y = f(x)}$



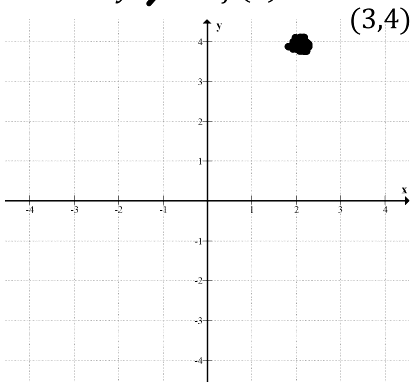
Point

$(3,2)\ is\ on\ f(x)$

Function Notation

$y = f(x)$

$y \cancel{-2} = f(x) + 2$



Operation

$\underline{(3,2)}$

$VT = +2 \quad (3,4)$

Add 2 to y-value

Mapping Notation

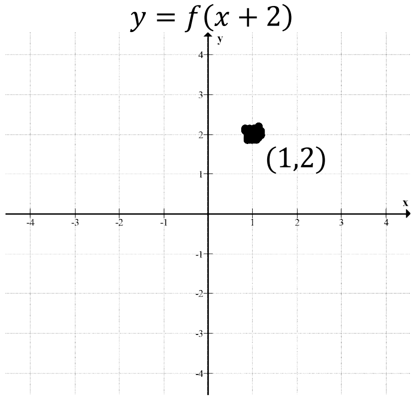
$(x, y + 2)$

UP TWO

A Vertical
Translation up 2

$y + 2$

$y = f(x + 2)$



$\underline{(3,2)}$

$HT = -2 \quad (1,2)$

Subtract 2 from x-value

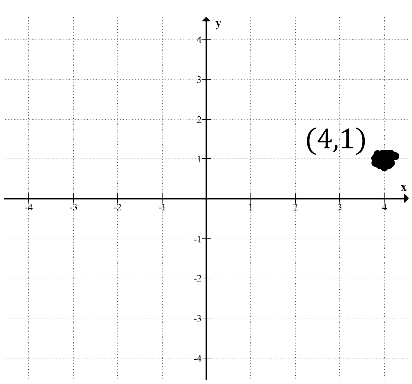
$(x - 2, y)$

LEFT 2

A Horizontal
Translation left 2

$x - 2$

$y + 1 = f(x - 1)$



$\underline{(3,2)}$

$HT = +1 \quad (4,2)$

$VT = -1 \quad (4,1)$

Add 1 to x-value
Subtract 1 from y-value

$(x + 1, y - 1)$

RIGHT 1
DOWN 1

A Horizontal Translation right 1
A Vertical Translation down 1

$x + 1 \qquad y - 1$

Do exactly what you see outside of the brackets on the right-hand side to the **y-value**
Do the **Opposite** of what you see inside the brackets to the **x-value**. Attached to the variable.
Do the **Opposite** of what you see on the left hand side to the **y-value**. Attached to the variable.

C12 - 1.1 - VHT Function Notation f(x) Notes

$\boxed{y = f(x)}$

$\boxed{f(x) = x^2}$ *Given*

$f(3) = ?$ $(3, y)$ *What is y when x is 3.*

$f(x) = x^2$
$f(x) = (x)^2$
$f(3) = (3)^2$

Put 3 in for x.

$f(3) = 9$ (3,9)

Put whatever is inside the brackets in for x. Substitute with Brackets

$y = x^2$
$y = (3)^2$
$y = 9$

x	y
3	9

$\boxed{f(x) = x^2}$

$f(x+2) = ?$

$f(x) = x^2$

$f(x+2) = (x+2)^2$

Put $(x+2)$ in for x.

Let's call it $g(x)$

Function Notation

$g(x) = ?$

$g(x) = f(x+2)$

$g(x) = (x+2)^2$

$HT = -2$

$f(x) + 1 = ?$

$f(x) = x^2$

$f(x) + 1 = x^2 + 1$ $f(x) + 1$

Let's call it $m(x)$

$m(x) = ?$

$m(x) = f(x) + 1$

$m(x) = x^2 + 1$

$VT = +1$

$f(x)$ does not mean $f \times x$
$f(x)$ is one thing
We dont divide by any part of $f(x)$ or $f(\#)$
Cant Distribute into/Factor out of a function $f(x)$

y is a variable
f is a function

$y = f(x)$ $y = m(x)$ $y = g(x)$

$g(x) \neq f(x) \neq m(x)$
Unless they do

C12 - 1.1 - VHT Graph y= Notes

Vertical Translation Up One

$$VT = +1$$

$$y = x^2$$
$$y - 1 = x^2 \qquad y \to y - 1$$
$$y = x^2 + 1$$

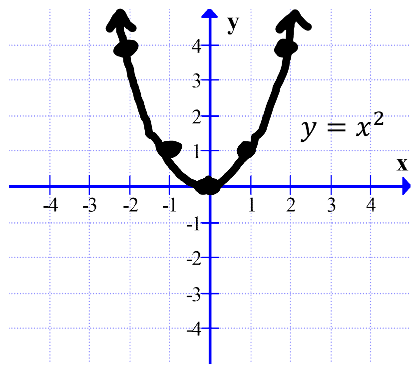
$$\boxed{Put\ y - 1\ in\ for\ y}$$

<u>Substitute the Opposite Operation for the Variable</u>

$$\boxed{g(x) = x^2 + 1}$$

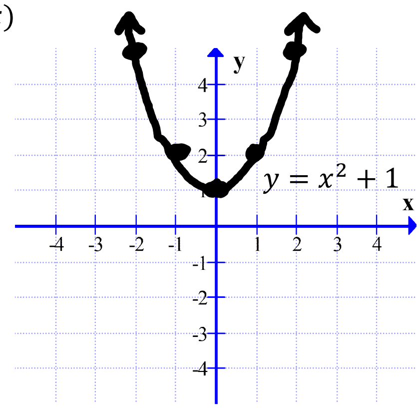
Let's call it $g(x)$

$y = f(x)$



x	y
-2	4
-1	1
0	0
1	1
2	4

$g(x) = x^2 + 1$



x	y
-2	5
-1	2
0	1
1	2
2	5

Add 1 to the y-value

$Up\ 1$

Horizontal Translation Left One

$$HT = -1$$

$$y = \sqrt{x} \qquad x \to x + 1$$
$$y = \sqrt{x+1}$$

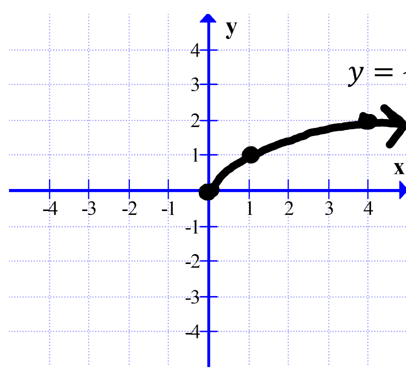
$$\boxed{Put\ x + 1\ in\ for\ x}$$

<u>Substitute the Opposite Operation for the Variable</u>

$$\boxed{m(x) = \sqrt{x+1}}$$

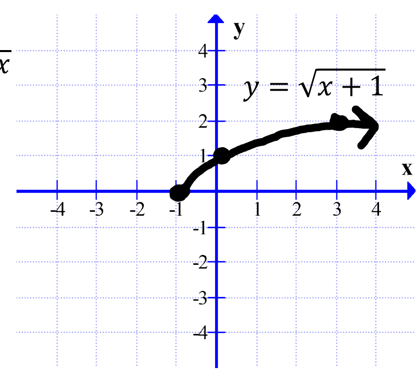
Let's call it $m(x)$

$y = f(x)$



x	y
-1	und
0	0
1	1
4	2

$m(x) = f(x+1)$



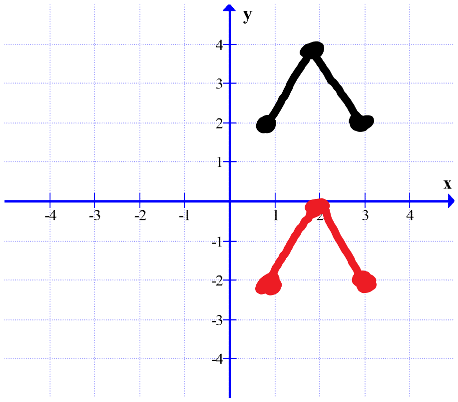
x	y
-2	und
-1	0
0	1
3	2

Subtract 1 from the x-value

$Left\ 1$

C12 - 1.1 - VHT Graphs f(x) Notes

Find the transformed equation of $f(x)$ in all forms.



$y = f(x)$

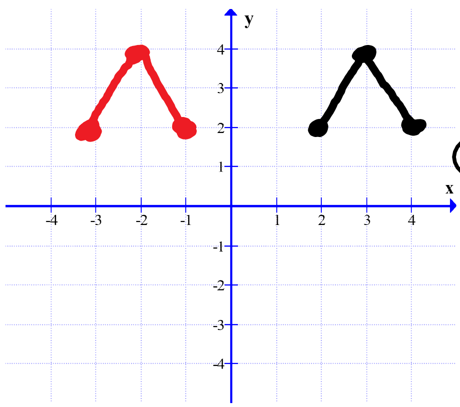
$y = f(x) + k$ $\quad$ $y - k = f(x)$

$y = f(x) + 4$ $\quad$ $y - 4 = f(x)$

$$y = f(x)$$
$$y - 4 = f(x)$$
$$y = f(x) + 4$$

$VT = 4 \quad y \rightarrow y - 4$

$y = f(x)$



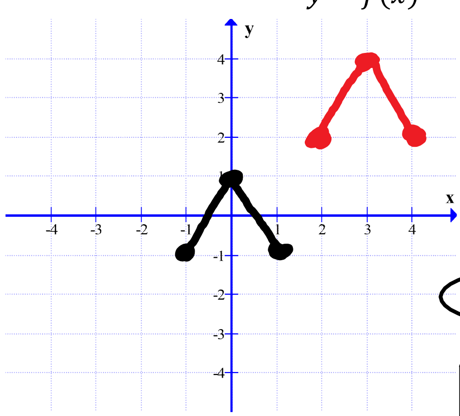
$y = f(x - h)$

$y = f(x - 5)$

$$y = f(x)$$
$$y = f(x - 5)$$

$HT = +5 \quad x \rightarrow x - 5$

$y = f(x)$



$y = f(x - h) + k$

$y = f(x + 3) - 3$

$$y = f(x)$$
$$y = f(x + 3)$$
$$y + 3 = f(x + 3)$$
$$y = f(x + 3) - 3$$

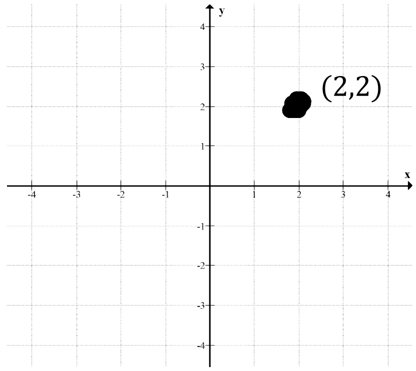
$HT = -3 \quad x \rightarrow x + 3$

$VT = -3 \quad y \rightarrow y + 3$

C12 - 1.2 - VHCE Point Notes

Find new point

$\boxed{y = f(x)}$



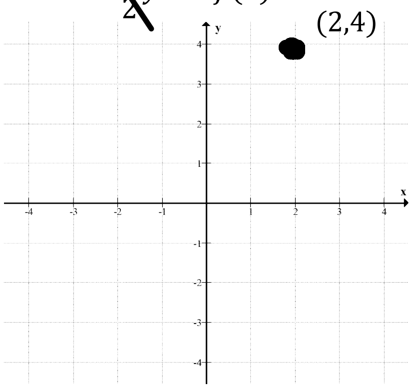
Point

$(2,2)$ *is on* $f(x)$

Function Notation

$y = f(x)$

$\cancel{\frac{1}{2}}y = 2f(x)$



Operation

Mapping Notation

$(x, 2y)$

$\underline{(2,2)}$

$VE = 2$ $(2,4)$

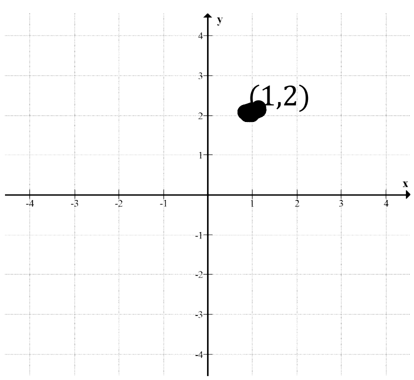
Multiply y-value by 2

$\boxed{y \; times \; 2}$

A Vertical Expansion by a Factor of 2

$2y$

$g(x) = f(2x)$



$\underline{(2,2)}$

$HC = \frac{1}{2}$ $(1,2)$

Multiply x-value by $\frac{1}{2}$

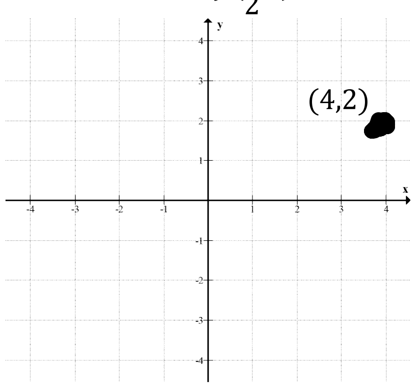
$(\frac{1}{2}x, y)$

$\boxed{x \; times \; \frac{1}{2}}$

A Horizontal Compression by a Half

$\frac{1}{2}x$

$h(x) = f(\frac{1}{2}x)$



$\underline{(2,2)}$

$(2x, y)$

$HE = 2$ $(4,2)$

Multiply x-value by 2

$\boxed{x \; times \; 2}$

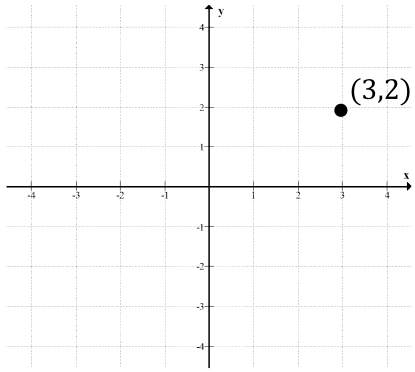
A Horizontal Expansion by 2

$2x$

Do exactly what you see outside of the brackets on the right-hand side to the **<u>y-value</u>**
Do the **<u>Opposite</u>** of what you see inside the brackets to the **<u>x-value</u>**. Attached to the variable.
Do the **<u>Opposite</u>** of what you see on the left hand side to the **<u>y-value</u>**. Attached to the variable.

C12 - 1.2 - VHR Point Notes

Find $g(x)$

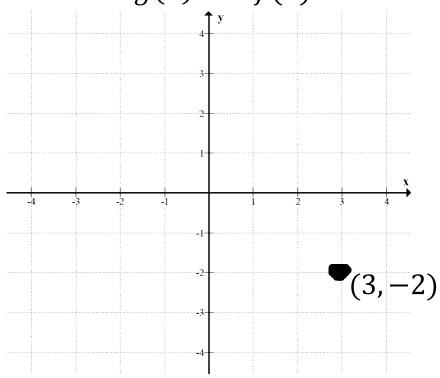


Point

$(3,2)$ *is on* $f(x)$

Function Notation

$g(x) = -f(x)$



Operation

$\underline{(3,2)}$

VR $(3,-2)$

Multiply y-value by -1

$\boxed{y \text{ times } -1}$

A Vertical Reflection

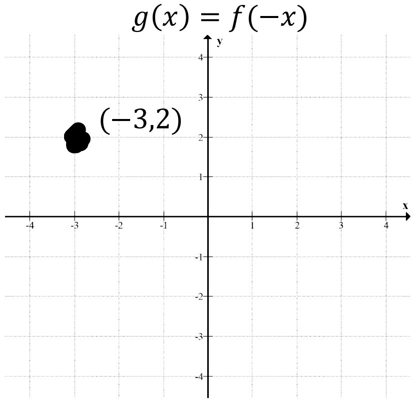
$-y$

Mapping Notation

$(x,-y)$

Reflection

$g(x) = f(-x)$



$\underline{(3,2)}$

HR $(-3,2)$

Multiply x-value by -1

$\boxed{x \text{ times } -1}$

A Horizontal Reflection

$-x$

$(-x,y)$

Reflection

Remember: beDMAS. Function Operations 1st. Inside Out.

C12 - 1.2 - VHCER Function Notation f(x) Notes

$\boxed{y = f(x)}$

$\boxed{f(x) = x^2}$ *Given*

$f(3) = ?$ $(3, y)$ *What is y when x is 3.*

$f(x) = x^2$
$f(x) = (x)^2$ *Put 3 in for x.*
$f(3) = (3)^2$

$f(3) = 9$ $(3,9)$

Put whatever is inside the brackets in for x. Substitute with Brackets

$y = x$
$y = (3)^2$
$y = 9$

x	y
3	9

$f(2x) = ?$

$f(x) = x^2$

$f(2x) = (2x)^2$ *Put $2x$ in for x*

Let's call it y

Function Notation

$y = ?$

$y = f(2x)$

$y = (2x)^2$

$$HC = \frac{1}{2}$$

$2f(x) = ?$

$f(x) = x^2$

$2f(x) = 2x^2$ $2 \times f(x)$

Let's call it $k(x)$

$k(x) = ?$

$k(x) = 2f(x)$

$k(x) = 2x^2$

$$VE = 2$$

$-f(x) = ?$

$f(x) = x^2$

$-f(x) = -x^2$ $-ve\ f(x)$

Let's call it $n(x)$

$n(x) = ?$

$n(x) = -f(x)$

$n(x) = -x^2$

$$VR$$

Vertical Reflection

C12 - 1.2 - VHCE Graph y= Notes

Vertical Expansion
by a factor of 2

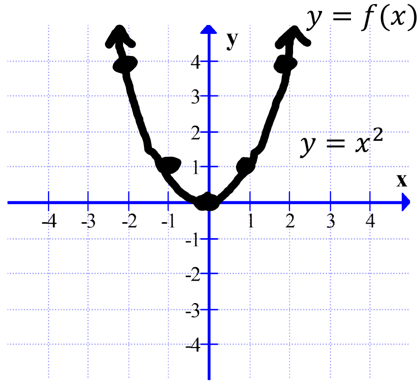
$VE = 2$

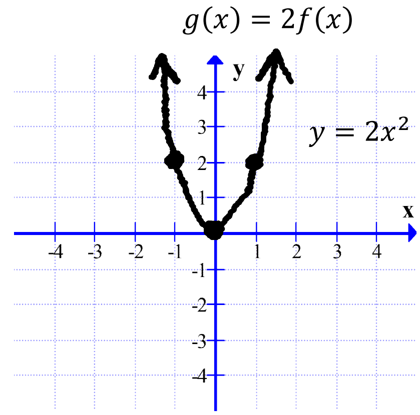
$$y = x^2$$
$$\frac{1}{2}y = x^2$$
$$y = 2x^2$$

$y \to \frac{1}{2}y$

$\boxed{Put\,\frac{1}{2}y\ in\ for\ y}$

<u>Substitute the Opposite Operation for the Variable</u>





x	y
-2	4
-1	1
0	0
1	1
2	4

x	y
-2	8
-1	2
0	0
1	2
2	8

Multiply y values by 2

Horizontal Compression
by a factor of $\frac{1}{2}$

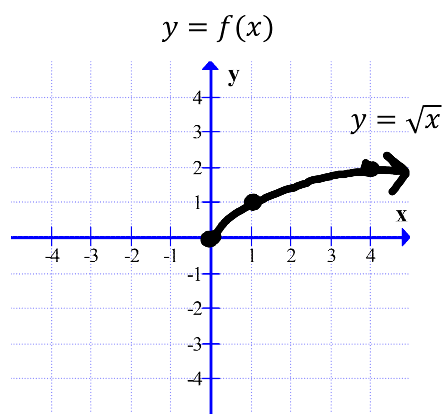
$HC = \frac{1}{2}$

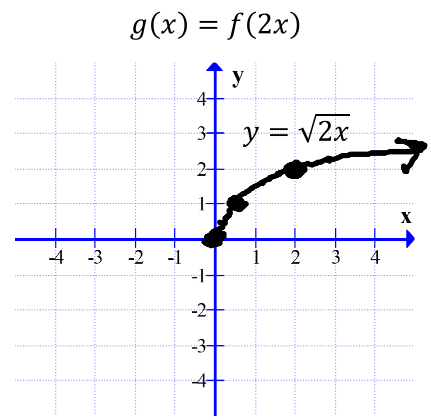
$$y = \sqrt{x}$$
$$y = \sqrt{2x}$$

$x \to 2x$

$\boxed{Put\ 2x\ in\ for\ x}$

<u>Substitute the Opposite Operation for the Variable</u>





x	y
-1	und
0	0
1	1
4	2

x	y
-1	und
0	0
$\frac{1}{2}$	1
2	2

Multiply x values by $\frac{1}{2}$

C12 - 1.2 - VHR Graph y= Notes

Vertical Reflection

VR

$$y = x^2$$
$$-y = x^2$$
$$y = -x^2$$

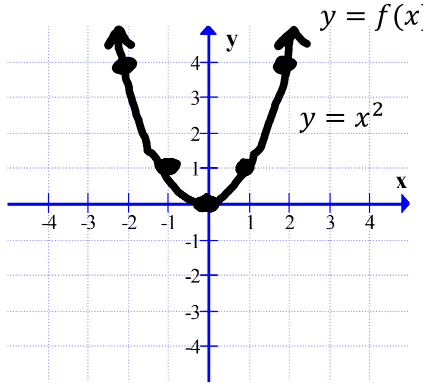
$y \to -y$

$\boxed{Put\ -y\ in\ for\ y}$

<u>Substitute the Opposite Operation for the Variable</u>

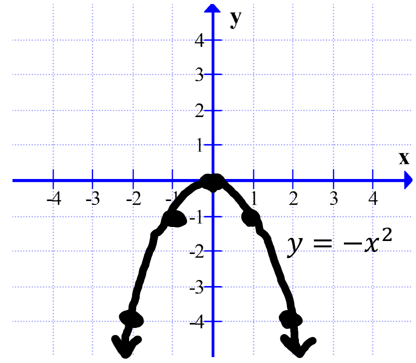
Over the x-axis

$y = f(x)$



x	y
-2	4
-1	1
0	0
1	1
2	4

$g(x) = -f(x)$



x	y
-2	-4
-1	-1
0	0
1	-1
2	-4

Multiplying y by negative 1

Horizontal Reflection

$$y = \sqrt{x}$$
$$y = \sqrt{-x}$$

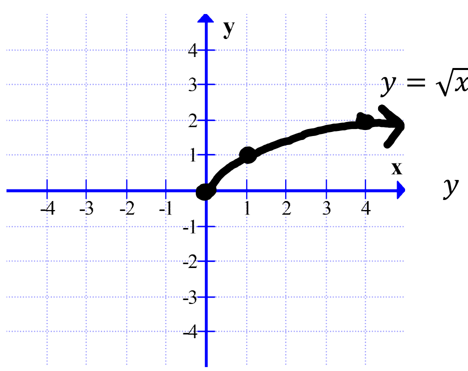
HR $\quad x \to -x$

$\boxed{Put\ -x\ in\ for\ x}$

<u>Substitute the Opposite Operation for the Variable</u>

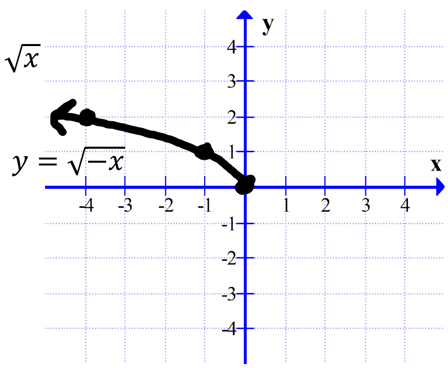
Over the y-axis

$y = f(x)$



x	y
-1	und
0	0
1	1
4	2

$g(x) = f(-x)$

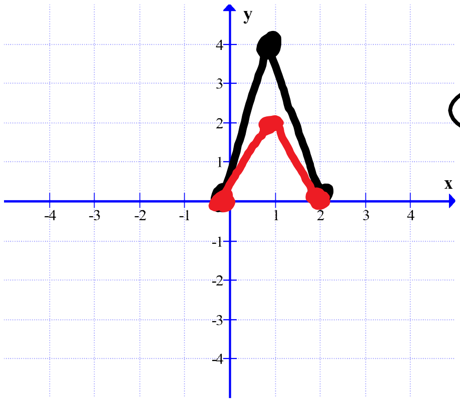


x	y
1	und
0	0
-1	1
-4	2

Multiplying x by negative 1

C12 - 1.2 - VHCER Graphs f(x) Notes

Find the transformed equation of $f(x)$ in all forms.



$y = f(x)$

$y = af(x)$

$y = 2f(x)$

$ay = f(x)$

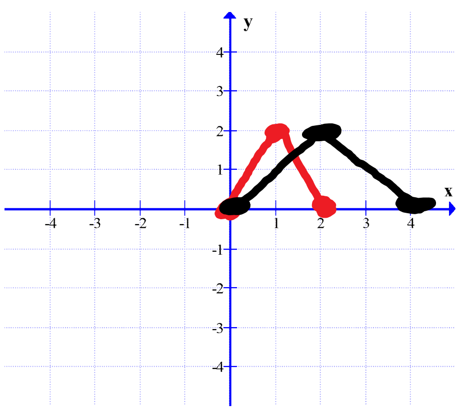
$\frac{1}{2}y = f(x)$

$y = f(x)$

$\frac{1}{2}y = f(x)$ $\quad VE = 2 \quad y \to \frac{1}{2}y$

$y = 2f(x)$

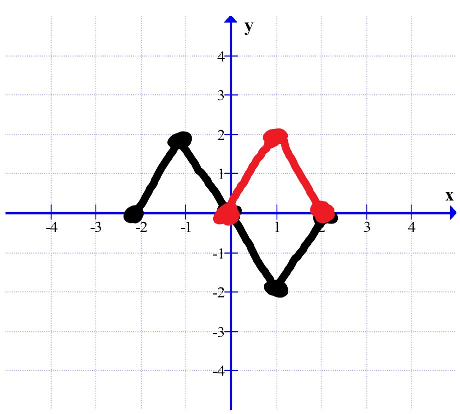
$y = f(x)$



$y = f(bx)$

$y = f(\frac{1}{2}x)$ $\quad HE = 2 \quad x \to \frac{1}{2}x$

$y = f(x)$



$y = f(bx)$

$HR \quad x \to -x$ $\quad y = f(x)$

$y = f(-x)$

$y = af(x)$

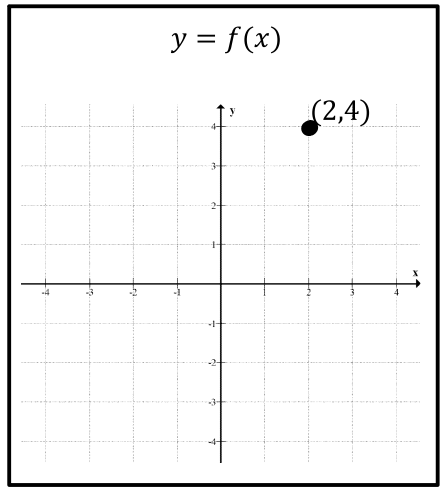
$y = f(x)$

$-y = f(x)$ $\quad VR \quad y \to -y$

$y = -f(x)$

C12 - 1.3 - VHTCER Point/s/Algebra/Factor/Order Notes

$(2,4)$ *is on* $f(x)$. *Find the point on* $g(x)$ *if*: $g(x) = f(x-2)+3$



	$\underline{(2,4)}$	
$HT = +2$	$(4,4)$	Add 2 to x-value
$VT = +3$	$(4,7)$	Add 3 to y-value

$(2,4)$ *is on* $f(x)$. *Find the point on* $g(x)$ *if*: $g(x) = -2f(x+1)-1$

	$\underline{(2,4)}$	
VR	$(2,-4)$	Multiply y-value by -1
$VE = 2$	$(2,-8)$	Multiply y-value by 2
$HT = -1$	$(1,-8)$	Subtract 1 from x-value
$VT = -1$	$(1,-9)$	Subtract 1 from y-value

$(2,4)$ *is on* $f(x)$. *Find the point on* $g(x)$ *if*: $g(x) = f\left(-\frac{1}{2}x\right)$

	$\underline{(2,4)}$	
HR	$(-2,4)$	Multiply x-value by -1
$HE = 2$	$(-4,4)$	Multiply x-value by 2

$(2,4)$ *and* $(4,6)$ *are on* $f(x)$. *Find the point on* $g(x)$ *if*: $g(x) = f(2(x-2))$

	$\underline{(2,4)}$	$\underline{(4,6)}$	
$HC = \frac{1}{2}$	$(1,4)$	$(2,6)$	Multiply x-value by a half
$HT = +2$	$(3,4)$	$(4,6)$	Add 2 to x-value

Two Points

$$g(x) = f(2x-4)$$
$$g(x) = f(2(x-2))$$

$HC = \frac{1}{2}$
$HT = +2$

$$y = f(1-x)$$
$$y = f(-(-1+x))$$
$$y = f(-(x-1)$$

HR
$HT = +1$

Factor Brackets

; so x has a coefficient of 1

$$2g(x) - 4 = f(x)$$
$$2g(x) = f(x) + 4$$
$$g(x) = \frac{1}{2}f(x) + 2$$

Algebra
$VC = \frac{1}{2}$
$VT = +2$

$(2,4)$ *is on* $f(x)$. *Find the point on* $g(x)$ *if*: $g(x) = f^{-1}(x+2)$

1.4

	$\underline{(2,4)}$	
f^{-1}	$(4,2)$	Function operations 1st
$HT = -2$	$(2,2)$	Subtract 2 from x

C12 - 1.3 - VHTCER Function Notation f(x) Notes

$\boxed{y = f(x)}$

$\boxed{f(x) = x^2}$

$3f(-x) + 2 = ?$

$f(x) = x^2$

$3 \times f(-x) + 2$

$3f(-x) + 2 = 3(-x)^2 + 2$

Let's call it $d(x)$

Function Notation

$d(x) = ?$

$d(x) = 3f(-x) + 2$

$d(x) = 3(-x)^2 + 2$

$2f(x-1) + 5 = ?$

$f(x) = x^2$

Put $x - 1$ in for x
$+5$ to $2f(x-1)$

$2f(x-1) + 5 = 2(x-1)^2 + 5$

Let's call it $n(x)$

$n(x) = ?$

$n(x) = 2f(x-1) + 5$

$n(x) = 2(x-1)^2 + 5$

C12 - 1.3 - VHTCER y= Notes

Find the new equation.

$y = x^2 + x$

A Horizontal Reflection
A vertical expansion by a factor of 2
A vertical translation up 1
A horizontal translation left 5

$y = x^2 + x$

$y = (-x)^2 + (-x) \longrightarrow HR \longrightarrow x \to -x$

Algebra

$y = x^2 - x$

$\frac{1}{2}y = x^2 - x \longrightarrow VE = 2 \longrightarrow y \to \frac{1}{2}y$

Algebra

$y = 2x^2 - 2x$

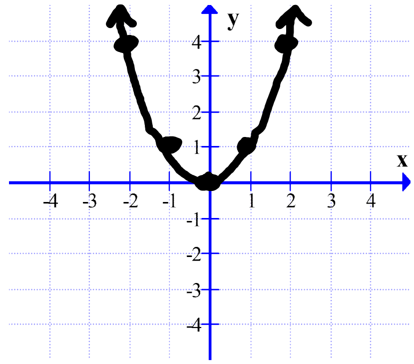
$y - 1 = 2x^2 - 2x \longrightarrow VT = +1 \longrightarrow y \to y - 1$

$y = 2x^2 - 2x + 1$

$y = 2(x+5)^2 - 2(x+5) + 1 \longrightarrow HT = -5 \longrightarrow x \to x + 5$

Foil?

C12 - 1.3 - VHTCER Graph y= Notes



$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

Vertical Expansion by a factor of 2 **<u>AND A</u>** Vertical Translation Up One

$$y = x^2$$
$$\frac{1}{2}y = x^2$$
$$y = 2x^2$$

$VE = 2$ $\quad y \to \frac{1}{2}y$

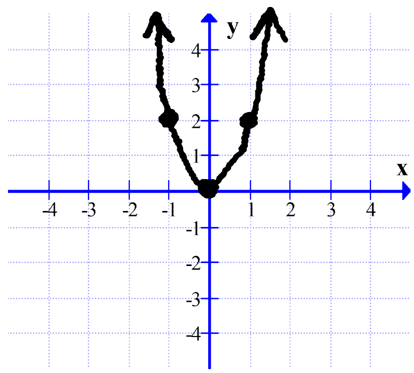
$\boxed{Put\ \frac{1}{2}y\ in\ for\ y}$

<u>Substitute the Opposite Operation for the Variable</u>

$VE = 2$

$y \times 2$

Multiply y values by 2



$y = 2x^2$

x	y
-2	8
-1	2
0	0
1	2
2	8

$$y = 2x^2$$
$$y - 1 = 2x^2$$

$VT = +1$ $\quad y \to y - 1$

$\boxed{Put\ y - 1\ in\ for\ y}$

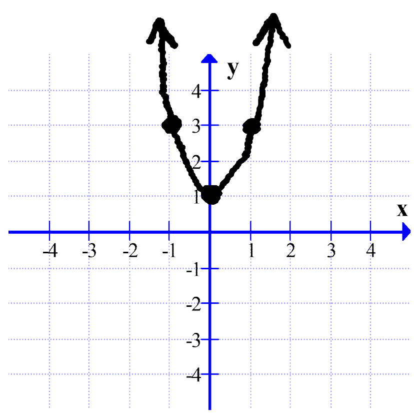
<u>Substitute the Opposite Operation for the Variable</u>

$$y = 2x^2 + 1$$

$VT = +1$

$y + 1$

$Up\ 1$



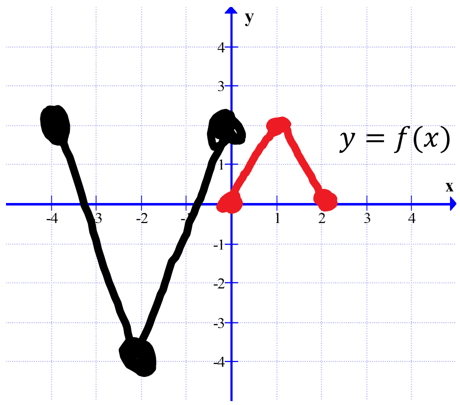
$y = 2x^2 + 1$

x	y
-2	9
-1	3
0	1
1	3
2	9

Add 1 to the y − values

C12 - 1.3 - VHTCER Graph f(x) Notes

Find the transformed equation.



$y = af(b(x-h)) + k$

Or do multiple intercepts to make sure.

$y = f(x)$

How wide is it?

2 units

How wide is it now?

4 units

What happened?

HE=2

$\boxed{x \to \frac{1}{2}x}$

$y = f\left(\frac{1}{2}x\right)$

How tall is it?

2 units

How tall is it now?

6 units

What happened?

VE=3

$\boxed{y \to \frac{1}{3}y}$

$\frac{1}{3}y = f\left(\frac{1}{2}x\right)$

$y = 3f\left(\frac{1}{2}x\right)$

Any reflections?

VR

$\boxed{y \to -y}$

$-y = 3f\left(\frac{1}{2}x\right)$

$y = -3f\left(\frac{1}{2}x\right)$

Pick a point, not an intercept,
do expansions, compressions,
and reflections.

	$(1,2)$
$HE = 2$	$(2,2)$
$VE = 3$	$(2,6)$
VR	$(2,-6)$

Has it moved?

	$(2,-6)$
$HT = -4$	$(-2,-6)$
$VT = +2$	$(-2,-4)$

$\boxed{\begin{array}{l} x \to x+4 \\ y \to y-2 \end{array}}$

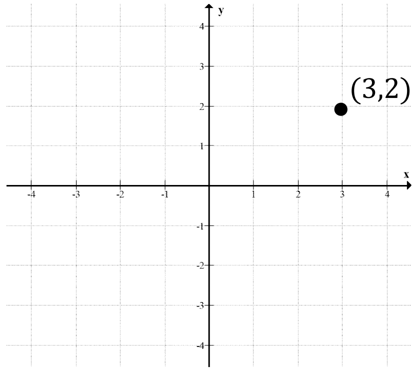
$y = -3f\left(\frac{1}{2}(x+4)\right)$

$y - 2 = -3f\left(\frac{1}{2}(x+4)\right)$

$y = -3f\left(\frac{1}{2}(x+4)\right) + 2$

C12 - 1.4 - Point $f^{-1}(x)$ Inverse Notes

$Find\ g(x)$

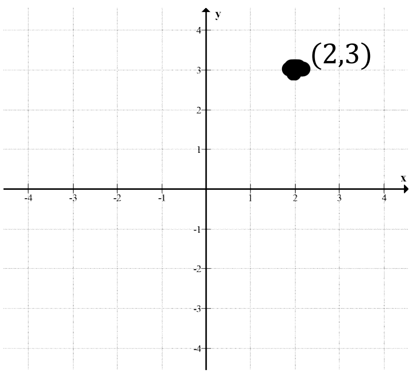


Point

$(3,2) is\ on\ f(x)$

Function Notation

$g(x) = f^{-1}(x)$



Operation

$\underline{(3,2)}$

$f^{-1}(x)$ $(2,3)$

$Switch\ x\ and\ y$

Inverse

$\boxed{x <\text{-}> y}$

(y, x)

Mapping Notation

(y, x)

Inverse 1st. Function Operations 1st. Inside Out.

C12 - 1.4 - Graph/Algebra $f^{-1}(x)$ Inverse Notes

$f(x) = 2x + 2$

$$y = 2x + 2$$
$$x = 2y + 2$$

$\boxed{\text{Switch x and y}}$

$$x - 2 = 2y$$
$$\frac{x}{2} - 1 = y$$

$\boxed{y = f(x)}$

$$y = \frac{1}{2}x - 1$$

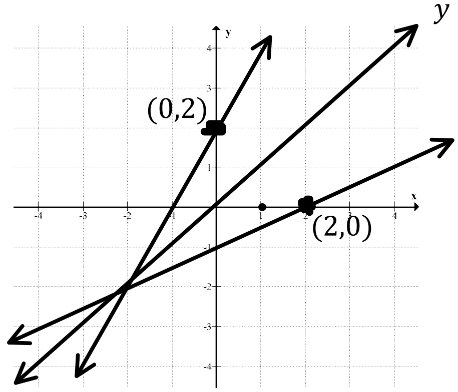
Solve for y

$$f^{-1}(x) = \frac{1}{2}x - 1$$

Write in Function Notation

$f(x) = 2x + 2$

x	2x+2
-2	-2
-1	0
0	2
1	4
2	6



$f^{-1}(x) = \frac{1}{2}x - 1$

x	2x-4
-2	-2
0	-1
2	0
4	1
6	2

Remember: The inverse is a diagonal reflection over the line $y = x$

$\longleftarrow$ *Switch* x and y $\longrightarrow$

Check your answer $f^{-1}(f(x)) = ?$

$$f^{-1}(x) = \frac{1}{2}x - 1$$
$$f^{-1}(2x-4) = \frac{1}{2}(2x+2) - 1$$
$$f^{-1}(2x-4) = x$$

$f^{-1}(f(x)) = x$ ✓

$f(f^{-1}(x)) = ?$

$$f(x) = 2x + 2$$
$$f\left(\frac{1}{2}x + 2\right) = 2\left(\frac{1}{2}x - 1\right) + 2$$
$$f\left(\frac{1}{2}x + 2\right) = x$$

$f(f^{-1}(x)) = x$ ✓

$$f(x) = \frac{x}{x+1}$$
$$y = \frac{x}{x+1}$$
$$x = \frac{y}{y+1}$$
$$x(y+1) = y$$
$$xy + x = y$$
$$x = y - xy$$
$$x = y(1-x) \qquad GCF = y$$
$$\frac{x}{1-x} = y$$
$$y = \frac{x}{1-x}$$
$$f^{-1}(x) = \frac{x}{1-x}$$

Switch x and y
Multiply
Distribute
Combine like terms (y's on one side)
Factor
Divide

A function has an inverse function if it is One-to-One, Or if you restrict the domain.

C12 - 1.5 - Order Matters Point/Functions Notes $\boxed{y = f(x)}$

Find the new point.

$(x, f(x)) = (2,4)$

x	y
2	4

A vertical expansion by a factor of 2
A vertical translation up 2

$$\begin{array}{ll} & \underline{(2,4)} \\ VE = 2 & (2,8) \\ VT = +2 & (2,10) \end{array}$$

x	y
2	10

A vertical translation up 2
A vertical expansion by a factor of 2

$$\begin{array}{ll} & \underline{(2,4)} \\ VT = +2 & (2,6) \\ VE = 2 & (2,12) \end{array}$$

Find the new equation.

$f(x) = x^2$

x	y
2	4

A vertical expansion by a factor of 2
A vertical translation up 2

$$\begin{aligned} f(x) &= x^2 \\ y &= x^2 \\ \frac{1}{2}y &= x^2 \\ y &= 2x^2 \\ y - 2 &= 2x^2 \end{aligned}$$

$Put \frac{1}{2}y \ in \ for \ y$

$Put \text{ "}y - 2\text{" } in \ for \ y$

$$y = 2x^2 + 2$$

x	y
2	10

A vertical translation up 2
A vertical expansion by a factor of 2

$$\begin{aligned} f(x) &= x^2 \\ y &= x^2 \\ y - 2 &= x^2 \\ y &= x^2 + 2 \\ \frac{1}{2}y &= x^2 + 2 \end{aligned}$$

$Put \text{ "}y - 2\text{" } in \ for \ y$

$Put \frac{1}{2}y \ in \ for \ y$

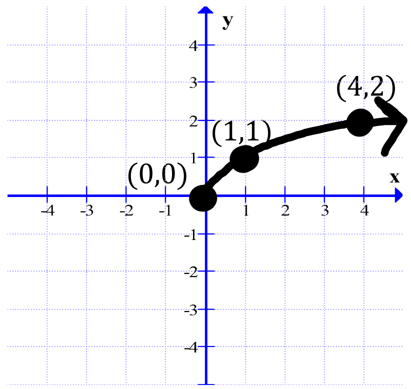
$$y = 2x^2 + 4$$

Remember: We always substitute the opposite operation for the variable.
Remember: Order matters. An addition then a multiplication is far different from the same multiplication and then the same addition. **Think about it!**
Remember: Do the operations in the order you are asked or follow DMAS

C12 - 2.1 - Radical Translations Notes

$y = \sqrt{x}$

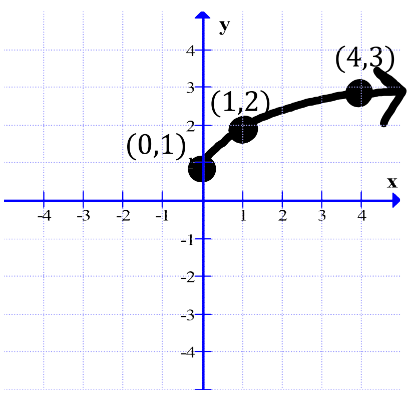
x	y
-1	und
0	0
1	1
4	2
9	3



$y = \sqrt{x}$

Notice it's half a parabola!

Remember: Choose increments of x in your table of values that square root easily.



$y = \sqrt{x} + 1$

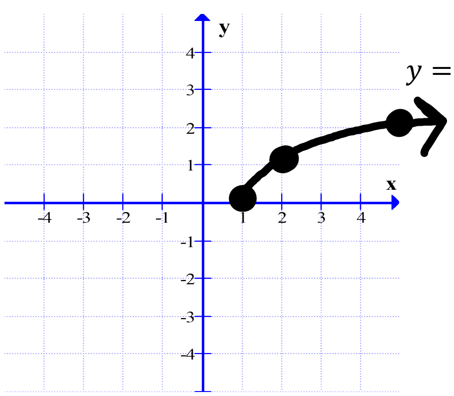
Up 1

Domain: Set Underneath root ≥ zero and solve

Range

$x \geq 0$

$y \geq 1$



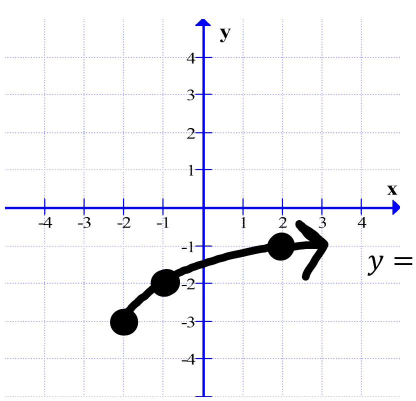
$y = \sqrt{x - 1}$

Right 1

$x - 1 \geq 0$

$x \geq 1$

$y \geq 0$



$y = \sqrt{x + 2} - 3$

Left 2, Down 3

$x + 2 \geq 0$

$x \geq -2$

$y \geq -3$

$\boxed{y = \sqrt{x - h} + k}$

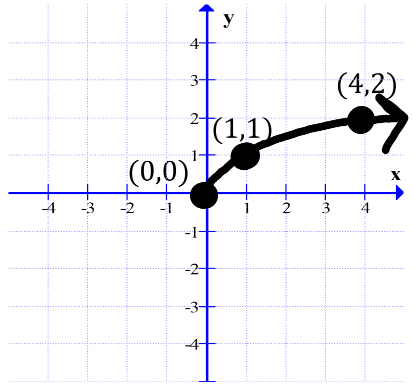
Vertex (h, k)

Vertex $(-2, -3)$

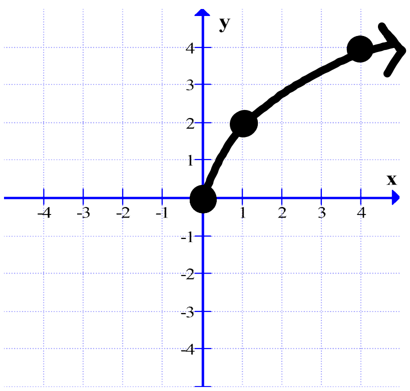
C12 - 2.2 - Radical Transformations Notes

$y = \sqrt{x}$

x	y
-1	und
0	0
1	1
4	2
9	3



$y = \sqrt{x}$

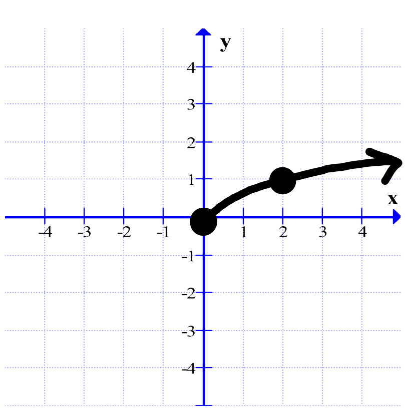


$y = 2\sqrt{x}$

Vertical Expansion = 2

Domain: $x \geq 0$

$Range$ $y \geq 0$



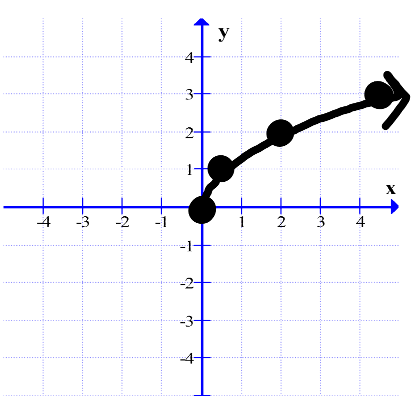
$y = \sqrt{\frac{1}{2}x}$

Horizontal Expansion = 2

$\frac{1}{2}x \geq 0$

$x \geq 0$

$y \geq 0$



$y = \sqrt{2x}$

$Horizontal\ Compression\ = \frac{1}{2}$

$2x \geq 0$

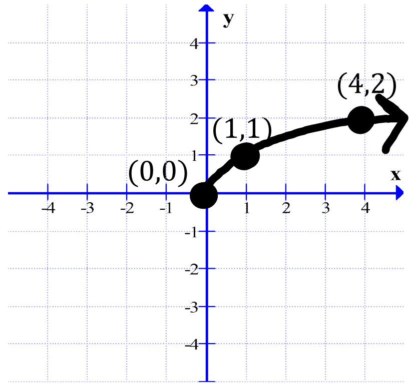
$x \geq 0$

$y \geq 0$

C12 - 2.3 - Radical Reflections Notes

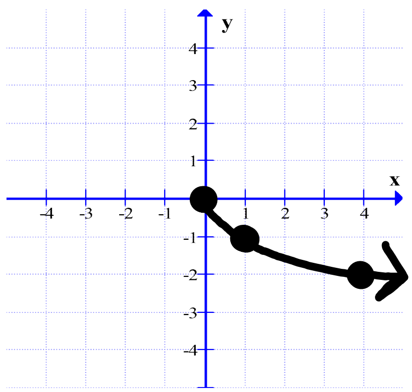
$y = \sqrt{x}$

x	y
-1	und
0	0
1	1
4	2
9	3



$y = \sqrt{x}$

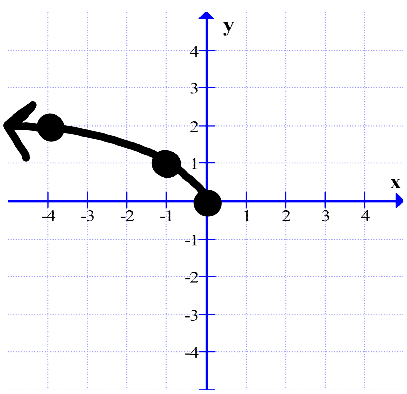
	Domain:	$Range$
Vertical Reflection		



$y = -\sqrt{x}$

Domain: $x \geq 0$, $x \geq 0$

Range: $y \leq 0$

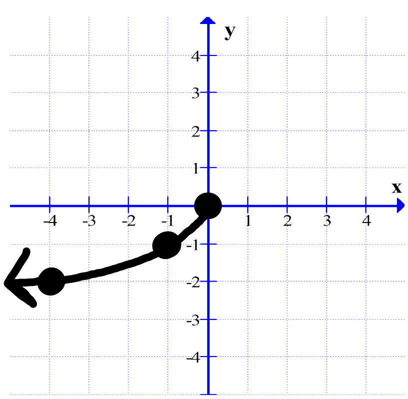


$y = \sqrt{-x}$

Horizontal Reflection

Domain: $-x \geq 0$, $x \leq 0$

Range: $y \geq 0$



$y = -\sqrt{-x}$

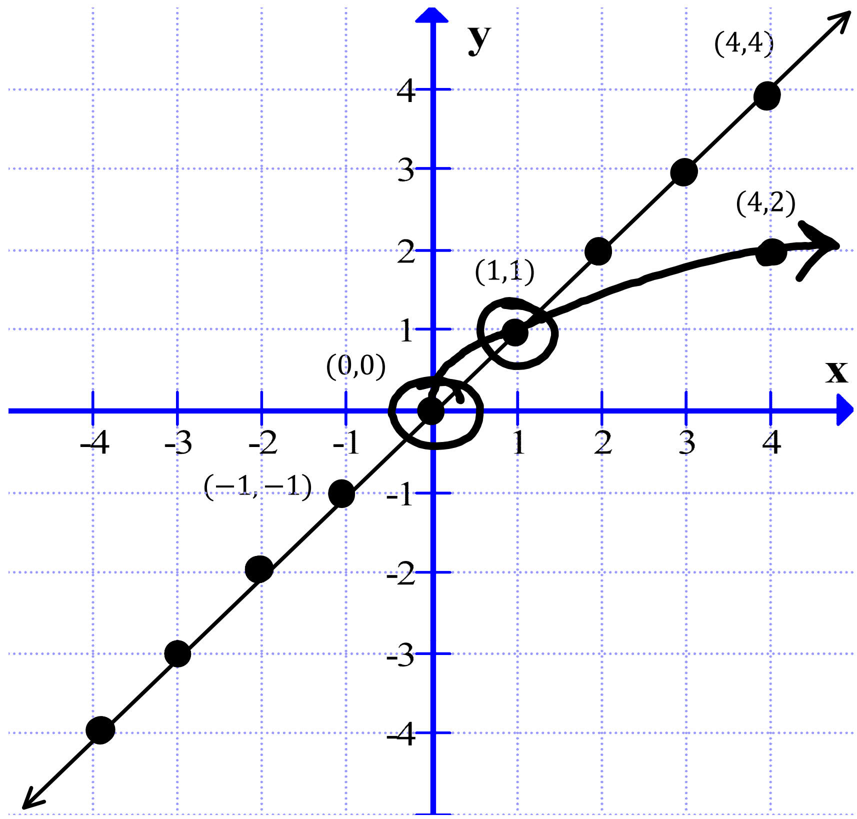
Vertical and Horizontal Reflection

Domain: $-x \geq 0$, $x \leq 0$

Range: $y \leq 0$

C12 - 2.4 - Square Root Functions Notes

Draw the graph of $\sqrt{x}$ *from the graph of* $f(x)$ *and label the invariant points and state the domain and range.*



x	$f(x)$	$\sqrt{f(x)}$
-1	-1	und
0	0	0
1	1	1
4	4	2

Pick an x value on $f(x)$.
Square root the $y - value$
Draw the new point.

$y = x$

x	$y = f(x)$
-1	-1
0	0
1	1
4	4

Invariant Points:

$(0,0)$

$(1,1)$

$y = \sqrt{x}$

x	$\sqrt{f(x)}$
-1	und
0	0
1	1
4	2

Domain: $x \in \mathbb{R}$

Range: $y \in \mathbb{R}$

Domain: $x \geq 0$

Range: $y \geq 0$

Remember: Cant square root a negative
Remember: Choose x-values whose y values can square root evenly if possible
Remember: Invariant points are on the line $y = 1$ *and* $y = 0$
Remember: Any point with a $y - value\ of$ "1" or "0" is *invariant*. $(x, 1)$ *and* $(x, 0)$

C12 - 3.1 - Long/Synthetic Division $R = 0$ Notes

$\frac{64}{4} = ?$

Goes Into
Multiply
Subtract
Bring Down
Repeat

$$\begin{array}{r} 16 \\ 4\overline{)64} \\ -\ 4\downarrow \\ \hline 24 \\ -\ 24 \\ \hline 0 \end{array}$$

Bring down

remainder

$$\text{divisor}\ \overline{)\ \text{dividend}}^{\ \text{quotient}}$$

$\frac{64}{4} = 16$

$\boxed{\frac{\text{dividend}}{\text{divisor}} = \text{quotient}}$

$64 = 4 \times 16$

$\boxed{\text{dividend} = (\text{quotient})(\text{divisor})}$

$\frac{x^2 + 5x + 6}{x + 3} = ?$

$$\begin{array}{r} x + 2 \\ x + 3\ \overline{)\ x^2 + 5x + 6} \\ -\quad x^2 + 3x \quad\downarrow \\ \hline 2x + 6 \\ -\quad 2x + 6 \\ \hline 0 \end{array}$$

remainder

$\frac{x^2 + 5x + 6}{x + 3} = x + 2$

$\boxed{\frac{P(x)}{x - a} = Q(x)}$

$x^2 + 5x + 6 = (x + 2)(x + 3)$

$\boxed{P(x) = Q(x)(x - a)}$

Synthetic Division

$\frac{x^2 + 5x + 6}{x + 3} = ?$

$x + 3 = 0$
$x = -3$

$1x^2 + 5x + 6$

$$\begin{array}{r|rrr} -3 & 1 & 5 & 6 \\ + & \downarrow & -3 & -6 \\ \hline & 1 & 2 & 0 \end{array}$$

$\leftarrow$ remainder

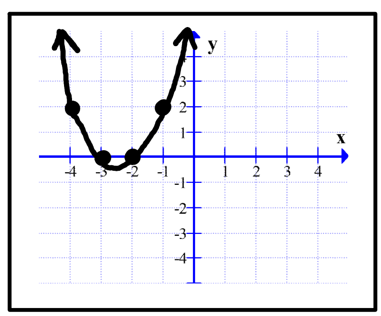
$1x + 2$

The exponents of x go down by one.

Factor Theorem

$f(x) = x^2 + 5x + 6$
$f(-3) = (-3)^2 + 5(-3) + 6$
$f(-3) = 0$

$(-3,0)$



C12 - 3.1 - Long/Synthetic Division Notes

$\frac{65}{4} = ?$

$$\begin{array}{r l} & \underline{\;16\;} \\ 4\,) & 65 \\ - & \underline{4\;\;} \\ & 25 \quad \textit{Bring down} \\ - & \underline{24} \\ & \;\;1 \end{array}$$

remainder

$$\begin{array}{r|l} divisor & \overline{\;dividend\;} \end{array} \quad \text{with } quotient \text{ on top}$$

$\frac{65}{4} = 16 + \frac{1}{4}$

$65 = 4 \times 16 + 1$

$$\boxed{\frac{dividend}{divisor} = quotient + \frac{remainder}{divisor}}$$

$$\boxed{dividend = (quotient)(divisor) + remainder}$$

$\frac{x^2 + 5x + 9}{x + 3} = ?$

$$\begin{array}{r l} & \underline{\;x + 2\;} \\ x + 3\,) & x^2 + 5x + 9 \\ - & \underline{x^2 + 3x\;\;\;\;} \\ & \;\;\;\;\;\;2x + 9 \\ - & \underline{\;\;\;\;\;\;2x + 6} \\ & \;\;\;\;\;\;\;\;\;\;\;\;\;3 \end{array}$$

remainder

$\frac{x^2 + 5x + 9}{x + 3} = x + 2 + \frac{3}{x + 3}$

$x^2 + 5x + 9 = (x + 2)(x + 3) + 3$

$$\boxed{\frac{P(x)}{x - a} = Q(x) + \frac{R}{x - a}}$$

$$\boxed{P(x) = Q(x)(x - a) + R}$$

Synthetic Division

$\frac{x^2 + 5x + 9}{x + 3} = ?$

$x + 3 = 0$
$x = -3$

$1x^2 + 5x + 9$

$$\begin{array}{r|rrr} -3 & 1 & 5 & 9 \\ + & \downarrow & -3 & -6 \\ \hline & 1 & 2 & 3 \end{array}$$

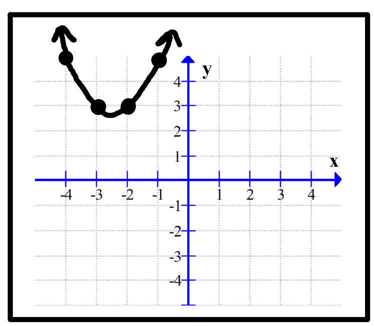
remainder

$1x + 2 \quad R: 3$

Remainder Theorem

$f(x) = x^2 + 5x + 6$
$f(-3) = (-3)^2 + 5(-3) + 9$
$f(-3) = 3$

$(-3, 3)$



C12 - 3.1 - Synthetic Division $R = 0$ Notes

$$\frac{x^3 + x^2 - 8x + 4}{x - 2}$$

$x - 2 = 0$
$x = 2$

Set denominator equal to zero and solve.
$Denominator = 0$

$$\begin{array}{c|cccc} 2 & 1 & 1 & -8 & 4 \\ + & & & & \\ \hline \end{array}$$

Place that number to the left.
Write the coefficients. $1x^3 + 1x^2 - 8x + 4$

$$\begin{array}{c|cccc} 2 & 1 & 1 & -8 & 4 \\ + & \downarrow & 2 & 6 & -4 \\ \hline & 1 & 3 & -2 & 0 \end{array}$$

1) Bring down the first coefficient
2) $(2) \times 1 = 2$
3) $1 + 2 = 3$
4) Repeat last two steps.

$1x^2 + 3x - 2 \qquad R = 0$

$$\frac{x^3 + x^2 - 8x + 4}{x - 2} = x^2 + 3x - 2$$

$$x^3 + x^2 - 8x + 4 = (x^2 + 3x - 2)(x - 2)$$

$$\boxed{\frac{dividend}{divisor} = quotient}$$

$$\boxed{dividend = (quotient)(divisor)}$$

$$\frac{x^3 + 2x^2 - 5x - 6}{x + 1}$$

$x + 1 = 0$
$x = -1$

Set denominator equal to zero and solve.
$Denominator = 0$

$$\begin{array}{c|cccc} -1 & 1 & 2 & -5 & -6 \\ + & & & & \\ \hline \end{array}$$

Place that number to the left.
Write the coefficients. $1x^3 + 2x^2 - 5x - 6$

$$\begin{array}{c|cccc} -1 & 1 & 2 & -5 & -6 \\ + & \downarrow & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$$\begin{array}{rl} & x^2 + x - 6 \\ \hline x + 1 \,) & x^3 + 2x^2 - 5x - 6 \\ - & x^3 + x^2 \\ \hline & x^2 - 5x \\ - & x^2 + x \\ \hline & -6x - 6 \\ - & -6x - 6 \\ \hline & 0 \qquad R = 0 \end{array}$$

$1x^2 + 1x - 6 \qquad R = 0$

$x^2 + x - 6$
$(x + 3)(x - 2)$

Factor

$$\frac{x^3 + 2x^2 - 5x - 6}{x + 1} = (x + 3)(x - 2)$$

$$x^3 + 2x^2 - 5x - 6 = (x + 3)(x - 2)(x + 1)$$

$$\boxed{\frac{P(x)}{x - a} = Q(x)}$$

$$\boxed{P(x) = Q(x)(x - a)}$$

C12 - 3.1 - Synthetic Division Remainder/Gap Notes

$$\frac{x^3+x^2-8x+7}{x-2}$$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 7 \\ + & & & & \\ \hline & & & & \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 7 \\ + & \downarrow & 2 & 6 & -4 \\ \hline & 1 & 3 & -2 & 3 \end{array}$$

remainder

$1x^2+3x-2 \qquad R=3$

$$\begin{array}{r} x^2+3x-2 \\ x-2\,\overline{)\,x^3+x^2-8x+7} \\ -\quad \underline{x^3-2x^2 \qquad\quad} \\ +3x^2-8x \\ -\quad \underline{3x^2-6x \quad} \\ -2x+7 \\ -\quad \underline{-2x+4} \\ 3 \end{array} \qquad R=3$$

The remainder is the y value when $x=2$ $(2,3)$

$f(2)=(2)^3+(2)^2-8(2)+7$

$f(2)=8+4-16+7$

$f(2)=3$

$$\frac{x^3+x^2-8x+6}{x-2}=x^2+3x-2+\frac{3}{x-2}$$

$$x^3+x^2-8x+6=(x^2+3x-2)(x-2)+3$$

$$\boxed{\frac{dividend}{divisor}=quotient+\frac{remainder}{divisor}}$$

$$\boxed{dividend=(quotient)\times(divisor)+remainder}$$

$$\frac{x^3+2x-12}{x-2}$$

$$\boxed{1x^3+0x^2+2x-12}$$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 2 & -12 \\ + & & & & \\ \hline & & & & \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 2 & -12 \\ + & \downarrow & 2 & 4 & 12 \\ \hline & 1 & 2 & 6 & 0 \end{array}$$

$1x^2+2x+6 \qquad R=0$

$$\frac{x^3+2x-12}{x-2}=x^2+2x+6$$

$$x^3+2x-12=(x^2+2x+6)(x-2)$$

$$\frac{x^3+2x^2-6x-12}{x+2}$$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -6 & -12 \\ + & & & & \\ \hline & & & & \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -6 & -12 \\ + & \downarrow & -2 & 0 & 12 \\ \hline & 1 & 0 & -6 & 0 \end{array}$$

$1x^2+0x-6 \qquad R:0$

$x^2-6 \qquad R:0$

$$\frac{x^3+2x^2-4x+8}{x+2}=x^2-6$$

$$x^3+2x^2-4x+8=(x^2-6)(x+2)$$

C12 - 3.2 - Factor/Remainder Theorem Notes

Factor Theorem	If $(x-a)$ is a factor of $f(x)$, then:	$f(a)=0$

Is $(x-2)$ a factor of $f(x)=x^3+x^2-8x+4$?

$f(x)=x^3+x^2-8x+4$
$f(x)=(2)^3+(2)^2-8(2)+4$
$f(2)=8+4-16+4$
$f(2)=0$
$(2,0)$

$$\boxed{\begin{aligned} x-2&=0\\ x&=2\end{aligned}}$$

$f(a)=0$
$(x-a)$
Is a Factor

$x-intercept$

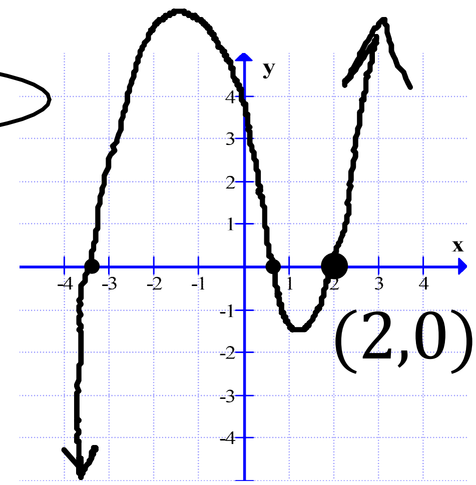
Synthetic Division

$$\frac{x^3+x^2-8x+4}{x-2}$$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 4 \\ & & 2 & 6 & -4 \\ \hline + & 1 & 3 & -2 & 0 \end{array}$$

Remainder = 0

$(x-2)$Is a Factor



Remainder Theorem	If $(x-a)$ is not a factor of $f(x)$, then:	$f(a)=remainder$

Is $(x-2)$ a factor of $f(x)=x^3+x^2-8x+5$?

$f(x)=x^3+x^2-8x+5$
$f(x)=(2)^3+(2)^2-8(2)+5$
$f(2)=8+4-16+5$
$f(2)=1$

$$\boxed{\begin{aligned} x-2&=0\\ x&=2\end{aligned}}$$

Remainder = 1

$f(a)\neq 0 \leftarrow$ R
$(x-a)$
Is Not a Factor

$(2,1)$

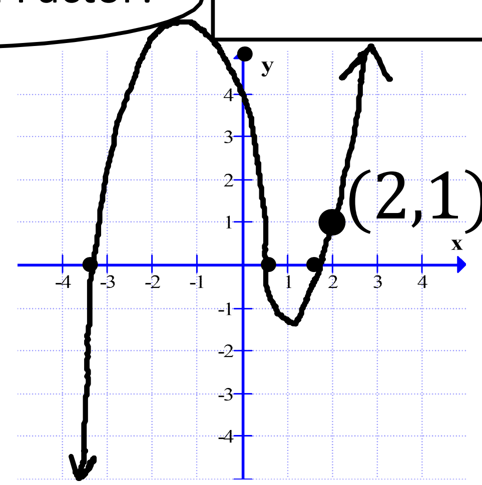
$(x-2)$ *is* Not a Factor!

Synthetic Division

$$\frac{x^3+x^2-8x+4}{x-2}$$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 5 \\ & & 2 & 6 & -4 \\ \hline + & 1 & 3 & -2 & 1 \end{array}$$

Remainder = 1



C12 - 3.2 - Find K Notes/HW

$Find\ k\ if\ (x+3)\ is\ a\ factor.$ $\boxed{f(-3)=0}$

$f(x)=x^3+2x^2+kx-6$

$f(-3)=(-3)^3+2(-3)^2+k(-3)-6=0$

$-15-3k=0$

$k=-5$

$f(x)=x^3+2x^2-5x-6$

$Find\ k\ if\ f(x)\ is\ divided\ by\ (x-1)\ and\ the\ remainder\ is\ -8.$ $\boxed{f(1)=-8}$

$f(x)=x^3+2x^2-5x+k$

$f(x)=(1)^3+2(1)^2-5(1)+k=-8$

$-2+k=-8$

$k=-6$

$f(x)=x^3+2x^2-5x-6$

$Find\ k\ if\ (x-3)\ is\ a\ factor.$

$f(x)=x^3-6x^2+kx-6$ k=11

$Find\ k\ if\ f(x)\ is\ divided\ by\ (x+3)\ and\ the\ remainder\ is\ 25.$

$f(x)=x^3+kx^2-4x-8$ k=2

$Find\ k\ if\ when\ divided\ by\ (x-5)\ the\ remainder\ is\ 24\ if\ (x-2)\ is\ a\ factor.$

$f(x)=x^3-6x^2+11x+k$ k=-6

$Find\ k\ if\ when\ divided\ by\ (x-2)\ the\ remainder\ is\ the\ same\ as\ if\ divided\ by\ (x-3).$

$f(x)=x^3+2x^2-4x+k$ k=-8

C12 - 3.3 - Factoring Trinomials Notes

$\boldsymbol{f(x) = x^2 - 6x + 5}$

Potential Factors: Factors of c = ± 5 *and* ± 1

$f(x) = x^2 \ldots\ldots\ldots + 5$ $\pm 1{,}5$

Solve by inspection.

$f(1) = 1^2 - 6(1) + 5$
$f(1) = 0$

$(x-1)$ *is a factor.*
$(1{,}0)$ $x - int$

$\boxed{\text{Stop here if you want}}$

$f(-1) = (-1)^2 - 6(-1) + 5$
$f(-1) = 12$

$(x+1)$ *is NOT a factor*
$(-1{,}12)$ (x, y)

$f(5) = 5^2 - 6(5) + 5$
$f(5) = 0$

$(x-5)$ *is a factor*
$(5{,}0)$ $x - int$

$f(x) = x^2 \ldots\ldots\ldots + 5$

Examples:

$f(x) = (x-5)(x-1)$

$f(x) = (x+5)(x+1)$

$\boxed{(x+a)(x+b) = x^2 \ldots + ab}$

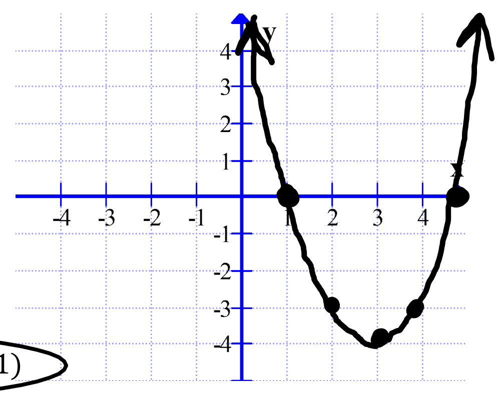
x	*y*
1	*0*
-1	*12*
5	*0*

Do synthetic division with 1

$$\begin{array}{r|rrr} 1 & 1 & -6 & 5 \\ + & \downarrow & 1 & -5 \\ \hline & 1 & -5 & 0 \end{array}$$

$x^2 - 6x + 5$

$x - 5$



$$\frac{x^2 - 6x + 5}{x - 1} = x - 5$$

$$x^2 - 6x + 5 = (x-5)(x-1)$$

Or Do synthetic division with 5!

$$\begin{array}{r|rrr} 5 & 1 & -6 & 5 \\ + & \downarrow & 5 & -5 \\ \hline & 1 & -1 & 0 \end{array}$$

$x - 1$

$$\frac{x^2 - 6x + 5}{x - 5} = x - 1$$

$$x^2 - 6x + 5 = (x-1)(x-5)$$

$\Updownarrow$ $(x-1)$*is a factor ?*
$f(1) = 0,$ *if you put* $+1$ *in for x it must equal zero,* (*or it is not a factor*)
$(+1{,}0)$ *is an* $x - intercept$

C12 - 3.3 - Factoring Quadomials Notes

$f(x) = x^3 + 2x^2 - 5x - 6$

Potential Factors: Factors of c = $\pm 1, \pm 2, \pm 3, \pm 6$

$f(x) = x^3 \ldots\ldots\ldots\ldots\ldots\ldots - 6$ $\quad \pm 1,2,3,6,$

$f(x) = x^3 \ldots\ldots. -6$

Examples:

$f(x) = (x-2)(x-3)(x-1)$

$f(x) = (x+2)(x+3)(x-1)$

$f(x) = (x+2)(x-3)(x+1)$

$\boxed{(x-a)(x+b)(x-c) = x^3 \ldots + \; abc}$

Solve by inspection.

$f(1) = (1)^3 + 2(1)^2 - 5(1) - 6$
$f(1) = 1 + 2 - 5 - 6$
$f(1) = -8$ $\quad (x-1)\ is\ NOT\ a\ factor$

$f(-1) = (-1)^3 + 2(-1)^2 - 5(-1) - 6$
$f(-1) = -1 + 2 + 5 - 6$
$f(-1) = 0$ $\quad (x+1)\ is\ a\ factor$

$\boxed{6^3 = 216, its\ not\ going\ to\ be\ 6!}$

x	y
1	-8
-1	0
6	252

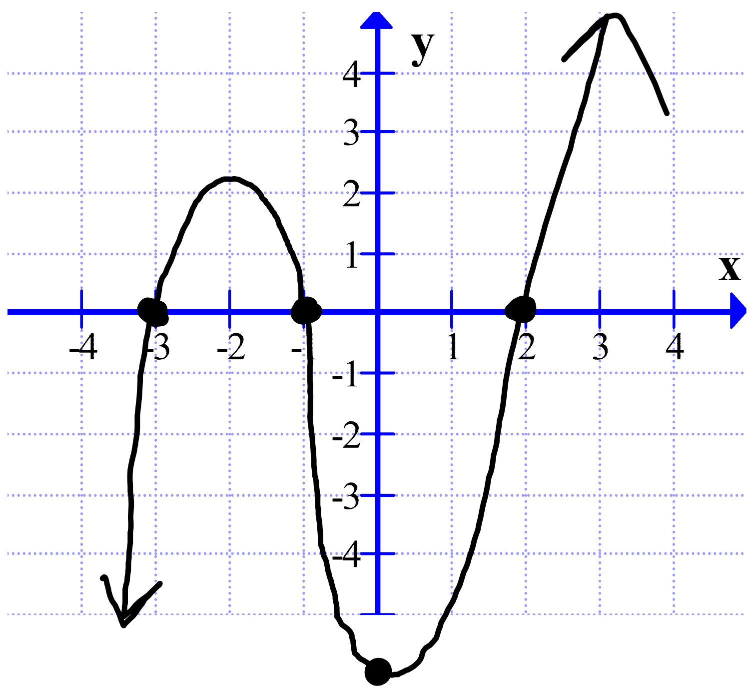
Do synthetic division with -1

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ + & \downarrow & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$1x^2 + 1x - 6$
$(x+3)(x-2)$ $\quad$ *Factor*

$f(x) = (x+3)(x-2)(x+1)$

$f(-3) = 0$
$f(2) = 0$
$f(-1) = 0$



y-int: $(0, -6)$

C12 - 3.3 - Potential Factors Notes $\pm\frac{d}{a}$

$f(x) = x^3 + x^2 - 8x + 4$

$Potential\ Factors:\ \pm1, \pm2, \pm4$

$\boxed{factors\ of\ "d"}$

Solve by inspection

$$
\begin{aligned}
f(1) &= (1)^3 + (1)^2 - 8(1) + 4 = -2 \\
f(-1) &= (-1)^3 + (-1)^2 - 8(-1) + 4 = 12 \\
f(2) &= (2)^3 + (2)^2 - 8(2) + 4 = 0
\end{aligned}
$$

$(x-1)\ is\ NOT\ a\ factor$

$(x+1)\ is\ NOT\ a\ factor$

$(x-2)\ is\ a\ factor$ (2,0)

$$
\begin{array}{c|cccc}
2 & 1 & 1 & -8 & 4 \\
+ & & 2 & 6 & -4 \\
\hline
 & 1 & 3 & -2 & 0
\end{array}
$$

$\boldsymbol{f(x) = 3x^2 + 5x - 2}$

$Potential\ Factors:\ \pm2, \pm1, \pm\frac{2}{3}, \pm\frac{1}{3}$

$factors\ of\ "c"$

$and\ \dfrac{factors\ of\ "c"}{factors\ of\ "a"}$

Solve by inspection

$$
\begin{aligned}
f(-1) &= 3(-1)^2 + 5(-1) - 2 = -4 \\
f(1) &= 3(1)^2 + 5(1) - 2 = 6 \\
f(2) &= 3(2)^2 + 5(2) - 2 = 20 \\
f(-2) &= 3(-2)^2 + 5(-2) - 2 = 0
\end{aligned}
$$

$(x+1)\ is\ NOT\ a\ factor$

$(x-1)\ is\ NOT\ a\ factor$

$(x-2)\ is\ NOT\ a\ factor$

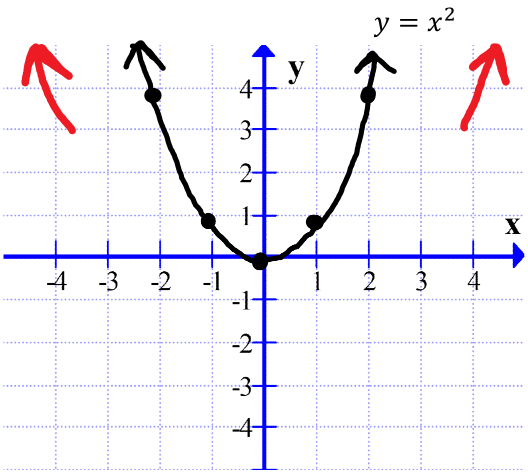
$(x+2)\ is\ a\ factor$ $(-2,0)$

$$
\begin{array}{c|ccc}
-2 & 3 & 5 & -2 \\
+ & & -6 & 2 \\
\hline
 & 3 & -1 & 0
\end{array}
$$

C12 - 3.4 - End Behaviour Polynomials Notes

Leading Term
Table of Values

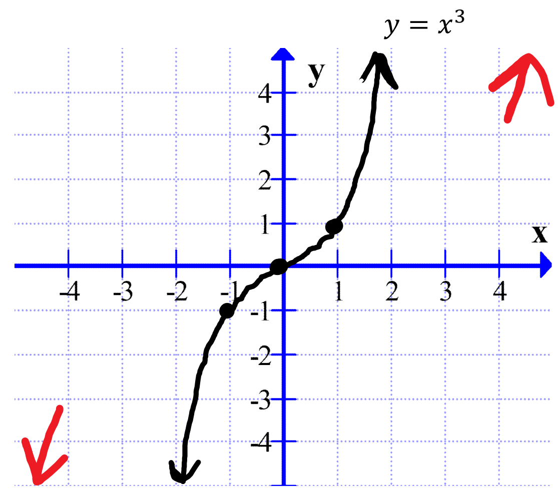
$+\#x^{even}$



Q2,Q1

x	y
-10	+
+10	+

$+\#x^{odd}$

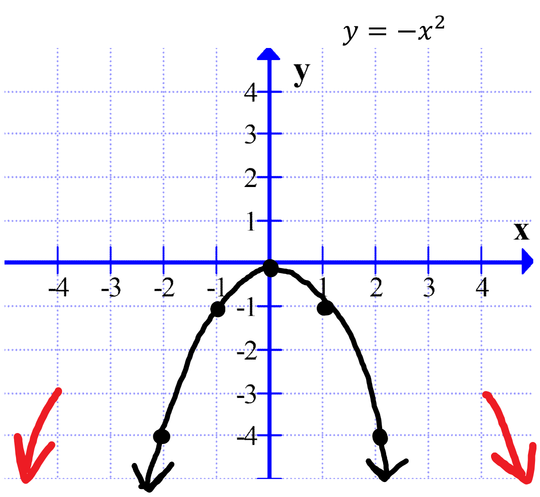


Q3,Q1

x	y
-10	-
+10	+

$y \geq \#$	
$Range$	yER
$y \leq \#$	

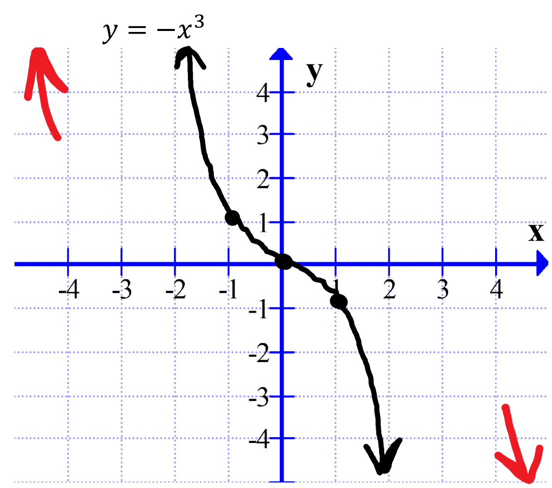
$-\#x^{even}$



Q3,Q4

x	y
-10	-
+10	-

$-\#x^{odd}$

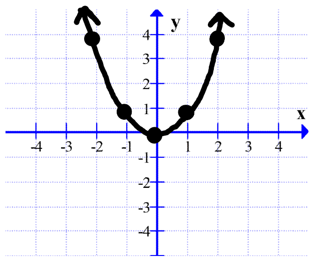


Q2,Q4

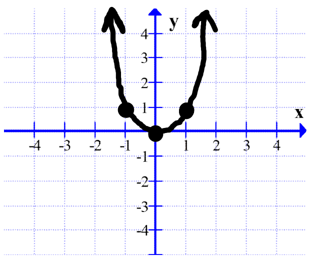
x	y
-10	+
+10	-

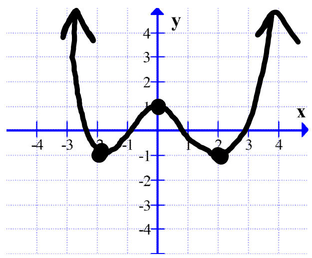
C12 - 3.4 - End Behaviour Polynomials Notes

Leading Term
Table of Values

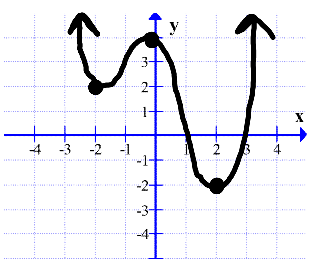


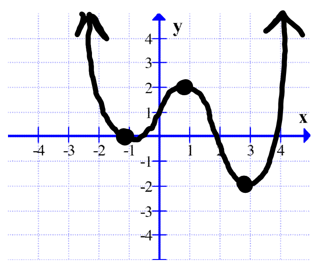
$y = x^2$

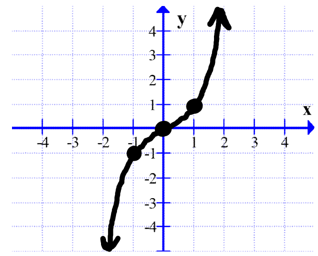




$y = x^4 \ldots$

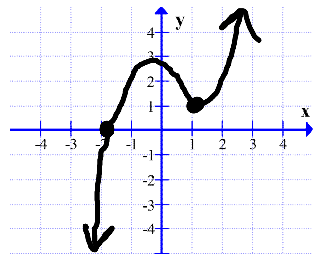


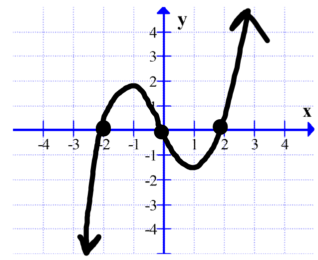


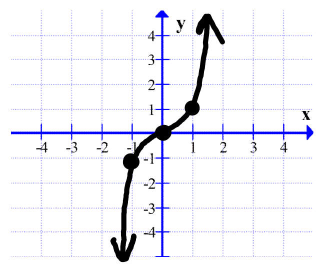


$y = x^3$

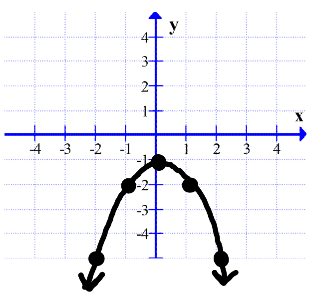
$y = x^3 \ldots$



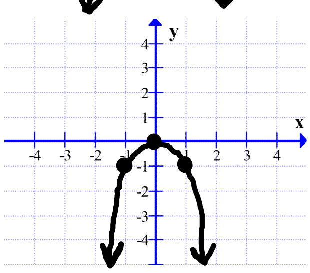




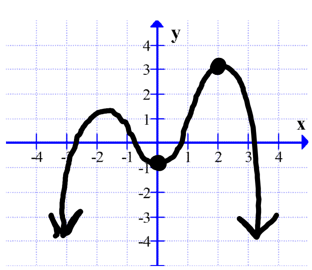
$y = x^5$



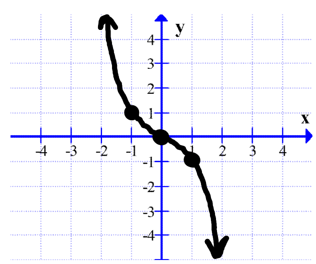
$y = -x^2$



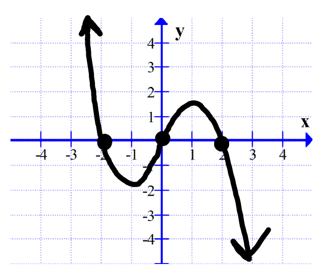
$y = -x^4$



$y = -x^4 \ldots$

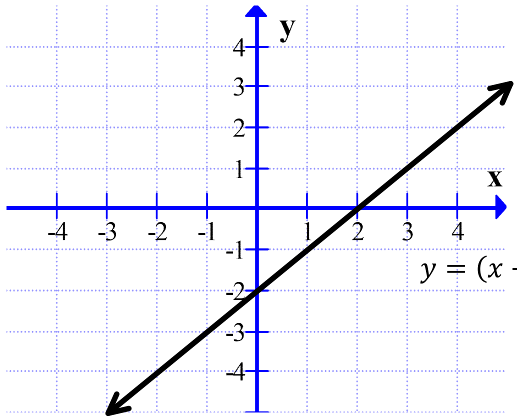


$y = -x^3$



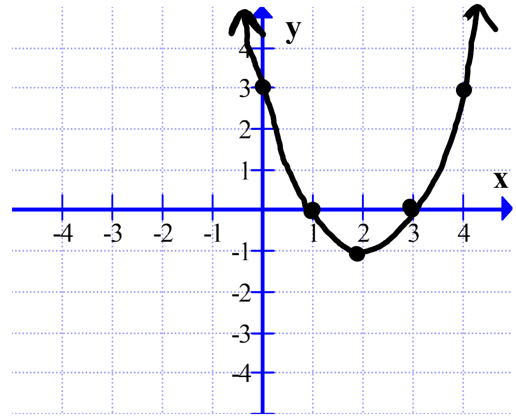
$y = -x^3 \ldots$

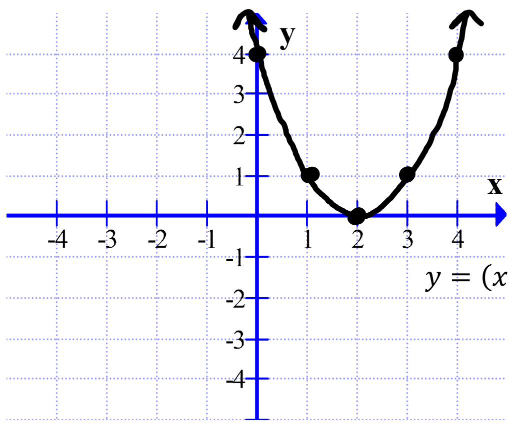
C12 - 3.4 - Multiplicity (Factor Exponents) Graph Notes



$y = (x - 2)^1$

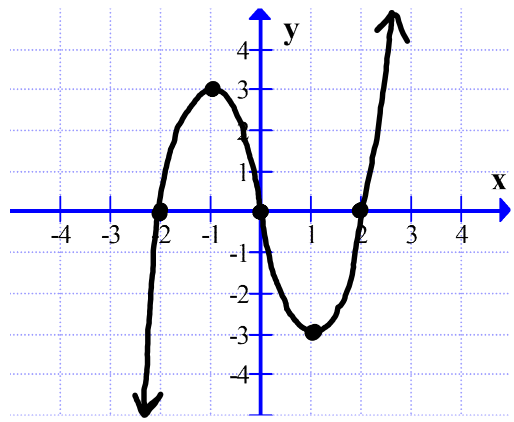
$y = (x - 1)^1(x - 3)^1$

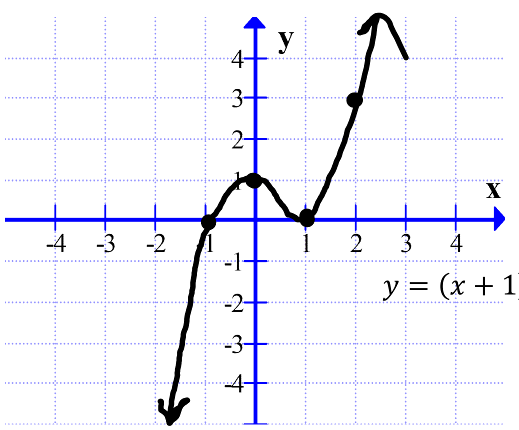




$y = (x - 2)^2$

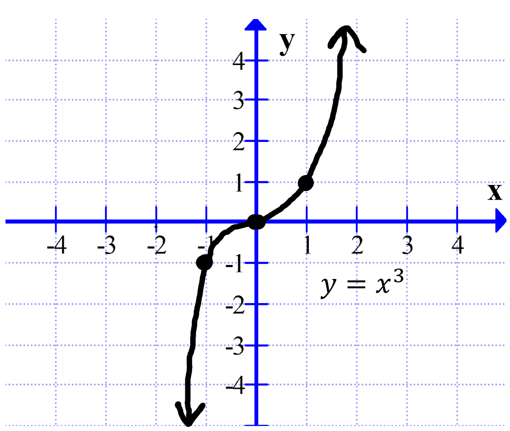
$y = x^1(x - 2)^1(x + 2)^1$



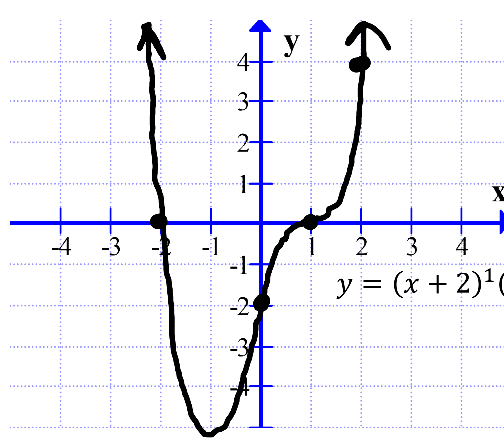


$y = (x + 1)^1(x - 1)^2$

$Degree\ 1$: $Straight\ through\ x - intercept$
$Degree\ 2$: $Bounce\ off\ x - intercept$
$Degree\ 3$: $Chair\ Shape\ through\ x - intercept$

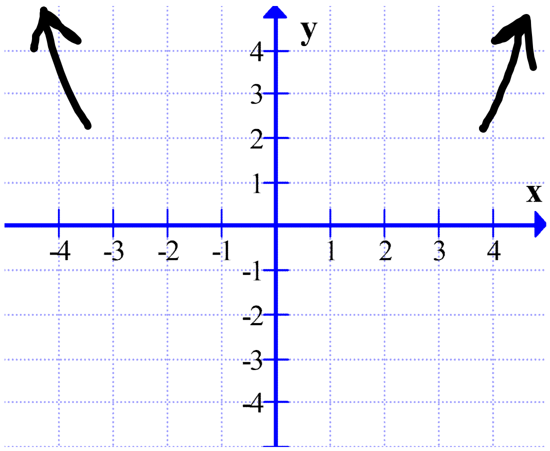


$y = x^3$



$y = (x + 2)^1(x - 1)^3$

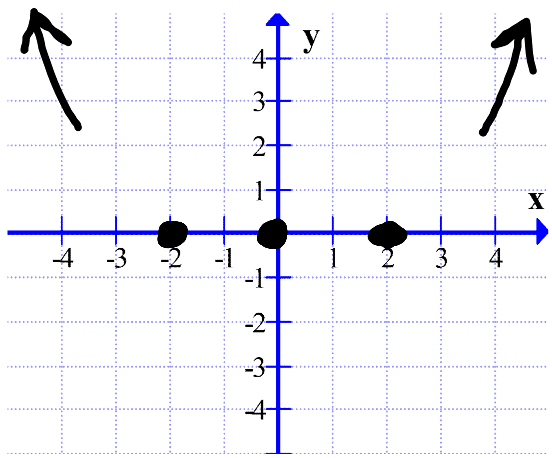
C12 - 3.4 - Graph $y = x(x-2)^2(x+2)^3$ Notes



$\boxed{y = x(x-2)^2(x+2)^3}$

1) *End Behavior*

$y = x(x-2)^2(x+2)^3$
$y = x(x)^2(x)^3$
$y = +x^6 \longrightarrow y = +x^{even}$
Q3,Q1

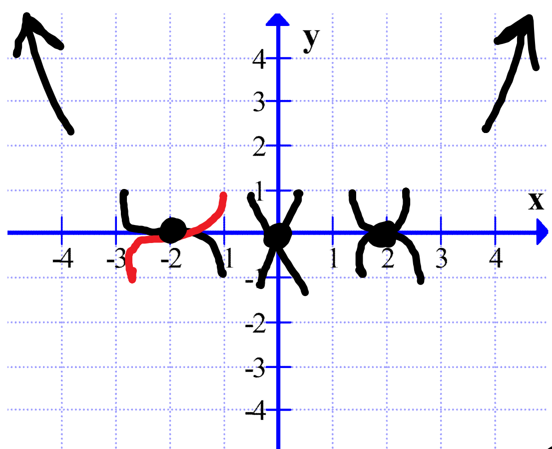


2) *x − intercepts, y intercept*

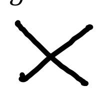
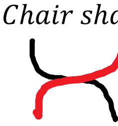
$x - 2 = 0$	$x = 0$	$x + 2 = 0$
$x = 2$		$x = -2$
$(0,2)$	$(0,0)$	$(0,-2)$

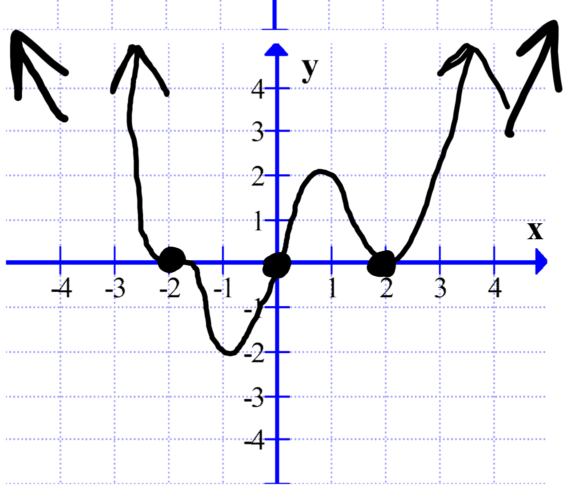
$y = x(x-2)^2(x+2)^3$
$y = 0(0-2)^2(0+2)^3$
$y = 0(-2)^2(2)^3$
$y = 0(-1)(8)$
$y = 0$

y-int: $(0,0)$



3) *Multiplicity*

$(x-2)^2$	x^1	$(x+2)^3$
$x = 2$ *Degree* 2	$x = 0$ *Degree* 1	$x = -2$ *Degree* 3
U − shape	*Straight through*	*Chair shape*
		



4) *Graph*

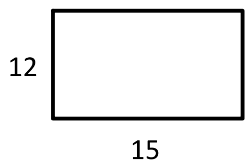
$y = x(x-2)^2(x+2)^3$

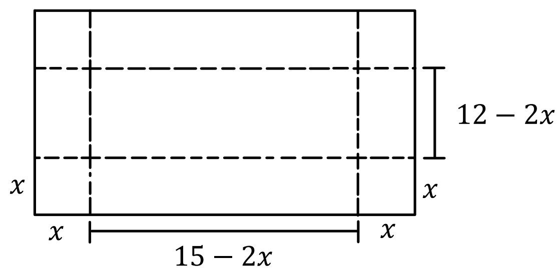
Start from an arrow
Chair at $x = -2$
Straight through at $x = 0$
Bounce at $x = 2$
End at an arrow

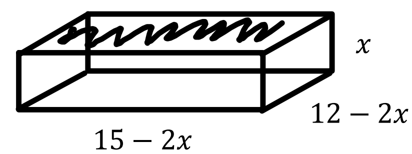
C12 - 3.5 - Open Rectangular Box Cut Side x Notes

An open rectangular box is made by cutting equal integer lengths from each corner of a 12 cm by 15 cm rectangular piece of cardboard, then folding up the sides. Find the length of the square that must be cut from each corner so the box has a volume of 162 cm^3. And find length to cut for Max Volume and find Max Volume.

$let\ x = length\ to\ cut$







$\boxed{v = lwh}$

$$
\begin{aligned}
Volume &= length \times width \times height \\
V &= (12-2x)(15-2x)x \\
162 &= (12-2x)(15-2x)(x) \\
162 &= 180x - 54x^2 + 4x^3 \\
0 &= 4x^3 - 54x^2 + 180x - 162 \\
0 &= 2x^3 - 27x^2 + 90x - 81
\end{aligned}
$$

Potential Factors: The factors of 81: $\cancel{\pm 27}, \cancel{\pm 9}, \pm 3, \pm 1$

Solve by inspection: $Check\!: x = 1, 3$

$$
\begin{aligned}
f(x) &= 2x^3 - 27x^2 + 90x - 81 \\
f(3) &= 2(3)^3 - 27(3)^2 + 90(3) - 81 \\
f(3) &= 54 - 243 + 270 - 81 = 0
\end{aligned}
$$

$$
\begin{array}{c|cccc}
3 & 2 & -27 & 90 & -81 \\
+ & \downarrow & 6 & -63 & 81 \\
\hline
 & 2 & -21 & 27 & 0
\end{array}
$$

$2x^2 - 21x + 27$

$\therefore x = 3\ cm$

$2x^2 - 21x + 27$

$(2x-3)(x-9)$

$\cancel{x = 1.5}$ $\cancel{x = 9}$ $\boxed{x < 6}$

Reject non-integers

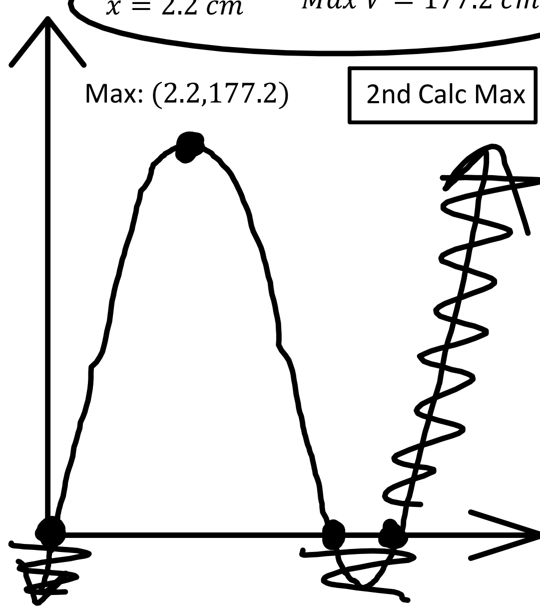
$Domain\!:\ x > 0,\quad x\ cant\ be\ negative!$

$x < 6,\quad Cant\ cut\ 2\ 6's\ off\ a\ 12!$

We need to reject 6 and greater so we don't get negatives lengths.

$\boxed{V = (12-2x)(15-2x)x}$

$x = 2.2\ cm \qquad Max\ V = 177.2\ cm^2$



$Check\ Answer$

$l = 15 - 2x$	$w = 12 - 2x$	$h = x$
$l = 15 - 2(3)$	$w = 12 - 2(3)$	$h = 3$
$l = 9$	$w = 6$	

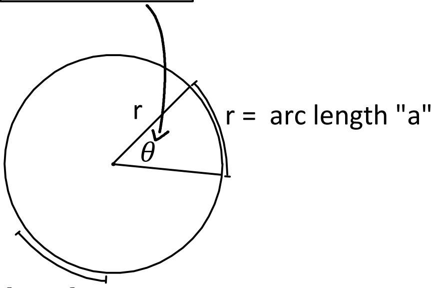
$$
\begin{aligned}
V &= lwh \\
V &= 9 \times 6 \times 3 \\
V &= 162\ cm^2
\end{aligned}
$$

C12 - 4.1 - Degree/Radian Notes

"One radian is equal to the length of the arc of a circle with radius = 1.

1 Radian is the central angle whose arc is equal to the radius

$\boxed{1\,rad = 57.3^o}$



Arc length

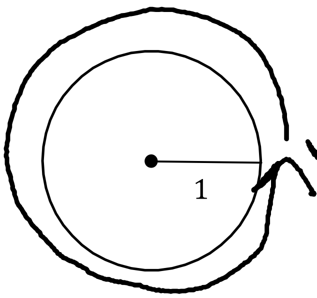
$$\boxed{\begin{array}{l} 1\,rad = 57.3^o \\ 1\,inch = 2.54\,cm \end{array}}$$

One Radian equals 57.3^o

$$\boxed{\theta_{rad} = \frac{a}{r}}$$

$$\theta_{rad} = \frac{r}{r}$$

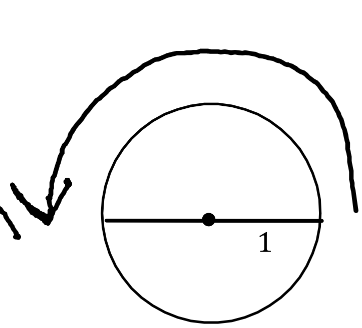
$$\theta_{rad} = 1\,rad$$



$\theta = 360^o = 2\pi_{rad}$

$C = 2\pi r$
$C = 2\pi(1)$

$C = 2\pi$
$C = 6.28$

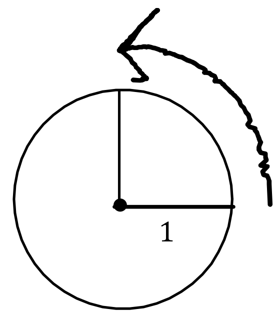


$\boxed{\theta = 180^0 = \pi_{rad}}$

$C = 2\pi r$
$C = 2\pi(1)$
$C = 2\pi$

$\frac{C}{2} = \frac{2\pi}{2}$

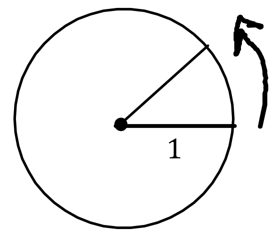
$C = \pi$



$\theta = 90^0 = \frac{\pi}{2_{rad}}$

$C = 2\pi r$
$C = 2\pi(1)$
$C = 2\pi$

$\frac{C}{4} = \frac{2\pi}{4}$
$C = \frac{\pi}{2}$



$\theta = 45^0 = \frac{\pi}{4_{rad}}$

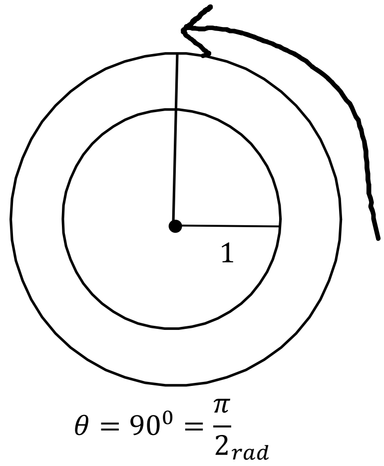
$C = 2\pi r$
$C = 2\pi(1)$
$C = 2\pi$

$\frac{C}{8} = \frac{2\pi}{8}$

$C = \frac{\pi}{4}$

Notice the size of the circle does not matter.

$90^0 = \frac{\pi}{2}$



$\theta = 90^0 = \frac{\pi}{2_{rad}}$

C12 - 4.1 - Degree/Radian Conversion Notes

$\boxed{\dfrac{180^o}{\pi_{rad}}}$ $\boxed{\dfrac{\pi_{rad}}{180^o}}$

Degrees to Radians: $\times \dfrac{\pi}{180^o}$

Radians to Degrees: $\times \dfrac{180^0}{\pi}$

$\pi \text{ and } 180^o \text{ are the same thing, just in different units}$

Find θ in radians

$30^0 = ?$ $\quad 30^0 \times \dfrac{\pi}{180^o} = \dfrac{30\pi}{180} = \dfrac{\pi}{6} = 0.52$

$120^0 = ?$ $\quad 120^0 \times \dfrac{\pi}{180^o} = \dfrac{120\pi}{180} = \dfrac{2\pi}{3}$

$99^0 = ?$ $\quad 99^0 \times \dfrac{\pi}{180^o} = \dfrac{99\pi}{180} = \dfrac{11\pi}{20}$

Find θ in degrees

$\dfrac{\pi}{3_{rad}} = ?$ $\quad \dfrac{\pi}{3_{rad}} \times \dfrac{180^0}{\pi} = \dfrac{180\pi}{3\pi} = 60^0$

$\dfrac{2\pi}{5_{rad}} = ?$ $\quad \dfrac{2\pi}{5_{rad}} \times \dfrac{180^0}{\pi} = \dfrac{360\pi}{5\pi} = 72^0$

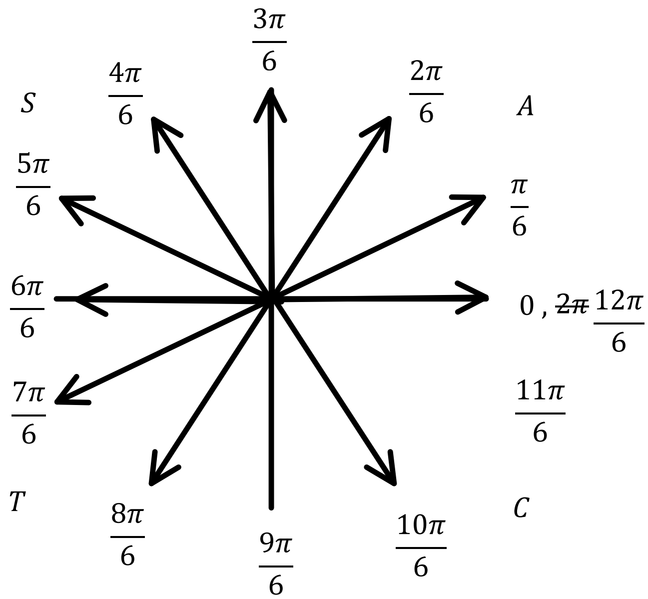
$1.57_{rad} = ?$ $\quad 1.57_{rad} \times \dfrac{180^0}{\pi} = 90^0$

$3 = ?$ $\quad 3_{rad} \times \dfrac{180^0}{\pi} = \dfrac{540}{\pi} = 171.89^0$

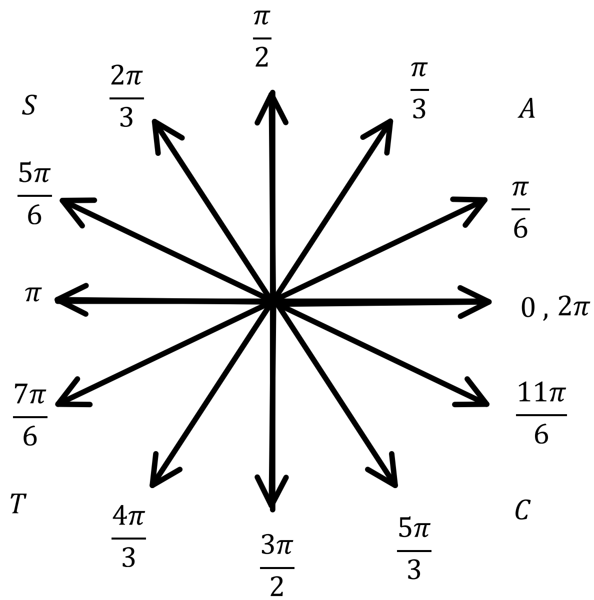
Degrees	*Radians*	*Radians*	*Radians*
0^0	0_{rad}	0_{rad}	0_{rad}
15^0	$\dfrac{\pi}{12_{rad}}$	$\dfrac{\pi}{12_{rad}}$	0.26_{rad}
30^0	$\dfrac{2\pi}{12_{rad}}$	$\dfrac{\pi}{6_{rad}}$	0.52_{rad}
45^0	$\dfrac{3\pi}{12_{rad}}$	$\dfrac{\pi}{4_{rad}}$	0.79_{rad}
60^0	$\dfrac{4\pi}{12_{rad}}$	$\dfrac{\pi}{3_{rad}}$	1.05_{rad}
75^0	$\dfrac{5\pi}{12_{rad}}$	$\dfrac{5\pi}{12_{rad}}$	1.31_{rad}
90^0	$\dfrac{6\pi}{12_{rad}}$	$\dfrac{\pi}{2_{rad}}$	1.57_{rad}
180^0	$\dfrac{12\pi}{12} = \pi_{rad}$	π_{rad}	3.14_{rad}
270^0	$\dfrac{3\pi}{2}_{rad}$	$\dfrac{3\pi}{2}_{rad}$	4.71_{rad}
360^0	$2\pi_{rad}$	$2\pi_{rad}$	6.28_{rad}
720^0	$4\pi_{rad}$	$4\pi_{rad}$	12.56_{rad}

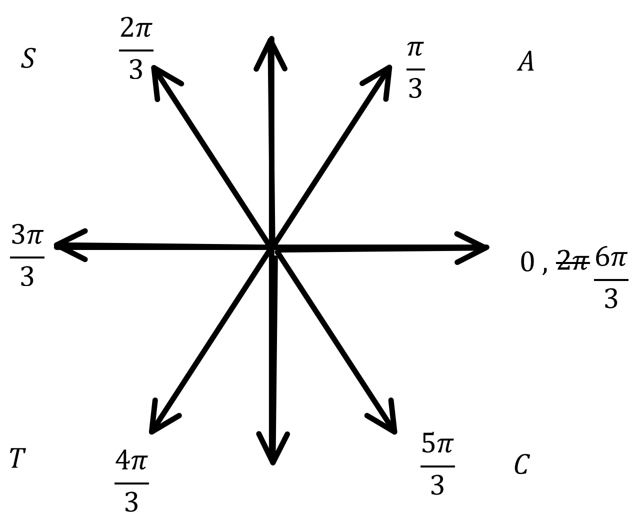
If there are no units it is in radians.

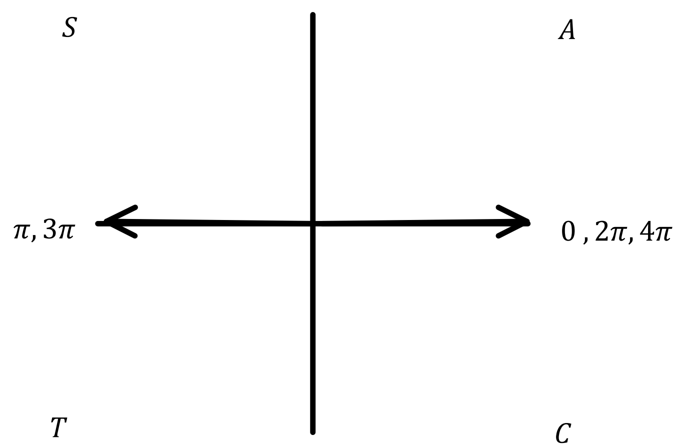
C12 - 4.1 - $\frac{\#\pi}{\#}$ *Notes*

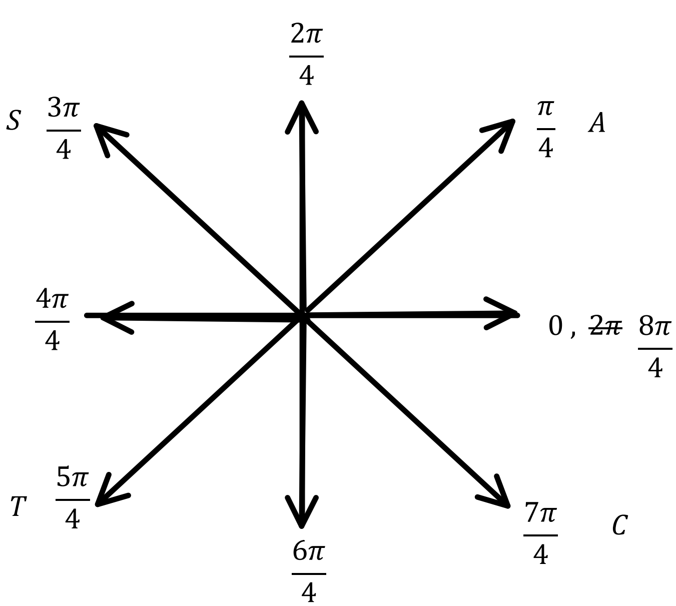


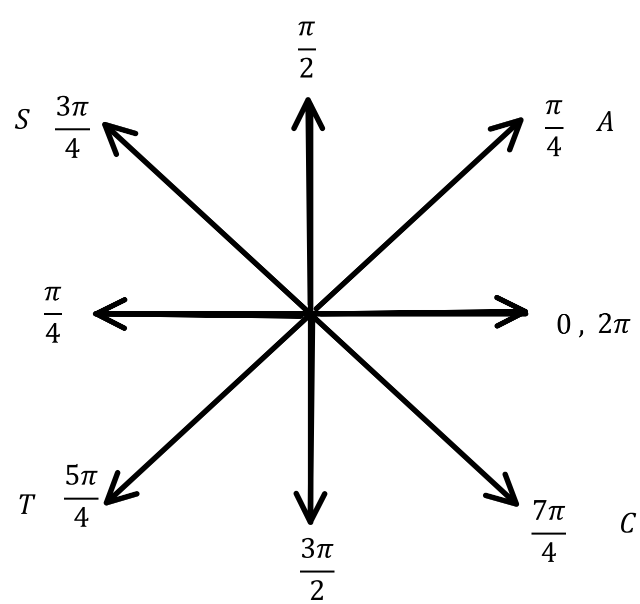
Simplify









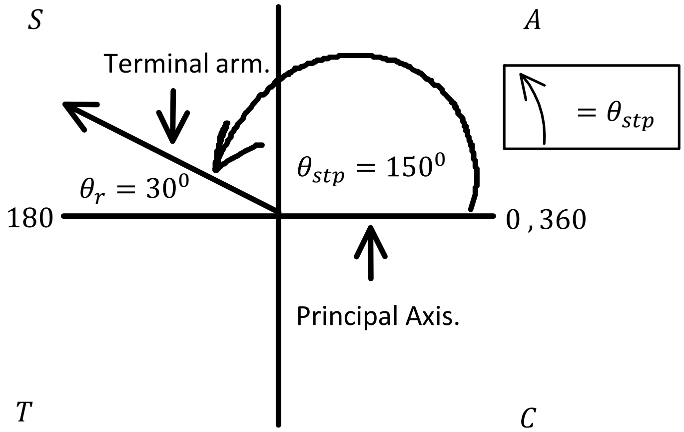


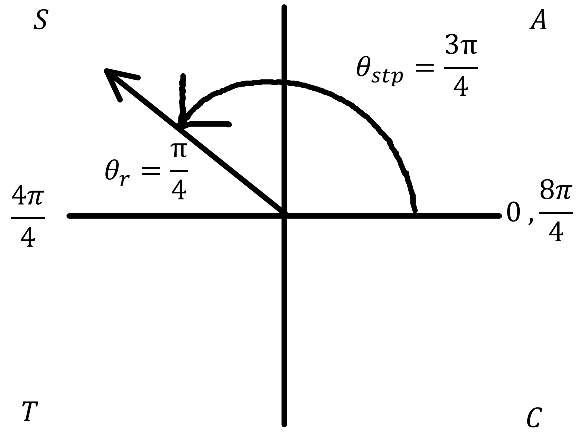
C12 - 4.2 - θ_r, θ_{stp} Notes

(always positive, between 0 and $\pi/2$)

θ_r: the "reference angle" is the angle between the terminal arm and the x-axis.

θ_{stp}: the "angle in standard position" from the principal axis (+ x-axis) to the terminal arm.





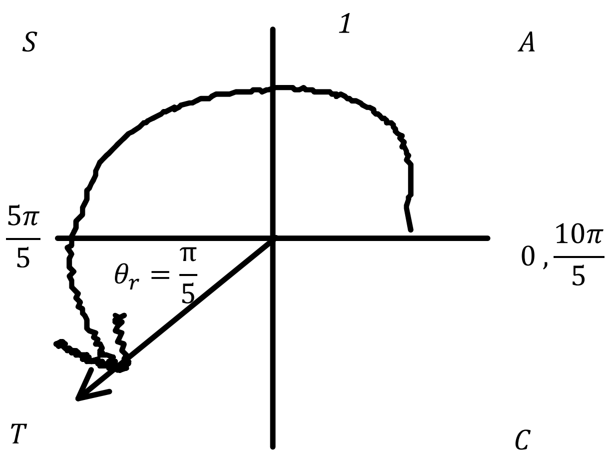
$$\theta_r = 180 - 150$$
$$\theta_r = 30^0$$

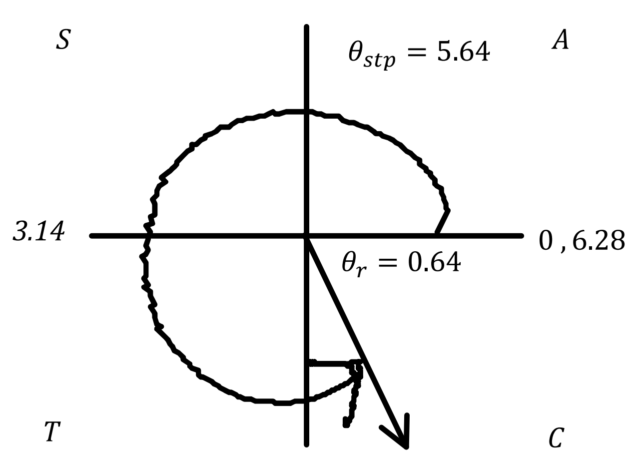
$$\theta_{stp} = 180 - 30$$
$$\theta_{stp} = 150^0$$

LCD

$$\theta_r = \pi - \theta_{stp}$$
$$\theta_r = \pi - \frac{3\pi}{4}$$
$$\theta_r = \frac{4\pi}{4} - \frac{3\pi}{4}$$
$$\theta_r = \frac{\pi}{4}$$

$$\theta_{stp} = \pi - \theta_r$$
$$\theta_{stp} = \pi - \frac{\pi}{4}$$
$$\theta_{stp} = \frac{4\pi}{4} - \frac{\pi}{4}$$
$$\theta_{stp} = \frac{3\pi}{4}$$





$$\theta_r = \pi + \theta_{stp}$$
$$\theta_r = \pi + \frac{6\pi}{5}$$
$$\theta_r = \frac{5\pi}{5} + \frac{6\pi}{5}$$
$$\theta_r = \frac{\pi}{5}$$

$$\theta_{stp} = \pi + \theta_r$$
$$\theta_{stp} = \pi + \frac{\pi}{5}$$
$$\theta_{stp} = \frac{5\pi}{5} + \frac{\pi}{5}$$
$$\theta_{stp} = \frac{6\pi}{5}$$

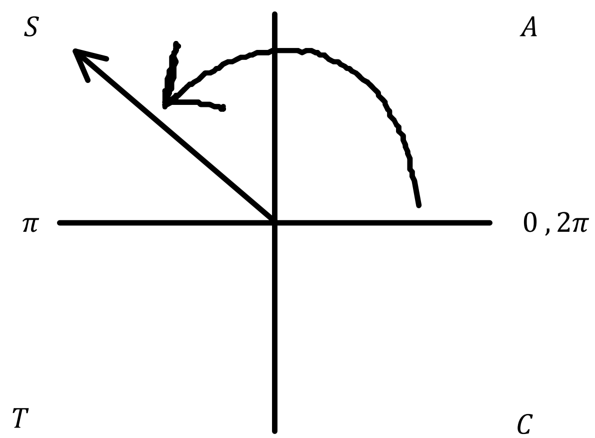
$$\theta_r = 2\pi - \theta_{stp}$$
$$\theta_r = 2\pi - 5.64$$
$$\theta_r = 0.64$$

$$\theta_{stp} = 2\pi - \theta_r$$
$$\theta_{stp} = 2\pi - 0.64$$
$$\theta_{stp} = 5.64$$

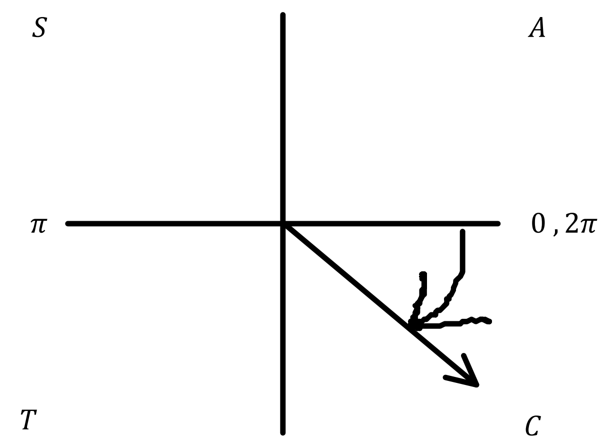
$\boxed{\theta_{cot} = \theta_{stp} \pm 2\pi n, nEI}$

C12 - 4.2 - $\pm\,\theta_{stp}, \theta_{cot}\,, \theta_{gen}$ Notes

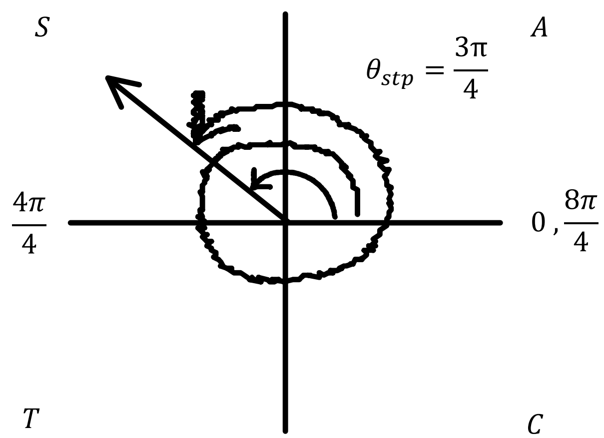
Counter-clockwise rotation is a positive $\boldsymbol{\theta_{stp}}$



Clockwise rotation is a negative $\boldsymbol{\theta_{stp}}$



θ_{cot}: the "co-terminal angle"is any angle with the same terminal arm.



$$\theta_{cot} = \theta_{stp} \pm 2\pi$$
$$\theta_{cot} = \frac{3\pi}{4} + 2\pi$$
$$\theta_{cot} = \frac{3\pi}{4} + \frac{8\pi}{4}$$
$$\theta_{cot} = \frac{11\pi}{4}$$

$$\theta_{cot} = \theta_{stp} \pm 2\pi$$
$$\theta_{cot} = \frac{3\pi}{4} - 2\pi$$
$$\theta_{cot} = \frac{3\pi}{4} - \frac{8\pi}{4}$$
$$\theta_{cot} = -\frac{5\pi}{4}$$

θ_{gen}: the "general solution"is all angles with the same terminal arm.

$$\theta_{gen} = \theta_{stp} \pm 2\pi n, nEI$$
$$\theta_{gen} = \frac{3\pi}{4} \pm 2\pi n, nEI$$

Basic logic will calculate θ_{stp} *and* θ_r much more easily than using these formulas.

$$\frac{9\pi}{2}$$

$$\frac{9\pi}{2} - 2\pi \qquad \frac{9\pi}{2} - \frac{4\pi}{2} \qquad \frac{5\pi}{2}$$

$$\frac{5\pi}{2} - 2\pi \qquad \frac{5\pi}{2} - \frac{4\pi}{2} \qquad \frac{\pi}{2}$$

OR

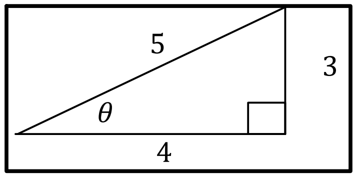
$$\frac{9\pi}{2} \div 2\pi \qquad \frac{9\pi}{2} \times \frac{1}{2\pi} \qquad \frac{9}{4} = 2.25$$

$$\frac{9\pi}{2} - 2(2\pi) \qquad \frac{9\pi}{2} - 4\pi \qquad \frac{9\pi}{2} - \frac{8\pi}{2} \qquad \frac{\pi}{2}$$

OR $\quad 0.25 \times 2\pi = \frac{\pi}{2}$

You may need to add or subtract 2π more than once.

C12 - 4.3 - Find Ratio/Type in Calc Notes



Degrees are for children, unless you are taking physics.

$sin\theta = \frac{O}{H}$ $csc\theta = \frac{H}{O}$

$sin\theta = \frac{3}{5}$ $csc\theta = \frac{5}{3}$

The first letters switch s<->c

$cos\theta = \frac{A}{H}$ $sec\theta = \frac{H}{A}$

$cos\theta = \frac{4}{5}$ $sec\theta = \frac{5}{4}$

The first letters switch c<->s

$sec\theta = \frac{1}{cos\theta}$

$sec\theta = \frac{1}{\left(\frac{4}{5}\right)}$

$sec\theta = 1 \times \frac{5}{4}$

No one would do this! $sec\theta = \frac{5}{4}$

$tan\theta = \frac{O}{A}$ $cot\theta = \frac{A}{O}$

$tan\theta = \frac{3}{4}$ $cot\theta = \frac{4}{3}$

The ones with the t's

Type in Calculator (Degrees or Radians)

$sin25^0 = 0.42$

$cos180^0 = -1$

$sin30^0 = \frac{1}{2}$

$\tan(-980^0) = -5.67$

$csc140^0 =$

$csc\theta = \frac{1}{sin\theta}$

$csc140^0 = \frac{1}{sin140^0}$

$csc140^0 = -1.56$

$csc140 \neq \frac{1}{\sin\left(\frac{1}{140}\right)}$

$sec65^0 =$

$sec\theta = \frac{1}{cos\theta}$

$sec65^0 = \frac{1}{cos65^0}$

$sec\theta = 2.37$

$cot25^0 =$

$cot\theta = \frac{1}{tan\theta}$

$cot25^0 = \frac{1}{tan25^0}$

$cot25^0 = 2.14$

$sin2.5 = 0.60$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\tan(5\pi) = 0$

$cos\pi = -1$

$csc3.4 =$

$csc\theta = \frac{1}{sin\theta}$

$csc3.4 = \frac{1}{sin3.4}$

$csc3.4 = -3.91$

$\sec\left(\frac{3}{5}\right) =$

$sec\theta = \frac{1}{cos\theta}$

$\sec\left(\frac{3}{5}\right) = \frac{1}{\cos\left(\frac{3}{5}\right)}$

$\sec\left(\frac{3}{5}\right) = 1.21$

$\sec\left(\frac{3}{5}\right) \neq \cos\left(\frac{5}{3}\right)$

$\sec\left(\frac{3}{5}\right) \neq \frac{1}{\cos\left(\frac{5}{3}\right)}$

$cot250 =$

$cot\theta = \frac{1}{tan\theta}$

$cot250 = \frac{1}{tan250}$

$cot250 = -0.25$

Find θ in Degrees

$sin\theta = \frac{3}{5}$

$\theta = \sin^{-1}\left(\frac{3}{5}\right)$

$\theta = 36.9^0$

$sec\theta = \frac{2}{1}$

$cos\theta = \frac{1}{2}$

$\theta = \cos^{-1}\left(\frac{1}{2}\right)$

$\theta = 60^0$

$sec\theta = \frac{H}{A}$

$cos\theta = \frac{A}{H}$

Find θ in Radians

$cos\theta = \frac{3}{5}$

$\theta = \cos^{-1}\left(\frac{3}{5}\right)$

$\theta = 0.93$

$cot\theta = 3$

$cot\theta = \frac{3}{1}$

$tan\theta = \frac{1}{3}$

$\theta = \tan^{-1}(\frac{1}{3})$

$\theta = 0.32$

$cot\theta = \frac{A}{O}$

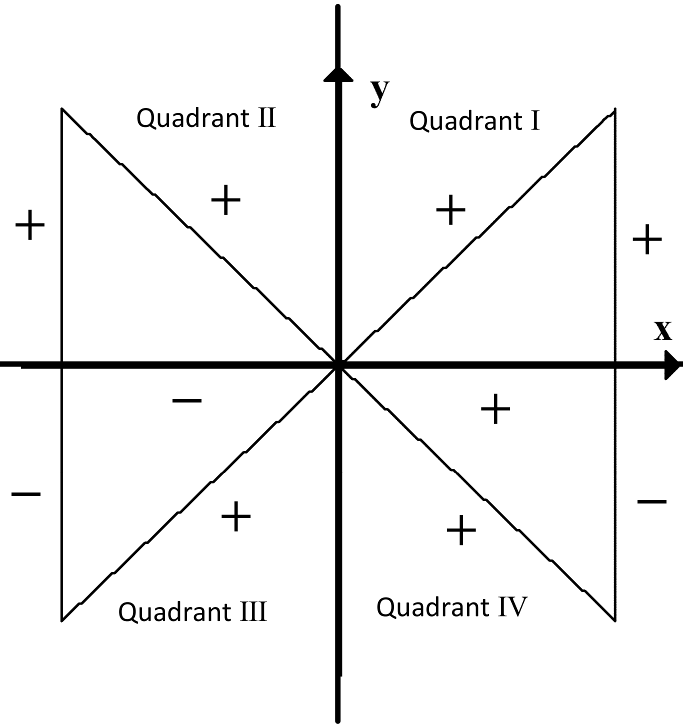
$tan\theta = \frac{O}{A}$

C12 - 4.3 - ASTC Notes

$(+)^2+(-)^2=+$ $\sqrt{+}=+$	Remember: the hypotenuse is always positive.	$(+)^2+(+)^2=+$ $\sqrt{+}=+$

$$sin\theta=\frac{+}{+}=+$$
$$cos\theta=\frac{-}{+}=-$$
$$tan\theta=\frac{+}{-}=-$$

$$sin\theta=\frac{+}{+}=+$$
$$cos\theta=\frac{+}{+}=+$$
$$tan\theta=\frac{+}{+}=+$$



$$sin\theta=\frac{-}{+}=-$$
$$cos\theta=\frac{-}{+}=-$$
$$tan\theta=\frac{-}{-}=+$$

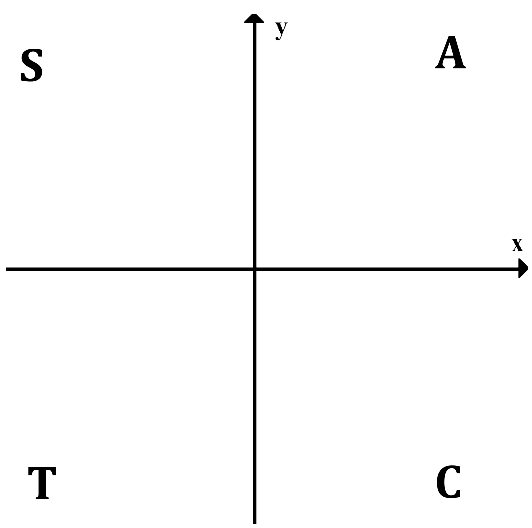
$$sin\theta=\frac{-}{+}=-$$
$$cos\theta=\frac{+}{+}=+$$
$$tan\theta=\frac{-}{+}=-$$

Students

Only **S**in positive.

All

All (sin, cos, tan) positive



Only **T**an positive.

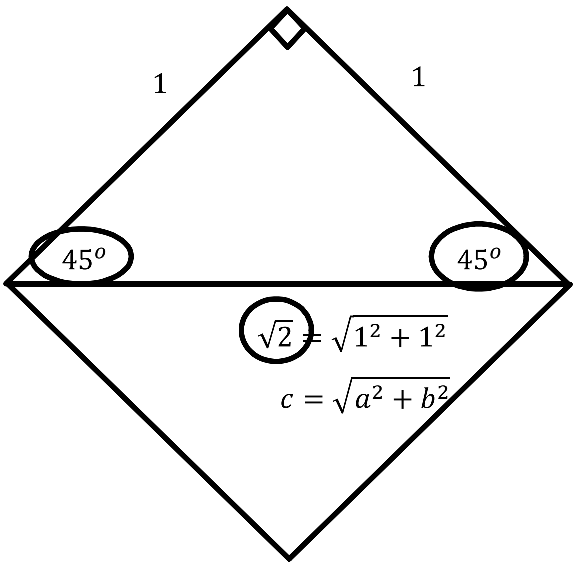
Take

Calculus

Only **C**os positive.

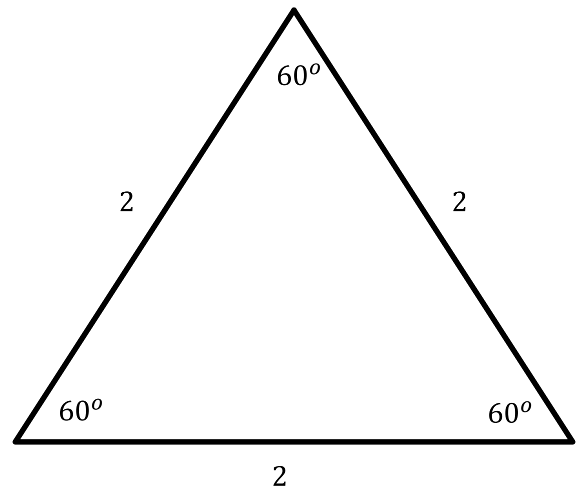
C12 - 4.3 - Special Triangles 30,45,60 sin/cos/tan Notes

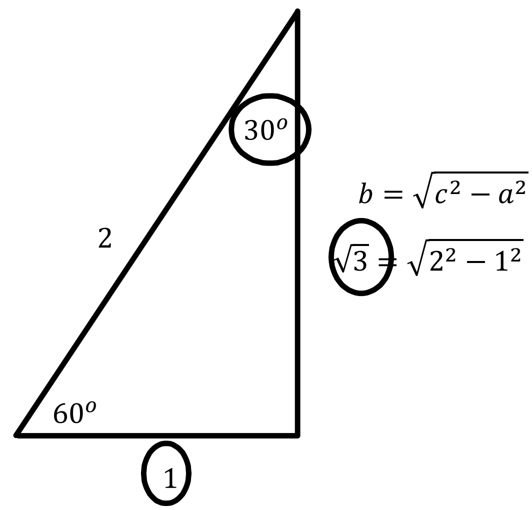
Right Isosceles, with sides =1

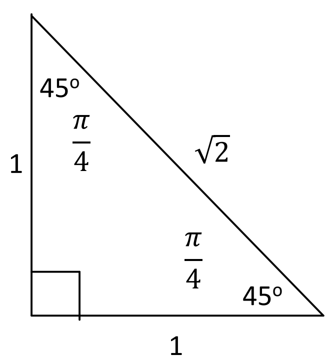


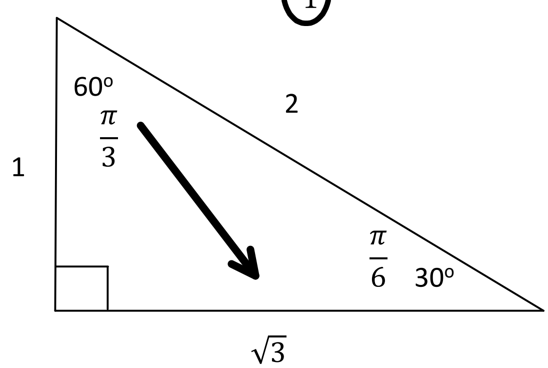
Diagonal of a square with sides lengths of 1

Half an equilateral with sides 2









$60 > 30$

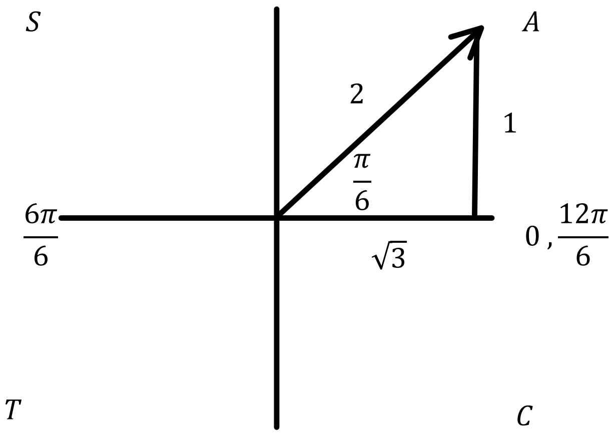
$\sqrt{3} > 1$

60 must open up to the root 3. And Vice Versa

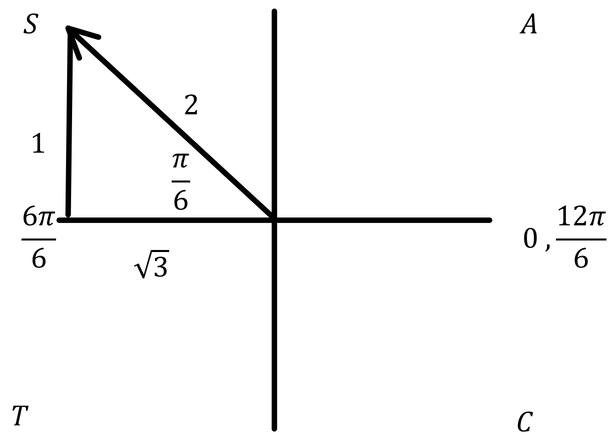
$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$	$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$	$\sin\frac{\pi}{6} = \frac{1}{2}$
$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$	$\cos\frac{\pi}{3} = \frac{1}{2}$	$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$
$\tan\frac{\pi}{4} = \frac{1}{1}$	$\tan\frac{\pi}{3} = \frac{\sqrt{3}}{1}$	$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

C12 - 4.3 - $sin\theta = ?$ *Notes*

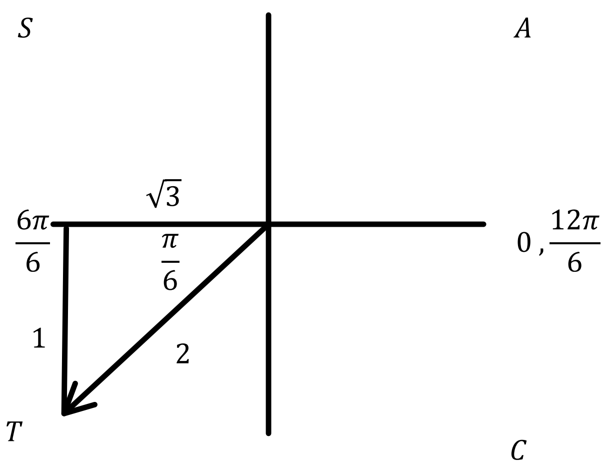
$\sin\frac{\pi}{6} = ?$ $\sin\frac{\pi}{6} = \frac{1}{2}$



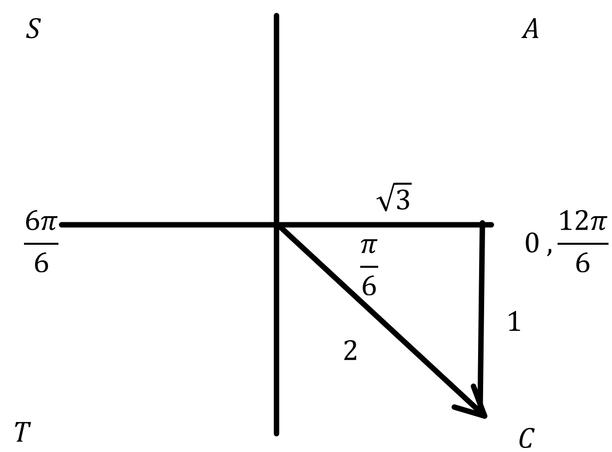
$\sin\frac{5\pi}{6} = ?$ $\sin\frac{5\pi}{6} = \frac{1}{2}$



$\sin\frac{7\pi}{6} = ?$ $\sin\frac{7\pi}{6} = -\frac{1}{2}$



$\sin\frac{11\pi}{6} = ?$ $\sin\frac{11\pi}{6} = -\frac{1}{2}$

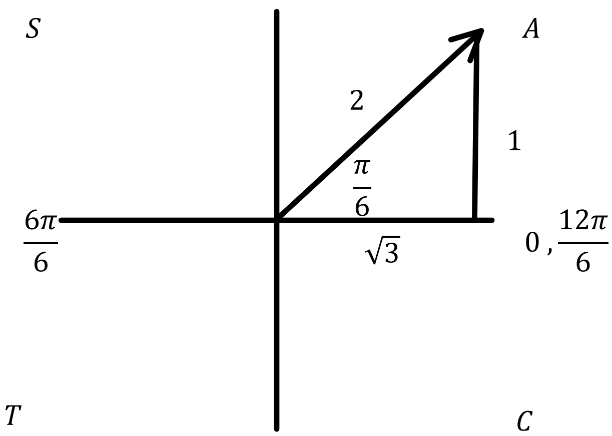


SOH - CAH - TOA

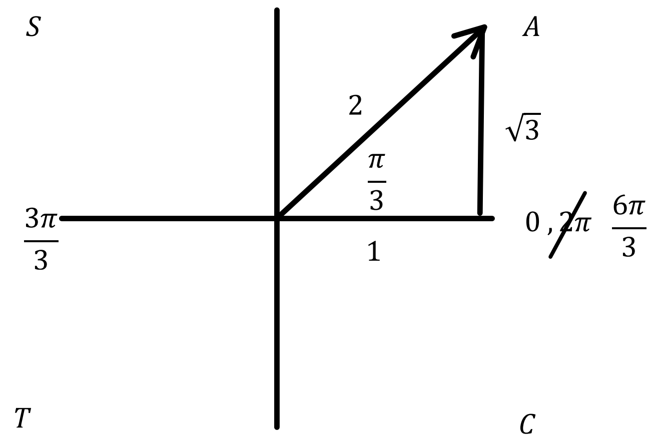
$sin\theta = \frac{O}{H}$	$cos\theta = \frac{A}{H}$	$tan\theta = \frac{O}{A}$
$csc\theta = \frac{1}{sin\theta} = \frac{H}{O}$	$sec\theta = \frac{1}{cos\theta} = \frac{H}{A}$	$cot\theta = \frac{1}{tan\theta} = \frac{A}{O}$

C12 - 4.3 - $sin\theta, cos\theta, tan\theta = ?\, Notes$

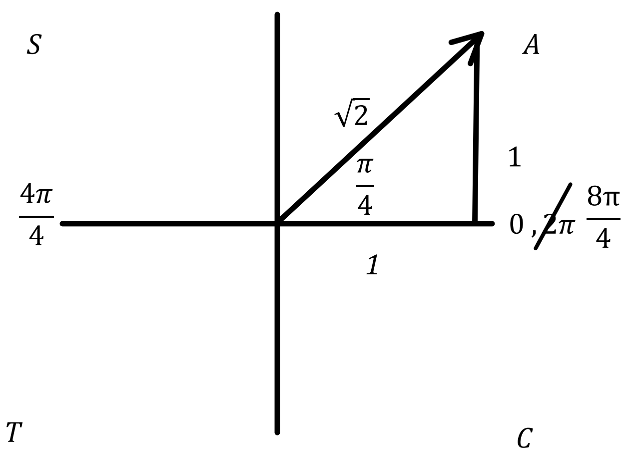
$\cos\frac{\pi}{6} = ?$ $\quad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$



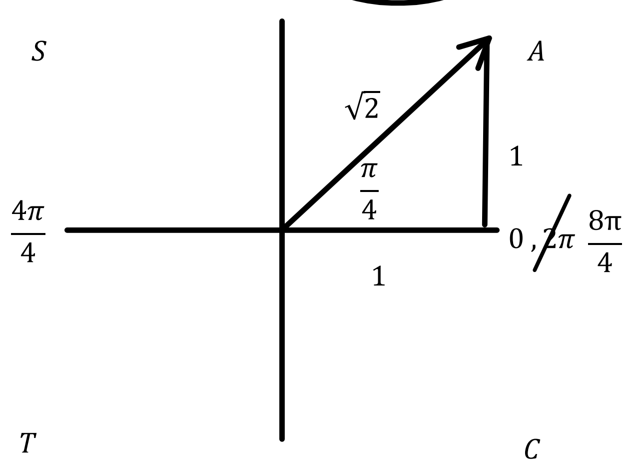
$\sin\frac{\pi}{3} = ?$ $\quad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$



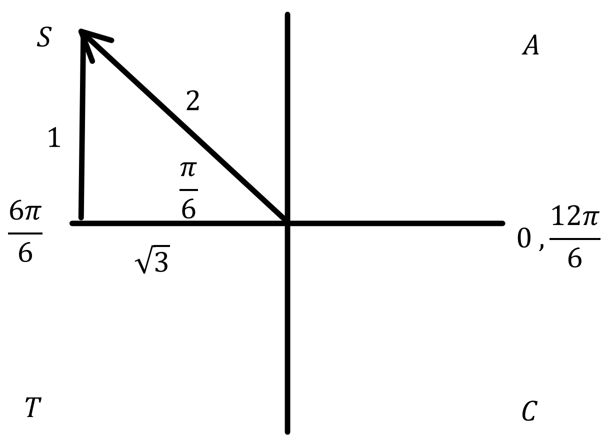
$\sin\frac{\pi}{4} = ?$ $\quad \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$



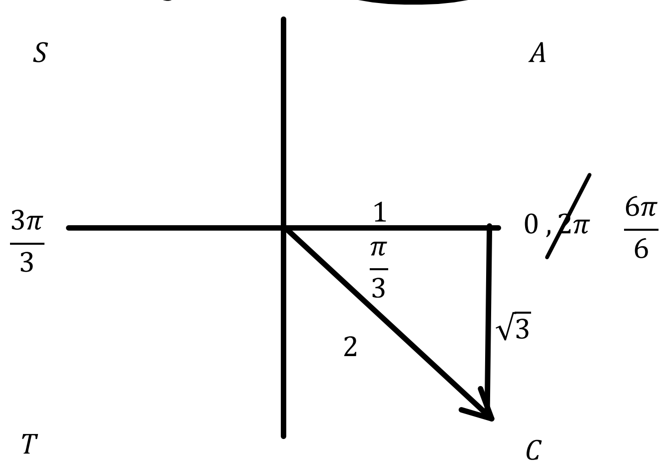
$\tan\frac{\pi}{4} = ?$ $\quad \tan\frac{\pi}{4} = 1$



$\cos\frac{5\pi}{6} = ?$ $\quad \cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$



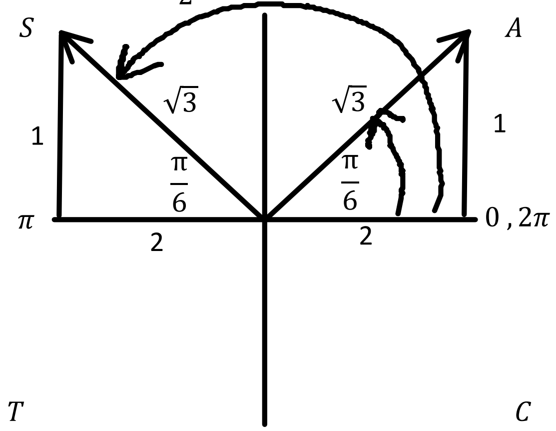
$\tan\frac{5\pi}{3} = ?$ $\quad \tan\frac{5\pi}{3} = -\sqrt{3}$



C12 - 4.3 - $sin\theta = \frac{1}{2}$ Notes

Solve for $\theta, 0^o \leq \theta < 2\pi$.

$sin\theta = \frac{1}{2}$



$\theta_{stp} = \frac{\pi}{6}$

$\theta_{stp} = \pi - \frac{\pi}{6}$
$= \frac{6\pi}{6} - \frac{\pi}{6}$
$= \frac{5\pi}{6}$

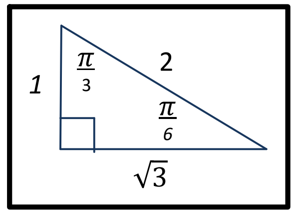
$\theta_{stp} = \frac{\pi}{6}, \frac{5\pi}{6}$

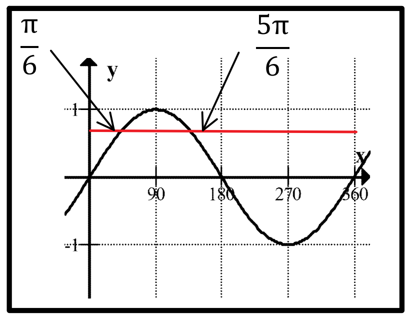
Solve for the arrows θ_{stp}

Draw two triangles where sin θ is positive: ASTC Quadrant I, II

Label triangles based on special triangles/SOH CAH TOA
Label the reference angle according to special triangles.

Draw an arrow from the principal axis to:
The first and second terminal arm.

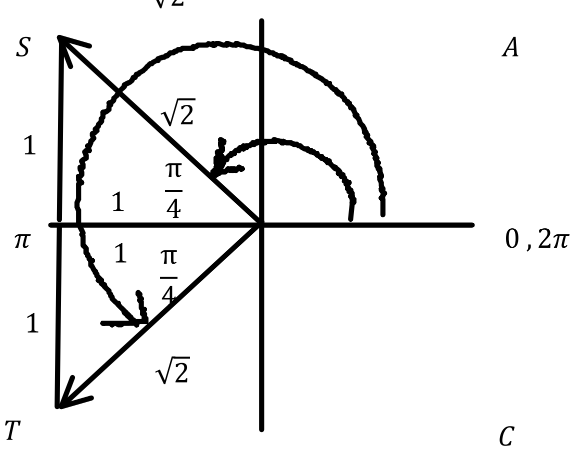


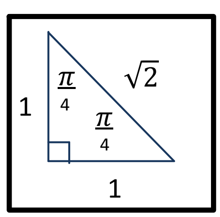


Check your answer: $\sin(\frac{\pi}{6}) = \frac{1}{2}$ $\sin(\frac{5\pi}{6}) = \frac{1}{2}$

Solve for $\theta, 0^o \leq \theta < 2\pi$ *and state the General Solution*.

$cosx = -\frac{1}{\sqrt{2}}$





$\theta_{stp} = \pi - \frac{\pi}{4}$

$\theta_{stp} = \frac{4\pi}{4} - \frac{\pi}{4}$

$\theta_{stp} = \frac{3\pi}{4}$

$\theta_{stp} = \pi + \frac{\pi}{4}$

$\theta_{stp} = \frac{4\pi}{4} + \frac{\pi}{4}$

$\theta_{stp} = \frac{5\pi}{4}$

$\theta_{stp} = \frac{3\pi}{4}, \frac{5\pi}{4}$

General Solution: $\theta = \theta_{stp} \pm pn, n \in I$

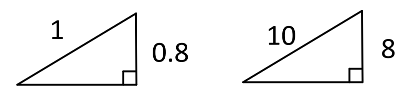
$\theta = \frac{3\pi}{4} \pm 2\pi n, n \in I$ $\theta = \frac{5\pi}{4} \pm 2\pi n, n \in I$

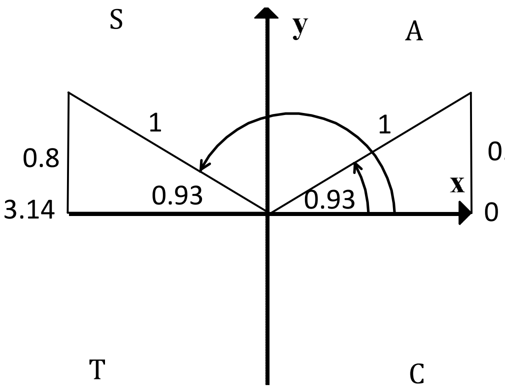
C12 - 4.3 - $sin\theta = .8$ & $Point$ Notes

Solve for $\theta, 0^o \le \theta < 2\pi$ and general solution

$\sin\theta = 0.8$

$$sin\theta = \frac{0.8}{1} = \frac{8}{10}$$





Draw two triangles where $\sin\theta$ is positive:
ASTC Quadrant I, II

Label the triangles according to SOH CAH TOA

$Solve\ for\ \theta_r$: $\boxed{\theta_r = \sin^{-1}(+\frac{O}{H})}$

$$\boxed{\begin{aligned} sin\theta &= \frac{0.8}{1} \\ \theta_r &= \sin^{-1}\left(+\frac{0.8}{1}\right) \\ \theta_r &= 0.93 \end{aligned}}$$

Draw an arrow from the principal axis to the first terminal arm,
draw an arrow from the principal axis to the second terminal arm.

$\boxed{\text{Only inverse positives} = \theta_r}$

Solve for the arrows θ_{stp}

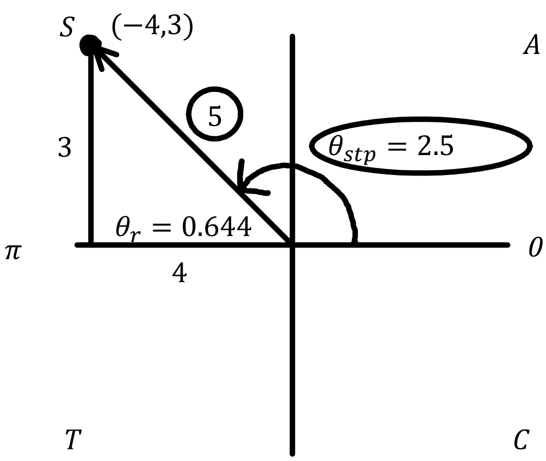
$\theta_{stp} = 0.93$ $\quad$ $\theta_{stp} = \pi - 0.93 = 2.21$

Check your answer: $\quad sin0.93 = 0.8 \quad sin2.21 = 0.8$

$\theta_{stp} = 0.93, 2.21$

$General\ Solution$: $\theta = \theta_{stp} \pm pn, n \in I$	$\theta = \theta_{stp} \pm pn, n \in I$
$\theta = 0.93 \pm 2\pi n, n \in I$	$\theta = 2.21 \pm 2\pi n, n \in I$

Find sinx, cox, tanx, cscx, secx, and cotx for the following point. Find θ_{stp}



$sin\theta = +\frac{3}{5}$ $\quad$ $cscx = +\frac{5}{3}$

$cos\theta = -\frac{4}{5}$ $\quad$ $secx = -\frac{5}{4}$

$tan\theta = -\frac{3}{4}$ $\quad$ $cotx = -\frac{4}{3}$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 3^2 + 4^2 &= c^2 \\ 9 + 16 &= c^2 \\ 25 &= c^2 \\ \sqrt{25} &= \sqrt{c^2} \\ 5 &= c \end{aligned}$$

$$\begin{aligned} tan\theta &= \frac{O}{A} \\ tan\theta &= \frac{3}{-4} \\ tan\theta &= -0.75 \\ \theta &= \tan^{-1}(+0.75) \\ \theta &= 0.644 \end{aligned}$$

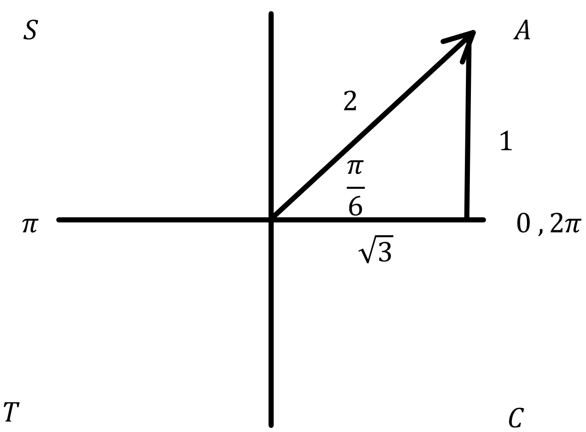
$\boxed{\text{Only inverse positives} = \theta_r}$

$\pi - 0.644 = 2.50$

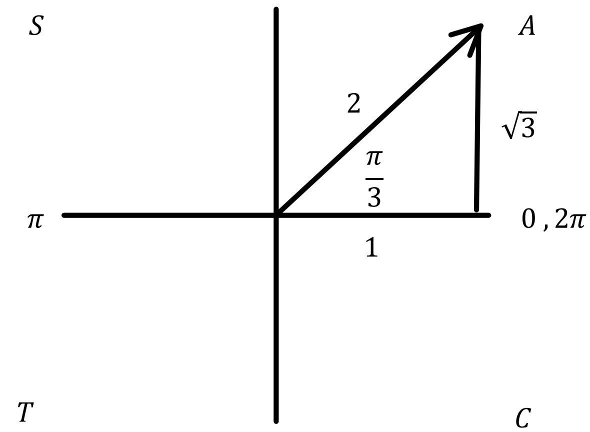
$\theta_{stp} = 2.50$

C12 - 4.3 - $csc\theta, sec\theta, cot\theta = ?$ *Notes*

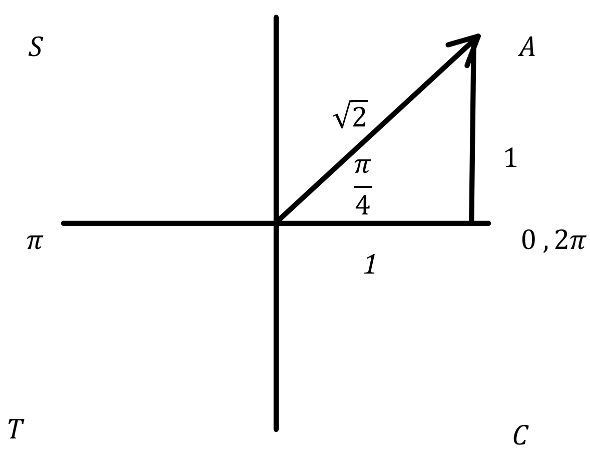
$\sec\frac{\pi}{6} = ?$ $\sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}$ $\boxed{\sec\frac{\pi}{6} \neq \cos\left(\frac{6}{\pi}\right)}$



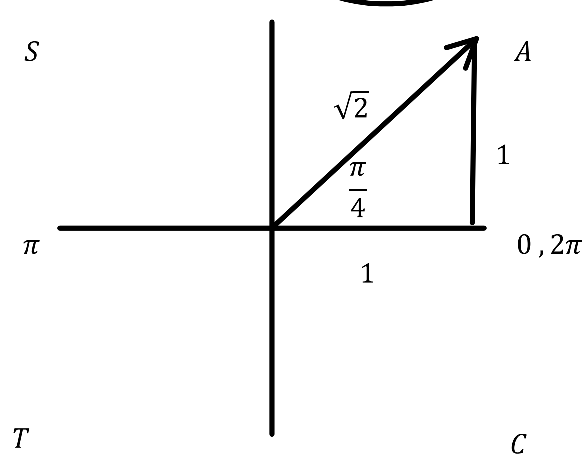
$\csc\frac{\pi}{3} = ?$ $\csc\frac{\pi}{3} = \frac{2}{\sqrt{3}}$



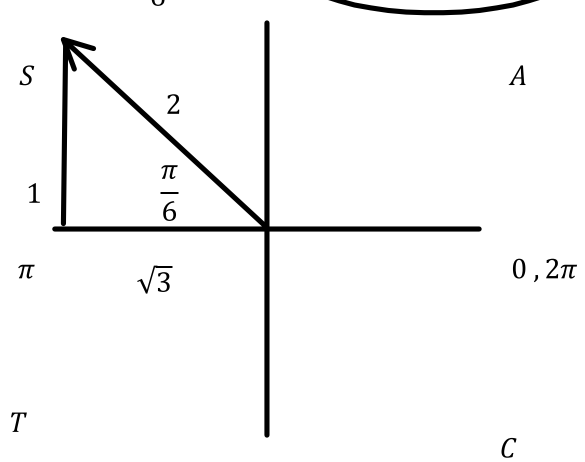
$\csc\frac{\pi}{4} = ?$ $\csc\frac{\pi}{4} = \sqrt{2}$



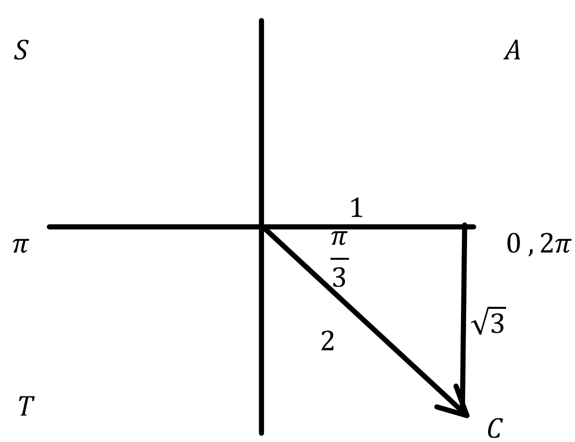
$\cot\frac{\pi}{4} = ?$ $\cot\frac{\pi}{4} = 1$



$\sec\frac{5\pi}{6} = ?$ $\sec\frac{5\pi}{6} = -\frac{2}{\sqrt{3}}$



$\cot\frac{5\pi}{3} = ?$ $\cot\frac{5\pi}{3} = -\frac{1}{\sqrt{3}}$



C12 - 4.3 - $csc\theta = 2$ Notes

Solve for $\boldsymbol{\theta}, \mathbf{0^o} \leq \boldsymbol{\theta} < \mathbf{2\pi}$.

$$csc\theta = \frac{2}{1}$$

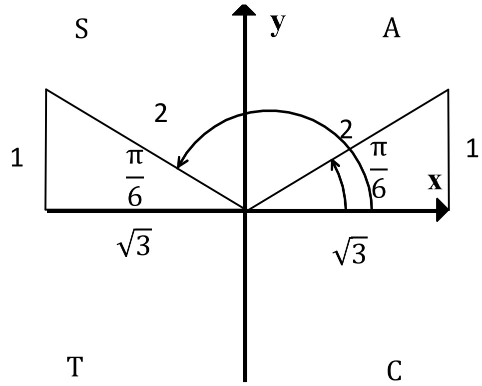
$$\boxed{OR \qquad sin\theta = \frac{1}{2}}$$

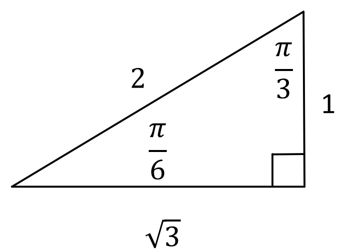
Draw two triangles where csc θ is positive:
ASTC Quadrant I, II

Label the triangles according to special triangles/SOH CAH TOA

Label the reference angle according to special triangles.

Draw an arrow from the principal axis to the first terminal arm
Draw an arrow from the principal axis to the second terminal arm.





$$\theta_{stp} = \frac{\pi}{6} \qquad \theta_{stp} = \pi - \frac{\pi}{6}$$
$$= \frac{6\pi}{6} - \frac{\pi}{6}$$
$$= \frac{5\pi}{6}$$

$$\theta_{stp} = \frac{\pi}{6}, \frac{5\pi}{6}$$

Solve for the arrows θ_{stp}

Check your answer:

$cot\theta$=0.1

$$cot\theta = \frac{0.1}{1} = \frac{1}{10}$$

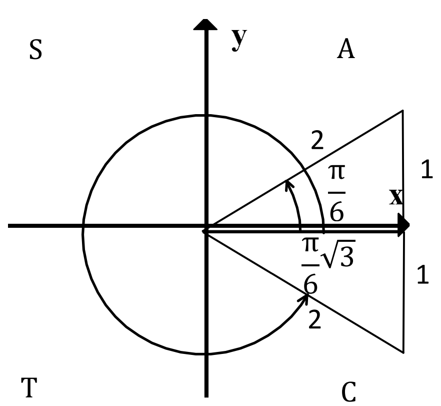
$$tan\theta = \frac{1}{0.1} = \frac{10}{1}$$
$$\theta = \tan^{-1}(10)$$
$$\theta = 84.24^0$$

...

$$\theta = 84.24, 264.29^0$$

Solve for $\boldsymbol{\theta}, \mathbf{0^o} \leq \boldsymbol{\theta} < \mathbf{2\pi}$ *and state the General Solution*.

$$secx = \frac{2}{\sqrt{3}} \qquad \boxed{cos\theta = \frac{\sqrt{3}}{2}}$$



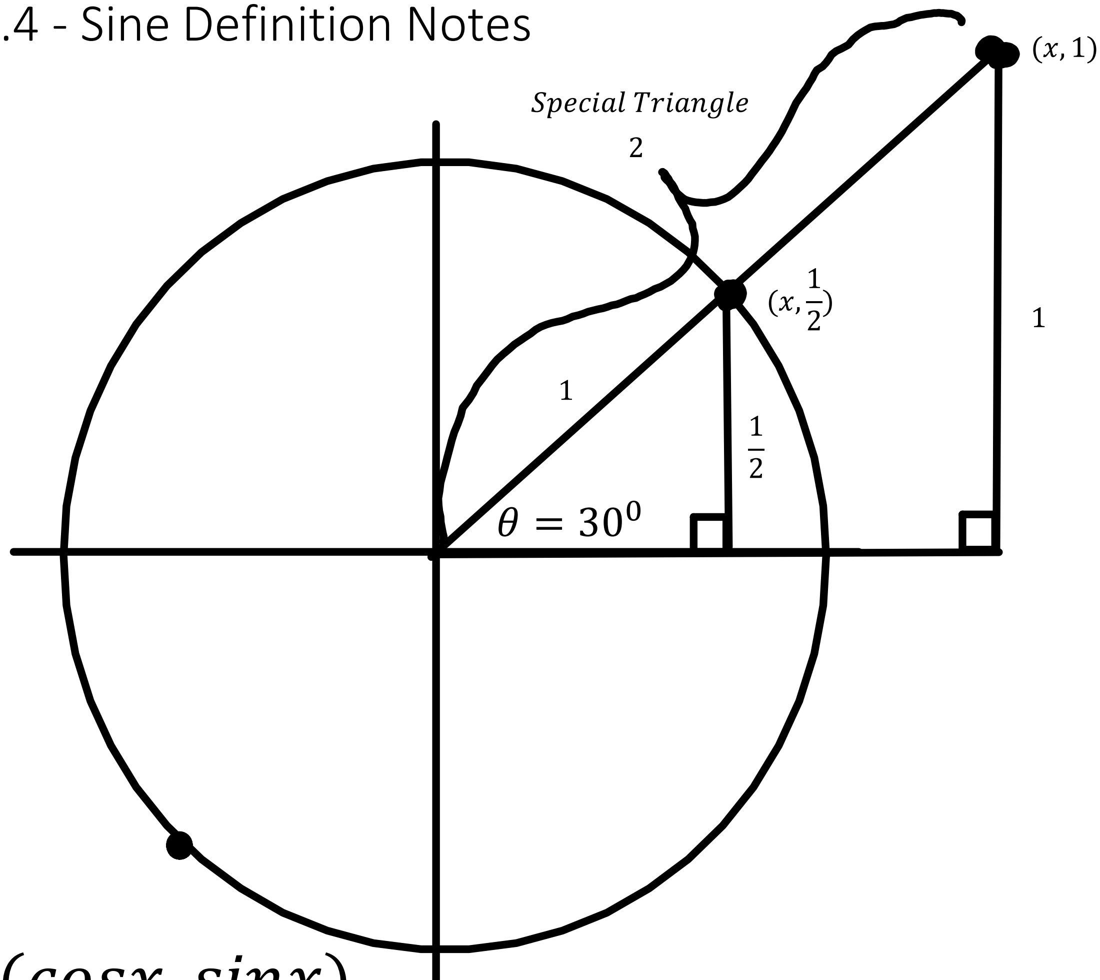
$$\theta_{stp} = \frac{\pi}{6} \qquad \theta_{stp} = 2\pi - \frac{\pi}{6}$$
$$= \frac{11\pi}{6}$$

$$\theta_{stp} = \frac{\pi}{6}, \frac{11\pi}{6}$$

General Solution: $\theta = \theta_{stp} \pm pn, n \in I$

$$\theta = \frac{\pi}{6} \pm 2\pi n, n \in I \qquad \theta = \frac{11\pi}{6} \pm 2\pi n, n \in I$$

C12 - 4.4 - Sine Definition Notes



$(cosx, sinx)$

$$sin\theta = \frac{O}{H}$$

$$sin\theta = \frac{1}{2}$$

$$sin\theta = \frac{O}{H}$$

$$sin\theta = \frac{\frac{1}{2}}{1}$$

$$sin\theta = \frac{1}{2}$$

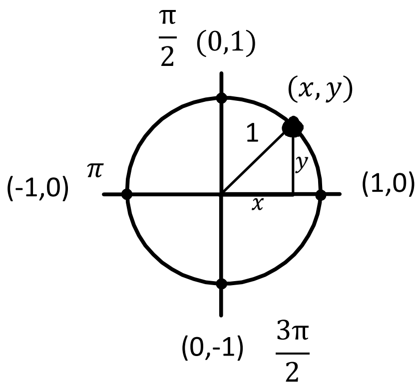
$$(x, y)$$

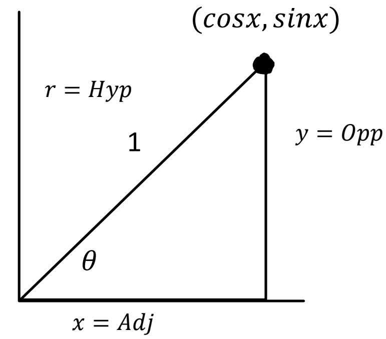
$$(cosx, sinx)$$

$$tanx = m$$

$$sin\theta = y$$

C11 - 4.4 - Unit Circle Quadrantal Angle Notes





$Radius\ of\ unit\ circle = 1$
$Hyp = 1$

$$sin\theta = \frac{Opp}{Hyp}$$
$$sin\theta = \frac{y}{1}$$
$$\boxed{sin\theta = y}$$

$$cos\theta = \frac{Adj}{Hyp}$$
$$cos\theta = \frac{x}{1}$$
$$\boxed{cos\theta = x}$$

$$tan\theta = \frac{Opp}{Adj}$$
$$\boxed{tan\theta = \frac{y}{x}}$$

$$sin0 = \frac{0}{1}$$
$$sin0 = 0$$
$$sin(\frac{3\pi}{2}) = \frac{-1}{1}$$
$$sin(\frac{3\pi}{2}) = -1$$

$$cos\left(\frac{3\pi}{2}\right) = \frac{0}{1}$$
$$cos(\frac{3\pi}{2}) = 0$$
$$cos2\pi = \frac{1}{1}$$
$$cos2\pi = 1$$

$$tan0 = \frac{0}{1}$$
$$tan0 = 0$$
$$\tan(\frac{3\pi}{2}) = \frac{-1}{0}$$
$$tan(\frac{3\pi}{2}) = UND$$

$$csc\theta = \frac{Hyp}{Opp}$$
$$\boxed{csc\theta = \frac{1}{y}}$$
$$csc\theta = \frac{1}{0}$$
$$csc0 = und$$

$$\boxed{csc\theta = \frac{1}{sin\theta}}$$
$$csc0 = \frac{1}{sin0}$$
$$csc0 = \frac{1}{0}$$
$$csc\theta = und$$

$$sec\theta = \frac{Hyp}{Adj}$$
$$\boxed{sec\theta = \frac{1}{x}}$$

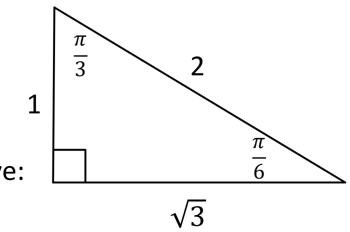
$$\boxed{sec\theta = \frac{1}{cos\theta}}$$
$$\sec\left(\frac{\pi}{2}\right) = \frac{1}{\cos\left(\frac{\pi}{2}\right)}$$
$$\sec\left(\frac{\pi}{2}\right) = \frac{1}{0}$$
$$\sec\left(\frac{\pi}{2}\right) = und$$

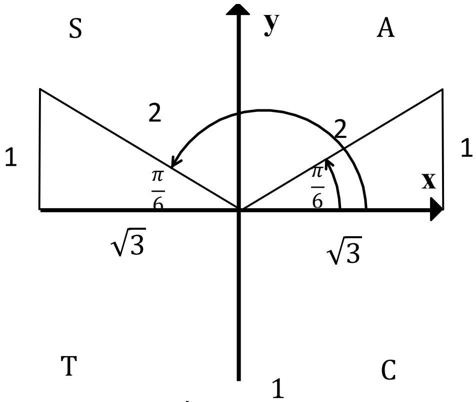
$$cot\theta = \frac{Adj}{Opp}$$
$$\boxed{cot\theta = \frac{x}{y}}$$
$$cot0 = \frac{0}{1}$$
$$cot0 = 0$$

C12 - 4.5 - $\sin 2\theta$ ASTC Special Triangles Notes

Solve for θ $0^o \le \theta < 2\pi$, and the general solution.

$\sin 2\theta = \frac{1}{2}$ $\qquad \sin m = \frac{1}{2}$ $\qquad \boxed{Let\ m = 2\theta}$





Draw two triangles where $\sin m$ is positive:
ASTC Quadrant I, II

Label the triangles according to special triangles and SOH CAH TOA

Label the reference angle according to special triangles.

Draw an arrow from the principal axis to the first terminal arm
Draw an arrow from the principal axis to the second terminal arm.

$$sinm = \frac{1}{2}$$

Solve for the arrows θ_{stp}

$$m_{stp} = \frac{\pi}{6} \qquad m_{stp} = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$m_{stp} = \frac{\pi}{6}, \frac{5\pi}{6}$$

Check your answer: $\sin\frac{\pi}{6} = \frac{1}{2}$ $\qquad \sin\frac{5\pi}{6} = \frac{1}{2}$

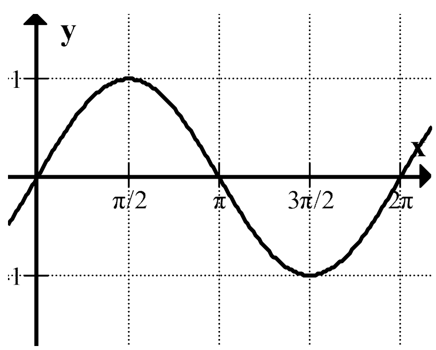
Substitute 2θ back in for m.

$$m = \frac{\pi}{6} \qquad m = \frac{5\pi}{6}$$

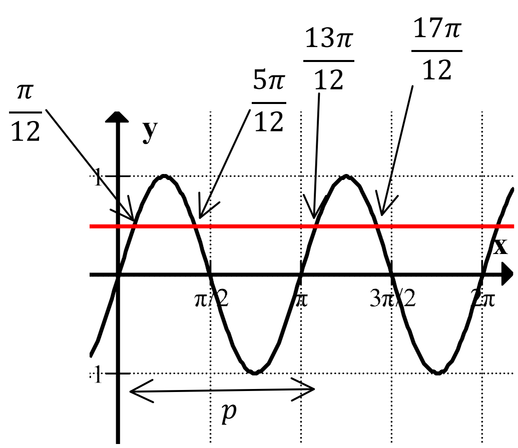
$$2\theta = \frac{\pi}{6} \qquad 2\theta = \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{6 \times 2} \qquad \theta = \frac{5\pi}{6 \times 2}$$

$$\theta = \frac{\pi}{12} \qquad \theta = \frac{5\pi}{12}$$



$y = sin\theta$



$y = sin2\theta$

$y = \frac{1}{2}$

$$\boxed{p = \frac{2\pi}{b} \quad p = \frac{2\pi}{2} = \pi}$$

$$\theta = \theta_{stp} \pm p \qquad \theta = \frac{\pi}{12} + \pi \qquad \theta = \frac{13\pi}{12}$$

$$\theta = \theta_{stp} \pm p \qquad \theta = \frac{5\pi}{12} + \pi \qquad \theta = \frac{17\pi}{12}$$

$$\cancel{\theta = \frac{13\pi}{12} + \pi} \qquad \cancel{\theta = \frac{25\pi}{12} > 2\pi}$$

$$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

Add/Subtract period until outside of the domain.

General Solution:	$\theta = \theta_{stp} \pm pn, n \in I$ $\theta = \frac{\pi}{12} \pm \pi n, n \in I$	$\theta = \theta_{stp} \pm pn, n \in I$ $\theta = \frac{5\pi}{12} \pm \pi n, n \in I$

The usual number of answers in the domain times b.

C12 - 4.5 - Algebra Period Equations Notes

$\boxed{0 \le \theta < 2\pi}$

$$2sinx + 1 = 0$$
$$2sinx = -1$$
$$sinx = -\frac{1}{2}$$

$$5 - 3cosx = 4$$
$$-3cosx = -1$$
$$cosx = \frac{1}{3}$$

$$sinx = x - 1$$

$$x = 1.93$$

$y = sinx$
$y = x - 1$

Find Intersection

$$\cos\left(\frac{\pi}{2}x\right) = 0$$
$$cosm = 0$$

$\boxed{let\ m = \frac{\pi}{2}x}$

...

$$m = \frac{\pi}{2} \qquad \frac{\pi}{2}x = \frac{\pi}{2} \qquad x = 1$$

$$x = 1 + 4 \qquad x = 5$$

$$m = \frac{3\pi}{2} \qquad \frac{\pi}{2}x = \frac{3\pi}{2} \qquad x = 3$$

$$x = 3 + 4 \qquad \cancel{x = 7}$$

Reject

$$\boxed{\begin{aligned} p &= \frac{2\pi}{b} \\ p &= \frac{2\pi}{\frac{\pi}{2}} \\ p &= 2\pi \times \frac{2}{\pi} \\ p &= 4 \end{aligned}}$$

$$\tan(x - 1) = -0.2$$
$$tanm = -0.2$$

$\boxed{let\ m = x - 1}$

...

$$m = 2.94 \qquad x - 1 = 2.94 \qquad x = 3.94$$

$$x = 3.94 - \pi \qquad x = 0.80$$

$$m = 6.09 \qquad x - 1 = 6.09 \qquad \cancel{x = 7.09}$$

Reject

$$x = 7.09 - \pi \qquad x = 3.94$$

$$\boxed{\begin{aligned} p &= \frac{\pi}{b} \\ p &= \frac{\pi}{1} \\ p &= \pi \end{aligned}}$$

$$\sin\left(\frac{\pi}{4}(x - 6)\right) = \frac{1}{2}$$
$$sinm = \frac{1}{2}$$

$\boxed{let\ m = \frac{\pi}{4}(x - 6)}$

Add/Subtract period until outside of the domain.

...

$$m = \frac{\pi}{6} \qquad \frac{\pi}{4}(x - 6) = \frac{\pi}{6} \qquad x - 6 = \frac{2}{3} \qquad x = \frac{20}{3} \qquad \cancel{x = 6.67}$$

$$x = 6.67 - 8 \qquad \cancel{x = -1.33}$$

$$m = \frac{5\pi}{6} \qquad \frac{\pi}{4}(x - 6) = \frac{5\pi}{6} \qquad x - 6 = \frac{10}{3} \qquad x = \frac{28}{3} \qquad \cancel{x = 9.33}$$

$$x = 9.33 - 8 \qquad x = 1.33$$

$$\boxed{\begin{aligned} p &= \frac{2\pi}{b} \\ p &= \frac{2\pi}{\frac{\pi}{4}} \\ p &= 8 \end{aligned}}$$

The usual number of answers in the domain times b.

C12 - 4.6 - Equations Algebra Notes

$sinx + sinx = 2sinx$

$5cosx - 3cosx = 2cosx$

Add/Subtract Like Terms

$$\begin{aligned} 3tanx &= 5 + tanx \\ 3m &= 5 + m \\ 2m &= 5 \\ m &= 2.5 \\ tanx &= 2.5 \end{aligned}$$

....

$\boxed{let\ m = tanx}$

$$\begin{aligned} 1 + sinx &= 4sinx \\ 1 + m &= 4m \\ 3m &= 1 \\ m &= \frac{1}{3} \\ sinx &= \frac{1}{3} \end{aligned}$$

.....

$\boxed{let\ m = sinx}$

$\boxed{Algebra}$

$$\begin{aligned} \frac{cosx}{cosx+1} &= -\frac{1}{3} \\ \frac{m}{m+1} &= -\frac{1}{3} \\ 3m &= -m - 1 \\ m &= -\frac{1}{4} \\ cosx &= -\frac{1}{4} \end{aligned}$$

$m = cosx$

$$\begin{aligned} 2sinx &= 4 \\ sinx &= 2 \end{aligned}$$

No Solution

$$\boxed{\begin{aligned} sinx &= cosx \\ \frac{sinx}{cosx} &= 1 \\ tanx &= 1 \\ &\ldots \\ x &= \frac{\pi}{4}, \frac{5\pi}{4} \end{aligned}}$$

$$\begin{aligned} 5 - 2cscx &= 0 \\ 5 - 2 \times \frac{1}{sinx} &= 0 \\ 5 - \frac{2}{m} &= 0 \\ 5 &= \frac{2}{m} \\ m &= \frac{2}{5} \\ m &= 0.4 \\ sinx &= 0.4 \end{aligned}$$

....

Identities

$\boxed{csc\theta = \frac{1}{sin\theta}}$

$\boxed{let\ m = sinx}$

$$\begin{aligned} sinx - cscx &= 0 \\ sinx - \frac{1}{sinx} &= 0 \\ m - \frac{1}{m} &= 0 \\ \left(m - \frac{1}{m} = 0\right) &\times m \\ m^2 - 1 &= 0 \\ (m+1)(m-1) &= 0 \end{aligned}$$

Identities

$\boxed{let\ m = sinx}$

$Factor$

$$\begin{aligned} m &= 1 \\ sinx &= 1 \end{aligned} \qquad \begin{aligned} m &= -1 \\ sinx &= -1 \end{aligned}$$

....

$sinx \neq 0$

C12 - 4.6 - Factoring/Distributing Notes

$cosx(cosx+1)$

$\cos^2 x + cosx$ *Distribution*

$\boxed{\begin{array}{l} m(m+1) \\ m^2+m \end{array}}$

$\sin x - \sin^2 x$

$sinx(1-sinx)$ *Factor*

$\boxed{\begin{array}{l} \sin x - \sin^2 x \\ m - m^2 \\ m(1-m) \\ sinx(1-sinx) \end{array}}$ $\boxed{m = sinx}$

$sinxcosx + cosx$

$cosx(sinx+1)$

$\boxed{\begin{array}{ll} nm+m & n = sinx \\ m(n+1) & m = cosx \end{array}}$

$(cosx+1)(cosx-2)$

$\cos^2 x - cosx - 2$

$\boxed{\begin{array}{l} (m+1)(m-2) \\ m^2-m-2 \end{array}}$

$(1+cosx)(1-cosx)$

$1 - \cancel{cosx} + \cancel{cosx} - \cos^2 x$ *Distribution*

$1-\cos^2 x$

$\boxed{\begin{array}{c} (m+n)(m-n) \\ m^2-n^2 \end{array}}$

$1-\sin^2 x$

$(1+sinx)(1-sinx)$

$\boxed{\begin{array}{c} 1-a^2 \\ (1-a)(1+a) \end{array}}$

$\cos^2 x - 1$

$(cosx+1)(cosx-1)$

$\boxed{\begin{array}{c} a^2-1 \\ (a+1)(a-1) \end{array}}$

Differences of squares

$\sin^2 x - \cos^2 x$

$(sinx+cosx)(sinx-cosx)$

$\boxed{\begin{array}{c} m^2-n^2 \\ (m+n)(m-n) \end{array}}$

$\cos^4 x - \sin^4 x$

$(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$

$(cos2x)(1)$

$cos2x$

$\boxed{\begin{array}{c} m^4 - n^4 \\ (m^2+n^2)(m^2-n^2) \end{array}}$

Identities

$\boxed{\begin{array}{l} \sin^2\theta + \cos^2\theta = 1 \\ cos2\theta = \cos^2\theta - \sin^2\theta \end{array}}$

$\sin^2 x + sinx - 2$ *Factor*

$(sinx+2)(sinx-1)$

$\boxed{\begin{array}{c} \sin^2 x + sinx - 2 \\ m^2+m-2 \\ (m+2)(m-1) \\ (sinx+2)(sinx-1) \end{array}}$ $\boxed{let\ m = sinx}$

$\sin^2\theta + 2sin\theta cos\theta + \cos^2\theta$

$(sin\theta + cos\theta)(sin\theta + cos\theta)$

$(cosx+sinx)^2$

$\boxed{\begin{array}{l} m^2+2mn+n^2 \\ (m+n)(m+n) \end{array}}$

$-sinx + 1 = 1 - sinx$ $\quad 2 + sinx + \sin^2 x = \sin^2 x + sinx + 2$ *Rearange order of Terms*

$\sin^2 x + tanx + \cos^2 x = \sin^2 x + \cos^2 x + tanx = 1 + tanx$

C12 - 4.6 - Solving Equations Notes

$$\cos^2 x + cosx = 0$$
$$cosx(cosx + 1) = 0$$

Factor

$cosx = 0$

....

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$cosx + 1 = 0$

$cosx = -1$

....

$x = \pi$

$$\cos^2 x + \cos x = 0$$
$$m^2 + m = 0$$
$$m(m + 1) = 0$$

$\boxed{let\ m = cosx}$

Factor

$m = 0$

$cosx = 0$

...

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$m = -1$

$cosx = -1$

...

$x = \pi$

$$\sin^2 x + sinx - 2 = 0$$
$$m^2 + m - 2 = 0$$
$$(m + 2)(m - 1) = 0$$

$\boxed{let\ m = sinx}$

$\cancel{m = -2}$

$\cancel{sinx = -2}$

Reject

$m = 1$

$sinx = 1$

..

$x = \frac{\pi}{2}$

$$\boxed{\begin{array}{c} -1 \leq sinx \leq 1 \\ -1 \leq cosx \leq 1 \end{array}}$$

$$2\sin^2 x + sinx - 1 = 0$$
$$2m^2 + m - 1 = 0$$

...

$$(2m - 1)(m + 1) = 0$$

$\boxed{let\ m = sinx}$

Factor

$m = \frac{1}{2}$

$sinx = \frac{1}{2}$

....

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$m = -1$

$sinx = -1$

...

$x = \frac{3\pi}{2}$

$$3cos^2x - 8cosx - 5 = 0$$
$$3m^2 - 8m - 5 = 0$$

$\boxed{let\ m = cosx}$

...

$\cancel{m = 3.18}$ $\quad m = -.52$

$\boxed{\text{Quadform}}$

$cosx = -.52$

...

$x = 2.12 \quad x = 4.16$

C12 - 4.6 - Identities Chapter 6 Notes

$$\begin{aligned} sin2x + cosx &= 0 \\ 2sinxcosx + cosx &= 0 \\ cosx(2sinx - 1) &= 0 \end{aligned}$$

Identities

$$\boxed{sin2\theta = 2sin\theta cos\theta}$$

$cosx = 0$

$$\begin{aligned} 2sinx - 1 &= 0 \\ sinx &= \frac{1}{2} \end{aligned}$$

.....

$$\begin{aligned} sinx - \cos^2 x - 1 &= 0 \\ sinx - (1 - \sin^2 x) - 1 &= 0 \\ sinx - 1 + \sin^2 x - 1 &= 0 \\ \sin^2 x + sinx - 2 &= 0 \end{aligned}$$

Identities

$$\boxed{\cos^2 x = 1 - \sin^2 x}$$

$$\begin{aligned} sinx + cos2x &= 0 \\ sinx - (1 - 2\sin^2 x) &= 0 \\ 2\sin^2 x + sinx - 1 &= 0 \end{aligned}$$

....

Identities

$$\boxed{cos2x = 1 - 2\sin^2 x}$$

$$\begin{aligned} \frac{cosx}{cosx + 1} &= -\frac{1}{3} \\ \frac{m}{m + 1} &= -\frac{1}{3} \\ 3m &= -m - 1 \\ m &= -\frac{1}{4} \\ cosx &= -\frac{1}{4} \end{aligned}$$

$$\begin{aligned} 1 + cosx &= sinx \\ (1 + cosx)^2 &= (sinx)^2 \\ 1 + 2cosx + \cos^2 x &= \sin^2 x \\ 1 + 2cosx + \cos^2 x &= 1 - \cos^2 x \\ 2\cos^2 x + 2cosx &= 0 \\ 2cosx(cosx + 1) &= 0 \end{aligned}$$

$cosx = 0$ $\quad$ $cosx = -1$

.... $\quad$ ....

$$\begin{aligned} cosxcos2x - sinxsin2x &= -1 \\ cosxcos2x - sinxsin2x &= -1 \\ \cos(2x + x) &= -1 \\ cos3x &= 1 \end{aligned}$$

....

$$\boxed{\cos(\alpha + \beta) = cos\alpha cos\beta - sin\alpha sin\beta}$$

Identities

Any Denominator ever can't equal zero!

C12 - 4.7 - NPV Trig Notes

$0 \le \theta < 2\pi$ General Solution

$\frac{1}{cos\theta}$ $\frac{\frac{tan\theta}{sin\theta}}{cos\theta}$ $\frac{\frac{sec\theta}{1}}{cos\theta}$ $cos\theta \neq 0$ $\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}$ $\theta \neq \frac{\pi}{2} + \pi n, nEI$ $\boxed{cos\theta = y}$

$\boxed{p = \frac{3\pi}{2} - \frac{\pi}{2} = \pi}$

$\frac{1}{sin\theta}$ $\frac{\frac{cot\theta}{cos\theta}}{sin\theta}$ $\frac{\frac{csc\theta}{1}}{sin\theta}$ $sin\theta \neq 0$ $\theta \neq 0, \pi$ $\theta \neq \pi n, nEI$ $\boxed{sin\theta = y}$

$\frac{1}{tan\theta}$

$\frac{1}{\frac{sin\theta}{cos\theta}}$ $sin\theta \neq 0$ $cos\theta \neq 0$ ...

$\frac{1}{cot\theta}$

$\frac{1}{\frac{cos\theta}{sin\theta}}$ $sin\theta \neq 0$ $cos\theta \neq 0$ ...

Any denominator or any part of a fraction that will make a denominator zero.

$\frac{1}{cos\theta + 1}$

$cos\theta + 1 \neq 0$

$cos\theta \neq -1$

$\theta \neq \pi$ $\theta \neq \pi + 2\pi n, nEI$

$\frac{1}{sin\theta - \frac{1}{2}}$

$sin\theta - \frac{1}{2} \neq 0$

$sin\theta \neq \frac{1}{2}$

...

$\frac{1}{\cos^2 x - 1}$

$\cos^2 x - 1 \neq 0$

$\sin^2 x \neq 1$

$sinx \neq \pm 1$

...

$\frac{1}{\sin^2 x + 1}$

$\sin^2 x + 1 \neq 0$

$\sin^2 x \neq -1$

$sinx \neq \sqrt{-1}$

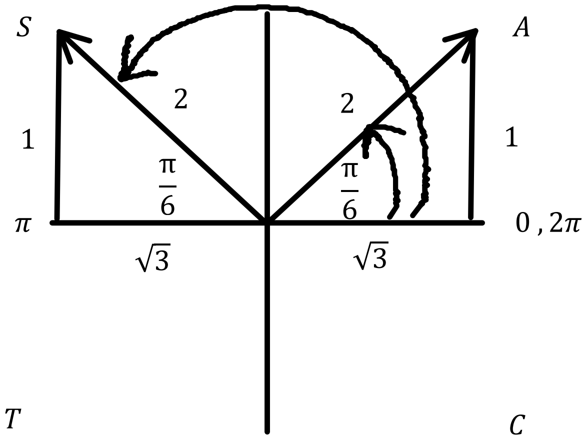
No Restrictions

C12 - 4.7 - ASTC General Solutions

$\boxed{\theta = \theta_{stp} \pm pn, n \in I}$

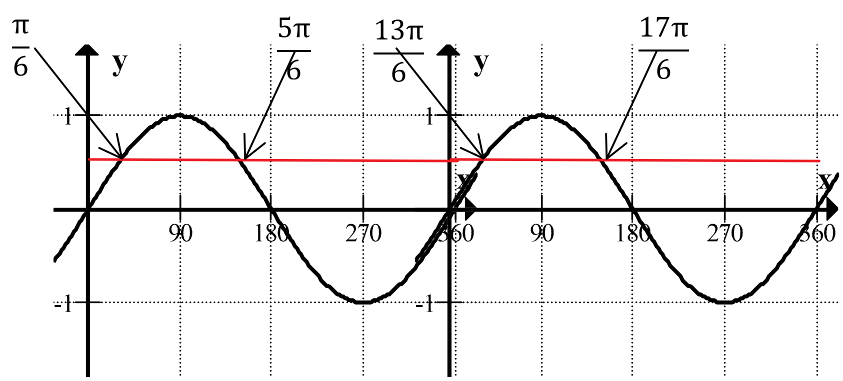
$Solve\ for\ \theta, 0 \leq \theta < 2\pi, and\ find\ general\ solution.$

$sin\theta = \frac{1}{2}$



$\theta = \frac{\pi}{6}$ $\theta = \frac{5\pi}{6}$

$\theta = \frac{\pi}{6} + 2\pi n, nEI$ $\theta = \frac{5\pi}{6} + 2\pi n, nEI$



$$\theta = \frac{\pi}{6} + 2\pi$$
$$\theta = \frac{\pi}{6} + \frac{12\pi}{6}$$
$$\theta = \frac{13\pi}{6}$$

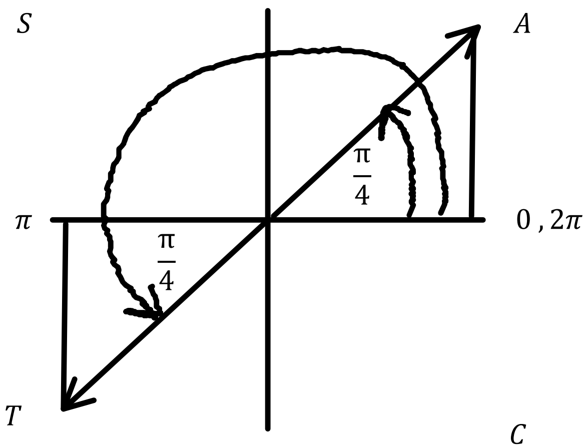
$$\theta = \frac{5\pi}{6} + 2\pi$$
$$\theta = \frac{5\pi}{6} + \frac{12\pi}{6}$$
$$\theta = \frac{17\pi}{6}$$

C12 - 4.7 - ASTC Reject General Solutions

$$\boxed{\theta = \theta_{stp} \pm pn, n \in I}$$

Solve for $\theta, 0 \leq \theta < 2\pi$, *and find general solution.*

$tan\theta = 1$

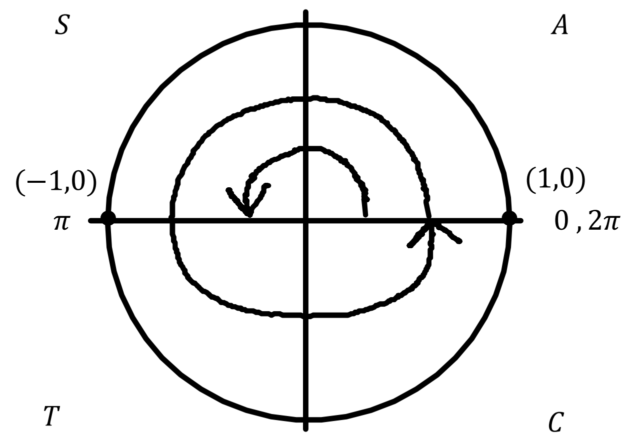


$\theta = \frac{\pi}{4}$ $\qquad$ $\theta = \frac{5\pi}{4}$

$\theta = \frac{\pi}{4} + \pi n, nEI$ $\qquad$ $\cancel{\theta = \frac{5\pi}{4} + \pi n, nEI}$

$\frac{\pi}{4}, \frac{5\pi}{4}, \ldots$ $\qquad$ $\frac{\pi}{4} + p + p + p \ldots$

$sin\theta = 0$

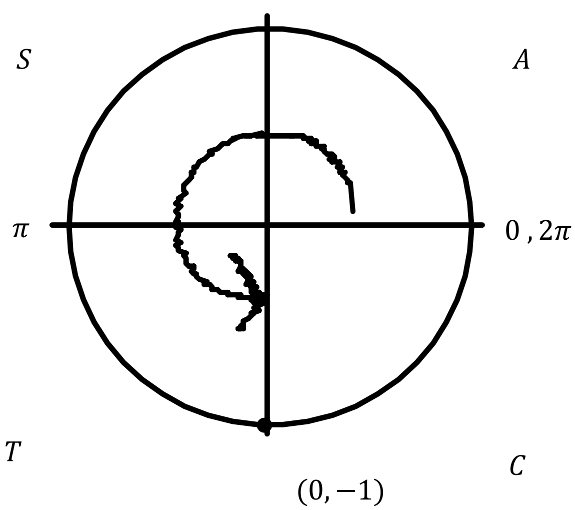


$\theta = 0$ $\qquad$ $\theta = \pi$

$\theta = 0 + 2\pi n, nEI$ $\qquad$ $\theta = \pi + 2\pi n, nEI$

$\theta = \pi n, nEI$

$sin\theta = -1$

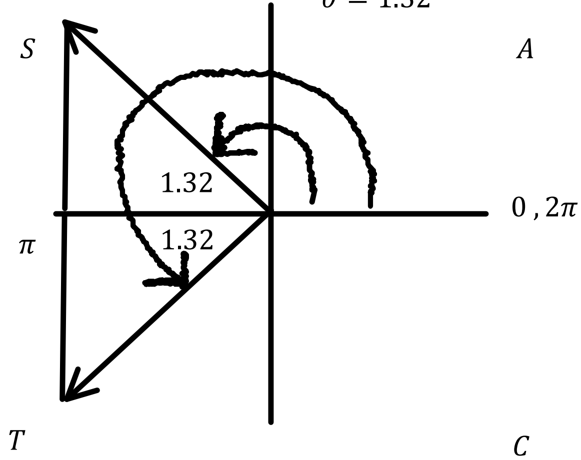


$\theta = \frac{3\pi}{2}$

$\theta = \frac{3\pi}{2} + 2\pi n, nEI$

$cos\theta = -\frac{1}{4}$ $\qquad$ $\theta = \cos^{-1}(\frac{1}{4})$

$\theta = 1.32$



$\theta = 1.82$ $\qquad$ $\theta = 4.46$

$\theta = 1.82 + 2\pi n, nEI$ $\qquad$ $\theta = 4.46 + 2\pi n, nEI$

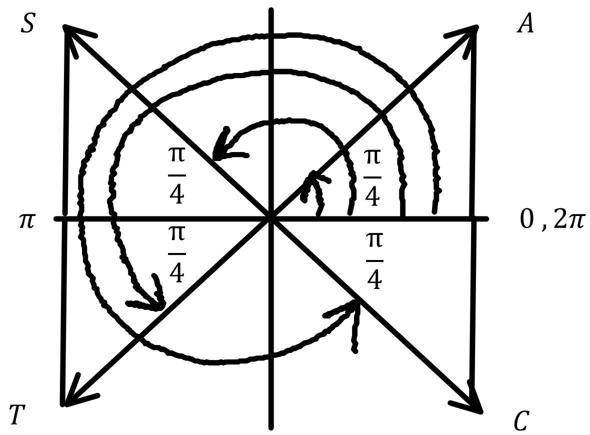
C12 - 4.7 - Square Root General Solutions

$\boxed{\theta = \theta_{stp} \pm pn, n \in I}$

Solve for $\theta, 0 \leq \theta < 2\pi$, and find general solution.

$$\sin^2\theta = \frac{1}{2}$$

$$sin\theta = \pm\frac{1}{\sqrt{2}}$$



$\theta = \frac{\pi}{4}$ $\theta = \frac{3\pi}{4}$ $\theta = \frac{5\pi}{4}$ $\theta = \frac{7\pi}{4}$

$$\theta = \frac{\pi}{4} + \frac{\pi}{2}n, nEI$$

$$\tan^2\theta = 1$$

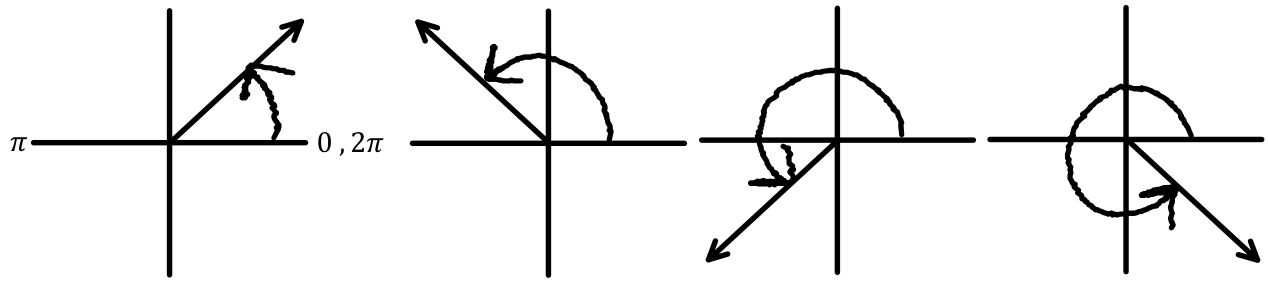
$$\frac{\sin^2\theta}{\cos^2\theta} = 1$$

$$\sin^2\theta = \cos^2\theta$$

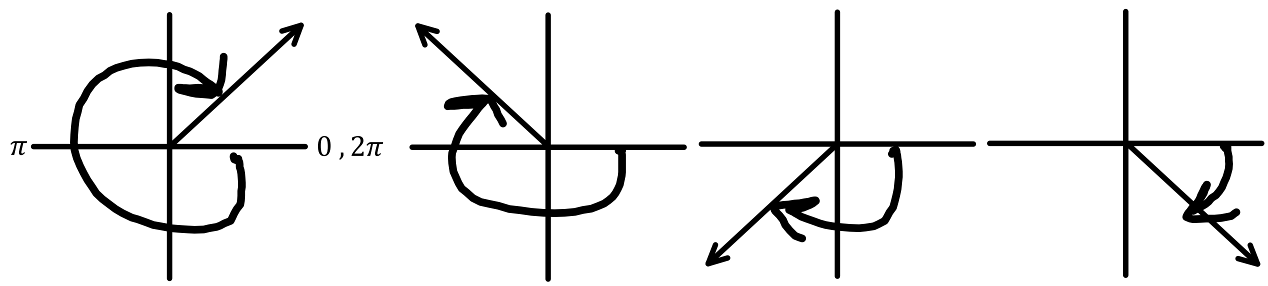
$$\theta = \frac{\pi}{4} + \frac{\pi}{2}n, nEI$$

C12 - 4.7 - Domain Change Notes

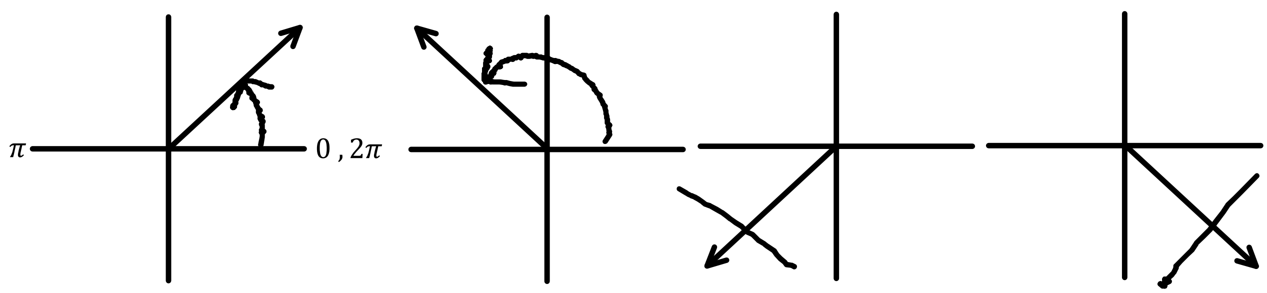
$0 \le \theta < 2\pi$



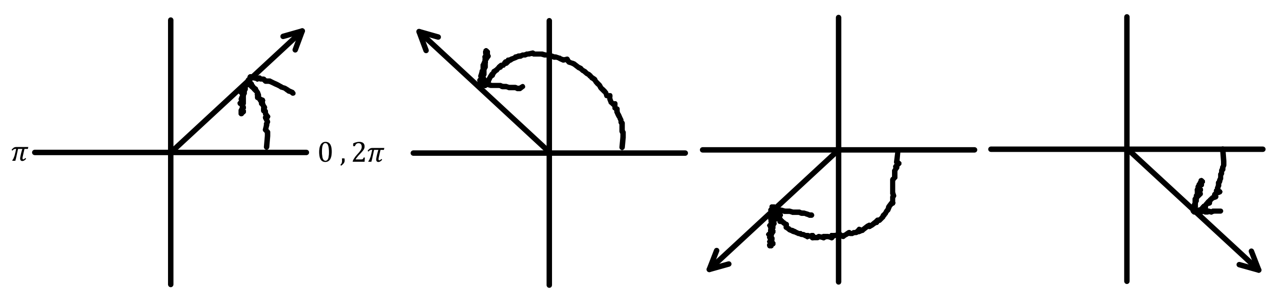
$-2\pi \le \theta < 0$



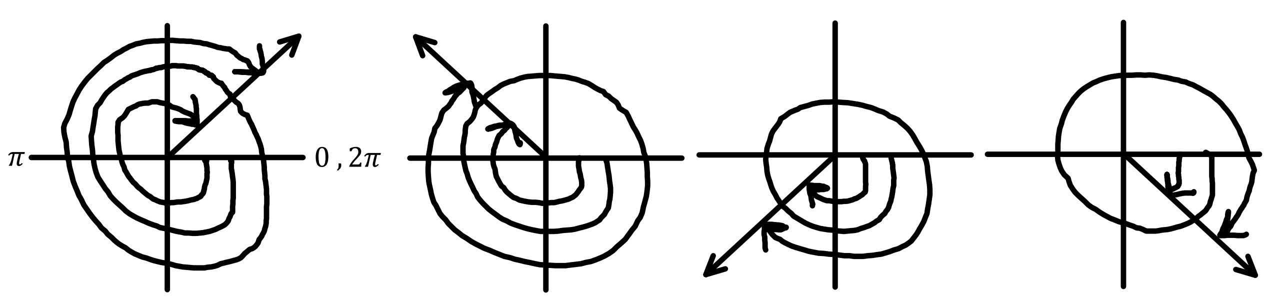
$0 \le \theta < \pi$



$-\pi \le \theta < \pi$

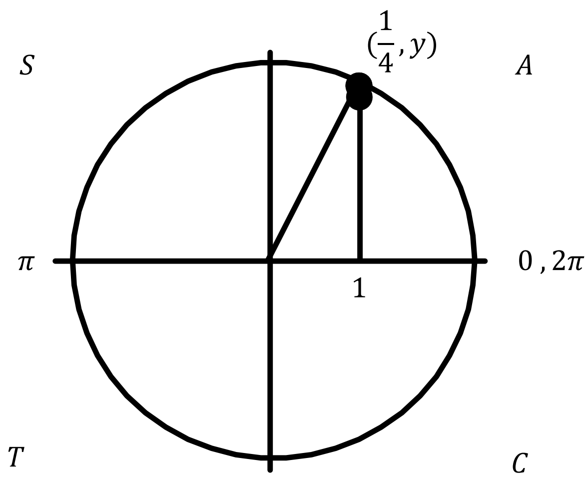


$-4\pi \le \theta < 0$

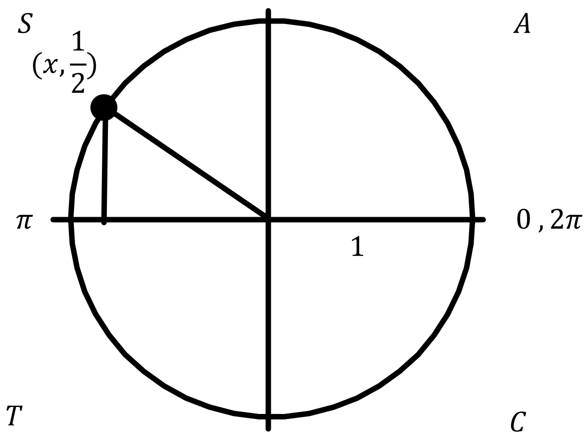


C12 - 4.8 - Solve (x,y) Unit Circle Notes

Solve the point on the unit circle



$$x^2 + y^2 = 1$$
$$\left(\frac{1}{4}\right)^2 + y^2 = 1$$
$$\frac{1}{16} + y^2 = \frac{16}{16}$$
$$y^2 = \frac{15}{16}$$
$$y = \pm\frac{\sqrt{15}}{4} \qquad \left(\frac{1}{4}, \frac{\sqrt{15}}{4}\right)$$



$$x^2 + y^2 = 1$$
$$x^2 + \left(\frac{1}{2}\right)^2 = 1$$
$$x^2 + \frac{1}{4} = \frac{4}{4}$$
$$x^2 = \frac{3}{4}$$
$$x = \pm\frac{\sqrt{3}}{2} \qquad \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

Is the point on the unit circle

$(-\frac{3}{4}, \frac{1}{4})$

$$x^2 + y^2 = 1$$
$$\left(-\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 \neq 1$$
$$\frac{9}{16} + \frac{1}{16} \neq 1$$
$$\frac{10}{16} \neq 1$$

Not on Unit Circle

$(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$

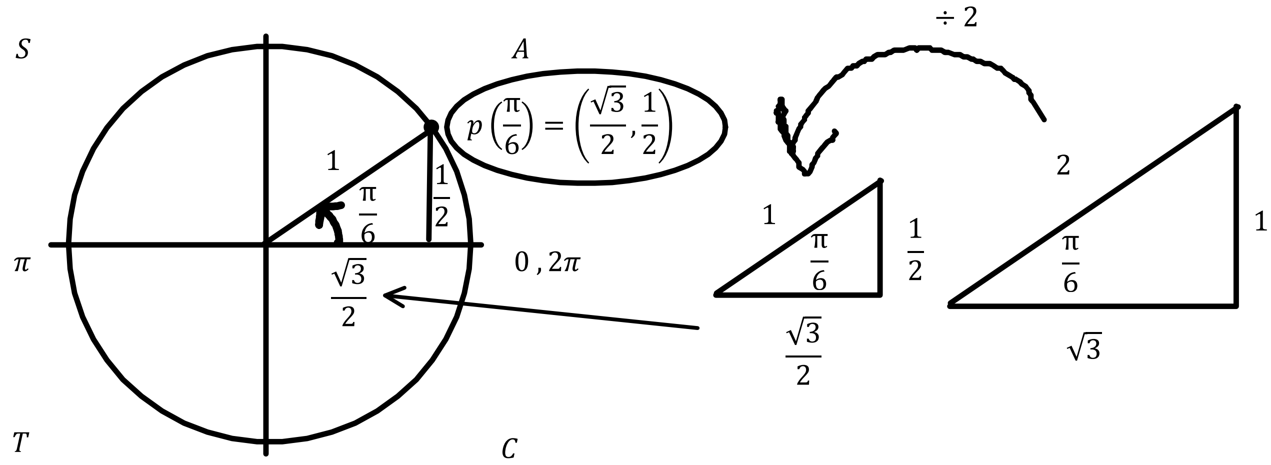
$$x^2 + y^2 = 1$$
$$\left(-\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 = 1$$
$$\frac{3}{4} + \frac{1}{4} = 1$$
$$1 = 1$$

On Unit Circle

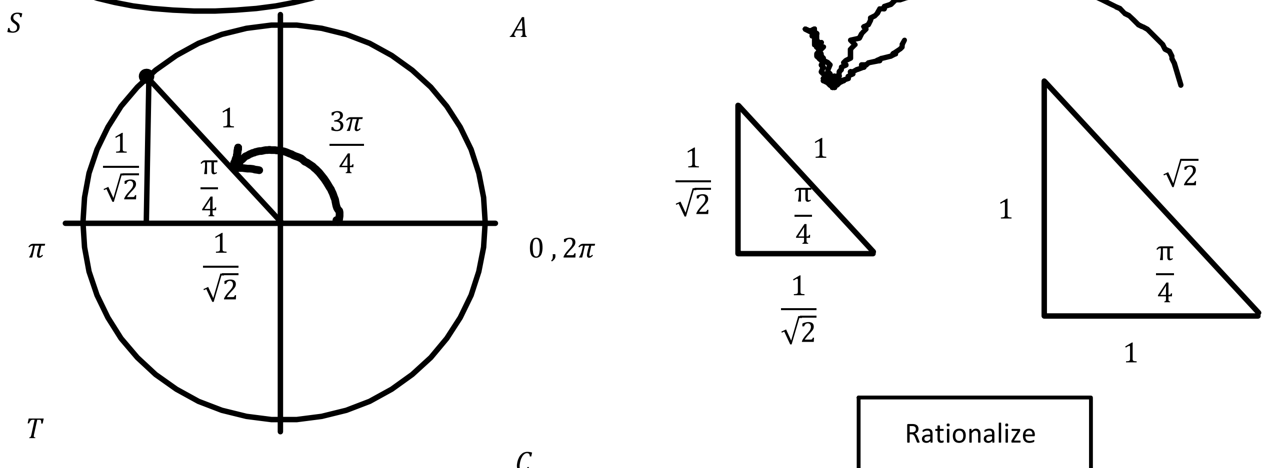
C12 - 4.8 - Solve p(θ) Unit Circle Notes

Solve the point on the unit circle

$$p\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$



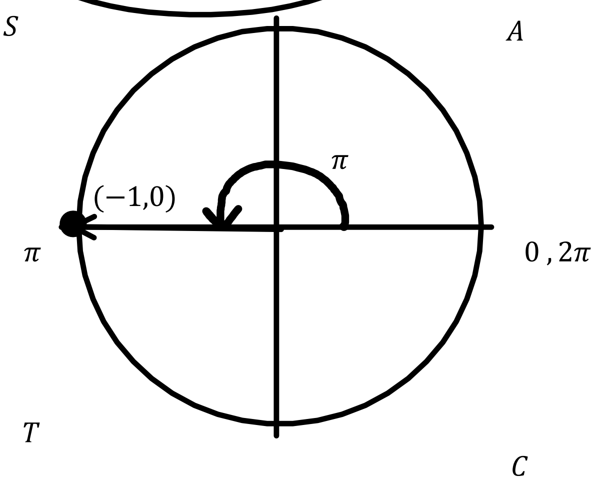
$$p\left(\frac{3\pi}{4}\right) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$



Rationalize

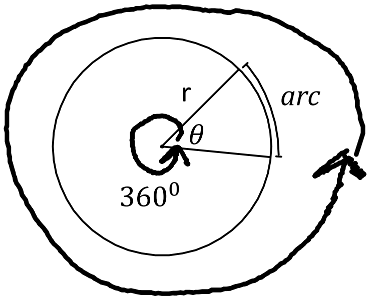
$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$p(\pi) = (-1,0)$$



C12 - 4.9 - Arc Length, Sector Area Notes

θ *in radians*



Circumference

$$\boxed{\frac{arc\ length}{Circumference} = \frac{\theta}{360^o}}$$ Grade 8-11

$$\frac{arc\ length}{Circumference} = \frac{\theta}{2\pi}$$

$$\frac{a}{2\pi r} = \frac{\theta}{2\pi}$$

$$\cancel{2\pi} \times \frac{a}{\cancel{2\pi} r} = \frac{\theta}{\cancel{2\pi}} \times \cancel{2\pi}$$

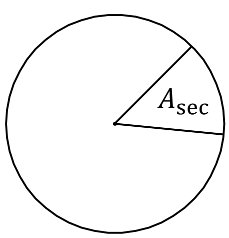
$$\frac{a}{r} = \theta$$

$$\cancel{r} \times \frac{a}{\cancel{r}} = \theta \times r$$

$$\boxed{a = \theta r}$$

$$\boxed{a = \theta r}$$ θ *must be in radians*

Sector Area



$$\frac{Area_{sector}}{Area_{Total}} = \frac{arc\ length}{Circumference}$$

$$\boxed{A = \frac{\theta r^2}{2}}$$

$$\frac{A_{\text{sec}}}{\pi r^2} = \frac{a}{2\pi r}$$

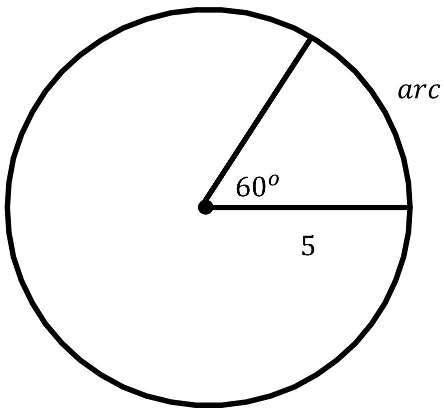
$$\boxed{A_{sec} = \frac{ar}{2}}$$

$$\boxed{A = \frac{ar}{2}}$$

$$\boxed{\frac{A_{\text{sec}}}{\pi r^2} = \frac{a}{2\pi r} = \frac{\theta}{360^o} = \frac{\theta}{2\pi}}$$ They are all equal to each other.

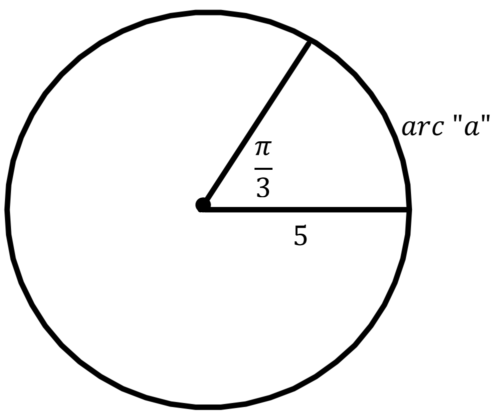
C12 - 4.9 - Arc Length Notes

Find the arc length



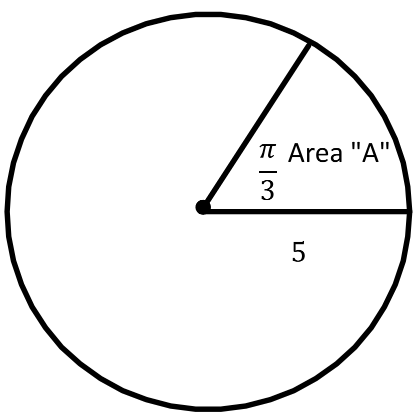
$$\frac{a}{C} = \frac{\theta}{360}$$
$$\frac{a}{2\pi r} = \frac{60}{360}$$
$$a = \frac{\pi(5)}{3}$$
$$a = 5.24$$

$$C = 2\pi r$$
$$C = 2\pi \times 5$$
$$C = 31.4$$
$$\frac{60^o}{360^o} = \frac{1}{6} \text{ of circle}$$
$$\frac{1}{6} \times 31.4 = 5.2$$



$$a = \theta r$$
$$a = \frac{\pi}{3} \times 5$$
$$a = 5.24$$

Find the Area pf the Sector

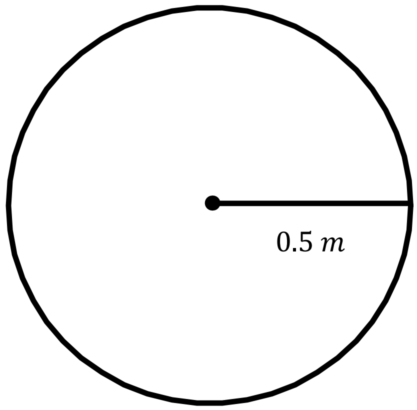


$$\frac{Area_{sector}}{Area_{Total}} = \frac{\theta}{2\pi}$$
$$\frac{A_{sec}}{\pi r^2} = \frac{\theta}{2\pi}$$
$$A_{sec} = \frac{\theta r^2}{2}$$
$$A_{sec} = \frac{\left(\frac{\pi}{3} \times 5^2\right)}{2}$$
$$A_{sec} = 13.09$$

$$A = \pi r^2$$
$$A = \pi(5)^2$$
$$A = 78.54$$
$$\frac{1}{6} \times 78.54 = 13.09$$

C12 - 4.9 - Angular Velocity Notes

Find the angular velocity of a wheel travelling 25 meters per second if the radius 0.5 meters. Find the arc in 0.1 seconds.



$$w = \frac{\theta}{t}$$

Angular Velocity "w"

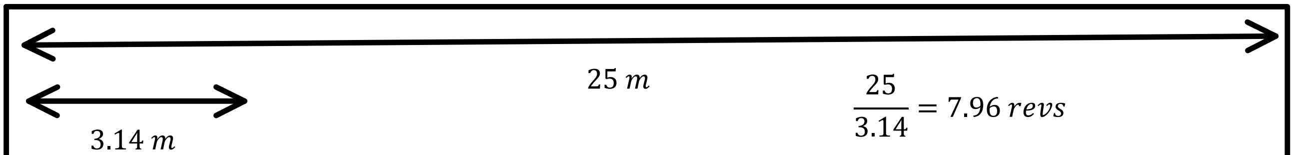
$$C = 2\pi r$$
$$C = 2\pi(0.5)$$
$$C = 3.14\ m$$

$$1\ Rev = 2\pi \qquad \boxed{w = \frac{v}{C} \times 2\pi}$$

$$\boxed{w = \frac{v}{r}}$$

$$\frac{25m}{s} \times \frac{1\ revolution}{3.14m} = 7.96\frac{Rev}{s}$$

$$7.96 \times \frac{2\pi}{s} = 15.92\frac{Rad}{s} \qquad w = 15.92\pi\frac{Rad}{s}$$



$$\frac{25}{3.14} = 7.96\ revs$$

$$25\frac{m}{s} \Rightarrow 0.04\frac{s}{m} \qquad \frac{25}{1} \Rightarrow \frac{1}{25} = 0.04$$

$$15.92\pi\frac{Rad}{s} \times 0.1\ s = 5\ Rad \qquad w = \frac{\theta}{t}$$

$$a = \theta r$$
$$a = 5(0.5)$$
$$a = 2.5\ m$$

$$\frac{25m}{s} \times \frac{1\ revolution}{3.14m} = 7.96\frac{Rev}{s} \qquad 1\ Rev = 360^0$$

$$7.96 \times \frac{360^0}{s} = 2865.6\frac{o}{s} \qquad w = 2865.6\frac{o}{s} \qquad \boxed{w = \frac{v}{C} \times 360^0}$$

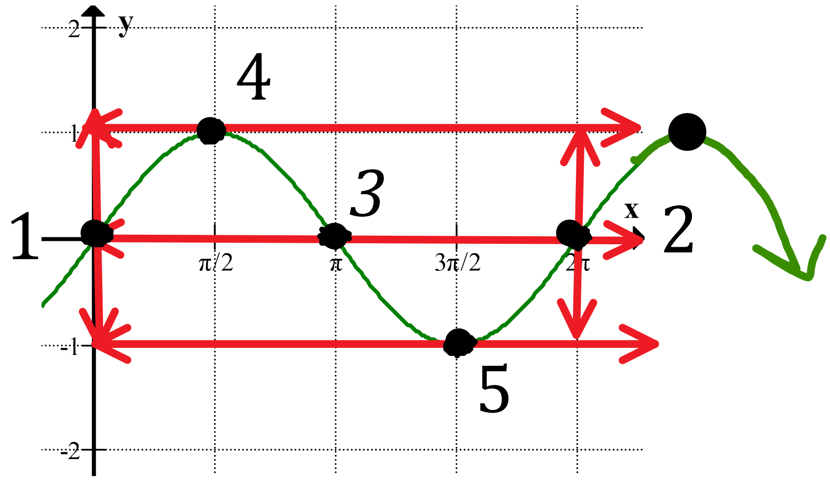
$$25\frac{m}{s} \times 0.1\ s = 2.5\ m \qquad \boxed{d = vt}$$

C12 - 5.1 - TOV Radians sinx,cosx,tanx TOV Graphs Notes

$y = \sin x$

x	y
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

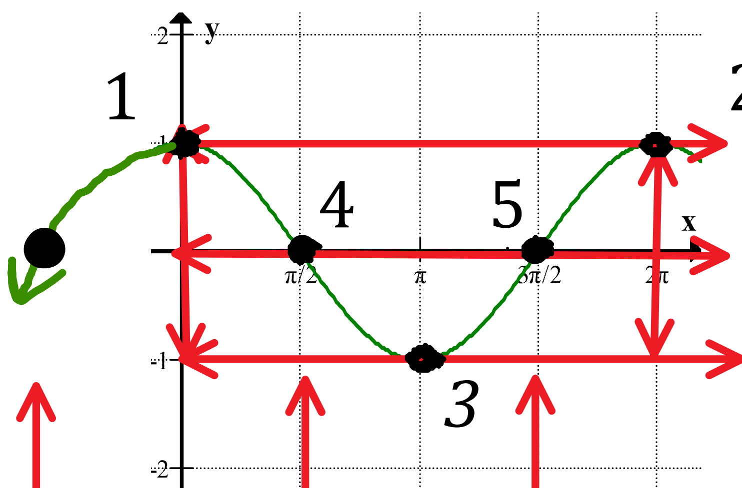
Pt.
(0,0)
$(\frac{\pi}{2},1)$
$(\pi,0)$
$(\frac{3\pi}{2},-1)$
$(2\pi,0)$



$y = \cos x$

x	y
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1

Pt.
(0,1)
$(\frac{\pi}{2},0)$
$(\pi,-1)$
$(\frac{3\pi}{2},0)$
$(2\pi,1)$

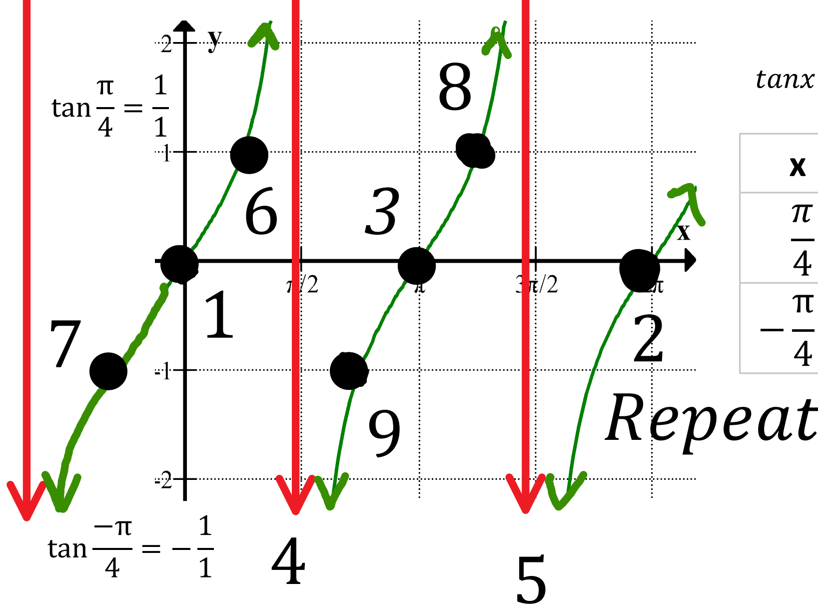


$y = \tan x$

x	y
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	und
$\frac{3\pi}{4}$	-1
π	0

Pt.
(0,0)
$(\frac{\pi}{4},1)$
$(\frac{\pi}{2},\text{und})$
$(\frac{3\pi}{4},-1)$
$(\pi,0)$

Tan is Zero when sin is zero
Tan is UND when cos is zero



$$\tan x = \frac{\sin x}{\cos x}$$

x	y
$\frac{\pi}{4}$	1
$-\frac{\pi}{4}$	-1

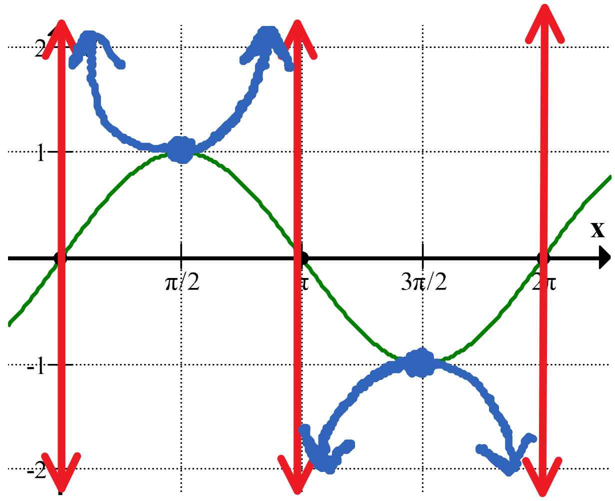
Special Triangles
ASTC

C12 - 5.1 - TOV Radians cscx,secx,cotx TOV Graphs Notes

$y = \csc x$

x	y
0	und
$\frac{\pi}{2}$	1
π	und
$\frac{3\pi}{2}$	-1
2π	und

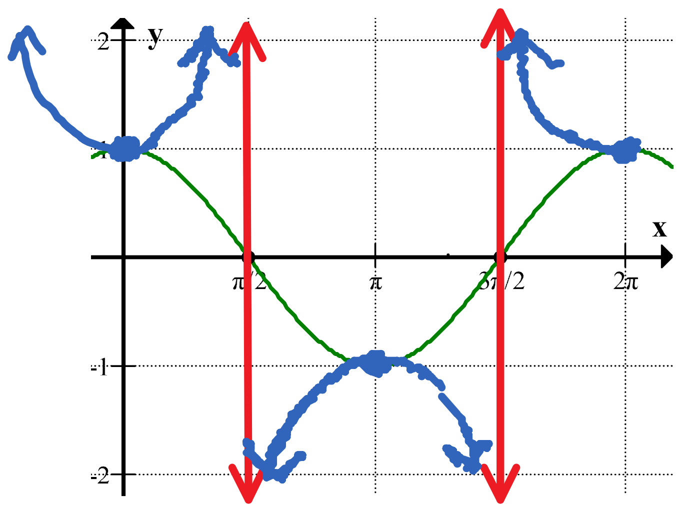
Pt.
(0,0)
$(\frac{\pi}{2},1)$
$(\pi,0)$
$(\frac{3\pi}{2},-1)$
$(2\pi,0)$



$y = \sec x$

x	y
0	1
$\frac{\pi}{2}$	und
π	-1
$\frac{3\pi}{2}$	und
2π	1

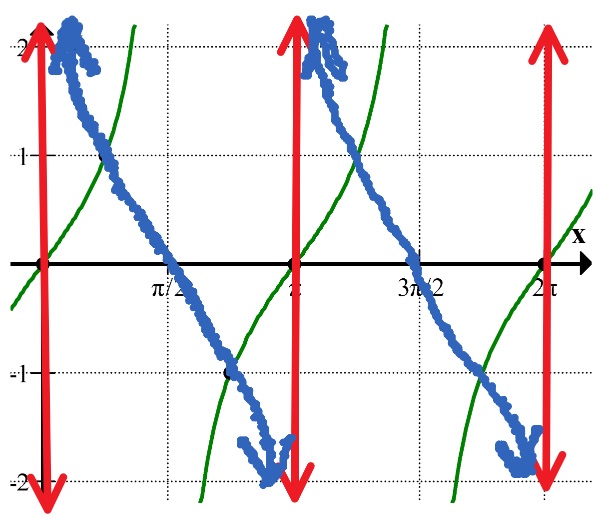
Pt.
(0,1)
$(\frac{\pi}{2},0)$
$(\pi,-1)$
$(\frac{3\pi}{2},0)$
$(2\pi,1)$



$y = \cot x$

x	y
0	und
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	-1
π	und

Pt.
(0,0)
$(\frac{\pi}{4},1)$
$(\frac{\pi}{2},\text{und})$
$(\frac{3\pi}{4},-1)$
$(\pi,0)$



Cot is Zero when cos is zero
Cot is UND when sin is zero

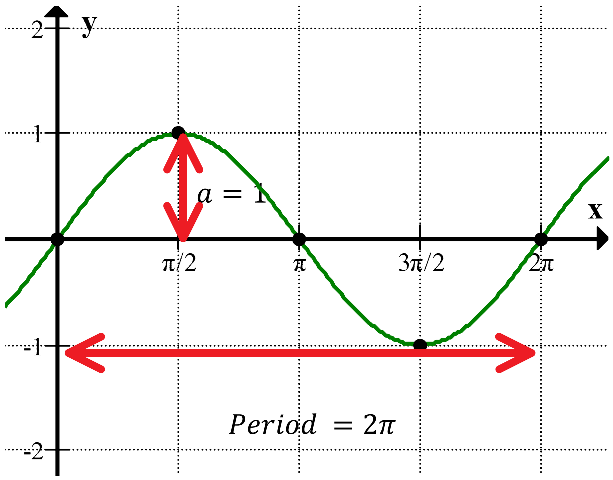
$$cotx = \frac{cosx}{sinx}$$

C12 - 5.2 - (a,b) Sine Transformations Notes

$$y = a\sin(b(x - c)) + d$$

Amplitude | Period $= \frac{2\pi}{b}$ | Phase Shift | Center Line

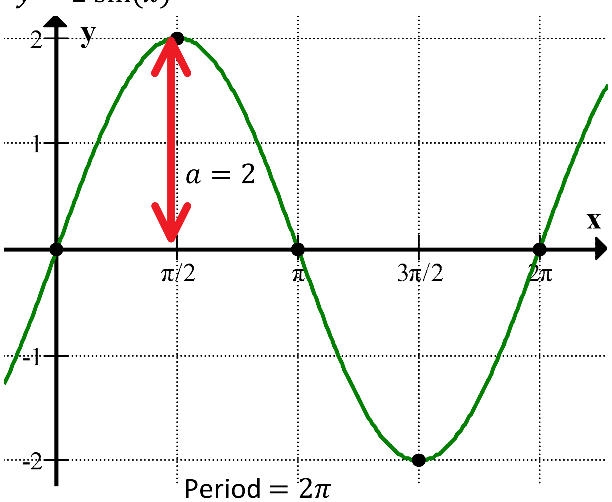
$y = \sin(x)$



Amplitude Change

$y = 2\sin(x)$

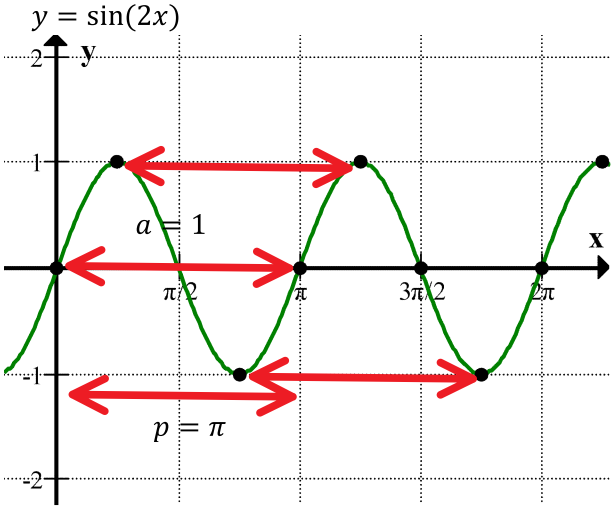
$VE = 2$



Period Change

$y = \sin(2x)$

$HC = \frac{1}{2}$



$p = \frac{2\pi}{b}$

$p = \frac{2\pi}{2}$

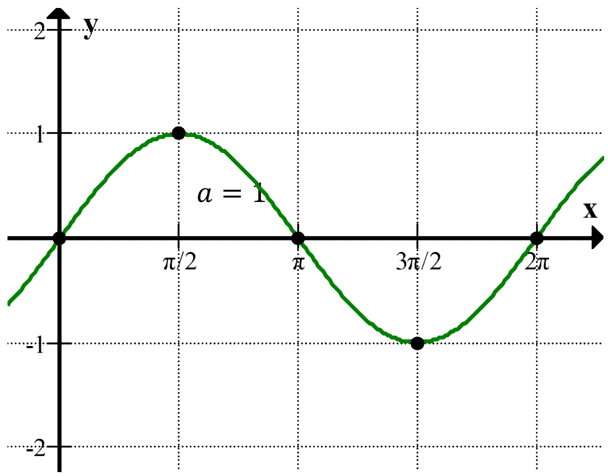
$p = \pi$

C12 - 5.3 - (d,c) Sinusoidal Transformations Notes

$$y = a\sin(b(x - c)) + d$$

Amplitude   Period $= \frac{2\pi}{b}$ Phase Shift   Center Line

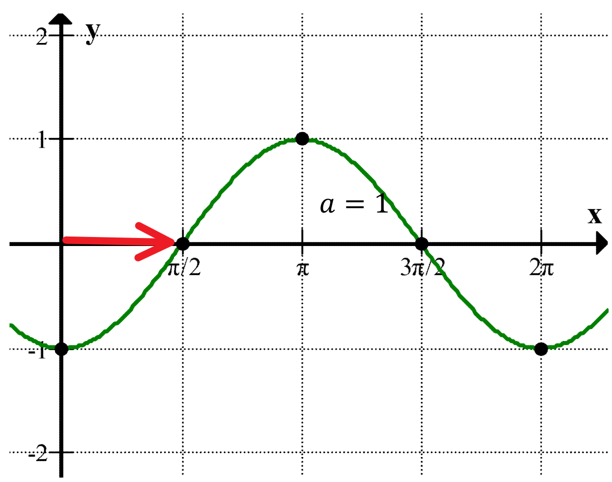
$y = \sin(x)$



Phase Shift

$y = \sin\left(x - \frac{\pi}{2}\right)$

$HT = \frac{\pi}{2}$

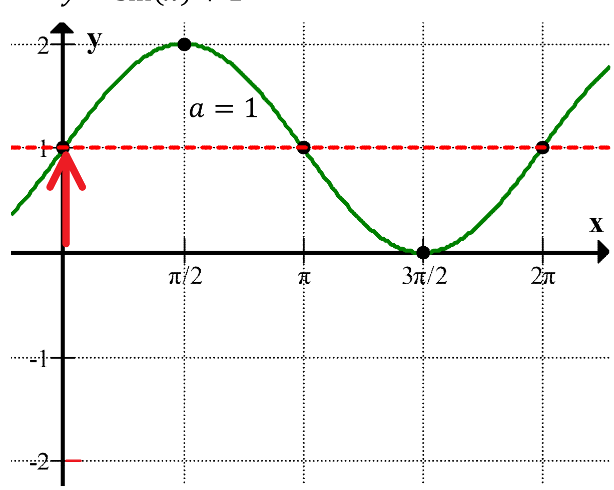


Phase shift $\frac{\pi}{2}$ units to the right

Center Line Shift

$y = \sin(x) + 1$

$VT = +1$



$up\ 1$

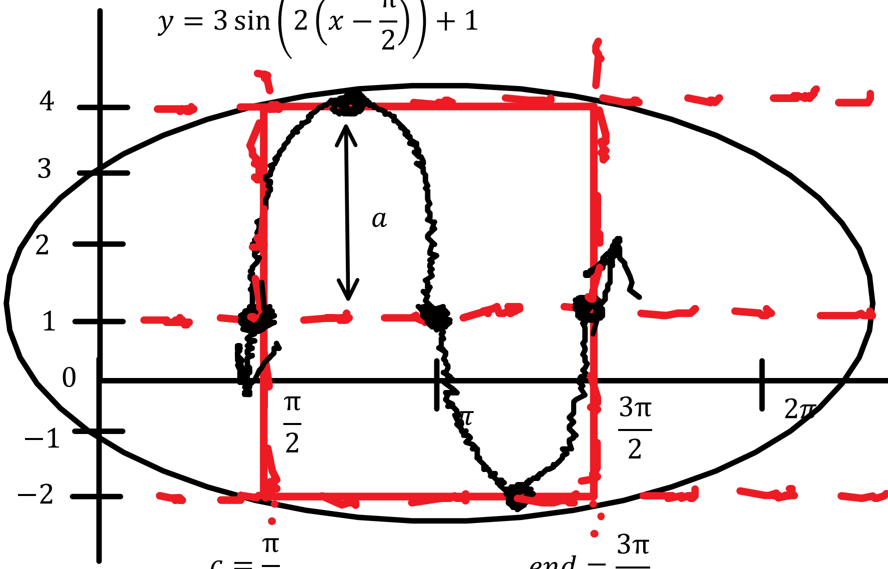
C12 - 5.4 - Trig Graphing Notes

$y = a\sin(b(x-c)) + d$

1) d
2) a
3) c
4) b

Graph

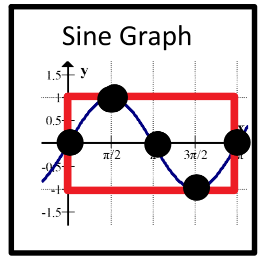
$$\overset{a}{y = 3}\sin\left(\overset{b}{2}\left(x - \overset{c}{\frac{\pi}{2}}\right)\right) + \overset{d}{1}$$



$d + a = 4$

$d = 1$

$d - a = -2$



$c = \frac{\pi}{2}$ $\quad end = \frac{3\pi}{2}$

$p = \frac{2\pi}{b}$

$p = \frac{2\pi}{2}$

$p = \pi$

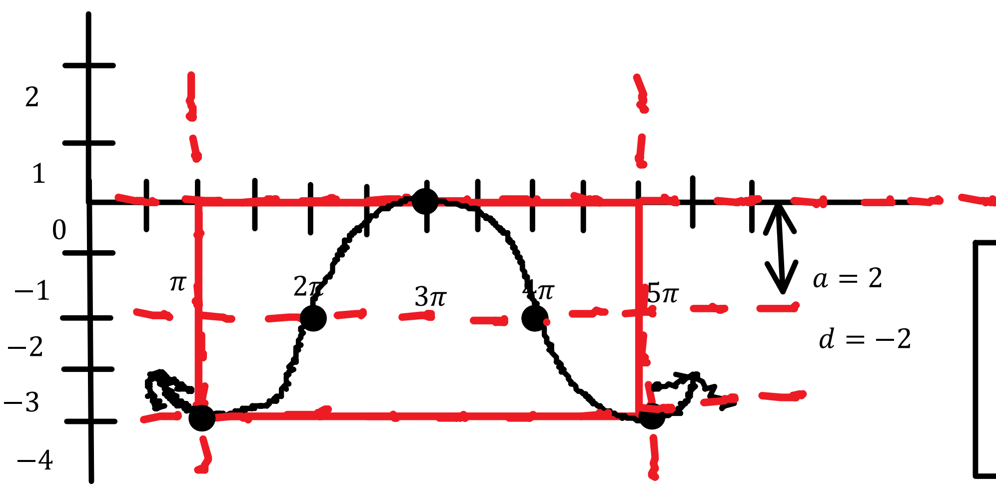
$end = c + p$

$= \frac{\pi}{2} + \pi$

$= \frac{\pi}{2} + \frac{2\pi}{2}$

$end = \frac{3\pi}{2}$

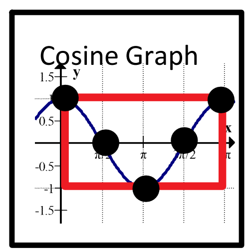
Find Equation



$a = 2$

$d = -2$

1) d
2) a
3) c
4) b



$c = \pi$

$p = 2nd - c$

$= 5\pi - \pi$

$= 4\pi$

$end = 5\pi$

$p = \frac{2\pi}{b}$

$b = \frac{2\pi}{p}$

$b = \frac{2\pi}{4\pi}$

$b = \frac{1}{2}$

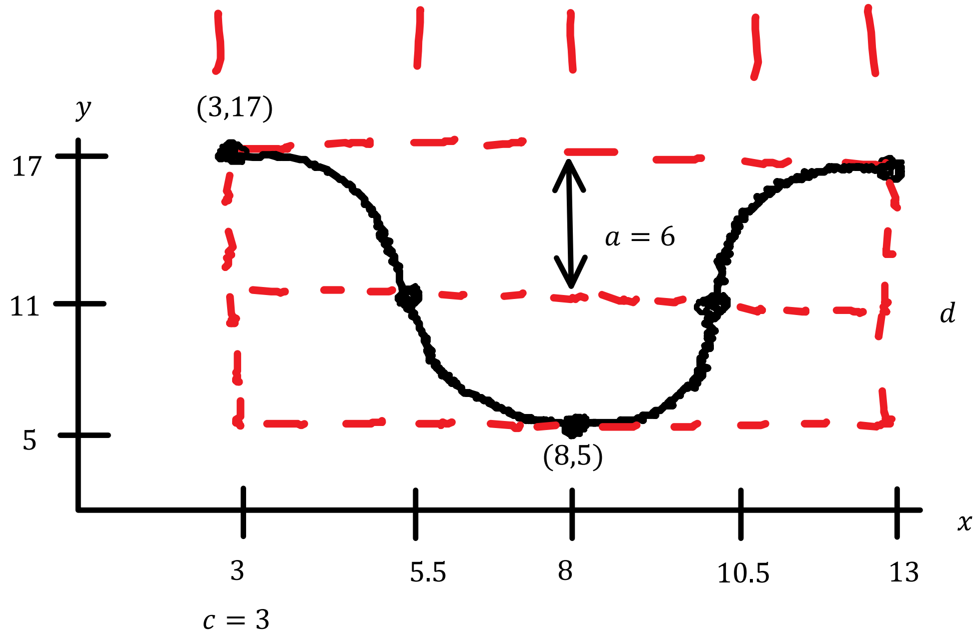
Equation

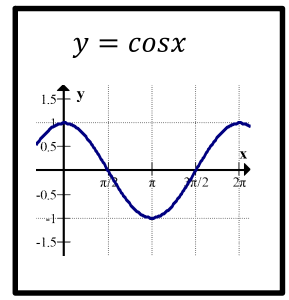
$$y = \overset{a}{-2}\cos\left(\overset{b}{\frac{1}{2}}(x - \overset{c}{\pi})\right) \overset{d}{-2}$$

$y = a\cos(b(x-c)) + d$

C12 - 5.4 - Max Min Points Notes

A sinusoidal function has a maximum at (3,17) and a minimum at (8,5). Find the equation.





$\frac{17+5}{2} = 11$ $\quad \frac{17-5}{2} = 6$ $\quad 8 - 3 = 5$ $\quad 5 \times 2 = 10$ $\quad p = 10$

$17 - 6 = 11$

$5 + 6 = 11$

$\frac{5}{2} = 2.5$ $\quad 3 + 2.5 = 5.5$

$$y = a\cos(b(x - c)) + d$$

$$p = \frac{2\pi}{b}$$

$$b = \frac{2\pi}{p}$$

$$b = \frac{2\pi}{10}$$

$$b = \frac{\pi}{5}$$

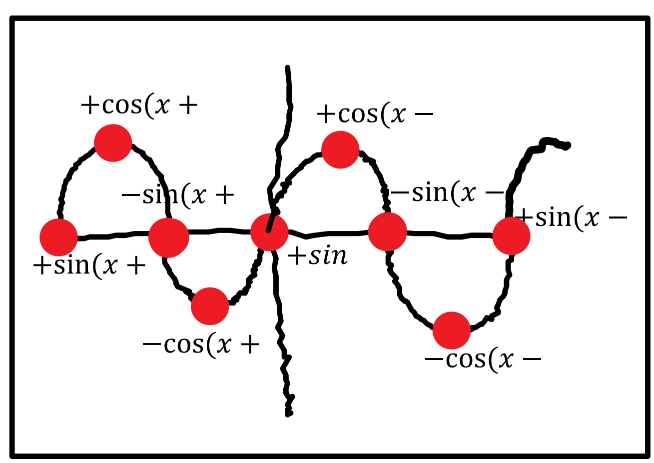
$$y = 6\cos\left(\frac{\pi}{5}(x - 3)\right) + 11$$

$$y = 6\sin\left(\frac{\pi}{5}(x - 10.5)\right) + 11$$

$$y = -6\sin\left(\frac{\pi}{5}(x - 5.5)\right) + 11$$

$$y = -6\cos\left(\frac{\pi}{5}(x - 8)\right) + 11$$

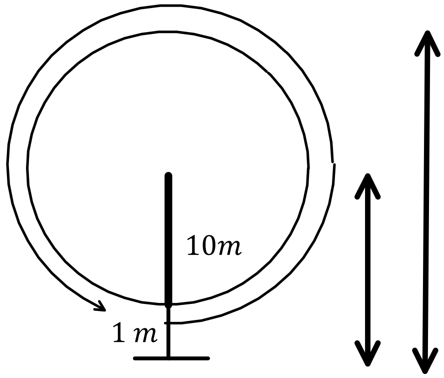
$\pm sin/\cos$ and "c"



C12 - 5.5 - Ferris Wheel Notes

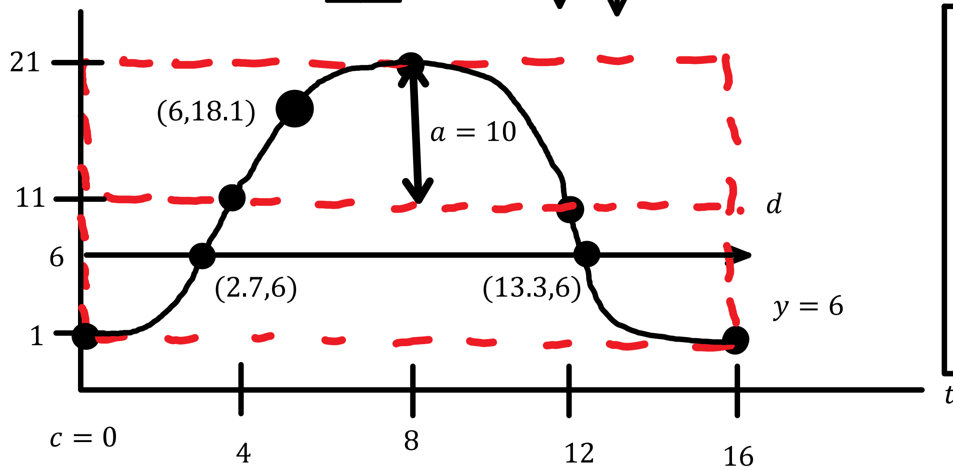
Make it 6m in one cycle!

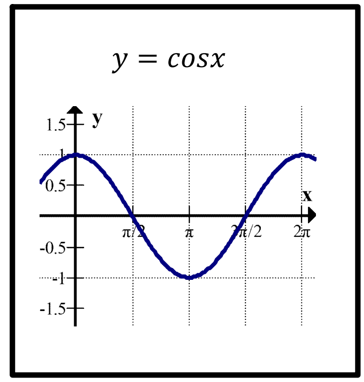
A Ferris wheel with radius 10 m is 1 m off the ground. It takes 16 seconds for one complete revolution. Draw a diagram of the Ferris wheel, graph the height of a passenger starting at the bottom and write the sinusoidal equation. How high 6 at second? How long above 6m in one cycle? No Calculator!



t	h
0	1
4	11
8	21
12	11
16	1

$$\boxed{\frac{16}{4} = 4}$$





$$p = \frac{2\pi}{b}$$

$$b = \frac{2\pi}{p}$$

$$b = \frac{2\pi}{16}$$

$$b = \frac{\pi}{8}$$

$$y = a\cos(b(x - c)) + d$$

$$h = -10\cos\left(\frac{\pi}{8}(t)\right) + 11$$

$$p = 16\ seconds$$

$$\boxed{h = +10\sin\left(\frac{\pi}{8}(t - 4)\right) + 11}$$

$$h = -10\cos\left(\frac{\pi}{8}(6)\right) + 11$$

$$h = 18.1m$$

Sub 6 in for t. Or. Graph and 2nd Calc Value

$$\boxed{\frac{10 + 11\sqrt{2}}{\sqrt{2}} = 18.1 \quad Exact\ Value}$$

$$y_1 = -10\cos\left(\frac{\pi}{8}(t)\right) + 11$$

$$y_2 = 6$$

$$10.7\ seconds$$

Find Intersection, and Subtract, (or Algebra and Inverse)

$$13.333 - 2.666 = 10.666$$

$$6 = -10\cos\left(\frac{\pi}{8}(t)\right) + 11$$

$$cosm = \frac{1}{2}$$

$$\boxed{m = \frac{\pi t}{8}}$$

$$t = \frac{8}{3}, \frac{40}{3}$$

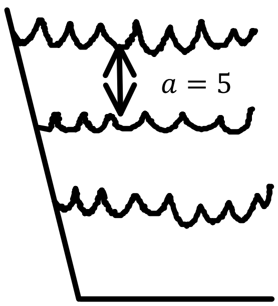
Exact Value

C12 - 5.5 - Tide Notes

$$\frac{24min}{60min} = 0.4\ hr$$

Graph and find Equation. High tide depth 18m at 8 am. Low tide depth 8 m at 1:24 pm.

(8,18) (13.4,8)



$d = 18\ m$

$d = 13$

$d = 8\ m$

t	h
8	18
10.7	13
13.4	8
16.1	13
18.8	18

$$\frac{8+13.4}{2} = 10.7$$

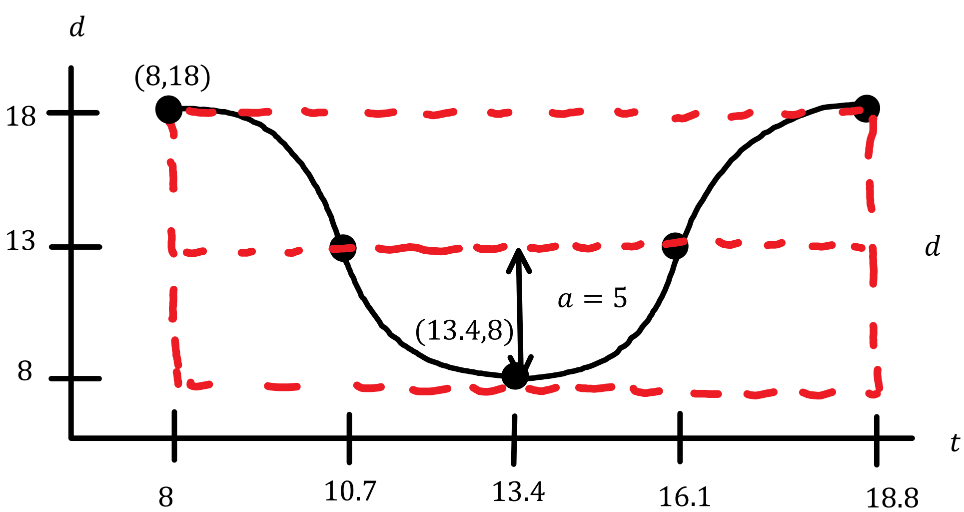
$$\frac{13.4-8}{2} = 2.7$$

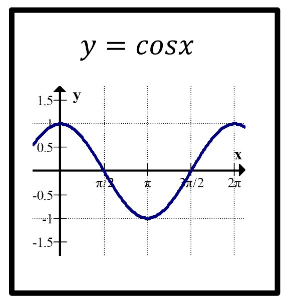
$$8 + 2.7 = 10.7$$
$$13.4 + 2.7 = 16.1$$
$$16.1 + 2.7 = 18.8$$

$$\frac{8+18}{2} = 13$$

$$\frac{18-8}{2} = 5$$

$$18 - 5 = 13$$
$$8 + 5 = 13$$





$c = 8$

$$y = a\cos\big(b(x-c)\big) + d$$

$$d = +10\cos\left(\frac{\pi}{5.4}(x-8)\right) + 13$$

$$p = \frac{2\pi}{b}$$

$$b = \frac{2\pi}{p}$$

$$b = \frac{2\pi}{10.8}$$

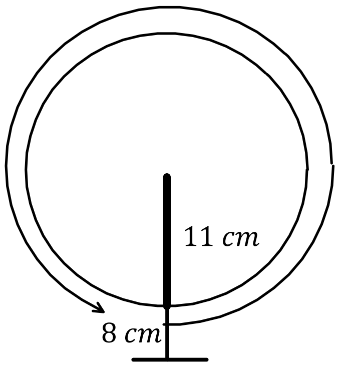
$$b = \frac{\pi}{5.4}$$

$$p = 18.8 - 8 = 10.8$$

(13.4,8)

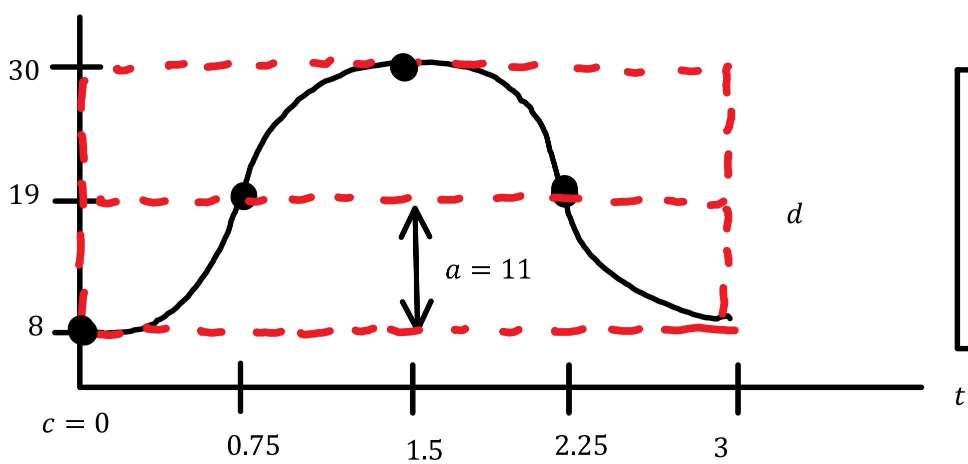
C12 - 5.5 - Bike Pedal Notes

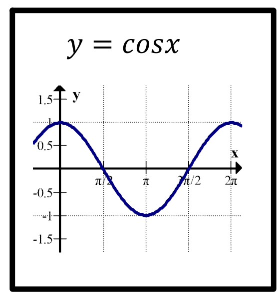
A bicycle pedal does 20 revolutions per minute and has a radius of 11 cm and the 8 cm off the ground at its lowest point. Find the sinusoidal equation.



t	h
0	8
0.75	19
1.5	30
2.25	19
3	8

$$\boxed{\frac{3}{4} = 0.75}$$





$$y = a\cos(b(x-c)) + d$$

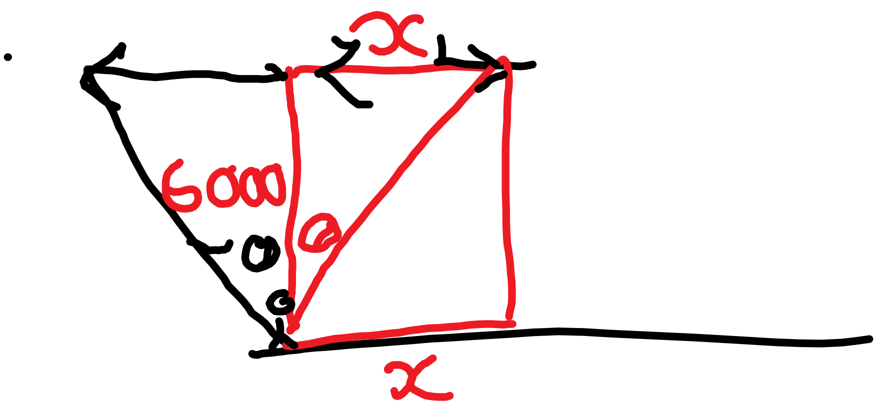
$$h = -11\cos\left(\frac{2\pi}{3}(t)\right) + 19$$

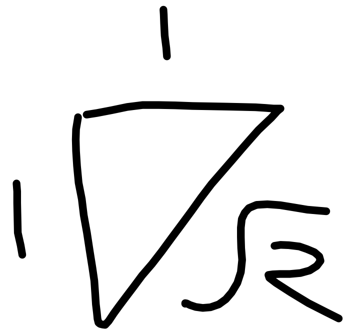
$$p = \frac{2\pi}{b}$$

$$b = \frac{2\pi}{3}$$

$$\frac{20rev}{min} = \frac{20rev}{60s} = \frac{1rev}{3s}$$

$$Period = 3s$$



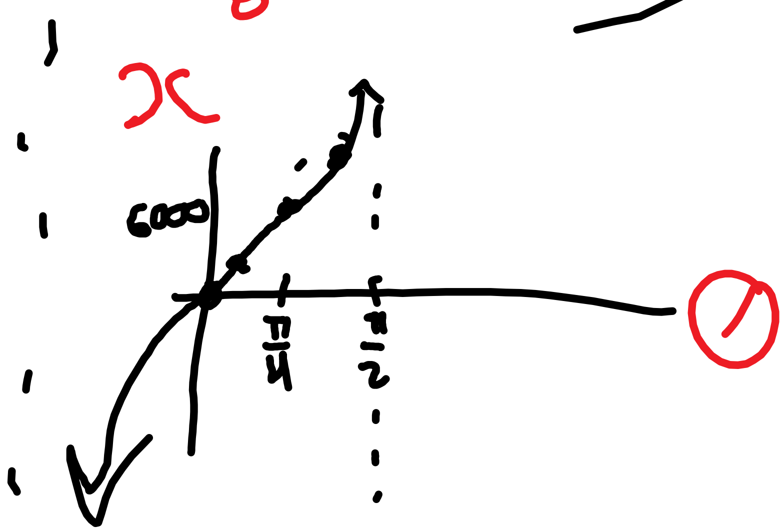


θ	x
0	0
$\frac{2\pi}{8} = \frac{\pi}{4}$	6000
$\frac{\pi}{8}$	2485
$\frac{3\pi}{8}$	14485

$\tan\theta = \frac{o}{a}$

$6000\tan\frac{\pi}{4} = \frac{x}{\cancel{6000}}$

$x = 6000$



C12 - 6.1 - Ratios $cscx\ secx\ cotx$ Notes

$\frac{sinx}{sinx} = 1$ $\quad \frac{\sin^2 x}{sinx} = sinx$ $\quad \frac{\sin^3 x}{sinx} = \sin^2 x$

$\frac{\cos x}{cosx} = 1$ $\quad \frac{\cos^2 x}{cosx} = cosx$ $\quad \frac{\cos^3 x}{\cos^2 x} = \cos x$

$$\boxed{\begin{array}{l}\sin^2 x = (sinx)(sinx) \neq sinx^2 \\ \cos^2 x = (cosx)(cosx) \neq cosx^2\end{array}}$$

$\frac{sinx}{1} \times \frac{1}{cosx}$

$\frac{sinx}{cosx}$

$= tanx$ $\quad \boxed{tanx = \frac{sinx}{cosx}}$

$sinxtanx$

$sinx \times \frac{sinx}{cosx}$

$= \frac{\sin^2 x}{cosx}$

$cosxtanx$

$cosx \times \frac{sinx}{cosx}$

$= sinx$

$\frac{sinxcosx}{sinx} = cosx$

$\frac{cosxsinx}{cosx} = sinx$

$\frac{sinx}{tanx}$

$\frac{sinx}{\left(\frac{sinx}{cosx}\right)}$

$sinx \div \frac{sinx}{cosx}$

$sinx \times \frac{cosx}{sinx}$

$= cosx$

$\frac{cosx}{tanx}$

$\frac{cosx}{\left(\frac{sinx}{cosx}\right)}$

$cosx \div \frac{sinx}{cosx}$

$cosx \times \frac{cosx}{sinx}$

$= \frac{\cos^2 x}{sinx}$

$\frac{tanx}{cosx}$

$\frac{\left(\frac{sinx}{cosx}\right)}{\frac{cosx}{1}}$

$\frac{sinx}{cosx} \div \frac{cosx}{1}$

$\frac{sinx}{cosx} \times \frac{1}{cosx}$

$= \frac{sinx}{\cos^2 x}$

$\frac{tanx}{sinx}$

$\frac{\left(\frac{sinx}{cosx}\right)}{\frac{sinx}{1}}$

$\frac{sinx}{cosx} \div \frac{sinx}{1}$

$\frac{sinx}{cosx} \times \frac{1}{sinx}$

$\frac{1}{cosx}$

$= secx$ $\quad \boxed{secx = \frac{1}{cosx}}$

Flip and Multiply

$secxcosx =$

$\frac{1}{cosx} \times cosx =$

$\frac{cosx}{cosx} = 1$ $\quad \boxed{secx = \frac{1}{cosx}}$

$secxsinx =$

$\frac{1}{cosx} \times sinx =$

$\frac{sinx}{cosx} = tanx$ $\quad \boxed{\frac{sinx}{cosx} = tanx}$

$secxtanx =$

$\frac{1}{cosx} \times \frac{sinx}{cosx} = \frac{sinx}{\cos^2 x}$

$cscxsinx = \frac{1}{sinx} \times sinx = \frac{sinx}{sinx} = 1$ $\quad \boxed{cscx = \frac{1}{sinx}}$

$csxcosx =$

$\frac{1}{sinx} \times cosx =$

$\frac{cosx}{sinx} = cotx$ $\quad \boxed{\frac{cosx}{sinx} = cotx}$

$cscxtanx =$

$\frac{1}{sinx} \times \frac{sinx}{cosx} =$

$\frac{1}{\cos x} = secx$

C12 - 6.2 - Add Subtract Fractions Notes

$$\frac{1}{cosx} - \frac{\sin x}{cosx} = \frac{1 - sinx}{cosx}$$

Add Fractions: LCD

$$\frac{1}{sinx} - sinx$$
$$\frac{1}{sinx} - sinx \times \frac{sinx}{sinx}$$
$$\frac{1}{sinx} - \frac{\sin^2 x}{sinx}$$
$$\frac{1 - \sin^2 x}{sinx}$$
$$\frac{\cos^2 x}{sinx}$$

$$\boxed{\begin{aligned}\sin^2 x + \cos^2 x &= 1 \\ \cos^2 x &= 1 - \sin^2 x\end{aligned}}$$

$$\frac{1}{sinxcosx} - \frac{sinx}{cosx}$$
$$\frac{1}{sinxcosx} - \frac{sinx}{cosx} \times \frac{sinx}{sinx}$$
$$\frac{1}{sinxcosx} - \frac{\sin^2 x}{sinxcosx}$$
$$\frac{1 - \sin^2 x}{sinxcosx}$$
$$\frac{\cos^2 x}{sinxcosx}$$
$$\frac{cosx}{sinx}$$
$$cotx$$

$$\frac{sinx + cosx}{cosx}$$
$$\frac{sinx}{cosx} + \frac{cosx}{cosx}$$
$$tanx + 1$$

Separate Fractions

$$\frac{\frac{1}{cosx} - cosx}{1 - \frac{sinx}{cosx}}$$
$$\frac{\frac{1}{cosx} - \frac{\cos^2 x}{cosx}}{\frac{cosx}{cosx} - \frac{sinx}{cosx}}$$
$$\frac{\frac{1 - \cos^2 x}{cosx}}{\frac{cosx - sinx}{cosx}}$$
$$\frac{1 - \cos^2 x}{cosx} \times \frac{cosx}{cosx - siinx}$$
$$\frac{1 - \cos^2 x}{cosx - sinx}$$
$$\frac{\sin^2 x}{cosx - sinx}$$

Add Fractions: LCD
Flip and Multiply

$$\frac{\frac{1}{cosx} - cosx}{1 - \frac{sinx}{cosx}}$$

LDC = cosx

$$\left(\frac{\frac{1}{cosx} - cosx}{1 - \frac{sinx}{cosx}}\right) \times \frac{cosx}{cosx}$$
$$\frac{1 - \cos^2 x}{cosx - sinx}$$
$$\frac{\sin^2 x}{cosx - sinx}$$

Multiply top and bottom by LCD

$$\frac{\frac{1}{cosx} - cosx}{1 - \frac{sinx}{cosx}}$$
$$\frac{1 - \cos^2 x}{cosx - sinx}$$

C12 - 6.3 - Proofs Pythag Reciprocal Fractions Notes

$\tan x \csc x$	$\sec x$
$\left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x}\right)$	
$\frac{1}{\cos x}$	
$\sec x$	

$\frac{\cot x}{\csc x}$	$\cos x$
$\frac{\left(\frac{\cos x}{\sin x}\right)}{\left(\frac{1}{\sin x}\right)}$	
$\frac{\cos x}{\sin x} \times \frac{\sin x}{1}$	
$\cos x$	

$1 + \tan^2 x$	$\sec^2 x$
$1 + \frac{\sin^2 x}{\cos^2 x}$	$\frac{1}{\cos^2 x}$
$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$	
$\frac{\cos^2 x + \sin^2 x}{\cos^2 x}$	
$\frac{1}{\cos^2 x}$	

$\csc x \cos^2 x + \sin x$	$\csc x$
$\frac{1}{\sin x} \times \cos^2 x + \sin x$	$\frac{1}{\sin x}$
$\frac{\cos^2 x}{\sin x} + \sin x \times \frac{\sin x}{\sin x}$	
$\frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x}$	
$\frac{\cos^2 x + \sin^2 x}{\sin x}$	
$\frac{1}{\sin x}$	

$\cot x + \tan x$	$\csc x \sec x$
$\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$	$\csc x \sec x$
$\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$	
$\frac{1}{\sin x \cos x}$	
$\left(\frac{1}{\sin x}\right)\left(\frac{1}{\cos x}\right)$	
$\csc x \sec x$	

$\frac{1 + \cos x}{1 + \sec x}$	$\cos x$
$\left(\frac{1 + \cos x}{1 + \frac{1}{\cos x}}\right)$	$\cos x$
$\frac{(1 + \cos x)}{\left(\frac{\cos x + 1}{\cos x}\right)}$	
$(1 + \cos x) \times \frac{\cos x}{\cos x + 1}$	
$\frac{\cos x(1 + \cos x)}{\cos x + 1}$	
$\cos x$	

C12 - 6.4 - Proofs Conjugate Notes

Conjugate:

$a+b \longleftrightarrow a-b$

$a-b \longleftrightarrow a+b$

Conjugate:

$1-sinx \longleftrightarrow 1+sinx$

$1+sinx \longleftrightarrow 1-sinx$

Conjugate:

$1+cosx \longleftrightarrow 1-cosx$

$1-cosx \longleftrightarrow 1+cosx$

$$\frac{\square}{1+cosx} \times \frac{1-cosx}{1-cosx} \qquad \frac{\square}{1+sinx} \times \frac{1-sinx}{1-sinx}$$

$$\frac{\square}{1-cosx} \times \frac{1+cosx}{1+cosx} \qquad \frac{\square}{1-sinx} \times \frac{1+sinx}{1+sinx}$$

Prove that the two sides are equal.

$$\frac{sinx}{1+cosx} = \frac{1-cosx}{sinx}$$

The conjugate

$$\times \frac{1-cosx}{1-cosx}$$

$$\frac{sinx}{1+cosx} \times \boxed{\frac{1-cosx}{1-cosx}} \qquad \frac{(1-cosx)}{\sin x}$$

1) Multiply the top and bottom by the conjugate of the denominator
2) FOIL the bottom
3) Pythagorean Identity
4) Simplify

$$\frac{sinx\,(1-cosx)}{(1+cosx)(1-cosx)}$$

$$\frac{sinx(1-cosx)}{1-\cancel{cosx}+\cancel{cosx}-\cos^2 x}$$

$(1+cosx)(1-cosx)$

$1-\cancel{cosx}+\cancel{cosx}-\cos^2 x$

$1-\cos^2 x$

FOIL (FL)

$(a+b)(a-b)$

$a^2-\cancel{ab}+\cancel{ab}+b^2$

a^2-b^2

$$\frac{sinx(1-cosx)}{1-\cos^2 x}$$

$\sin^2 x - \cancel{\cos^2} x = 1$

$\sin^2 x = 1-\cos^2 x$

Now we have the Pythagorean identity

$$\frac{sinx(1-cosx)}{\sin^2 x}$$

$$\frac{(1-cosx)}{\sin x} \qquad RHS \checkmark$$

Conj
FL
Pythag
Simp

C12 - 6.4 - Proofs Foil Conjugate Fact Frac Notes

$$\frac{1-cosx}{sinx} = \frac{sinx}{1+cosx}$$

$\frac{(1-cosx)}{sin\,x}$	$\frac{sinx}{1+cosx} \times \frac{1-cosx}{1-cosx}$ $\boxed{Conjugate!}$
	$\frac{sinx(1-cosx)}{1-\cos^2 x}$
	$\frac{sinx(1-cosx)}{\sin^2 x}$
	$\frac{(1-cosx)}{sin\,x}$

Foil

$$(sinx-1)(sinx+1) = -\cos^2 x$$

$sin^2 x - 1$	
$-\cos^2 x$	

Factor

$$\frac{1+cosx}{\sin^2 x} = \frac{1}{1-cos\,x}$$

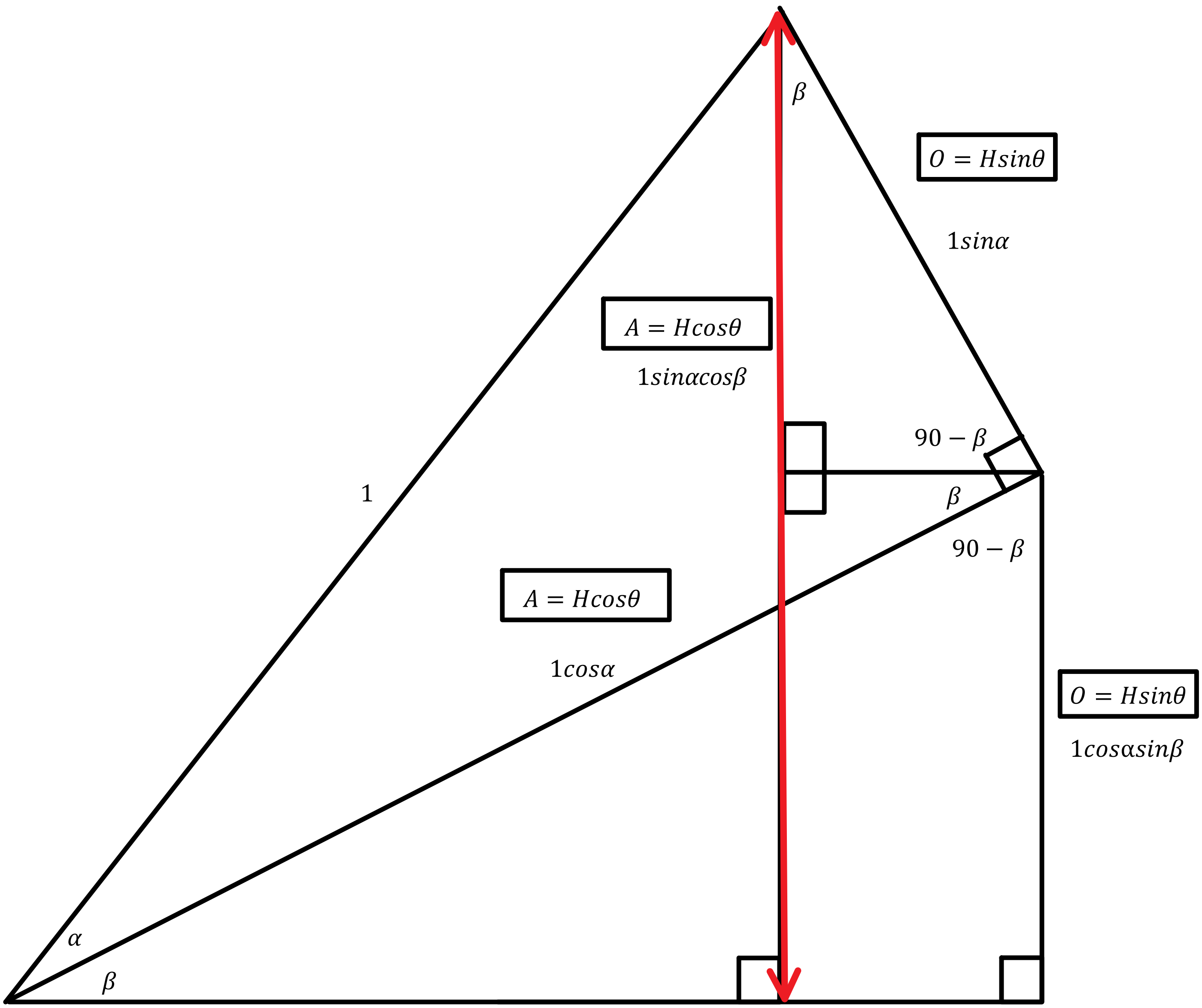
$\frac{1+cosx}{1-cos^2 x}$	$\frac{1}{1-cos\,x}$
$\frac{1+cosx}{(1-cos\,x)(1+cosx)}$	
$\frac{1}{1-cos\,x}$	

Add and Subtract Fractions

$$\frac{1}{1+cosx} + \frac{1}{1-cosx} = 2\csc^2 x$$

$\frac{1}{1+cosx} \times \frac{1-cosx}{1-cosx} + \frac{1}{1-cosx} \times \frac{1+cosx}{1+cosx}$	$2 \times \frac{1}{\sin^2 x}$
$\frac{(1-cosx)+(1+cosx)}{(1-cosx)(1+cosx)}$	$\frac{2}{\sin^2 x}$
$\frac{1-cosx+1+cosx}{1-\cos^2 x}$	
$\frac{2}{\sin^2 x}$	

C12 - 6.5 - Sum and Differences Angle Theory



$$\boxed{\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta}$$

C12 - 6.5 - Simplify/Expand Sum Difference Notes

$\boxed{tan\theta = \frac{sin\theta}{cos\theta}}$

$\boxed{\sin(a+b) = sinacosb + sinbcosa}$

$$\sin(x+\pi) = sinxcos\pi + sin\pi cosx$$
$$= sinx \times -1 + 0 \times cosx$$
$$= -sinx$$

$\boxed{\cos(a+b) = cosacosb - sinasinb}$

$$cos45cos30 + sin45sin30 = \cos(45^o - 30^o)$$
$$= cos15$$

$$\sin\left(\frac{\pi}{12}\right) =$$
$$sin15^0 =$$
$$\sin(45^0 - 30^0) = sin45cos30 - sin30cos45$$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$\boxed{\frac{\pi}{12} = 15^0}$

$15 = 45 - 30$ Or $\frac{\pi}{12} = \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{4} - \frac{\pi}{3}$

$\boxed{\sin(a-b) = sinacosb - sinbcosa}$

Sin is the same sign sincos:cossin
Cos is the opposite sign coscos:sinsin

Rationalize!

$$\cos(-75) = \cos(-45-30)$$
$$= \cos(-45)\cos(30) + \sin(-45)\cos(30)$$

$\boxed{\cos(-x) = cosx \qquad \sin(-x) = -sinx}$

OR

$$cos75^o =$$
$$\cos(45^o + 30^o) = cos45cos30 - sin45sin30$$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$\cos(a-b) = cosacosb + sinasinb$

$$sec15^o =$$
$$\frac{1}{cos15^o} =$$
$$\frac{1}{\cos(45^o - 30^o)} = \frac{1}{(cos45cos30 + sin45sin30)}$$
$$= \frac{1}{\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}}$$
$$= \frac{1}{\left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)}$$
$$= \frac{1}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$
$$= 1 \times \frac{2\sqrt{2}}{\sqrt{3}+1}$$
$$= \frac{2\sqrt{2}}{\sqrt{3}+1}$$

$$\cos\left(\frac{\pi}{6} + x\right)\cos\left(\frac{\pi}{6} - x\right) + \sin\left(\frac{\pi}{6} + x\right)\sin\left(\frac{\pi}{6} - x\right) =$$
$$\cos\left(\frac{\pi}{6} + x - (\frac{\pi}{6} - x)\right) = \cos(2x)$$

$sin255 = -sin105$

$\sin255 = \sin(360 - 105) = \sin 360cos105 - sin105cos360 = 0 - sin105(1) = -sin105 = -\sin(45+60)$

A combination of special and quadrantal angles…^*	$255 = 180 + 75$ $255 = 180 + (45 + 30)$	$285 = 180 + 105$ $285 = 180 + (60 + 45)$	$195 = 180 + 15$ $195 = 90 + 105$

C12 - 6.6 - Double Angle Notes

$4\sin 6x = 8\sin 3x\cos 3x$

$2\sin x = 4\sin\frac{1}{2}x\cos\frac{1}{2}x$

$\frac{1}{2}\sin 4x = 1\sin 2x\cos 2x$

$2sin\pi = 4\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right) = 0$

$\boxed{sin2x = 2sinxcosx}$

Double the number in front.
Half the angle. Add a Cos

$8\sin 3x\cos 3x = 4\sin 6x$

$4\sin\frac{1}{2}x\cos\frac{1}{2}x = 2\sin x$

$4\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right) = 2\sin\left(\frac{\pi}{3}\right) = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$\boxed{2sinxcosx = sin2x}$

Half the number in front.
Double the angle. Cos goes away

$cos4x = \cos^2 2x - \sin^2 2x$

$cos4x = 2\cos^2 2x - 1$

$$\boxed{\begin{aligned} cos2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}}$$

Half the angle

$1 - 2\sin^2 2x = cos4x$

Double the angle

$2\cos^2 3x - 2\sin^2 3x =$
$2(\cos^2 3x - \sin^2 3x) = 2cos6x$

GCF

$4\cos^2 5 - 2 =$
$2(2\cos^2 5 - 1) = 2cos10$

$1 - 2\sin^2 \pi = cos2\pi = 1$

$1 - 2\sin^2\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

$Simplify\ to\ sinx\ or\ cosx$

$1 - cos2x$
$1 - (1 - 2\sin^2 x)$
$1 - 1 + 2\sin^2 x$
$2\sin^2 x$

$1 + cos2x$
$1 + (2\cos^2 x - 1)$
$1 + 2\cos^2 x - 1$
$2\cos^2 x$

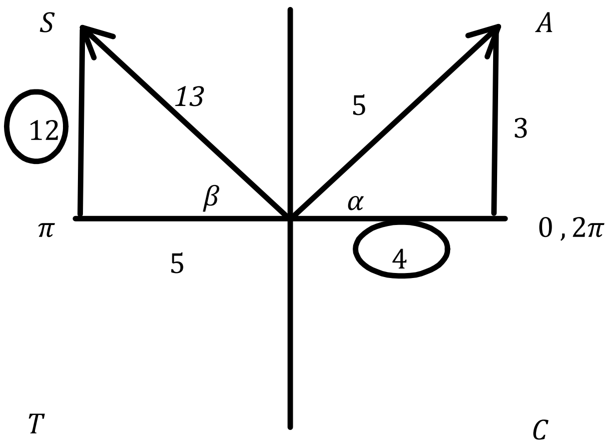
C12 - 6.6 - Proofs Double Angle Notes

$tanx$	$\frac{sin2x}{1 + cos2x}$
$\frac{sinx}{cosx}$	$\frac{sin2x}{1 + (2\cos^2 x - 1)}$
	$\frac{sin2x}{1 + (2\cos^2 x - 1)}$
	$\frac{2sinxcosx}{2\cos^2 x}$
	$\frac{2sinxcosx}{2\cos^2 x}$
	$\frac{2sinxcosx}{2\cos^2 x}$
	$\frac{sinx}{\cos x}$

C12 - 6.6 - CosA= SinB= Sum/Double Angles Notes

Solve: $sin\alpha = \frac{3}{5}$; QI $\quad$ $cos\beta = -\frac{5}{13}$; QII $\quad$ $\sin(\alpha+\beta) = ?$ $\quad$ $sin2\alpha = ?$

$cos2\beta = ?$



$a^2 + b^2 = c^2$
$a = 4$

$a^2 + b^2 = c^2$
$b = 12$

$$\sin(\alpha+\beta) = sin\alpha cos\beta + cos\alpha sin\beta$$
$$= \frac{3}{5} \times -\frac{5}{13} + \frac{4}{5} \times \frac{12}{13}$$
$$= -\frac{3}{13} + \frac{48}{65}$$
$$= \frac{33}{65}$$

$$sin2\alpha = 2sin\alpha cos\alpha$$
$$= 2 \times \frac{3}{5} \times \frac{4}{5}$$
$$= \frac{24}{25}$$

$$cos2\beta = 1 - 2\sin^2\beta$$
$$= 1 - 2\left(\frac{12}{13}\right)^2$$
$$= -\frac{119}{169}$$

C12 - 7.1 - Exponent Laws Notes

Simplify

$5^2 \times 5^3 = 5^5$ Add Exponents

$\frac{3^5}{3^2} = 3^3$ Subtract Exponents

$(2^2)^3 = 2^6$

$(3 \times 4)^2 = 3^2 \times 4^2$

$(2x)^3 = 2^3x^3 = 8x^3$

$\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2}$

Multiply/Distribute Exponents

$5^{-2} = \frac{1}{5^2}$ $\quad \frac{1}{3^{-2}} = \frac{3^2}{1}$ $\quad 3a^{-2} = \frac{3}{a^2}$ $\quad (2x)^{-3} = \frac{1}{(2x)^3}$

$3^{-1} = \frac{1}{3}$ $\quad \frac{1}{3^1} = 3^{-1}$ $\quad 3^{-3}a^{-2} = \frac{1}{3^3a^2}$ $\quad \left(\frac{5}{3}\right)^{-2} = \frac{3^2}{5^2}$ $\quad \frac{1}{25} = \frac{1}{5^2} = 5^{-2}$

Negative Exponents

$9 = 3^2$ $\quad 25 = 5^2$ $\quad 4^2 = (2^2)^2 = 2^4$ $\quad 27^4 = (3^3)^4 = 3^{12}$ Change Base

$5^{\frac{3}{4}} = \sqrt[4]{5^3}$ $\quad 8^{\frac{1}{3}} = \sqrt[3]{8}$

$$8^{\frac{2}{3}} = \sqrt[3]{8^2} = 2^2 = 4$$

$$\sqrt[4]{\frac{1}{16}} = \frac{\sqrt[4]{1}}{\sqrt[4]{16}} = \frac{1}{2}$$

Radicals

$$\frac{3^4 \times 3^{-3}}{9} = \frac{3^1}{3^2} = 3^{-1} = \frac{1}{3^1} = \frac{1}{3}$$

Add Exponents
Change Base
Subtract Exponents
Negative Exponents
Simplify

$$\frac{4^2 \times 16^3}{128^2} = \frac{\left((2^2)^2 \times (2^4)^3\right)}{(2^7)^2} = \frac{2^4 \times 2^{12}}{2^{14}} = \frac{2^{16}}{2^{14}} = 2^{(16-14)} = 2^2 = 4$$

Change of base
Multiply Exponents
Add Exponents

Subtract Exponents
Simplify

C12 - 7.1 - Simplifying/Separating Exponents Notes

Simplify

$3^x \times 3 =$
$3^x \times 3^1 = 3^{x+1}$

Add Exponents

$(5^2)^x = 5^{2x}$

Multiply Exponents

$\frac{6^x}{6} =$
$\frac{6^x}{6^1} = 6^{x-1}$

$\frac{3}{3^x} =$
$\frac{3^1}{3^x} = 3^{1-x}$

Subtract Exponents

Separate into a multiplication/division/or use brackets with the same base. $(Isolate\ \#^x)$

$6^{x+1} = 6^x(6^1) = 6(6^x)$

$7^{x-1} = 7^x \times 7^{-1} = \frac{7^x}{7^1}$

$4^{1-x} = 4^1(4^{-x})$
$= \frac{4}{4^x}$

$5^{2x} = (5^x)^2 = (5^2)^x$

$3^{2x+1} = 3^{2x}3^1$
$= (3^x)^2 3^1$
$= 3(3^x)^2$

$6^x = (2 \times 3)^x$
$= 2^x \times 3^x$

$\frac{2^{7x+5} \times 8^{x+1}}{4^{x-2}} =$
$\frac{2^{7x+5} \times (2^3)^{x+1}}{(2^2)^{x-2}} =$
$\frac{2^{7x+5} \times 2^{3x+3}}{2^{2x-4}} =$
$\frac{2^{10x+8}}{2^{2x-4}} = 2^{8x+12}$

Change Base
Multiply Exponents
Add Exponents
Subtract Exponents

Subtracting
Negative
Exponents!

$\frac{2^{7x+5} \times 8^{x+1}}{4^{x-2}} =$	$8^{(x+1)} =$ $(2^3)^{(x+1)} = 2^{3x+3}$
$\frac{2^{7x+5} \times 2^{3x+3}}{2^{2x-4}} =$	$4^{x-2} =$ $(2^2)^{x-2} = 2^{2x-4}$

C12 - 7.2 - Solving Exponential Equations Notes

Solve for x

$2^x = 4^2$
$2^x = (2^2)^2$
$2^x = 2^4$
$x = 4$

$4 = 2^2$

Same Base: Make exponents equal to each other

Check Answer:

$2^4 = 4^2$
$16 = 16$ ✓

$2^x 2^1 = 2^5$
$2^{x+1} = 2^5$
$x + 1 = 5$
$x = 4$

Add Exponents

$2^x 2^1 = 2^5$
$2^4 2^1 = 2^5$
$2^5 = 2^5$ ✓

$4^{x+1} = 8^{2x-2}$
$(2^2)^{x+1} = (2^3)^{2x-2}$
$2^{2x+2} = 2^{6x-6}$
$2x + 2 = 6x - 6$
$8 = 4x$
$x = 2$

$4 = 2^2$
$8 = 2^3$

Change of Base
Multiply Exponents

Solve

$4^{x+1} = 8^{2x-2}$
$4^{2+1} = 8^{2(2)-2}$
$4^3 = 8^2$
$64 = 64$ ✓

$2^{x^2-x} = 1$
$2^{x^2-x} = 2^0$
$x^2 - x = 0$
$x(x-1) = 0$
$x = 0$ $x = 1$

$2^0 = 1$

Change of Base

Factor

Solve

$2^{x^2-x} = 1$
$2^{0^2-0} = 1$
$2^0 = 1$
$1 = 1$ ✓

$2^{x^2-x} = 1$
$2^{1^2-1} = 1$
$2^0 = 1$
$1 = 1$ ✓

$2^{x^2-3x} = \frac{1}{4}$
$2^{x^2-3x} = 2^{-2}$
$x^2 - 3x = -2$
$x^2 - 3x + 2 = 0$
$(x-2)(x-1) = 0$
$x = 2$ $x = 1$

$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

Change of Base

Factor

Solve

$2^{x^2-3x} = \frac{1}{4}$
$2^{2^2-3(2)} = \frac{1}{4}$
$2^{-2} = \frac{1}{4}$
$\frac{1}{4} = \frac{1}{4}$ ✓

$2^{x^2-3x} = \frac{1}{4}$
$2^{1^2-3(1)} = \frac{1}{4}$
$2^{-2} = \frac{1}{4}$
$\frac{1}{4} = \frac{1}{4}$ ✓

$x^{\frac{2}{5}} = 3$
$\left(x^{\frac{2}{5}}\right)^{\frac{5}{2}} = (3)^{\frac{5}{2}}$
$x = 3^{\frac{5}{2}}$

$\frac{2}{5} \times \frac{5}{2} = 1$

$x = 15.5885$

Take both/sides to reciprocal exponent of variable

Brackets around the left side
Brackets around the right side

$x^{\frac{2}{5}} = 3$
$\left(3^{\frac{5}{2}}\right)^{\left(\frac{2}{5}\right)} = 3$
$3 = 3$ ✓

$(x+1)^{\frac{2}{3}} = 16$
$\left((x+1)^{\frac{2}{3}}\right)^{\left(\frac{3}{2}\right)} = (16)^{\frac{3}{2}}$
$x + 1 = \sqrt[2]{16^3}$
$x + 1 = 4^3$
$x + 1 = 64$
$x = 63$

$x^2 = 9$ $(x^2)^{\frac{1}{2}} = 9^{\frac{1}{2}}$ $x = \pm 3$	Square root both sides $\sqrt{x} = x^{\frac{1}{2}}$

$x^{\frac{m}{n}} = \sqrt[x]{x^m}$

$(x+1)^{\frac{2}{3}} = 16$
$(63+1)^{\frac{2}{3}} = 16$
$64^{\frac{2}{3}} = 16$
$\sqrt[3]{64^2} = 16$
$4^2 = 16$
$16 = 16$ ✓

C12 - 7.2 - Separate/Factoring/Solving Exponents Notes

Solve for x

$$2(3^x) + 3^x = 243 \qquad let\ m = 3^x$$
$$2m + m = 243$$
$$3m = 243$$
$$m = 81$$
$$3^x = 81$$
$$3^x = 3^4$$
$$x = 4$$

Check Answer:

$$2(3^x) + 3^x = 243$$
$$2(3^4) + 3^4 = 243$$
$$2(81) + 81 = 243$$
$$243 = 243 \checkmark$$

$$7^x + 7^{x+1} = 392 \qquad Let\ m = 7^x$$
$$7^x + 7^x 7^1 = 392$$
$$m + 7m = 392$$
$$8m = 392$$
$$m = 49$$
$$7^x = 49$$
$$7^x = 7^2$$
$$x = 2$$

$$7^x + 7^{x+1} = 392$$
$$7^2 + 7^{2+1} = 392$$
$$49 + 343 = 392$$
$$392 = 392 \checkmark$$

$$(2^x)^2 - 12(2^x) + 32 = 0 \qquad let\ m = 2^x$$
$$m^2 - 12m + 32 = 0$$
$$(m-4)(m-8) = 0$$

$$m - 4 = 0 \qquad m - 8 = 0$$
$$m = 4 \qquad m = 8$$
$$2^x = 4 \qquad 2^x = 8$$
$$2^x = 2^2 \qquad 2^x = 2^3$$
$$x = 2 \qquad x = 3$$

$$(2^x)^2 - 12(2^x) + 32 = 0$$
$$(2^2)^2 - 12(2^2) + 32 = 0$$
$$16 - 48 + 32 = 0 \checkmark$$

$$(2^x)^2 - 12(2^x) + 32 = 0$$
$$(2^3)^2 - 12(2^3) + 32 = 0$$
$$64 - 96 + 32 = 0 \checkmark$$

$$9^{2x} - 2(9^x) - 3 = 0 \qquad let\ m = 9^x$$
$$(9^x)^2 - 2(9^x) - 3 = 0$$
$$m^2 - 2m - 3 = 0$$
$$(m-3)(m+1) = 0$$

$$m - 3 = 0 \qquad m + 1 = 0$$
$$m = 3 \qquad m = -1$$
$$9^x = 3 \qquad 9^x = -1$$
$$(3^2)^x = 3^1 \qquad \textit{No Solution}$$
$$3^{2x} = 3^1$$
$$2x = 1$$
$$x = \frac{1}{2}$$

$$9^{2x} - 2(9^x) - 3 = 0$$
$$9^{2\left(\frac{1}{2}\right)} - 2\left(9^{\left(\frac{1}{2}\right)}\right) - 3 = 0 \qquad 9^{\frac{1}{2}} = \sqrt[2]{9^1} = 3$$
$$9^1 - 2(3) - 3 = 0$$
$$9 - 6 - 3 = 0$$
$$0 = 0 \checkmark$$

C12 - 7.2 - Separate/Factoring/Solving Exponents Notes

Solve for x

$$4^x - 4^{x-1} - 24 = 0$$
$$4^x - \frac{4^x}{4^1} - 24 = 0$$
$$\left(4^x - \frac{4^x}{4^1} - 24 = 0\right) \times 4$$
$$4(4^x) - 4^x - 96 = 0 \qquad let\ m = 4^x$$
$$4m - m - 96 = 0$$
$$3m = 96$$
$$m = 32$$

$$4^x = 32$$
$$(2^2)^x = 2^5$$
$$2^{2x} = 2^5$$
$$2x = 5$$
$$x = \frac{5}{2}$$

$$4^x - 4^{x-1} - 24 = 0$$
$$4^{\left(\frac{5}{2}\right)} - 4^{\left(\frac{5}{2}\right)-1} - 24 = 0$$
$$32 - 8 - 24 = 0$$

$$4^x + 4^{1-x} = 5$$
$$4^x + 4(4^{-x}) = 5$$
$$4^x + \frac{4}{4^x} = 5$$
$$Let\ m = 4^x \qquad m + \frac{4}{m} = 5$$
$$\left(m + \frac{4}{m} = 5\right) \times m$$
$$m^2 + 4 = 5m$$
$$m^2 + 4 = 5m$$
$$m^2 - 5m + 4 = 0$$
$$(m-1)(m-4) = 0$$

$$m - 1 = 0 \qquad m - 4 = 0$$
$$m = 1 \qquad m = 4$$

$$4^x = 1 \qquad 4^x = 4$$
$$4^x = 4^0 \qquad 4^x = 4^1$$
$$x = 0 \qquad x = 1$$

$$4^x + 4^{1-x} = 5 \qquad 4^x + 4^{1-x} = 5$$
$$4^0 + 4^{1-0} = 5 \qquad 4^1 + 4^{1-1} = 5$$
$$1 + 4 = 5 \qquad 4 + 1 = 5$$
$$5 = 5 \qquad 5 = 5$$

$$3^x - 3 = 4(3^{-x}) \qquad 3^{-x} = \frac{1}{3^x}$$
$$3^x - 3 = \frac{4}{3^x}$$
$$m - 3 = \frac{4}{m} \qquad let\ m = 3^x$$
$$\left(m - 3 = \frac{4}{m}\right) \times m$$
$$m^2 - 3m = 4$$
$$m^2 - 3m - 4 = 0$$
$$(m-4)(m+1) = 0$$

$$m - 4 = 0 \qquad m + 1 = 0$$
$$m = 4 \qquad m = -1$$

$$3^x = 4 \qquad 3^x = -1$$
$$x = 1.2619 \qquad No\ Solution$$

$$Calc\ y_1 = y_2$$

$$3^x - 3 = \frac{4}{3^x}$$
$$3^{1.2619} - 3 = \frac{4}{3^{1.2619}}$$
$$4 - 3 = \frac{4}{4}$$
$$1 = 1$$

$$2(2^x)^2 - 3(2^x) + 1 = 0$$
$$2m^2 - 3m + 1 = 0 \qquad let\ m = 2^x$$
$$(2m-1)(m-1) = 0$$

$$2m - 1 = 0 \qquad m - 1 = 0$$
$$m = \frac{1}{2} \qquad m = 1$$

$$2^x = \frac{1}{2} \qquad 2^x = 1$$
$$2^x = 2^{-1} \qquad 2^x = 2^0$$
$$x = -1 \qquad x = 0$$

$$2(2^x)^2 - 3(2^x) + 1 = 0$$
$$2(2^{-1})^2 - 3(2^{-1}) + 1 = 0$$
$$2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 1 = 0$$
$$2\left(\frac{1}{4}\right) - \frac{3}{2} + 1 = 0$$
$$0 = 0$$

$$2(2^x)^2 - 3(2^x) + 1 = 0$$
$$2(2^0)^2 - 3(2^0) + 1 = 0$$
$$2(1)^2 - 3(1) + 1 = 0$$
$$2 - 3 + 1 = 0$$
$$0 = 0$$

C12 - 7.2 - Separate/Factoring/Solving Exponents Notes

$Solve\ for\ x$

$$3^{2x+1} - 4(3^{x+1}) + 9 = 0$$
$$3^{2x}3^1 - 4(3^x3) + 9 = 0$$
$$(3^x)^2 3 - 4(3^x)3 + 9 = 0$$
$$3(3^x)^2 - 12(3^x) + 9 = 0$$
$$3m^2 - 12m + 9 = 0$$
$$m^2 - 4m + 3 = 0$$
$$(m-1)(m-3) = 0$$

Divide both sides by 3!

$let\ m = 3^x$

$$\boxed{\begin{array}{lll} 3^{2x+1}-4(3^{x+1})+9=0 & 3^{2x+1} = 3^{2x}3^1 & 4(3^{x+1}) = 4(3^x3^1) \\ 3(3^x)^2 - 12(3^x)+9=0 & = (3^x)^2 3^1 & = 12(3^x) \\ 3m^2 - 12m + 9 = 0 & = 3(3^x)^2 & \end{array}}$$

$m - 1 = 0$ $\quad$ $m - 3 = 0$

$m = 1$ $\quad$ $m = 3$

$3^x = 1$ $\quad$ $3^x = 3$

$3^x = 3^0$ $\quad$ $3^x = 3^1$

$x = 0$ $\quad$ $x = 1$

$$3^{2x+1} - 4(3^{x+1}) + 9 = 0$$
$$3^{2(0)+1} - 4\left(3^{(0)+1}\right) + 9 = 0$$
$$3^1 - 4(3) + 9 = 0$$
$$3 - 12 + 9 = 0$$
$$0 = 0 \ \checkmark$$

$$3^{2x+1} - 4(3^{x+1}) + 9 = 0$$
$$3^{2(1)+1} - 4\left(3^{(1)+1}\right) + 9 = 0$$
$$3^3 - 4(3^2) + 9 = 0$$
$$27 - 36 + 9 = 0$$
$$0 = 0 \ \checkmark$$

$$10^x - 4(5^x) - 5(2^x) + 20 = 0$$
$$2^x \times 5^x - 4(5^x) - 5(2^x) + 20 = 0$$
$$mn - 4n - 5m + 20 = 0$$
$$(mn - 4n)(-5m + 20) = 0$$
$$n(m-4) - 5(m-4) = 0$$
$$(n-5)(m-4) = 0$$

$let\ m = 2^x$ $\quad$ $let\ n = 5^x$ $\quad$ $10^x = (2 \times 5)^x$

$= 2^x \times 5^x$

$n - 5 = 0$ $\quad$ $m - 4 = 0$

$n = 5$ $\quad$ $m = 4$

$5^x = 5$ $\quad$ $2^x = 4$

$5^x = 5^1$ $\quad$ $2^x = 2^2$

$x = 1$ $\quad$ $x = 2$

$Group$

$$10^x - 4(5^x) - 5(2^x) + 20 = 0$$
$$10^1 - 4(5^1) - 5(2^1) + 20 = 0$$
$$10 - 20 - 10 + 20 = 0$$
$$0 = 0 \ \checkmark$$

$$10^x - 4(5^x) - 5(2^x) + 20 = 0$$
$$10^2 - 4(5^2) - 5(2^2) + 20 = 0$$
$$100 - 100 - 20 + 20 = 0$$
$$0 = 0 \ \checkmark$$

C12 - 7.3 - Half Life (HL) Theory

Bananas have a half life of 4 days.

Half Life: Time to decay to half of the remaining mass.

Fraction Remaining	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
Mass (grams)	$100\ g$	$50\ g$	$25\ g$	$12.5\ g$	$6.25\ g$
Time (days)	0	4	8	12	16
# Half Lives	0	1	2	3	4

How long till 3.125 g?

How much after 28 days?

How long till $\frac{1}{256}$ *th of original?*

$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{256}$
$6.25\ g$	$3.125\ g$	$1.5625\ g$	$0.78125\ g$	0.390625
16	20	24	28	32

$$F = P(r)^{\frac{t}{T}}$$

$$3.125 = 100\left(\frac{1}{2}\right)^{\frac{t}{4}}$$

$$t = 20\ d$$

$$F = P(r)^{\frac{t}{T}}$$

$$F = 100\left(\frac{1}{2}\right)^{\frac{28}{4}}$$

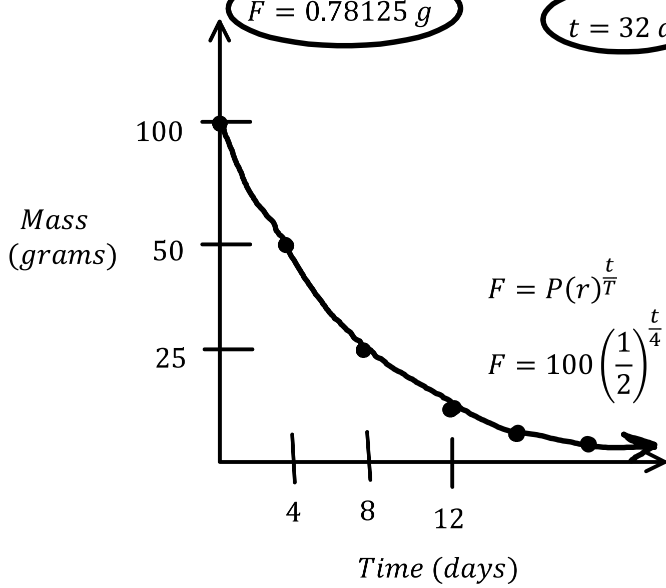
$$F = 0.78125\ g$$

$$F = P(r)^{\frac{t}{T}}$$

$$\frac{1}{256} = 1\left(\frac{1}{2}\right)^{\frac{t}{4}}$$

$$t = 32\ d$$

t	*g*
0	*100*
4	*50*
8	*25*
12	*12.5*
16	*6.25*
20	*3.125*
24	*1.5625*
28	*0.78125*
32	*0.390625*



$$F = P(r)^{\frac{t}{T}}$$

$$F = 100\left(\frac{1}{2}\right)^{\frac{t}{4}}$$

C12 - 7.3 - Word Problems Notes

If you deposit \$2000 in the bank at 12% interest how much will you have after 8 years?

$$F = P(1 \pm r)^t$$
$$F = 2000(1 + 0.12)^8$$
$$F = 4951.93$$

Find the rate to triple your money in 10 years.

$$F = P(1 + r)^t$$
$$3 = 1(1 + r)^{10}$$
$$(3)^{\frac{1}{10}} = \left((1 + r)^{10}\right)^{\frac{1}{10}}$$
$$1.116 = 1 + r$$
$$r = 0.1116 = 11.6\%$$

If you deposit \$5000 in the bank at 8% interest, compounded quarterly, how much will you have after 6 years?

$$F = P\left(1 \pm \frac{r}{n}\right)^{tn}$$
$$F = 5000\left(1 + \frac{0.08}{4}\right)^{6\times 4}$$
$$F = 8042.19$$

If a population starts at 1000 and triples every 4 hours, how large will the population grow in 25 hours?

$$F = P(r)^{\frac{t}{T}}$$
$$F = 1000(3)^{\frac{25}{4}}$$
$$F = 959417\ pop$$

If the population starts at 300 and grows continuously at a rate of 0.06, how large will it grow after 20 days?

$$F = Pe^{kt}$$
$$F = 300e^{0.06\times 20}$$
$$F = 996.03\ pop$$

How many times as intense is an earthquake of 6.0 than 3.0?

$$I = 10^{b-s}$$
$$I = 10^{6-3}$$
$$I = 10^3$$
$$I = 1000\ times$$

Find the present value of deposit worth \$2000 in the bank at 10% interest how much will you have after 4 years?

$$F = P(1 \pm r)^t$$
$$2000 = P(1 + 0.1)^4$$
$$2000 = P(1.4641)$$
$$P = \frac{2000}{1.1641}$$
$$P = \$1366.03$$

Find the rate of a \$1000 deposit worth \$1100 after 2 years.

$$F = P(1 \pm r)^t$$
$$1100 = 1000(1 + r)^2$$
$$\frac{1100}{1000} = (1 + r)^2$$
$$1.1 = (1 + r)^2$$
$$(1.1)^{\frac{1}{2}} = \left((1 + r)^2\right)^{\frac{1}{2}}$$
$$1.0488 = 1 + r$$
$$r = 0.0488$$
$$r = 4.9\%$$

How long to quadruple your money at 8%

$$F = P(1 \pm r)^t$$
$$400 = 100(1 + 0.08)^t$$
$$\frac{400}{100} = 1.08^t$$
$$4 = 1.08^t$$

$$y_1 = y_2$$

$$Calc\ Intersection\ or\ "logs"$$

$$t = 18.01\ yrs$$

If you deposit \$100 in the bank, how long will it take to grow to \$6400 if it doubles each year?

$$F = P(r)^{\frac{t}{T}}$$
$$6400 = 100(2)^{\frac{t}{1}}$$
$$\frac{6400}{100} = 2^t$$
$$64 = 2^t$$
$$2^6 = 2^t$$
$$t = 6s$$

An earth quake in California of Richter 8.5 Magnitude was 100 times as strong as an earth quake in Vancouver of what Richter Magnitude.

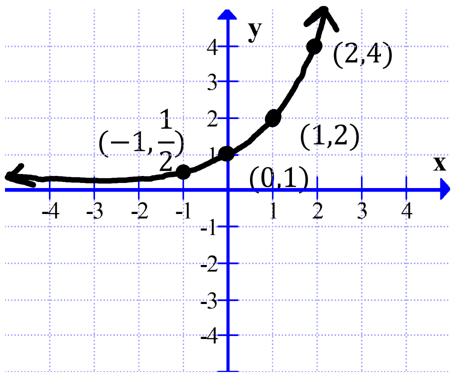
$$I = 10^{b-s}$$
$$100 = 10^{8.5-s}$$
$$10^2 = 10^{8.5-s}$$
$$2 = 8.5 - s$$
$$s = 6.5\ R$$

Light diminishes by 10% every 5 meters. Find the depth of 1% light.

$$F = P(1 \pm r)^{\frac{t}{T}}$$
$$1 = 100(1 - 0.1)^{\frac{d}{5}}$$
$$0.01 = 0.9^{\frac{d}{5}}$$
$$d = 218.5\ m$$

C12 - 7.4 - Exponent Reflections Graphs Notes

$y = 2^x$



x	y
-1	$\frac{1}{2}$
0	1
1	2
2	4

$2^{-1} = \frac{1}{2}$ $\quad (-1, \frac{1}{2})$

$2^0 = 1$ $\quad (0,1)$

$2^1 = 2$ $\quad (1,2)$

$2^2 = 4$ $\quad (2,4)$

$\boxed{Domain:\ xER}$

$\boxed{\text{Eg. Time* } t \geq 0}$

End Behavior

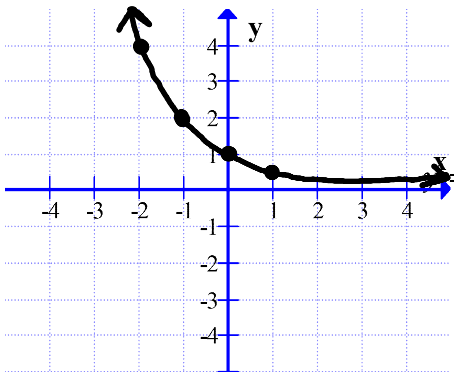
$x \to +\infty$
$y \to +\infty$

$x \to -\infty$
$y \to 0$

HA:

$y = 0$

$y = 2^{-x}$ Horizontal Reflection



$= \left(\frac{1}{2}\right)^x$

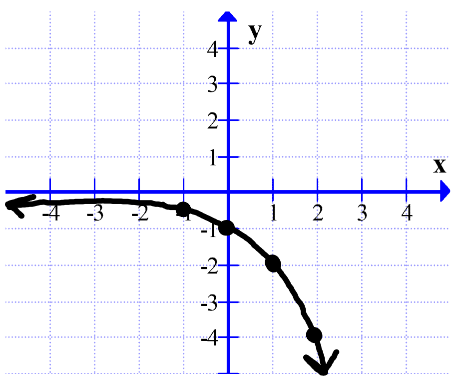
$$\left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x}$$

$x \to +\infty$
$y \to 0$

$x \to -\infty$
$y \to +\infty$

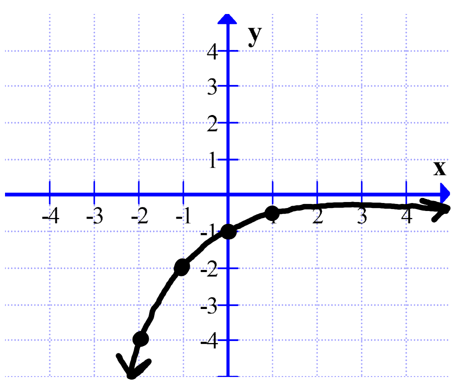
Remember: Positive Open up to the Right

Remember: Negative exponents and fractions with positive exponents Down to the Right



Vertical Reflection

$y = -2^x$

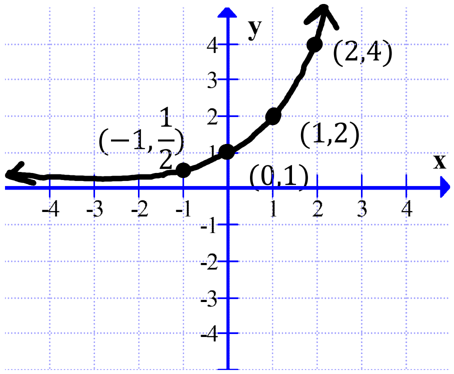


Vertical Reflection and
Horizontal Reflection

$y = -2^{-x}$

C12 - 7.4 - Exponent Transformations Graphs Notes

$y = 2^x$



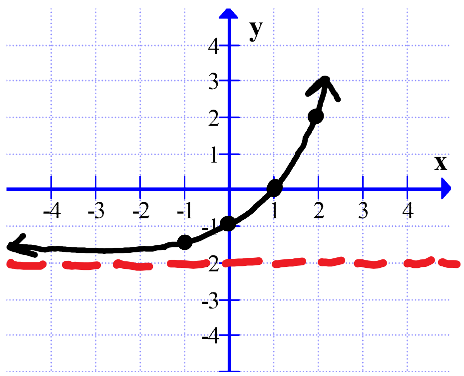
x	y
-1	$\frac{1}{2}$
0	1
1	2
2	4

$2^{-1} = \frac{1}{2}$ $\quad (-1,\frac{1}{2})$

$2^0 = 1$ $\quad (0,1)$

$2^1 = 2$ $\quad (1,2)$

$2^2 = 4$ $\quad (2,4)$



$y = 2^x - 2$

Down 2

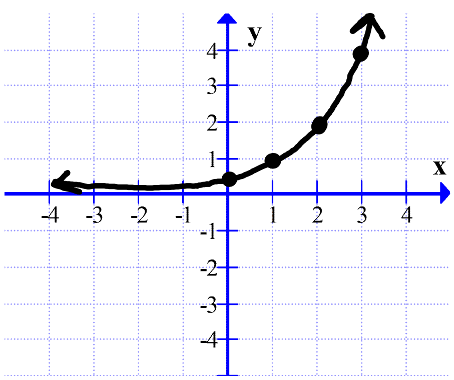
$y - 2$

$y = C^x \pm HA$

HA:

$y = -2$

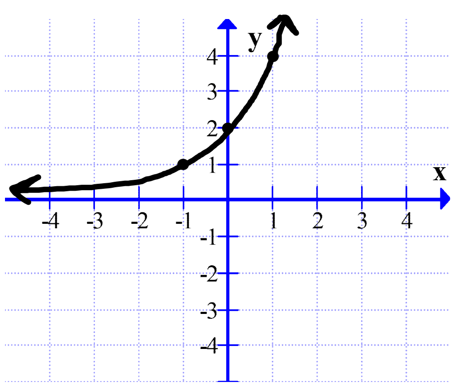
$$\boxed{y = a(C)^{b(x-h)} + k}$$



$y = 2^{x-1}$

Right One

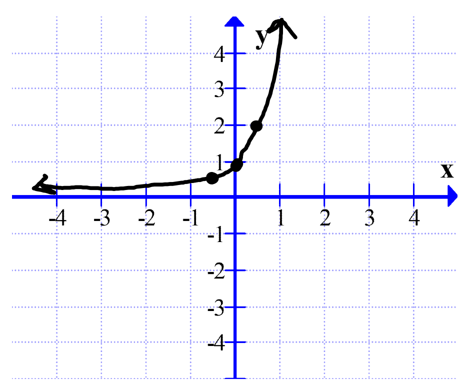
$x + 1$



$y = 2(2^x)$

Vertical Expansion = 2

$y \times 2$



$y = 2^{2x}$

Horizontal Compression = 1/2

$x \div 2$

C12 - 8.1 - $\log_b a = ?$ Definition Notes

The Definition of a Logarithm:

$\log_3 9 = ?$
$\log_3 9 = 2$

$? = 2$

Think: What power do you have to raise 3 to, to equal 9?

$\log_2 8 = ?$
$\log_2 8 = 3$

$? = 3$

8 equals 2 to what power?

$$\log_b a = c$$

'a' is the thing you are logging

'c' is the answer/exponent

'b' is the base.

Switching from Log Form to Exponential Form:

$$\log_b a = c$$

Log Form

$$a = b^c$$

Exponential Form

The thing you are Logging equals the Base to the other side.

Remember:
The base of the log is the base of the exponent.

The exponent is the Answer.

Log Form

$\log_2 16 = ?$
$\log_2 16 = 4$

$? = 4$

16 equals 2 to what power?

Exponential Form

$16 = 2^?$
$2^4 = 2^x$

$? = 4$

Log Form -> Exponential Form and Solve for x

$\log_2 16 = x$ — Set Log arbitrarily $= x$

$16 = 2^x$ — Exponential Form

$2^4 = 2^x$ — Change of Base

$x = 4$ — Same Base: Make exponents equal to each other

$$\boxed{\boxed{\log_2 16 = 4}}$$

$\log_{\frac{1}{2}} 16 = x$

$16 = \left(\frac{1}{2}\right)^x$ — Exponential Form

$2^4 = (2^{-1})^x$ — Change of Base

$2^4 = 2^{-x}$ — Exponent Laws

$4 = -x$ — Solve

$x = -4$

$\log_3 \left(\frac{1}{27}\right) = x$

$\frac{1}{27} = 3^x$ — Exponential Form

$\frac{1}{3^3} = 3^x$ — Change of Base

$3^{-3} = 3^x$ — Exponent Laws

$x = -3$

$\log_{2a} 16a^4 = x$

$16a^4 = (2a)^x$ — Exponential Form

$(2a)^4 = (2a)^x$ — Change of Base

$x = 4$

*C12 - 8.1 - $\log_b x = c, \log_x a = c, \log_b a = x$ *Notes*

Log Form -> Exponential Form and Solve for x

The base of the log is the base of the exponent

$\log_5 125 = x$

$125 = 5^x$ Exponential Form

$5^3 = 5^x$ Change of Base

$x = 3$ Same Base: Make exponents equal to each other

$\log_4 x = 3$

$x = 4^3$ Exponential Form

$x = 64$ Solve

$\log_6 x = 2$

$x = 6^2$

$x = 36$

$\log_5 x = -2$

$x = 5^{-2}$

$x = \frac{1}{5^2}$ Exponent Laws

$x = \frac{1}{25}$ Solve

$\log_9 x = \frac{1}{2}$

$x = 9^{\frac{1}{2}}$

$x = \sqrt{9}$

$x = 3$

$\log_x 64 = 3$

$64 = x^3$ Exponential Form

$4^3 = x^3$ Change of Base

$x = 4$ Solve

$\log_x 32 = 5$

$32 = x^5$ Exponential Form

$2^5 = x^5$ Change of Base

$\sqrt[5]{2^5} = \sqrt[5]{x^5}$ Fifth Root Both Sides

$x = 2$ Solve

$\log_x 27 = \frac{3}{2}$

$27 = x^{\frac{3}{2}}$

$27^{\frac{2}{3}} = \left(x^{\frac{3}{2}}\right)^{\frac{2}{3}}$

$27^{\frac{2}{3}} = x^1$

$\sqrt[3]{27^2} = x$

$x = 9$

Take both/ sides to reciprocal exponent

$\log_2(x-5) = 3$

$x - 5 = 2^3$

$x = 8 + 5$

$x = 13$

$x - 5 > 0$

$x > 5$

$\log_{x-2} 1 = 2$

$1 = (x-2)^2$

$1 = (x-2)(x-2)$

$1 = x^2 - 4x + 4$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$\cancel{x = 3}$ $x = 1$

$x - 2 > 0$

$x > 2$

$x - 2 \neq 1$

$x \neq 3$

$\log_{x-3} 2 = 1$

$2 = (x-3)^1$

$2 = x - 3$

$x = 5$

$x - 3 > 0$

$x > 3$

$x - 3 \neq 1$

$x \neq 4$

$\log_2 16 = x + 2$

$16 = 2^{x+2}$

$2^4 = 2^{x+2}$

$4 = x + 2$

$x = 2$

OR

$\log_2 16 = x + 2$

$\log_2 16 - 2 = x$

$4 - 2 = x$

$x = 2$

Do Algebra First!

$\log_2 16 = x$	$\log_2 16 = 4$
$16 = 2^x$	
$2^4 = 2^x$	
$x = 4$	

C12 - 8.2 - Log Restrictions Notes

State Restrictions: $\boxed{\log_b a \qquad a > 0 \qquad b > 0 \qquad b \neq 1}$

$\boxed{\begin{array}{c} \log x \\ x > 0 \end{array}}$

$\log 0 = und$

$\log(-3) = und$

$\boxed{\begin{array}{c} \log_x \# \\ x > 0,\, x \neq 1 \end{array}}$

$\log_0 \# = und$

$\log_{(-2)} \# = und$

$\log_1 \# = und$

State Restrictions and Solve

Domain: Set the thing you are logging to greater than or equal to zero, then solve.

$\log_2 x = 2$
$x = 2^2$

$x > 0$

$x = 4$

$\log_2(x-5) = 2$
$x - 5 = 2^2$
$x = 4 + 5$

$x = 9$

$x - 5 > 0$

$x > 5$

$\log_2(3-x) = 3$
$(3-x) = 2^3$
$3 - x = 8$

$x = -5$

$3 - x > 0$
$-x < 3$

$x < 3$

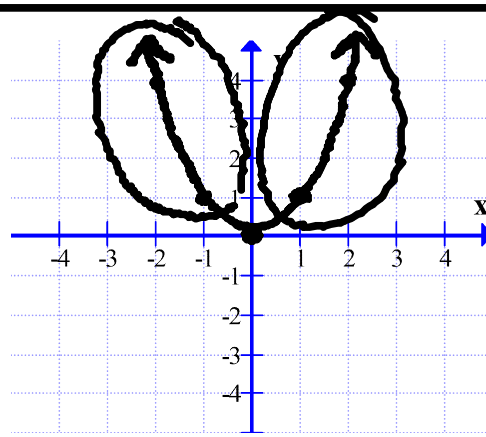
$\log_3 x^2 = 2$
$x^2 = 3^2$
$x^2 = 9$
$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

$x = 3$, $x = -3$

$x^2 > 0$
$x < 0, x > 0$

$x \neq 0$



$2\log_3 x = 2$
$\log_3 x = 1$
$x^2 = 1$
$x^2 = 9$
$\sqrt{x^2} = \sqrt{9}$
$x = \pm 3$

$x > 0$

$x = 3$, $\cancel{x = -3}$

$\log_{36}(5x - x^2) = \frac{1}{2}$
$5x - x^2 = 36^{\frac{1}{2}}$
$5x - x^2 = 6$
$x^2 - 5x + 6 = 0$
$(x-2)(x-3) = 0$

$x = 2$, $x = 3$

$5x - x^2 > 0$
$x(5-x) > 0$

$0 < x < 5$

$\log_9(x^2 - 1) = \frac{1}{2}$
$x^2 - 1 = 9^{\frac{1}{2}}$
$x^2 - 1 = 3$
$x^2 - 4 = 0$
$(x+2)(x-2) = 0$

$x = -2$, $x = 2$

$x^2 - 1 > 0$
$(x+1)(x-1) > 0$

$x < -1$, $x > 1$

$\log_{x-3} 16 = 2$
$16 = (x-3)^2$
$16 = (x-3)(x-3)$
$16 = x^2 - 6x + 9$
$0 = x^2 - 6x - 7$
$0 = (x-7)(x+1)$

$x = 7$, $\cancel{x = -1}$

$x - 3 > 0$

$x > 3$

$x - 3 \neq 1$

$x \neq 4$

$\boxed{\begin{array}{c} \log_3(-x) = 2 \\ -x = 3^2 \\ x = -9 \end{array}}$

Set the base of the log > 0 *and* $\neq 1$ *and solve.*

C12 - 8.3 - $\log a^m = m\log a$ Change Base Dist. Notes

$log5^2 \Updownarrow 2log5$

$logx^2 \Updownarrow 2logx$

$$3log4^2 \qquad 2\times 3log4 \qquad 6log4$$

OR

$$3log4^2 \qquad log4^{2\times3} \qquad log4^6 \qquad 6log4$$

Bring Exponent down in front and Vice Versa Multiply

$log\sqrt{x}$
$logx^{\frac{1}{2}}$
$\frac{1}{2}\log x$
$\frac{logx}{2}$

$\log\left(\frac{1}{2}\right)$
$log2^{-1}$
$-1log2$
$-log2$

$\log_5 5^4 = x$
$5^4 = 5^x$
$x = 4$

5 to what power is 5^4

$log_5 625 = x$
$\log_5 5^4 = x$
$4\log_5 5 = x$
$4 \times 1 = x$
$x = 4$

Change of Base
Bring Exponent down in front
Log Rules
Solve

$\boxed{\log_5 5 = 1}$

$log_5 625 = x$
$625 = 5^x$
$5^4 = 5^x$
$x = 4$

$logxy^2 =$
$logx + logy^2 =$
$logx + 2logy$

The exponent only applies to the y value

$logx^2y^2 =$
$logx^2 + logy^2 =$
$2logx + 2logy$

$\leftarrow$

$\log(xy)^2 =$
$2logxy =$
$2(logx + logy) =$
$2logx + 2logy$

$log3^{x+2}$
$(x+2)\log 3$
$xlog3 + 2log3$

Bring Exponent in front

Distribute

$3xlog7 - xlog2 =$
$x(3log7 - log2)$

$GCF = x$

Change of Base

$\frac{log16}{log4} =$
$\log_4 16 = 2$

$\frac{\log_2 16}{log_2 4} =$
$\log_4 16 = 2$

Exponential Form

$16 = 4^2$

$\log_2 2 = 2$
$\log_2 16 = 4$

$\frac{4}{2} = 2$

$\log_2 4 =$
$\frac{\log_5 4}{\log_5 2}$

Choose the Base you want!

$\log_8 16 =$
$\frac{\log_2 16}{\log_2 8} = \frac{4}{3}$

$\frac{1}{\log_8 2} =$
$\frac{1}{\left(\frac{log2}{log8}\right)} =$
$1 \times \frac{log8}{log2} = \frac{log8}{log2}$

Rule 6

$\log_3 9 + \log_9 2$
$\log_{(3)^2}(9)^2 + \log_9 2$
$\log_9 81 + \log_9 2$
$\log_9 81 \times 2$

$\log_9 162$

Take the base and the log to any exponent you like!

C12 - 8.4 - $\log_b m + \log_b n = \log_b mn$ $\;log_b m - log_b n = log_b \frac{m}{n}$ $\log_{b^n} a^n$ *Notes*

$\log_2 4 + \log_2 8 =$ $\boxed{2 + 3 = 5}$
$\log_2 4 \times 8 =$
$\log_2 32 = 5$

Add-Multiply

Exponential Form

$32 = 2^5$

$\boxed{\begin{array}{l}\log_2 4 = 2 \\ \log_2 8 = 3\end{array}}$

$\boxed{logA + \log B = logAB}$

$log1 + log5 + log7 =$
$log1 \times 5 \times 7 = \log 35$

$\boxed{logA + \log B + \log C = logABC}$

$\log_3 27 - \log_3 3 =$ $\boxed{3 - 1 = 2}$
$\log_3 \frac{27}{3} =$
$\log_3 9 = 2$

Subtract-Divide

$\boxed{LogA - LogB = \log\left(\frac{A}{B}\right)}$

Rearrange $\quad -logA + logB$
$logB - logA$
$\log\left(\frac{B}{A}\right)$

$log4 + log20 - log10 =$
$log \frac{4 \times 20}{10} = log\, 8$

Positives on top,
Negatives on Bottom
<u>Vice Versa</u>

$log5 - log2 + log10 =$
$log \frac{5 \times 10}{2} = log25$

$log5 - log2 - log10 =$
$log \frac{5}{2 \times 10} = \log \frac{1}{4}$

$LogA + LogB - LogC = \log\left(\frac{AB}{C}\right)$

$LogA - LogB - LogC = Log\left(\frac{A}{BC}\right)$

$Log\left(\frac{A}{BC}\right) = LogA - LogBC$
$Log\left(\frac{A}{BC}\right) = LogA - (LogB + LogC)$
$Log\left(\frac{A}{BC}\right) = LogA - LogB - LogC$

$logx + \log x =$
$\log x \times x = \log x^2$

$\log 3 + \log(x + 1) =$
$log3(x + 1) = \log(3x + 3)$

$\log(x - 2) + \log(x + 1) =$
$\log(x - 2)\,(x + 1) = \log(x^2 - x - 2)$

Add Multiply

$\log x^3 - \log x^2 =$
$\log \frac{x^3}{x^2} = logx$

$\log(x^2 - 1) - \log(x + 1) =$
$\log \frac{x^2 - 1}{x + 1} =$
$\log \frac{(x + 1)(x - 1)}{(x + 1)} = \log(x - 1)$

Subtract
Divide
Factor
Simplify

$\log_2 8 =$
$\log_{2^2} 8^2 =$
$\log_4 64 = 3$

Take the base and the log to any exponent you like!

Exponential Form

$64 = 4^3$

$\log_2 8 = 3$

$8 = 2^3$

$\log_4 16 =$
$\log_{\sqrt{4}} \sqrt{16} =$
$\log_2 4 = 2$

$\log_{\frac{1}{2}} 4 =$
$\log_{(\frac{1}{2})^{-1}} 4^{-1} =$
$\log_2 4^{-1} =$
$-1 \log_2 4 = -2$

$\left(\frac{1}{2}\right)^{-1} = 2$

$\log_2 4 = 2$

$-1 \times 2 = -2$

$\log_2 4 + \log_4 2 =$
$\log_{2^2} 4^2 + \log_4 2 =$
$\log_4 16 + \log_4 2 =$
$\log_4 32 \times 2 =$
$\log_4 64 = 3$

Take the base and the thing you are logging to an exponent to get like bases.

C12 - 8.4 - $log5 = m, log7 = n, Notes$

$Given$: $log5 = m$ $log7 = n$ $Solve\ in\ terms\ of\ m\ and\ n$:

$$log25 = log5^2$$
$$= 2log5$$
$$= 2m$$

$$log35 = log5 + log7$$
$$= m + n$$

$$log350 = log5 + log7 + log10$$
$$= m + n + 1$$

$$log5x = log5 + logx$$
$$= m + logx$$

$$log0.49 = \log\frac{49}{100}$$
$$= log49 - log100$$
$$= log7^2 - 2$$
$$= 2log7 - 2$$
$$= 2n - 2$$

$$\log_5 7 = \frac{log7}{log5}$$
$$= \frac{n}{m}$$

$Given$: $log4 = a$ $log6 = b$ $Solve\ in\ terms\ of\ a\ and\ b$:

$$log16 =$$
$$log4^2 =$$
$$2log4 =$$
$$2a$$

$$log16 =$$
$$log2^4 =$$
$$4log2 =$$
$$4a$$

$$log24 =$$
$$log6 + log4 =$$
$$\frac{b}{2} + \frac{a}{2}$$

$$log2 =$$
$$log\sqrt{4} =$$
$$log4^{\frac{1}{2}} =$$
$$\frac{1}{2}log4 =$$
$$\frac{1}{2}a$$

$$log3 =$$
$$\log\frac{6}{2} =$$
$$log6 - log2 =$$
$$b - \frac{1}{2}a$$

$$\log\frac{3}{2} =$$
$$log3 - log2 =$$
$$b - \frac{1}{2}a - \frac{1}{2}a =$$
$$b - a$$

$$log0.4 =$$
$$\log\left(\frac{4}{10}\right) =$$
$$log4 - log10 =$$
$$a - 1$$

C12 - 8.5 - De/Log Operation/Equation/Factoring Notes

$log8 = 0.9031$ $log_4 7 = 1.4037$

Calculator Math, Alpha, Math

$\log_5(x+1) = \log_5 7$ Delog both sides

$\cancel{\log_5}(x+1) = \cancel{\log_5} 7$

$x + 1 = 7$

$x = 6$

$\log_2(x-2) + \log_2(x+1) = 2$

$\log_2(x-2)(x+1) = 2$

$\log_2(x^2 - x - 2) = 2$

$x^2 - x - 2 = 2^2$

$x^2 - x - 2 = 4$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$ $\cancel{x = -2}$

$\log_2(x-2) + \log_2(x+1) = 2$

$\log_2(x^2 - x - 2) = \log_2 4$

$x^2 - x - 2 = 4$

$x^2 - x - 6 = 0$

See Left

Or Turn a number into a log!

$2 = \log_2 m$

$2^2 = m$

$m = 4$

$2 = \log_2 4$

$\log_2(x-2) - 2 = \log_2(x+1)$

$\log_2(x-2) + \log_2(x+1) = 2$ Algebra

See Above

$x - 2 > 0$ $x - 1 > 0$

$x > 2$ $\cancel{x > -1}$

Reject Redundant!

$\log_3(x-11) - \log_3(x-3) = 2$

$\log_3 \dfrac{x-11}{x-3} = 2$

$\dfrac{x-11}{x-3} = 3^2$

$\dfrac{x-11}{x-3} = 9$

$x - 11 = 9(x-3)$

$x - 11 = 9x - 27$

$16 = 8x$

$x = 2$ $x > 3$

$2\log_5 x + \log_5 x = 3$

$\log_5 x^2 + \log_5 x = 3$

$\log_5 x^2 \times x = 3$

$\log_5 x^3 = 3$

$x^3 = 5^3$

$x = 5$ $x > 0$

Must Bring exponents up 1st!

$(logx)^2 - logx^3 = 4$

$(logx)^2 - 3logx = 4$

$m^2 - 3m - 4 = 0$

$(m-4)(m+1) = 0$

$let\ m = logx$

$m = 4$ $m = -1$

$logx = 4$ $logx = -1$

$x = 10^4$ $x = 10^{-1}$

*C12 - 8.6 - Log Both Sides Notes

$$4 = 2^x$$
$$log4 = log2^x$$
$$log4 = xlog2$$
$$\frac{log4}{log2} = x$$
$$\log_2 4 = x$$
$$x = 2$$

Log Both Sides
Bring Exponents Down In Front
Divide
Change of base

Definition
Solve

$$3 = 5^x$$
$$log3 = log5^x$$
$$log3 = xlog5$$
$$\frac{log3}{log5} = x$$
$$\log_5 3 = x$$
$$x = 0.6826$$

Algebraic answer

Check Answer:

$$5^{0.6828} = 3$$

Before you log both sides!

$$3 = 2^x - 1$$
$$4 = 2^x$$

Add/Subtract First

$$8 = 2 \times 2^x$$
$$4 = 2^x$$

Divide First

Or

$$8 = 2 \times 2^x$$
$$log8 = \log(2 \times 2^x)$$
$$log8 = log2 + log2^x$$

$$4 = 7^{2x+1}$$
$$log4 = log7^{2x+1}$$
$$log4 = (2x+1)log7$$
$$log4 = 2xlog7 + \log 7$$
$$log4 - log7 = 2xlog7$$
$$\frac{log4 - log7}{2log7} = x$$
$$x = \frac{log4 - log7}{2log7}$$
$$x = -0.29$$

Distribute
Combine x's on one side
Everything else on other side
Factor out x
Divide

$$4 = 7^{2x+1}$$
$$\log_7 4 = 2x + 1$$
$$\log_7 4 - 1 = 2x$$
$$x = \frac{\log_7 4 - 1}{2}$$

$$2^{2x-5} = 9^{x+2}$$

$$\boxed{\log_2 9^{x+2} = 2x - 5}$$

$$log2^{2x-5} = log9^{x+2}$$
$$(2x-5)log2 = (x+2)log9$$
$$2xlog2 - 5log2 = xlog9 + 2log9$$
$$2xlog2 - xlog9 = 2log9 + 5log2$$
$$x(2log2 - log9) = 2log9 + 5log2$$
$$x = \frac{2log9 + 5log2}{2log2 - log9}$$

$$6 \times 3^x = 14^{2x-5}$$
$$\log(6 \times 3^x) = log14^{2x-5}$$
$$log6 + log3^x = log14^{2x-5}$$
$$log6 + xlog3 = (2x-5)log14$$
$$log6 + xlog3 = 2xlog14 - 5log14$$
$$2xlog14 - xlog3 = log6 + 5log14$$
$$x(2log14 - log3) = log6 + 5log14$$
$$x = \frac{log6 + 5log14}{2log14 - log3}$$

Rule 7 Proof

$$\boxed{b^{\log_b x} = x}$$

$$b^{\log_b x} = x$$
$$\log b^{\log_b x} = logx$$
$$\log_b x \log(b) = logx$$
$$\frac{\log_b x \log(b)}{logb} = \frac{logx}{\log b}$$
$$\log_b x = \frac{logx}{logb}$$
$$\log_b x = \log_b x$$
$$x = x$$

Remember: You may only log both sides if SAMD is complete. Bedmas backwards.
Remember: If you do log a product you must separate into an addition of logs.
Remember: If you log a sum you must use brackets
Remember: You may only de-log both sides if one log equals one log.

C12 - 8.6 - Word Problem Notes

How long to earn \$1500 on \$10000 at 10%/year?

$$F = P(1+r)^t$$
$$11500 = 10000(1+0.1)^t$$
$$\frac{11500}{10000} = 1.1^t$$
$$1.15 = 1.1^t$$
$$log1.15 = log1.1^t$$
$$log1.15 = tlog1.1$$
$$\frac{log1.15}{log1.1} = t$$
$$\log_{1.1} 1.15 = t$$
$$t = 1.47\ years$$

$$\boxed{\begin{array}{c} 10000 \\ \underline{+1500} \\ 11500 \\ \text{Logic} \end{array}}$$

$$\boxed{\begin{array}{r} 1.15 = 1.1^t \\ \log_{1.1} 1.15 = t \end{array}}$$

How long to triple your money at 10%/year?

$$F = P(1+r)^t$$
$$3 = 1(1+0.1)^t$$
$$3 = 1.1^t$$
$$\log_{1.1} 3 = t$$
$$t = 11.43\ years$$

$$\boxed{\begin{array}{c} P = 1 \\ \rightarrow \\ F = 3 \end{array}}$$

How long to grow \$10000 to \$12000 compounded quarterly at 10%?

$$F = P\left(1+\frac{r}{n}\right)^{tn}$$
$$12000 = 10000\left(1+\frac{0.1}{4}\right)^{4t}$$
$$1.2 = 1.025^{4t}$$
$$\log_{1.025} 1.2 = 4t$$
$$\frac{\log_{1.025} 1.2}{4} = t$$
$$t = 1.85\ years$$

An earthquake of magnitude 8 is 250 times as intense as an earth quake of what magnitude?

$$I = 10^{b-s}$$
$$250 = 10^{8-s}$$
$$\log_{10} 250 = 8 - s$$
$$s = 5.6\ magnitude$$

How long to grow 1000 Bacteria to 5000 at a continuous growth rate of 0.05?

$$F = Pe^{kt}$$
$$5000 = 1000e^{0.05t}$$
$$5 = e^{0.05t}$$
$$\frac{\ln_e 5}{0.05} = t$$
$$t = 32.2 \ldots$$

Find the half-life of a substance decaying to 20% of its original in 500 years?

$$F = P(r)^{\frac{t}{T}}$$
$$20 = 100\left(\frac{1}{2}\right)^{\frac{500}{T}}$$
$$0.2 = 0.5^{\frac{500}{T}}$$
$$\log_{0.5} 0.2 = \frac{500}{T}$$
$$T = \frac{500}{\log_{0.5} 0.2}$$

Cross Multiply

$$T = 215.34\ years$$

A substance has a half-life of 5 years. How long to be ten percent of its original?

$$F = P(r)^{\frac{t}{T}}$$
$$10 = 100\left(\frac{1}{2}\right)^{\frac{t}{5}}$$
$$0.1 = 0.5^{\frac{t}{5}}$$
$$\log_{0.5} 0.1 = \frac{t}{5}$$
$$t = 16.61\ years$$

Find the number of compounding periods to grow \$10000 to \$16288.95 at 10% in 5 years.

$$F = P\left(1+\frac{r}{n}\right)^{tn}$$
$$2 = 1\left(1+\frac{0.1}{n}\right)^{5n}$$

$y_1 = y_2$
Find Intersection

$$n = 2 \quad ; Semi-annually$$

C12 - 8.7 - Graph Log Notes

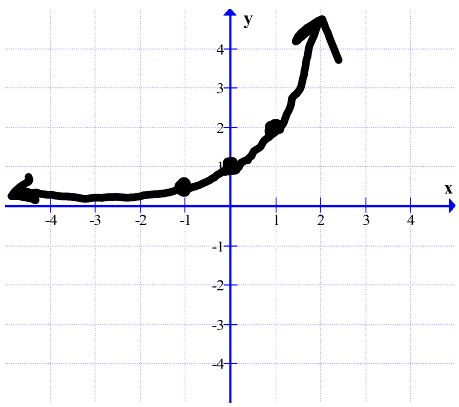
Graph: $y = \log_2 x$

$y = 2^x$

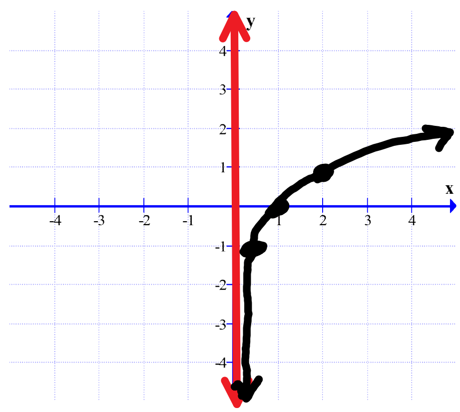
x	y
-1	$\frac{1}{2}$
0	1
1	2

x	y
$\frac{1}{2}$	-1
1	0
2	1

$y = 2^x$



$x <-> y$



$y = \log_2 x$

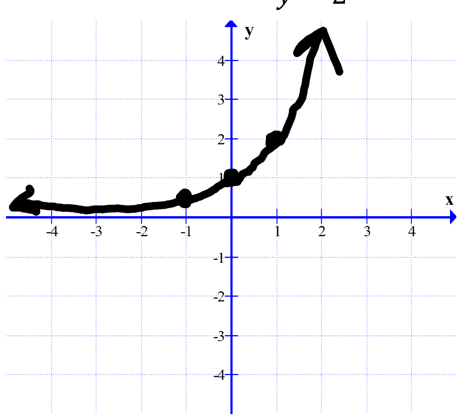
VA: $x = 0$ $\quad$ *Domain*: $x \geq 0$

Graph: $y = \log_2(x + 2) + 1$

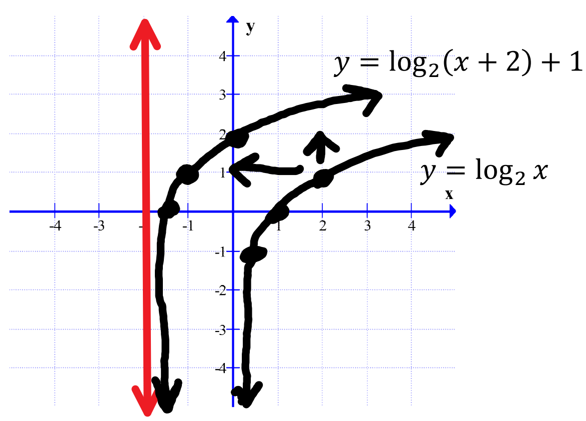
$y = 2^x$

$\boxed{\begin{array}{l} Left\ 2 \\ Up\ 1 \end{array}}$

$y = 2^x$



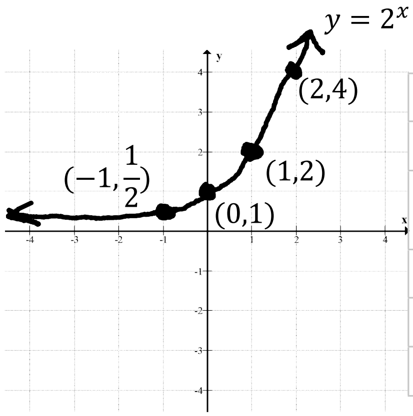
$x <-> y$



$x + 2 = 0$
$x = -2$
VA

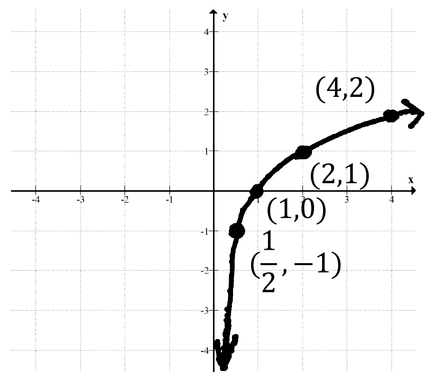
$x + 2 > 0$
$x > -2$
Domain

C12 - 8.8 - Inverse Log Graphs Notes



x	y
-1	$\frac{1}{2}$
0	1
1	2
2	4

x	y
0	und
$\frac{1}{2}$	-1
1	0
2	1
2	4

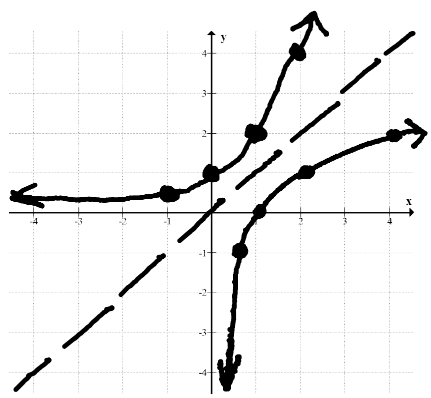


$y = 2^x$
$x = 2^y$ — *Switch x and y*
$logx = log2^y$ — Log Both Sides
$logx = ylog2$ — Bring Exponents Down In Front
$\frac{logx}{log2} = y$ — Divide
$\log_2 x = y$ — Change of base
$y = \log_2 x$ — Mirror
$f^{-1}(x) = \log_2 x$ — Inverse Function notation

$y = 2^x$
$x = 2^y$ — *Switch x and y*
$y = \log_2 x$ — Exponential to log Form
$f^{-1}(x) = \log_2 x$

Back the Other Way!

$y = \log_2 x$
$x = \log_2 y$
$2^x = y$
$y = 2^x$
$f^{-1}(x) = 2^x$



Remember: Inverse: Switch x and y
Remember: A diagonal reflection over the line $y = x$

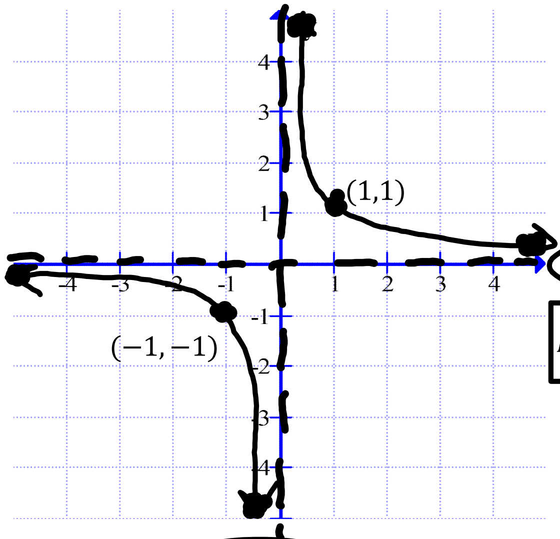
Inverse Proof

$y = 2^{x+1} - 3$
$x = 2^{y+1} - 3$
$x + 3 = 2^{y+1}$
$\log(x+3) = (y+1)log2$
$\frac{\log(x+3)}{log2} = y + 1$
$\log_2(x+3) = y + 1$
$\log_2(x+3) - 1 = y$
$y = \log_2(x+3) - 1$
$f^{-1}(x) = \log_2(x+3)$

$y = \log_2(x+3) - 1$
$x = \log_2(y+3) - 1$
$x + 1 = \log_2(y+3)$
$2^{x+1} = y + 3$
$2^{x+1} - 3 = y$
$y = 2^{x+1} - 3$
$f^{-1}(x) = 2^{x+1} - 3$

$$y = \frac{a}{x-h} + k$$

C12 - 9.1 - Graph TOV HT xy-int Notes



$y = \frac{1}{x}$

$HA: \ y = 0$

$HA: \ y = k$

x	y
-5	$-\frac{1}{5} = -0.2$
-1	-1
$-\frac{1}{10} = -0.1$	-10
0	und
$\frac{1}{10} = 0.1$	10
1	1
5	$\frac{1}{5}$

$$y = \frac{1}{x}$$
$$y = \frac{1}{\left(-\frac{1}{10}\right)}$$
$$y = 1\left(-\frac{10}{1}\right)$$
$$y = -10$$

$VA: \ x = 0$

$\text{VA: Set Denominator} = 0 \ and \ solve$

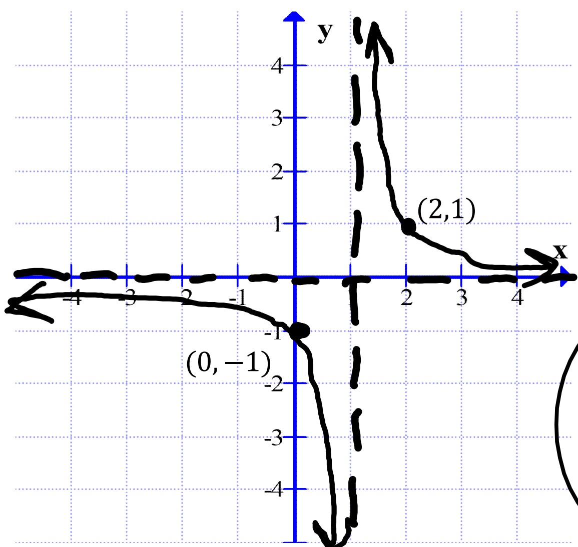
$x = 0 \qquad Domain: x \neq 0$

$x - int:$

$$y = \frac{1}{x}$$
$$0 = \frac{1}{x}$$
$$0 \neq 1$$

$y - int:$

$$y = \frac{1}{x}$$
$$y = \frac{1}{0}$$
$$y \neq$$



$HT = +1$

$y = \frac{1}{x-1}$

$HA: \ y = 0$

$Range$

$y \neq 0$

End Behavior

$x \to \infty, \qquad y \to 0^+$

$x \to -\infty, \qquad y \to 0^-$

As x gets close to …

Behavior near Asymptote

$x \to 1^+, \qquad y \to \infty$

$x \to 1^-, \qquad y \to -\infty$

$VA: \ x = 1 \qquad Domain: x \neq 1$

$VA:$

$$x - 1 = 0$$
$$x = 1$$

$x - int:$

$$y = \frac{1}{x-1}$$
$$0 = \frac{1}{x-1}$$

Careful! $\quad (x-1) \times 0 = \frac{1}{x-1} \times (x-1)$

$$0 \neq 1$$

$y - int:$

$$y = \frac{1}{x-1}$$
$$y = \frac{1}{0-1}$$
$$y = -1$$
$$(0, -1)$$

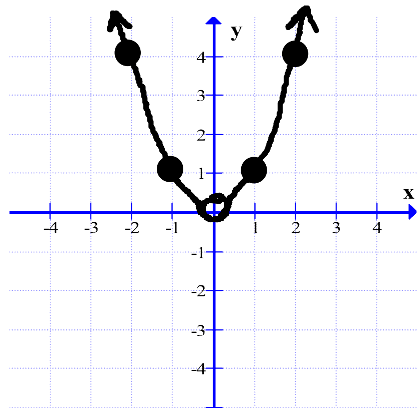
x	y
0	-1
1	und
2	1

C12 - 9.1 - Horizontal Asymptotes Cases Notes

$$y=\frac{ax^m}{bx^n}\qquad \begin{array}{llll} m>n & HA:\ none & & \\ m<n & HA:\ y=0 & or & HA:\ y=c \\ m=n & HA:\ y=\frac{a}{b} & or & HA:\ y=\frac{a}{b}+c \end{array}$$

Case 1:

$$y=\frac{x^3}{x^1}$$

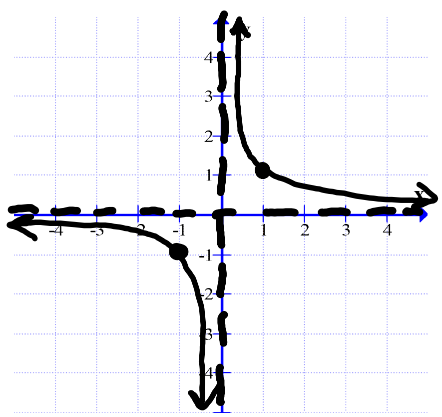


If the exponent of x is higher on the top than the bottom

$HA:\quad none$

Case 2:

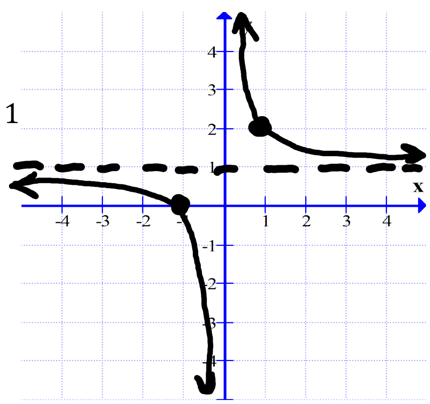
$$y=\frac{x^1}{x^2}$$



$$\frac{x^1}{x^2}+1=\frac{1x^1+1x^2}{1x^2}$$

LCD

$$y=\frac{x^1}{x^2}+1$$



If the exponent of x is higher on the bottom

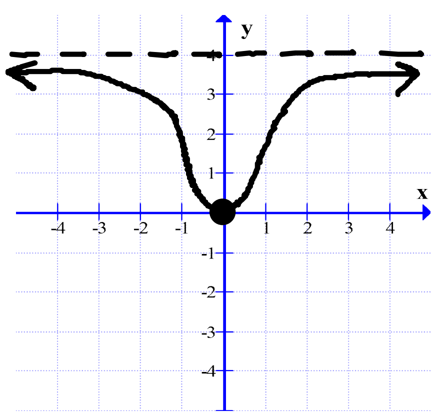
$HA:\ y=0$

If case 2 is shifted up or down $=\ c$

$HA:\ y=c \qquad y=1$

Case 3:

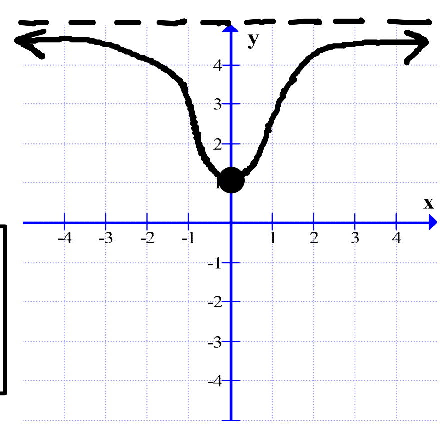
$$y=\frac{4x^2}{1x^2+2}$$



$$y=\frac{4x^2}{1x^2+1}+1$$

$$\frac{4x^2}{1x^2+1}+1=\frac{5x^2+1}{1x^2+1}$$

LCD



If the exponent of x is the same on the top as the bottom

$HA:\ y\ =\ fraction\ of\ coefficients$

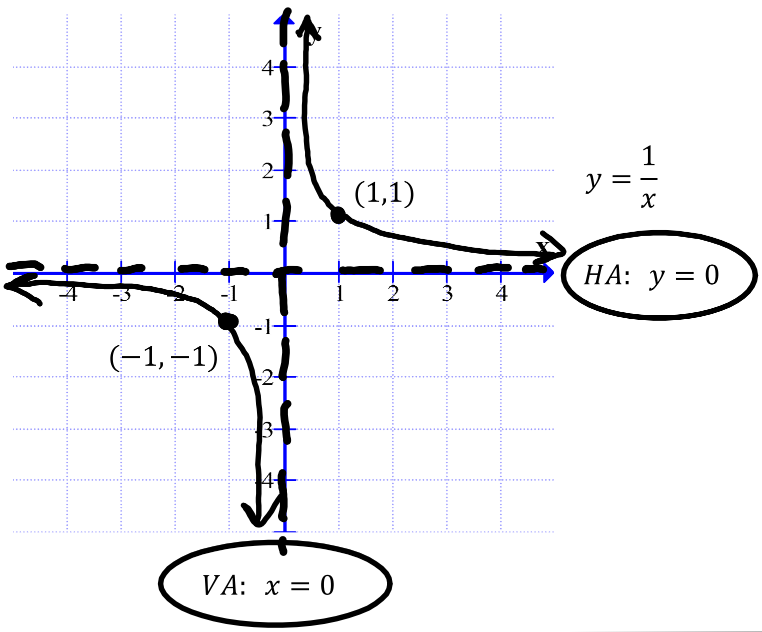
$$HA:\ y=\frac{4}{1}$$

If case 3 is shifted up or down $=c$

$HA:\ y=\ fraction\ of\ coefficients\ +\ c$

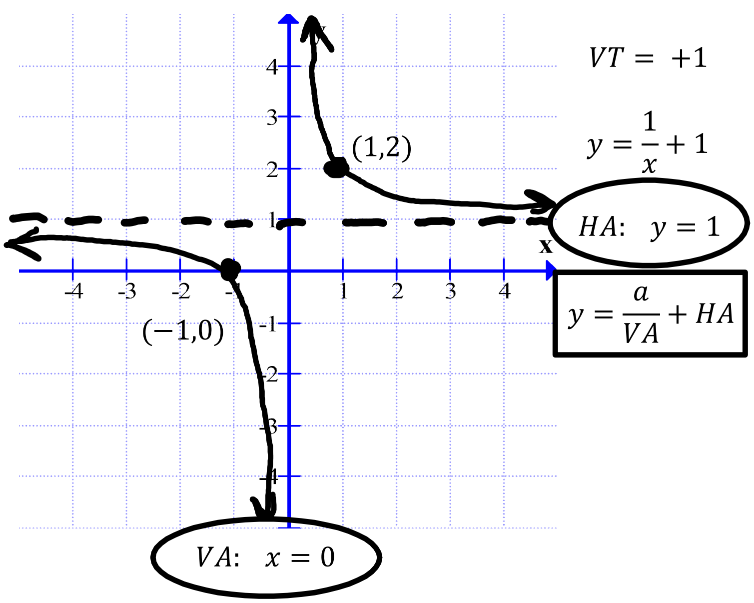
$$HA:\ y=\frac{4}{1}+1=5$$

C12 - 9.2 - Graph VT Add Fractions Long Division Notes



$$y = \frac{1}{x}$$

x	y
-1	-1
0	und
1	1



$VT = +1$

$$y = \frac{1}{x} + 1$$

$$\boxed{y = \frac{a}{VA} + HA}$$

Add Fractions

$$\frac{1}{x} + 1$$

LCD $\quad \frac{1}{x} + 1 \times \frac{x}{x}$

$$\frac{1}{x} + \frac{x}{x}$$

$$\frac{x+1}{x}$$

Long Division

$$x \overline{)\, x + 1} \quad \text{quotient } 1$$

$$-\; x$$

$$1$$

remainder

$$\frac{x+1}{x} = 1 + \frac{1}{x}$$

$$y = \frac{x+1}{x}$$

$$\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$$

$$\frac{x+1}{x} = \frac{x}{x} + \frac{1}{x} = 1 + \frac{1}{x}$$

Separate Fractions

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$x - int:$

$$y = \frac{1}{x} + 1$$
$$0 = \frac{1}{x} + 1$$
$$-1 = \frac{1}{x}$$
$$-1x = 1$$
$$x = -1$$

$(-1,0)$

$$y = \frac{x+1}{x}$$
$$0 = \frac{x+1}{x}$$
$$x \times 0 = \frac{x+1}{x} \times x$$
$$0 = x + 1$$
$$x = -1$$

$y - int:$

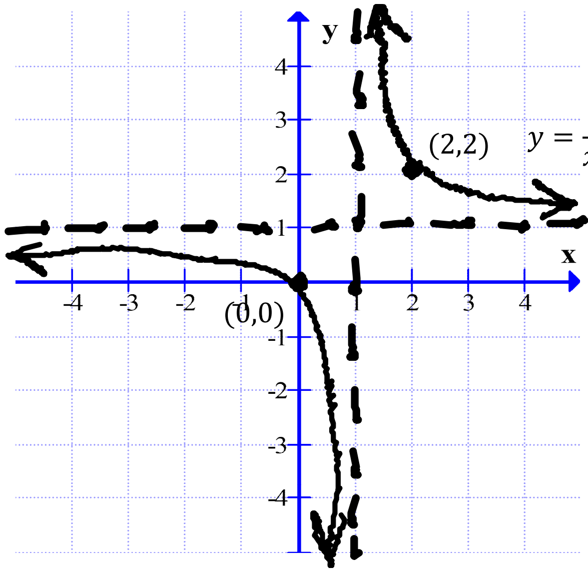
$$y = \frac{1}{x} + 1$$
$$y = \frac{1}{0} + 1$$

$y \neq$

$$y = \frac{x+1}{x}$$
$$y = \frac{0+1}{0}$$

$y \neq$

x	y
-1	0
0	und
1	2



$y = \frac{1}{x-1} + 1$

$HA: \; y = 1$

$VA: \; x = 1$

$$\frac{1}{x-1} + 1$$
$$\frac{1}{x-1} + 1 \times \frac{x-1}{x-1}$$
$$\frac{1}{x-1} + \frac{x-1}{x-1}$$
$$\frac{1 + x - 1}{(x-1)(x-1)}$$
$$\frac{x}{x-1}$$

$$\begin{array}{r} 1 \\ x-1 \overline{\smash{\big)}\, x+0} \\ \underline{-\; x-1} \\ 1 \end{array}$$

$$y = \frac{x}{x-1} \qquad \frac{x}{x-1} = \frac{1}{x-1} + 1$$

$x - int:$

$$y = \frac{1}{x-1} + 1$$
$$0 = \frac{1}{x-1} + 1$$
$$-1 = \frac{1}{x-1}$$
$$(x-1) \times -1 = \frac{1}{x-1} \times (x-1)$$
$$-x + 1 = 1$$
$$x = 0$$

(0,0)

Careful!

$$y = \frac{x}{x-1}$$
$$0 = \frac{x}{x-1}$$
$$(x-1) \times 0 = \frac{x}{x-1} \times (x-1)$$
$$0 = x$$
$$x = 0$$

$y - int:$

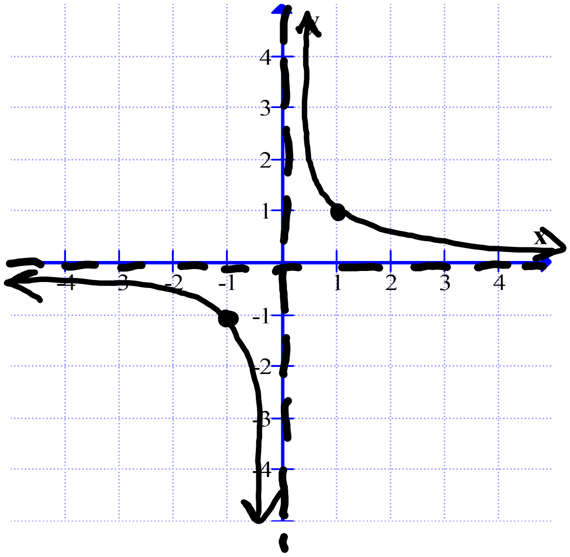
$$y = \frac{1}{x-1} + 1$$
$$y = \frac{1}{0-1} + 1$$
$$y = -1 + 1$$
$$y = 0$$

$$y = \frac{x}{x-1}$$
$$y = \frac{0}{0-1}$$
$$y = 0$$

(0,0)

x	y
0	0
1	und
2	2

C12 - 9.3 - Graph VE VR Notes



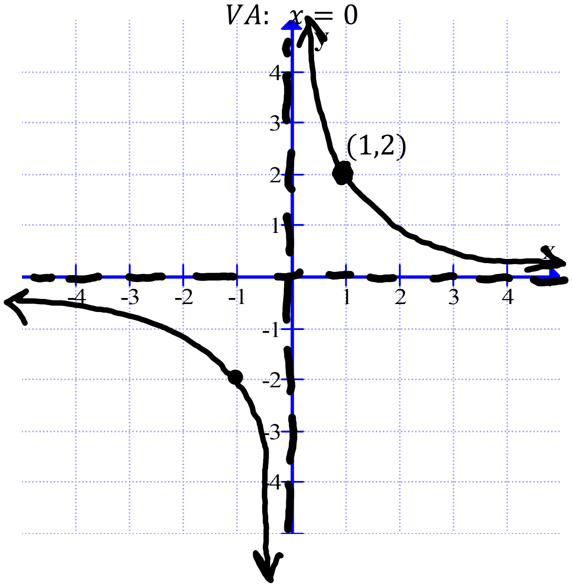
$$y = \frac{1}{x}$$

$HA:\ y = 0$

x	y
-1	1
0	und
1	1

$VA:\ x = 0$

$VE = 2$



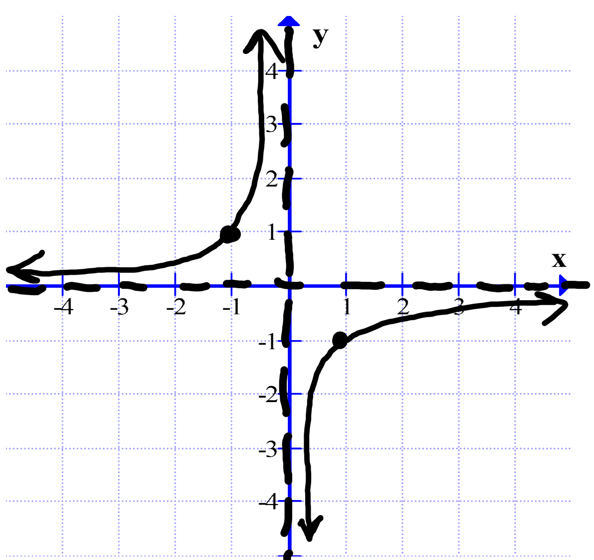
$$y = \frac{2}{x}$$

$HA:\ y = 0$

x	y
-1	-2
0	und
1	2

$VA:\ x = 0$

VR



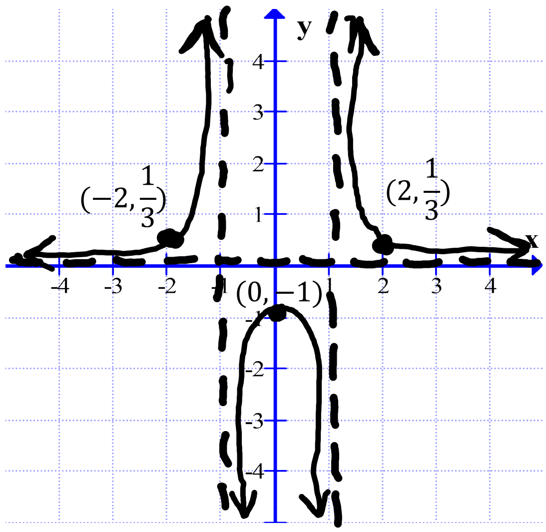
$$y = -\frac{1}{x}$$

$HA:\ y = 0$

x	y
-1	1
0	und
1	-1

$VA:\ x = 0$

C12 - 9.4 - Graph 2xVA's Notes



$$y=\frac{1}{x^2-1}$$

HA: $y=0$

x	y
-2	$\frac{1}{3}$
-1	und
0	-1
1	und
2	$\frac{1}{3}$

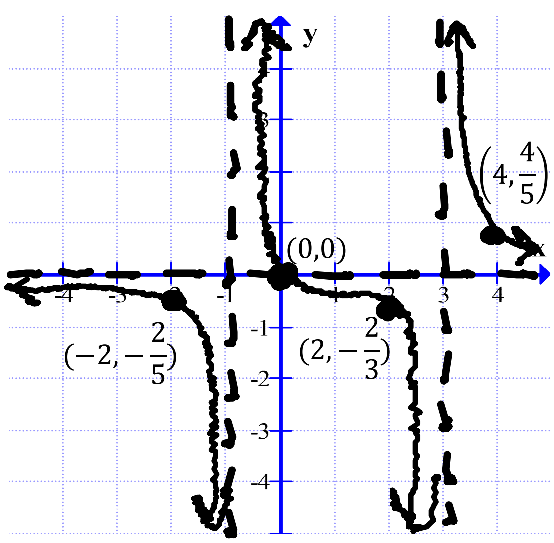
VA: $x=-1$ VA: $x=1$

VA:
$$x^2-1=0$$
$$(x+1)(x-1)=0$$
$$x+1=0 \qquad x-1=0$$
$$x=-1 \qquad x=1$$

$x-int$:
$$y=\frac{1}{x^2-1}$$
$$0=\frac{1}{x^2-1}$$
$$0\neq 1$$

$y-int$:
$$y=\frac{1}{x^2-1}$$
$$y=\frac{1}{0^2-1}$$
$$y=-1$$

$(0,-1)$



$$y=\frac{x}{x^2-2x-3}$$

HA: $y=0$

x	y
-2	$-\frac{2}{5}$
-1	und
0	0
2	$-\frac{2}{3}$
3	und
4	$\frac{4}{5}$

VA: $x=-1$ VA: $x=3$

VA:
$$x^2-2x-3=0$$
$$(x+1)(x-3)=0$$
$$x+1=0 \qquad x-3=0$$
$$x=-1 \qquad x=3$$

$x-int$:
$$0=\frac{x}{x^2-2x-3}$$
$$0=x$$
$$x=0$$

$y-int$:
$$y=\frac{0}{0^2-2(0)-3}$$
$$y=0$$

(0,0)

C12 - 9.5 - Holes Notes

$$y = \frac{(x-1)(x+2)}{x+2}$$

$$y = \frac{(x-1)\cancel{(x+2)}}{\cancel{x+2}}$$

$$y = x - 1$$

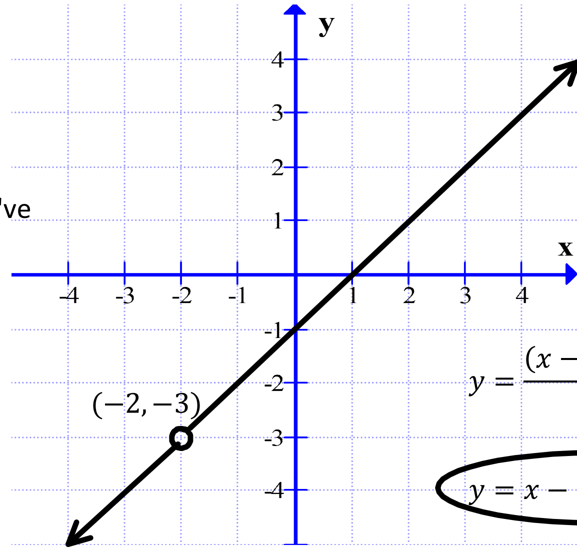
Hole: $x + 2 = 0$
$x = -2$

Set what you've crossed off equal to zero and solve.

$y = x - 1$
$y = -2 - 1$
$y = -3$

$(-2, -3)$

x	y
-2	-3



$$y = \frac{(x-1)(x+2)}{x+2}$$

$y = x - 1 \quad x \neq -2$

$x \neq -2$

$$y = \frac{x+3}{(x-1)(x+3)}$$

$$y = \frac{\cancel{x+3}}{(x-1)\cancel{(x+3)}}$$

$$y = \frac{1}{x-1}$$

Hole: $x + 3 = 0$
$x = -3$

$$y = \frac{1}{x-1}$$

$$y = \frac{1}{(-3)-1}$$

$$y = \frac{1}{-4}$$

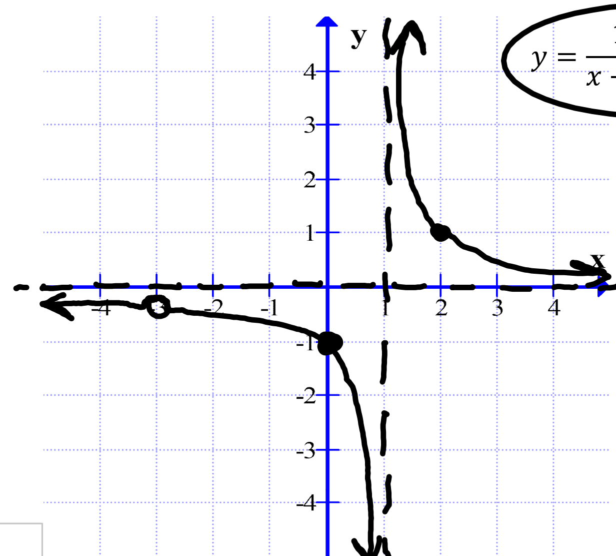
$\left(-3, -\frac{1}{4}\right)$

x	y
-3	$-\frac{1}{4}$

VA: $x - 1 = 0$
$x = 1$

$$y = \frac{x+3}{(x-1)(x+3)}$$

$y = \frac{1}{x-1} \quad x \neq -3$



HA: $y = 0$

VA: $x = 1$

C12 - 9.6 - Slant Asymptote Notes

$$y = \frac{x^2}{x+1}$$

$VA:$ $x + 1 = 0$, $x = -1$ $\qquad$ $HA:$ $\frac{x^2}{x}$ $\quad$ $none$

$Slant\ Asymptote$

$$\begin{array}{r|l} & x - 1 \\ \hline x+1 & x^2 + 0 + 0 \\ - & x^2 + x \\ \hline & -x + 0 \\ - & -x - 1 \\ \hline & +1 \end{array}$$

$x + 1 = 0$

$x = -1$

x^2

$$\begin{array}{r|rrr} -1 & 1 & 0 & 0 \\ + & \downarrow & -1 & +1 \\ \hline & 1 & -1 & +1 \end{array}$$

$x - 1 \quad R: +1$

$Slant\ Asymptote$

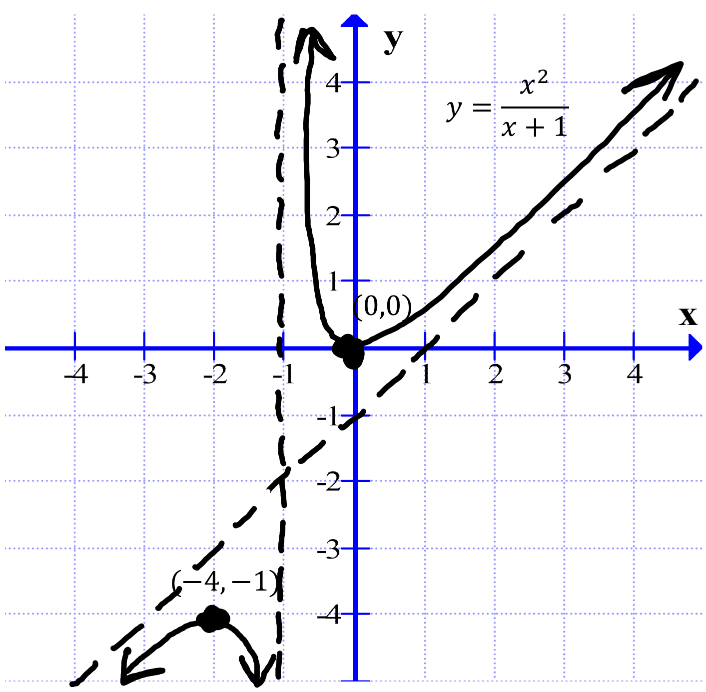
$y = x - 1$

$$\boxed{\frac{x^2}{x+1} = x - 1 + \frac{1}{x+1}}$$

$$x - 1 + \frac{1}{x+1}$$

$$x - 1 + \frac{1}{x+1}$$

$$\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a} \qquad Slant + \frac{R}{Divisor}$$



$Slant\ Asymptote$

$y = x - 1$

$VA:$ $x = -1$

x	y
-4	-1
-1	und
0	0

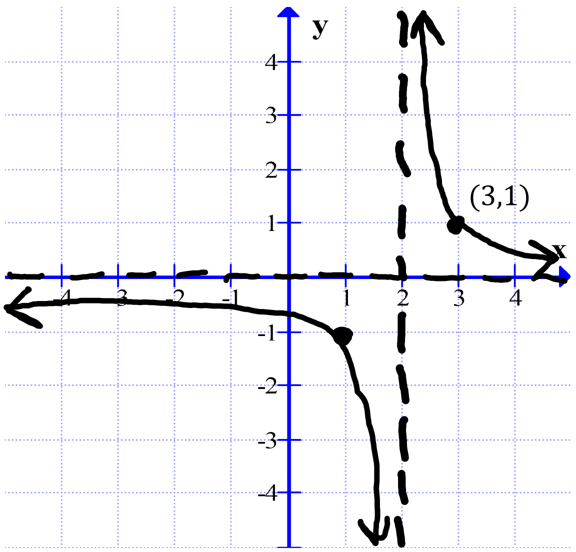
$$\frac{x^2}{x+1} = x - 1 + \frac{1}{x+1}$$

$$x - 1 \times \frac{x+1}{x+1} + \frac{1}{x+1}$$

$$\frac{x^2 - 1 + 1}{x+1}$$

$$\frac{x^2}{x+1}$$

C12 - 9.7 - HT/VT Graph -> Equation Notes



$HA:$ $y=0$

$VA:\quad x=2$

$$\boxed{y=\frac{a}{x-h}+k}$$

$$\boxed{y=\underline{\qquad\qquad}}$$

$$y=\frac{a}{x-2}+k$$

$x=2$

$x-2=0\qquad VA:\quad x=2$

$$y=\frac{a}{x-2}+0$$

$k=0\qquad HA:\quad y=0$

$$\boxed{HA:\quad y=k}$$

$$y=\frac{a}{x-2}$$

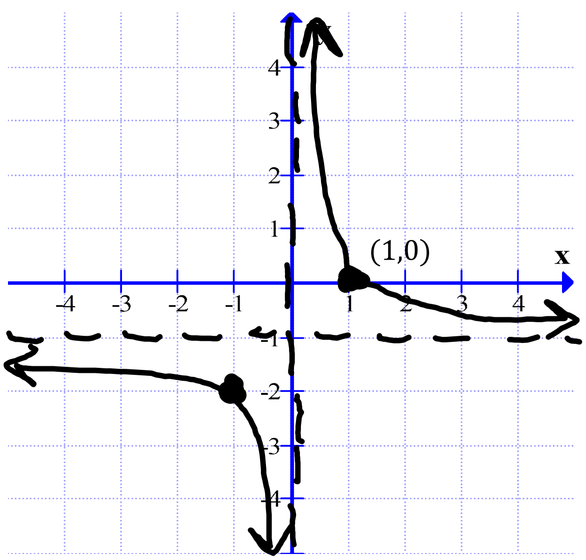
$$1=\frac{a}{3-2}$$

$(3,1)$

(x,y)

$a=1$

$$y=\frac{1}{x-2}$$



$HA:$ $y=-1$

$VA:\quad x=0$

$$\boxed{y=\frac{a}{x-h}+k}$$

$$\boxed{y=\frac{a}{VA's}+HA}$$

$$y=\frac{a}{x-0}+k$$

$x=0\quad VA:\quad x=0$

$$y=\frac{a}{x}+k$$

$$y=\frac{a}{x}-1$$

$k=-1\qquad HA:\quad y-1$

$$\boxed{HA:\quad y=k}$$

$$0=\frac{a}{1}-1$$

$(1,0)$

(x,y)

$a=1$

$$y=\frac{1}{x}-1$$

$$y=\frac{a(x-r)}{x-h}$$

$$\boxed{y=\frac{HA(x-int)}{VA's}}$$

$$\frac{1}{x}-1$$

$$\frac{1}{x}-1\times\frac{x}{x}$$

Add Fractions: LCD

$$\frac{1}{x}-\frac{x}{x}$$

$$\frac{1-x}{x}$$

$$y=\frac{a(x-r)}{x}$$

$VA:\quad x=0$

$$y=\frac{a(x-1)}{x}$$

$x=1$

$x-1=0$

$x:int:(1,0)$

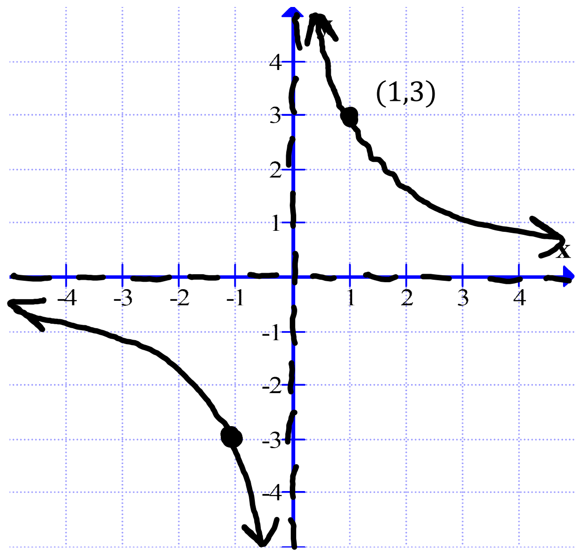
$$y=\frac{-(x-1)}{x}$$

$HA:\quad y=-1$

Case 3: $\frac{-1x}{1x}$

$$y=-\frac{x-1}{x}$$

C12 - 9.7 - E/C/R Graph -> Equation Notes



$HA:$ $y = 0$

$VA: \quad x = 0$

$$\boxed{y = \frac{a}{x-h} + k}$$

$y = \frac{a}{x} + k$ $\quad x = 0 \quad VA: \quad x = 0$

$y = \frac{a}{x} + 0$ $\quad k = 0 \quad HA: \quad y = 0$

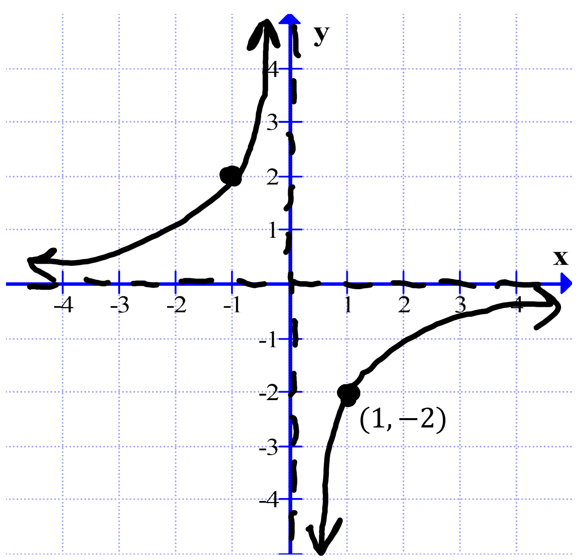
$\boxed{HA: \quad y = k}$

$y = \frac{a}{x}$

$3 = \frac{a}{1}$ $\quad (1,3)$ (x, y)

$a = 3$

$y = \frac{3}{x}$



$HA:$ $y = 0$

$VA: \quad x = 0$

$$\boxed{y = \frac{a}{x-h} + k}$$

$y = \frac{a}{x-0} + k$ $\quad x = 0 \quad VA: \quad x = 0$

$y = \frac{a}{x} + k$

$y = \frac{a}{x} + 0$ $\quad k = 0 \quad HA: \quad y = 0$

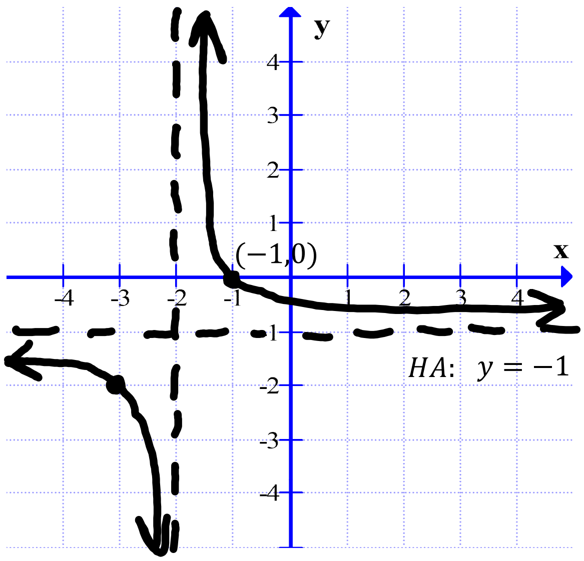
$\boxed{HA: \quad y = k}$

$-2 = \frac{a}{1}$ $\quad (1, -2)$ (x, y)

$a = -2$

$y = \frac{-2}{x}$

C12 - 9.7 - HT/VT Combo Graph -> Equation Notes



$VA: \quad x = -2$

$$\boxed{y = \frac{a}{x-h} + k}$$

$$y = \frac{a}{x+2} + k \qquad \begin{aligned} x &= -2 \\ x + 2 &= 0 \end{aligned} \qquad VA: \quad x = -2$$

$$y = \frac{a}{x+2} - 1 \qquad k = -1 \quad HA: \quad y = -1$$

$$\boxed{HA: \quad y = k}$$

$$y = \frac{a}{x+2} - 1$$

$$0 = \frac{a}{-1+2} - 1 \qquad \begin{matrix} (-1,0) \\ (x,y) \end{matrix}$$

$$a = 1$$

$$y = \frac{1}{x+2} - 1$$

$$y = \frac{a(x-r)}{x-h} \qquad \boxed{y = \frac{HA(x - int)}{VA's}}$$

$$y = \frac{a(x-r)}{x+2} \qquad VA: x = -2$$

$$y = \frac{a(x+1)}{x+2} \qquad \begin{aligned} x &= -1 \\ x + 1 &= 0 \end{aligned} \qquad x - int: (-1,0)$$

$$y = \frac{-(x+1)}{x+2} \qquad HA: y = -1 \qquad Case\ 3: \quad \frac{-1x}{1x}$$

$$\frac{1}{x+2} - 1$$

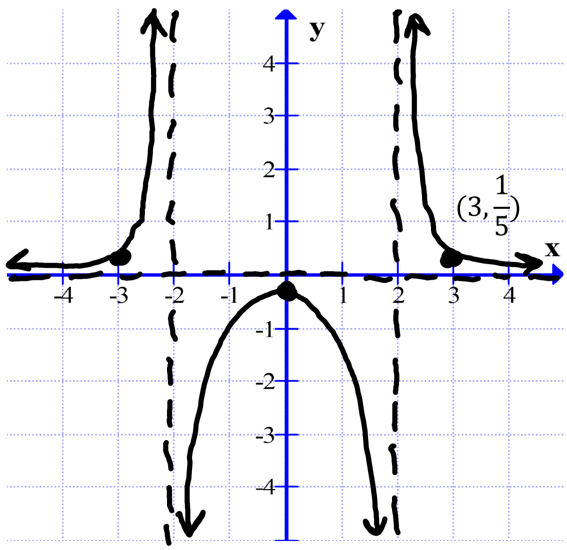
$$\frac{1}{x+2} - 1 \times \frac{x+2}{x+2}$$

$$\frac{1}{x+2} - \frac{x+2}{x+2}$$

$$\frac{-x-1}{x+2}$$

$$y = \frac{-(x+1)}{x+2}$$

C12 - 9.7 - HT/VT Combo Graph -> Equation Notes



$HA:$

$y = 0$

$VA: \quad x = -2 \qquad VA: \quad x = 2$

$$\boxed{y = \frac{a}{x-h} + k}$$

$$y = \frac{a}{(x+2)(x-2)} + k$$

$$y = \frac{a}{(x+2)(x-2)} + 0$$

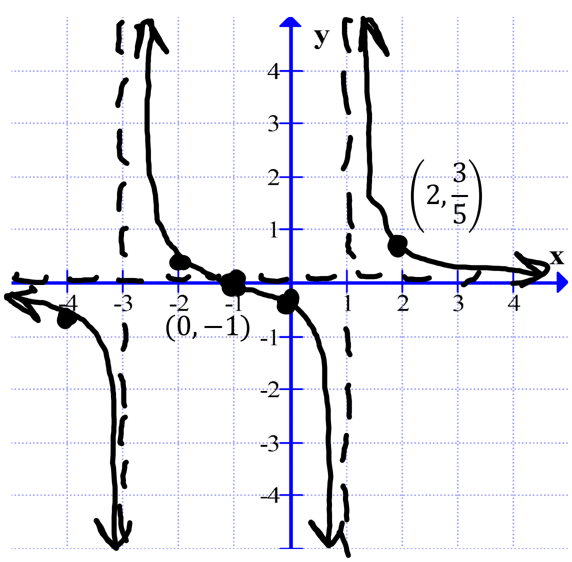
$$y = \frac{a}{(x+2)(x-2)}$$

$$\frac{1}{5} = \frac{a}{(3+2)(3-2)} \qquad (3, \frac{1}{5}) \quad (x, y)$$

$$a = 1$$

$$y = \frac{1}{(x+2)(x-2)}$$

$$y = \frac{1}{x^2 - 4}$$



$HA:$

$y = 0$

$VA: \quad x = -3 \qquad VA: \quad x = 1$

$$\boxed{y = \frac{a}{x-h} + k} \qquad \boxed{y = \frac{a(x - int)}{VA's} + HA}$$

$$y = \frac{a(x+1)}{(x+3)(x-1)} + k$$

$$y = \frac{a(x+1)}{(x+3)(x-1)} + 0$$

$$y = \frac{a(x+1)}{(x+3)(x-1)}$$

$$\frac{3}{5} = \frac{a(2+1)}{(2+3)(2-1)} \qquad (2, \frac{3}{5}) \quad (x, y)$$

$$\frac{3}{5} = \frac{3a}{5}$$

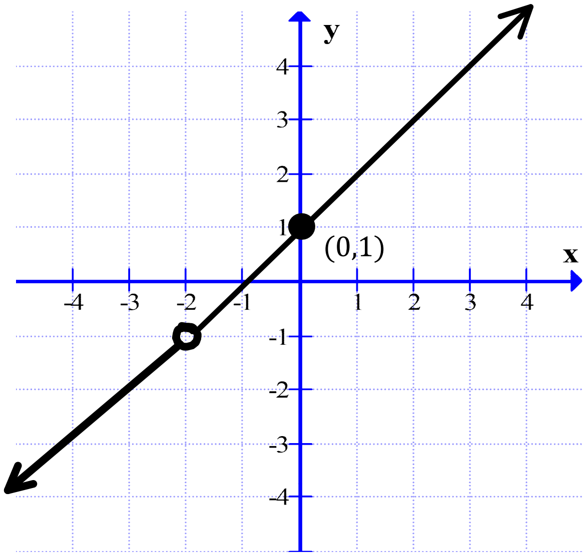
$$a = 1$$

$$y = \frac{x+1}{(x+3)(x-1)}$$

$$y = \frac{a(x-r)}{x-h} \qquad \boxed{y = \frac{HA(x - int)}{VA's}}$$

$$y = \frac{(x+1)}{(x+3)(x-1)} \qquad HA: \quad y = 0$$

Case 2: $\frac{x}{x^2}$



$\boxed{y = \dfrac{a}{x-h}}$ $\boxed{y = \dfrac{a(x-int)(holes)}{(VA's)(holes)}}$

$y = \dfrac{a(x+2)}{(x+2)} + k$ $x = -2$ $x + 2 = 0$ $hole: (-2,-1)$

$y = \dfrac{a(x+1)(x+2)}{(x+2)}$

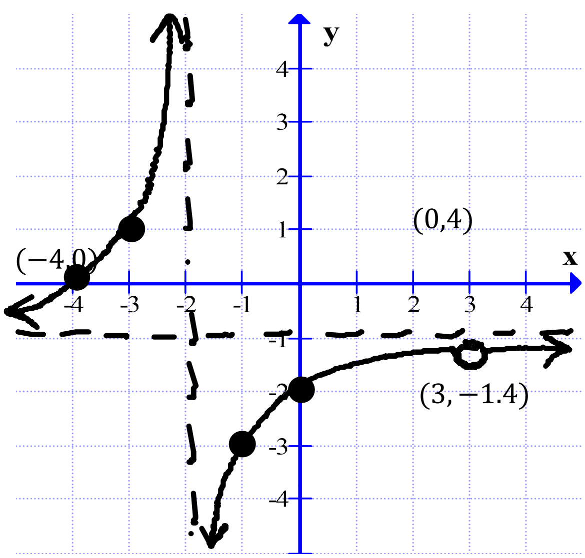
$y = \dfrac{a(x+1)\cancel{(x+2)}}{\cancel{(x+2)}}$ $x = -1$ $x + 1 = 0$ $x - int: (0,-1)$

$y = a(x+1)$ $(0,1)$ (x,y)

$1 = a(0+1)$

$a = 1$

$y = \dfrac{(x+2)(x+1)}{(x+2)}$ $HA: \ none$



$\boxed{y = \dfrac{a}{x-h}}$ $\boxed{y = \dfrac{a(x-int)(holes)}{(VA's)(holes)}}$

$y = \dfrac{a(x-3)}{(x-3)}$ $x = 3$ $x - 3 = 0$ $hole: (3,-1.4)$

$y = \dfrac{a(x-3)}{(x+2)(x-3)}$ $x = -2$ $x + 2 = 0$ $VA: \ x = -2$

$y = \dfrac{a(x+4)(x-3)}{(x+2)(x-3)}$ $x = -4$ $x + 4 = 0$ $x - int: (0,-4)$

$y = \dfrac{a(x+4)(x-3)}{(x+2)(x-3)}$

$-2 = \dfrac{a(0+4)}{(0+2)}$ $(0,-2)$ (x,y)

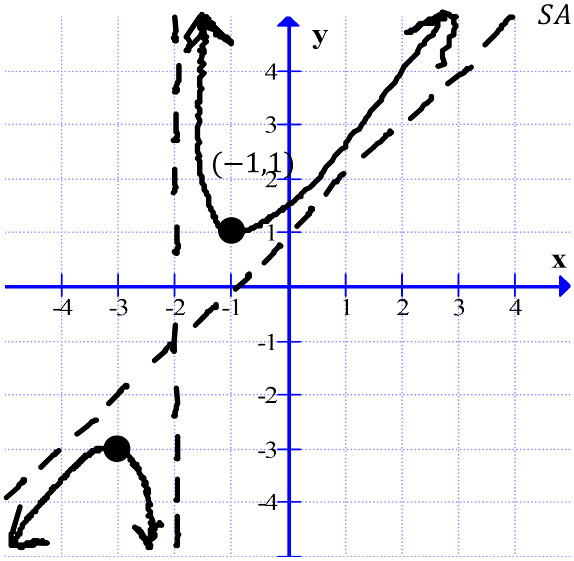
$-2 = \dfrac{4a}{2}$

$a = -1$

$y = \dfrac{-1(x+4)(x-3)}{(x+2)(x-3)}$ $HA:$ Case 3: $\dfrac{-1x^2}{1x^2}$

$y = -\dfrac{1}{1}$

C12 - 9.7 - Slant Graph -> Equation Notes



$SA: \quad y = x + 1$

$$y = \frac{a}{x-h} + Slant$$

$$\boxed{SA: \quad y = x + 1}$$

$$y = \frac{a}{x+2} + x + 1 \qquad y = Slant + \frac{R}{Divisor}$$

$$1 = \frac{a}{-1+2} - 1 + 1$$

$$(-1,1)$$
$$(x, y)$$

$$a = a$$

$$y = \frac{1}{x+2} + x + 1$$

$$\frac{1}{x+2} + x + 1$$
$$\frac{1}{x+2} + x + 1 \times \frac{x+2}{x+2}$$
$$\frac{1}{x+2} + x + 1 \times \frac{x+2}{x+2}$$
$$\frac{1}{x+2} + \frac{x^2 + 3x + 2}{x+2}$$
$$\frac{x^2 + 3x + 3}{x+2} \qquad y = \frac{x^2 + 3x + 3}{x+2}$$

C12 - 10.1 - Function Notation Notes

$\boxed{y = f(x) = y}$

$\boxed{f(x) = x + 2}$

$y = x + 2$

$y(3) = 3 + 2$

$f(3) = ?$ $(3, y)$ *What is y when x is 3. Put 3 in for x.*

$$f(x) = x + 2$$
$$f(3) = 3 + 2$$
$$f(3) = 5$$

$(3,5)$

Put whatever is inside the brackets in for x.

x	y
3	5

$f(x) = x + 2$

$f(x) = 6$ $(x, 6)$ *What is x when y is 6. Put 6 in for $f(x)$.* $x = ?$

$$y = x + 2$$
$$6 = x + 2$$
$$-2 \qquad -2$$
$$4 = x$$
$$x = 4$$

Put whatever $f(x)$ is equal to in for $f(x)$.

$$f(x) = x + 2$$
$$6 = x + 2$$
$$-2 \qquad -2$$
$$4 = x$$
$$x = 4$$

$(4,6)$

x	y
4	6

$f(x + 5) = ?$

$$f(x) = x + 2$$
$$f(x + 5) = (x + 5) + 2$$
$$f(x + 5) = x + 7$$

Put $x + 5$ in for f's x

$f(3x) = ?$

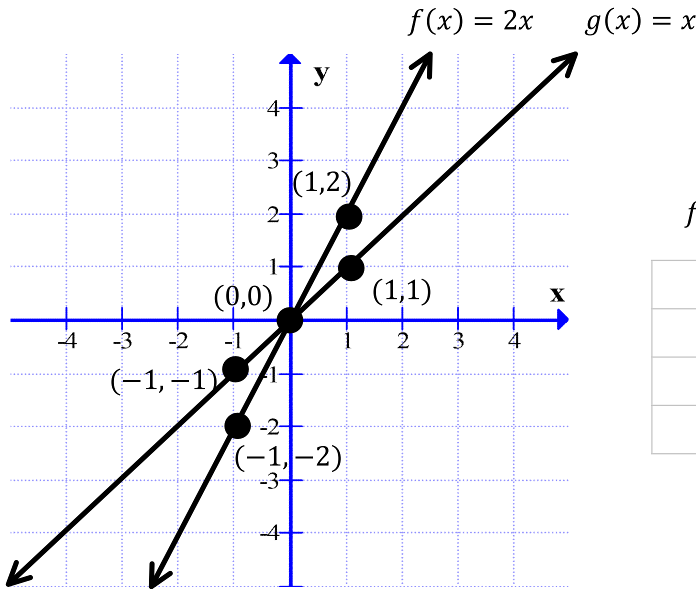
$$f(x) = x + 2$$
$$f(3x) = (3x) + 2$$
$$f(3x) = 3x + 2$$

Put $3x$ in for f's x

$f(x)$ does not mean $f \times x$
$f(x)$ is one thing
We dont divide by any part of $f(x)$ or $f(\#)$
Cant Distribute/Factor in/out of a function $f(x)$

$g(x) = y = f(x)$

C12 - 10.1 - Operation Graphs Notes



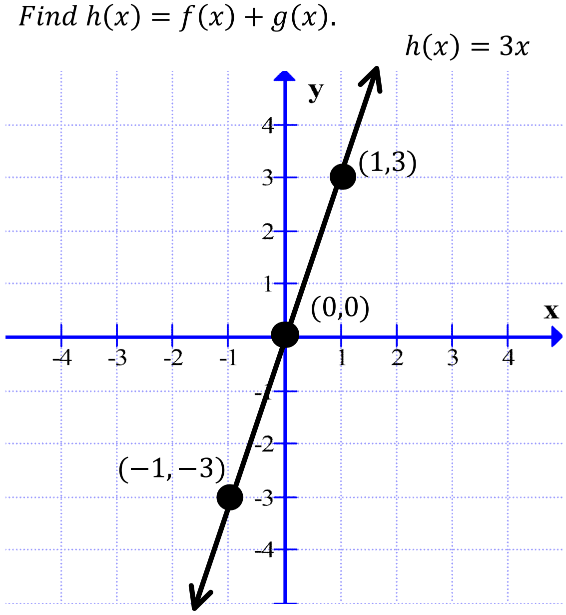
$f(x) = 2x$

x	f(x)
-1	-2
0	0
1	2

$g(x) = x$

x	g(x)
-1	-1
0	0
1	1

Find $h(x) = f(x) + g(x)$.



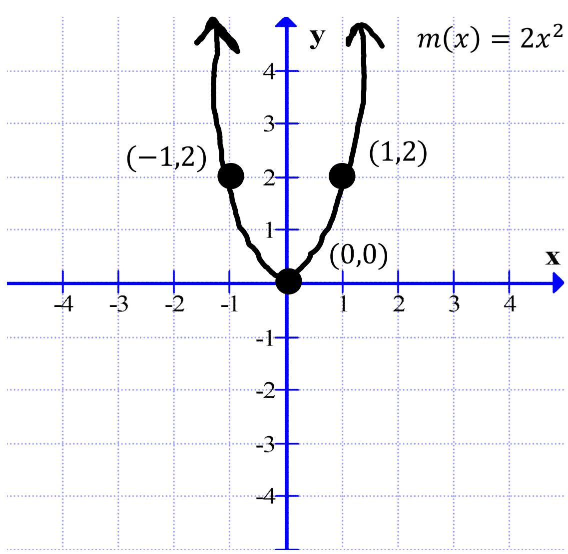
$$h(x) = f(x) + g(x)$$
$$= (2x) + (x)$$
$$h(x) = 3x$$

x	f(x)	g(x)	f(x)+g(x)
-1	-2	-1	-3
0	0	0	0
1	2	1	-3

Add $y-$ *values*

> *Pick an* x *value*
> *Add the* $y-$ *values of* $f(x)$ *and* $g(x)$
> *Draw the new point.*

Find $m(x) = f(x)g(x)$



$$m(x) = f(x)g(x)$$
$$= (2x)(x)$$
$$m(x) = 2x^2$$

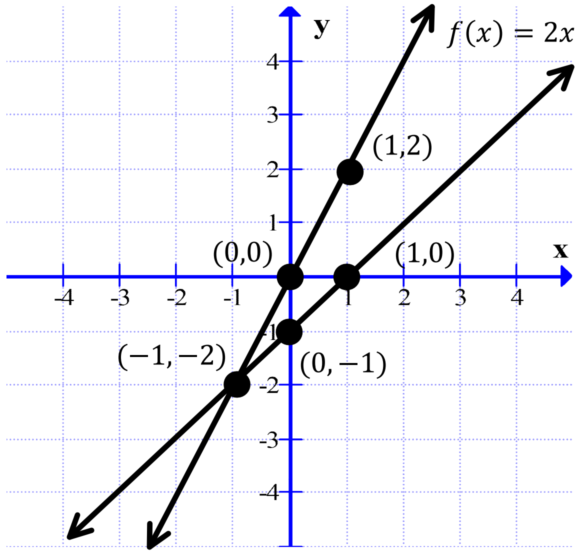
x	f(x)	g(x)	f(x)×g(x)
-1	-2	-1	2
0	0	0	0
1	2	1	2

Multiply $y-$ *values*

> *Pick an* x *value*
> *Multiply the* $y-$ *values of* $f(x)$ *and* $g(x)$
> *Draw the new point.*

$g(x) = y = f(x)$

C12 - 10.1 - Composit Graphs Notes



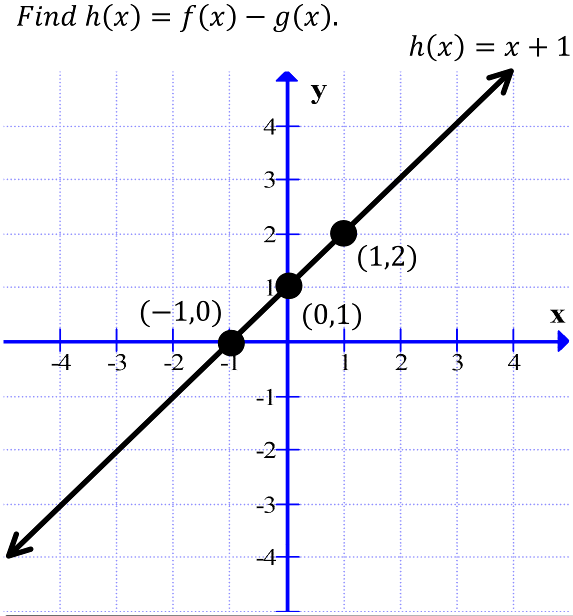
$f(x) = 2x$

x	f(x)
-1	-2
0	0
1	2

$g(x) = x - 1$

x	g(x)
-1	-2
0	-1
1	0

Find $h(x) = f(x) - g(x)$.



$$\begin{aligned} h(x) &= f(x) - g(x) \\ &= (2x) - (x - 1) \\ h(x) &= 2x - x + 1 \\ h(x) &= x + 1 \end{aligned}$$

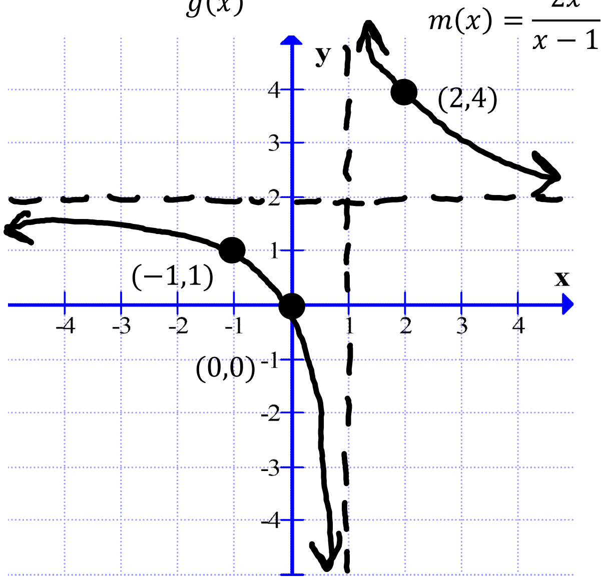
Substitute with brackets.
Distribute a negative

x	f(x)	g(x)	f(x)-g(x)
-1	-2	-2	0
0	0	-1	1
1	2	0	2

Subtract $y -$ values

Pick an x value
Subtract the $y -$ values of $f(x)$ and $g(x)$
Draw the new point.

Find $m(x) = \dfrac{f(x)}{g(x)}$



$$m(x) = \frac{f(x)}{g(x)} = \frac{2x}{x + 1}$$

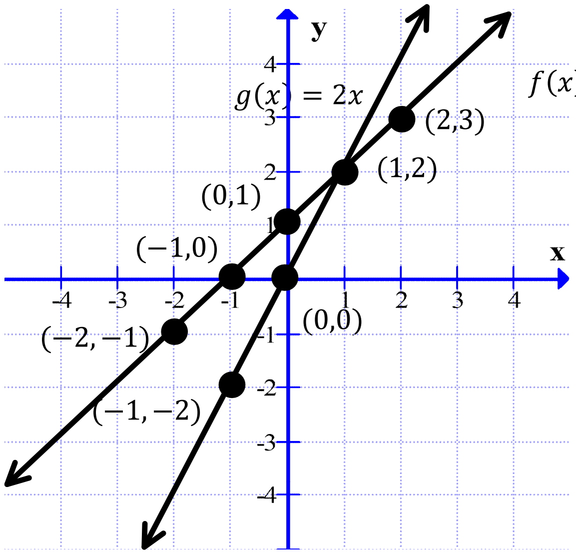
Divide $y -$ values

x	f(x)	g(x)	f(x)÷g(x)
-1	-2	-2	1
0	0	-1	0
1	2	0	Und
2	4	1	4

Pick an x value
Divide the $y -$ values of $f(x)$ and $g(x)$
Draw the new point.

C12 - 10.2 - Composite Function Notes

$\boxed{outside(inside)}$



x	f(x)
-1	0
0	1
1	2

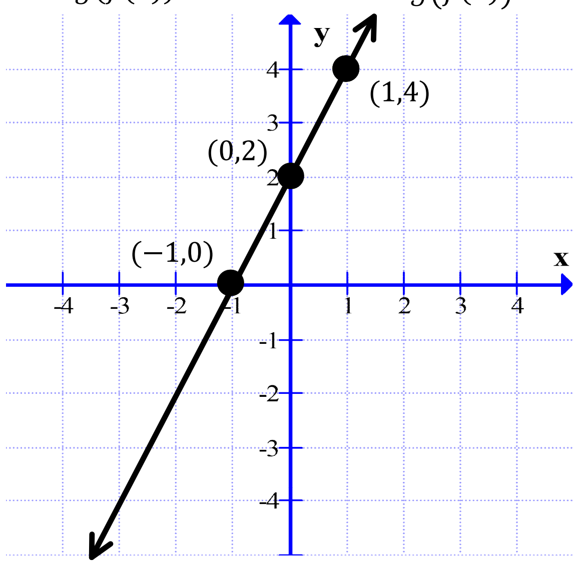
$f(x) = x + 1$

x	g(x)
-1	-2
0	0
1	2

$g(x) = 2x$

Find $g(f(x))$?

$g(f(x)) = 2x + 2$



$$g(x) = 2x$$
$$g(f(x)) = 2f(x)$$
$$g(x+1) = 2(x+1)$$
$$g(f(x)) = 2x + 2$$

Outside Function

Put $f(x)$ *into* g'*s* x.
$g(f(x)) = 2(x+1)$

x	f(x)
-1	0
0	1
1	2

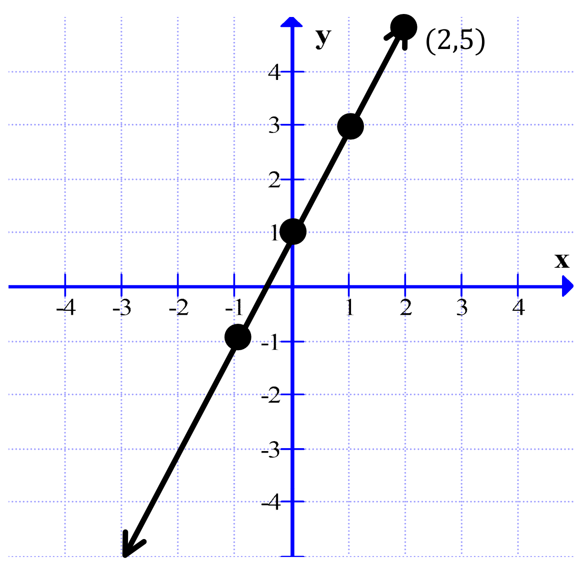
f(x)	g(f(x))
0	0
1	2
2	4

$g(-1) = 0$
$g(0) = 2$
$g(1) = 4$

x	g(f(x))
-1	0
0	2
1	4

Find $f(g(x))$?

$f(g(x)) = 2x + 1$



$$f(x) = x + 1$$
$$f(g(x)) = g(x) + 1$$
$$f(2x) = 2x + 1$$
$$f(g(x)) = 2x + 1$$

Outside Function

Put $g(x)$ *into* f'*s* x.
$f(g(x)) = 2x + 1$

Find $f(g(2))$.

$$g(x) = 2x$$
$$g(2) = 2(2)$$
$$g(2) = 4$$

Find $g(2)$
Find $f(4)$

$$f(x) = x + 1$$
$$f(4) = 4 + 1$$
$$f(4) = 5$$

OR

$$f(g(x)) = 2x + 1$$
$$f(g(2)) = 2(2) + 1$$
$$f(g(2)) = 5$$

Find $f(g(x))$
Put 2 *in for* x

$f(g(2)) = 5 \longrightarrow (2,5)$

C12 - 10.2 - Composite Function Notes

$\boxed{outside(inside)}$

Find $f(x)$ *and* $g(x)$ *if*:

$f(g(x)) = (x-1)^2$ $\quad$ $g(x) = ?$ $\quad$ $f(x) = ?$

$\boxed{outside(inside)}$

$g(x) = (x-1)$ $\quad$ $f(x) = x^2$ $\quad$ $f(x) = x^2$

$f(g(x)) = (g(x))^2$

$g(x) = inside$ $\quad$ $f(x) = outside$ $\quad$ $f(x-1) = (x-1)^2$

$\boxed{Or}$

$g(x) = x$ $\quad$ $f(x) = (x-1)^2$ $\quad$ cheeky

$f(g(x)) = x^2 - 6x + 9$

$f(g(x)) = (x-3)(x-3)$

$f(g(x)) = (x-3)^2$ $\quad$ $g(x) = x - 3$ $\quad$ $f(x) = x^2$

$f(g(x)) = x^2 - 6x + 13$

$f(g(x)) = (x-3)^2 + 4$ $\quad$ $g(x) = x - 3$ $\quad$ $f(x) = x^2 + 4$

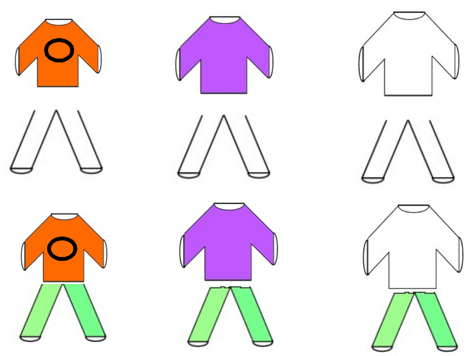
C12 - 11.1 - Fundamental Counting Principle Notes

Step 1: *a* choices
Step 2: *b* choices
Step 3: *c* choices

Total number of choices: $a \times b \times c$

Example: A person has 3 shirts and 2 pairs of pants. How many different outfits can they wear?

$3 \times 2 = 6$



Example: A woman has 4 pairs of shoes, 3 dresses and 5 hats. How many different outfits can she wear?

$4 \times 3 \times 5 = 60$

Example: A fashion designer has 4 different pairs of shoes, 3 different pairs of pants, 2 shirts, 5 necklaces, and 6 hats. How many different outfits can they prepare?

$4 \times 3 \times 2 \times 5 \times 6 = 720$

Example: How many 5 digit numbers are there?

$10\ digits\ to\ choose\ from\colon\ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

$$\underset{1\text{-}9}{\underline{\ 9\ }} \times \underset{0\text{-}9}{\underline{\ 10\ }} \times \underset{0\text{-}9}{\underline{\ 10\ }} \times \underset{0\text{-}9}{\underline{\ 10\ }} \times \underset{0\text{-}9}{\underline{\ 10\ }} = 90{,}000$$

A number can't start with a 0
i.e. 02345 = 2345, which is not a 5 digit number.

C12 - 11.2 - Factorials Notes

Factorial: The product of the consecutive numbers from n to 1.

Examples:

$5! = 5(4)(3)(2)(1) = 120$

$n! = n(n-1)(n-2)\ldots(3)(2)(1)$

$3! = 3 \times 2 \times 1$

$$\frac{7!}{4!} = \frac{7(6)(5)\cancel{(4)(3)(2)(1)}}{\cancel{4(3)(2)(1)}} = \frac{7(6)(5)}{1} = 7(6)(5) = 210$$

$$\boxed{\begin{array}{c} -3! = -3 \times 2 \times 1 \\ (-3)! = no\ solution \\ (0.5)! = no\ solution \end{array}}$$

Factorials with numbers

$0! = 1$
$1! = 1$
$2! = 2 \times 1 = 2$
$3! = 3 \times 2 \times 1 = 6$
$4! = 4 \times 3 \times 2 \times 1 = 24$
$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$
$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$
$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$

Factorials with variables

$n! = n(n-1)(n-2)(n-3)\ldots \times 3 \times 2 \times 1$

$(n-1)! = (n-1)(n-2)(n-3)\ldots \times 3 \times 2 \times 1$

$(n-2)! = (n-2)(n-3)\ldots \times 3 \times 2 \times 1$

$(n+2)! = (n+2)(n+1)(n)(n-1)\ldots \times 3 \times 2 \times 1$

$(n+1)! = (n+1)(n)(n-1)(n-2)\ldots \times 3 \times 2 \times 1$

You may close the factorial any time you want.

$6! = 6 \times 5 \times 4!$

$10! = 10 \times 9!$

$99! = 99 \times 98 \times 97!$

$n! = n(n-1)(n-2)!$

$(n-1)! = (n-1)(n-2)(n-3)!$

$(n+2)! = (n+2)(n+1)(n)!$

$$\frac{7!}{4!} = \frac{7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!}} = 7 \times 6 \times 5 = 210$$

$$\frac{n!}{(n-2)!} = \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = n(n-1) = n^2 - n$$

Expand the bigger one

$$\frac{10! - 9!}{8!} = \frac{10!}{8!} - \frac{9!}{8!}$$

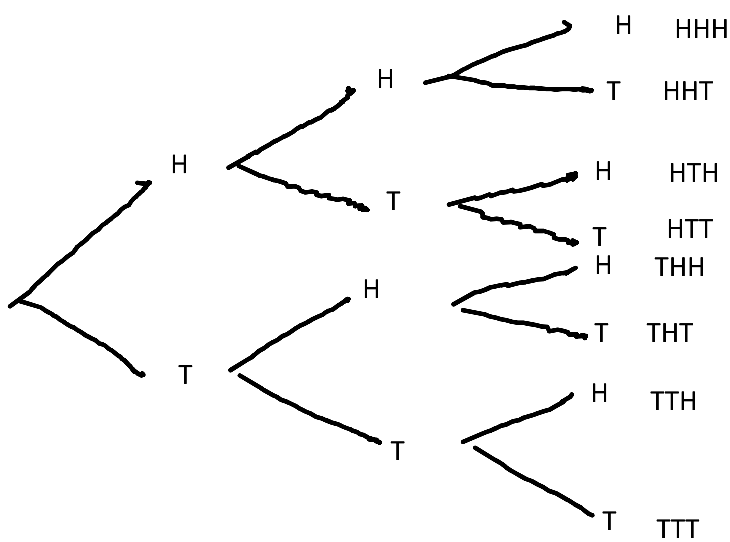
Separate Fractions

C12 - 11.3 - $(outcomes\ per\ trial)^{\#\ of\ trials}$ $Notes$

If you flip a coin three times what is the total number of outcomes? Draw a tree diagram to confirm.

$$\underset{\text{H,T}}{\underline{\quad 2 \quad}} \times \underset{\text{H,T}}{\underline{\quad 2 \quad}} \times \underset{\text{H,T}}{\underline{\quad 2 \quad}} = 2^3 = 8$$

$(outcomes\ per\ trial)^{\#\ of\ trials}$
$2^3 = 8$



If a test has 10 true and false questions how many answer keys are there possible?

$(outcomes\ per\ trial)^{\#\ of\ trials}$
$2^{10} = 1024$

If a test has A, B, C, D, multiple-choice answers with six questions how many answer keys are there possible?

$(outcomes\ per\ trial)^{\#\ of\ trials}$
$4^6 = 4096$

If a family has 8 children what is the number of combinations of boys and girls?

$(outcomes\ per\ trial)^{\#\ of\ trials}$
$2^8 = 256$

C12 - 11.4 - ABC nPr, n!; nCr Notes

Arranging All the Letters of ABC

No restrictions (repeats allowed)

$$\underset{\text{Eg.(A, B or C)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg.(A, B or C)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg. (A, B or C)}}{\underline{\quad 3 \quad}} = 3^3 = 27$$

27

AAA	AAB	ABA	BAA	ABC
BBB	AAC	ACA	CAA	ACB
CCC	BBA	BAB	ABB	BAC
	BBC	BCB	CBB	BCA
	CCA	CAC	ACC	CAB
	CCB	CBC	BCC	CBA

No repeats

Permutation <u>*Particular Order matters*</u>

$$\underbrace{ABC \quad ACB \quad BAC \quad BCA \quad CBA \quad CAB}_{6}$$

$$\underset{\text{Eg.(A or B or C)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg.(A or C)}}{\underline{\quad 2 \quad}} \times \underset{\text{Eg. (C)}}{\underline{\quad 1 \quad}} = 3! = 6$$

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_3P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1} = 3! = 6$$

No repeats

Combination <u>*Order doesn't matter*</u>

$$\underbrace{ABC = ACB = BAC = BCA = CBA = CAB}_{1}$$

$${}_nC_r = \frac{n!}{r!\,(n-r)!}$$

$${}_3C_3 = \frac{3!}{3!\,(3-3)!} = \frac{3!}{3! \times 0!} = \frac{3!}{3!} = \frac{3 \times 2 \times 1}{3 \times 2 \times 1} = 1$$

C12 - 11.4 - ABC ## nPr, n!, nCr Notes

Arranging Two of the Letters of ABC

No restrictions (repeats allowed)

$$\underset{(A,\ B \text{ or } C)}{\underline{\quad 3 \quad}} \times \underset{(A,\ B \text{ or } C)}{\underline{\quad 3 \quad}} = 9$$

AA	AB	AC	CB	9
BB	BA	CA	BC	
CC				

No repeats

Permutation *Particular Order matters*

~~AA~~	AB	AC	CB	6
~~BB~~	BA	CA	BC	
~~CC~~				

$$\underset{(A \text{ or } B \text{ or } C)}{\underline{\quad 3 \quad}} \times \underset{(B \text{ or } C)}{\underline{\quad 2 \quad}} = 6$$

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_3P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{3!}{1} = 3! = 3 \times 2 \times 1 = 6$$

No repeats

Combination *Order doesn't matter*

$AB = BA$ $AC = CA$ $BC = CB$ 3

$${}_nC_r = \frac{n!}{r!\,(n-r)!}$$

$${}_3C_2 = \frac{3!}{2!\,(3-2)!} = \frac{3!}{2! \times 1!} = \frac{3!}{2!} = \frac{3 \times 2 \times 1}{2 \times 1} = \frac{6}{2} = 3$$

C12 - 11.4 - Combination ABC Cases Notes

Rearranging All the Letters of ABC two at a time

No restrictions (repeats allowed)

$$\underset{\text{(A, B or C)}}{\underline{\quad 3 \quad}} \times \underset{\text{(A, B or C)}}{\underline{\quad 3 \quad}} = 9$$

AA	AB	AC	CB
BB	BA	CA	BC
CC			
Case 1: 2 *same*	*Case* 2: 2 *different*		

9

$Case\ 1: 2\ same \quad + \quad Case\ 2:\ 2\ different$

$$_3C_1 + {}_3C_2 \times 2!$$
$$3 \quad + \quad 3 \times 2!$$

$$= 3 + 6$$
$$= 9$$

Rearranging All the Letters of ABC

No restrictions (repeats allowed)

$$\underset{\text{Eg.(A, B or C)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg.(A, B or C)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg. (A, B or C)}}{\underline{\quad 3 \quad}} = 3^3 = 27$$

27

AAA	AAB	ABA	BAA	ABC
BBB	AAC	ACA	CAA	ACB
CCC	BBA	BAB	ABB	BAC
	BBC	BCB	CBB	BCA
	CCA	CAC	ACC	CAB
	CCB	CBC	BCC	CBA
Case 1: 3 *same*	*Case* 2: 2 *same* 1 *different*			*Case* 3: 3 *different*

$Case\ 1:\ 3\ same \quad + \quad Case\ 2:\ 2\ same, 1\ different \quad + \quad Case\ 3:\ 3\ different$

$$_3C_1 + {}_3C_2 \times 2 \times 1 \times 1 + {}_3C_2 \times 2 \times 1 \times 1 + {}_3C_2 \times 2 \times 1 \times 1 + {}_3C_3 \times 3!$$
$$3 \quad + \quad 3 \times 2 \quad + \quad 3 \times 2 \quad + \quad 3 \times 2 \quad + 1 \times 3!$$

$$= 3 + 6 + 6 + 6$$
$$= 27$$

C12 - 11.4 - 1,2,3 nPr, n! Notes

How many 3 digit numbers can we make from the numbers 1,2,3 with no repeats?

Permutation

Order matters

6
123
132
231
213
312
321

$$\underset{\text{Eg.(1 or 2 or 3)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg.(1 or 3)}}{\underline{\quad 2 \quad}} \times \underset{\text{Eg. (3)}}{\underline{\quad 1 \quad}} = 3! = 6$$

$${}_nP_r = \frac{n!}{(n-r)!}$$

Permutation: ${}_3P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1} = 3! = 3 \times 2 \times 1 = 6$

How many 2 digit numbers can we make from the numbers 1,2,3 with no repeats?

Permutation

Order matters

6
12
21
13
31
23
32

$$\underset{\text{Eg.(1 or 2 or 3)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg.(1 or 3)}}{\underline{\quad 2 \quad}} = 6$$

$${}_nP_r = \frac{n!}{(n-r)!}$$

Permutation: ${}_3P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{3!}{1} = 3! = 3 \times 2 \times 1 = 6$

How many 3 digit numbers can we make from the numbers 1,2,3 with no restrictions?

27

111	112	221	331	123
222	113	223	332	132
333	121	212	313	231
	131	232	323	213
	122	211	311	321
	133	233	322	312

$$\underset{\text{Eg.(1 or 2 or 3)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg.(1 or 2 or 3)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg.(1 or 2 or 3)}}{\underline{\quad 3 \quad}} = 3^3 = 27$$

C12 - 11.4 - Cases 0,1,2,3 nPr, n! Notes

How many four digit numbers can we make from the numbers 0,1,2,3 with no restrictions?

$$\underline{\;3\;} \times \underline{\;4\;} \times \underline{\;4\;} \times \underline{\;4\;} = 3 \times 4^3 = 192$$

Eg. (1,2,3) NOT '0'	Eg. (0,1 ,2,3)	Eg. (0,1,2,3)	Eg. (0,1,2,3)

How many 4 digit numbers can we make from the numbers 0,1,2,3 without repeating numbers?

Permutation: Order matters

18

1230	2130	3210
1203	2103	3201
1320	2310	3120
1302	2301	3102
1023	2013	3021
1032	2031	3012

(A number may not begin with a zero. Hence, there are only three choices (1,2,3) for the first digit, not four.)

$$\underline{\;3\;} \times \underline{\;3\;} \times \underline{\;2\;} \times \underline{\;1\;} = 3 \times 3! = 18$$

Eg. (1,2,3) NOT '0'	Eg. (0,2,3)	Eg. (0,3)	Eg. (3)

Permutation: ${}_3P_1 \times {}_3P_3 = 18$

How many 4 digit **EVEN** numbers can we make from the numbers 0,1,2,3 with no repeats?

Permutation: Order Matters

1230
1320
2130
2310
3120
3210

$+$

1302
1032
3012
3102

$= 10$

An even number must have a 0 or 2 last. If the 0 is last, we can use the 2 first. But, if we use the 2 last, the 0 cannot come first (Not a 4 digit number with the 0 first). Therefore, 2 cases.

Case 1: $\underline{\;3\;} \times \underline{\;2\;} \times \underline{\;1\;} \times \underline{\;1\;} = 6$

Eg. (1,2,3)	Eg. (2,3)	Eg. (3)	Eg. (0 or ~~2~~)

$$6 + 4 = 10$$

Case 2: $\underline{\;2\;} \times \underline{\;2\;} \times \underline{\;1\;} \times \underline{\;1\;} = 4$

Eg. (1,3)	Eg. (0,3)	Eg. (0)	2

Add cases.

If the last number affects the first numbers you can choose from: multiple cases.

C12 - 11.4 - President vs. Committee Notes

How many ways can you organize 3 people?

$$\underset{1,2,3}{\underline{\quad 3 \quad}} \times \underset{1,2}{\underline{\quad 2 \quad}} \times \underset{1}{\underline{\quad 1 \quad}} = 3! = 6$$

President Example:

A class is voting on a president, secretary and treasurer out of the 10 people running. How many different choices are there?

A president, secretary, and treasurer are all different positions. Particular Order matters.

$$\underset{1\text{-}10}{\underline{\quad 10 \quad}} \times \underset{1\text{-}9}{\underline{\quad 9 \quad}} \times \underset{1\text{-}8}{\underline{\quad 8 \quad}} = 720$$

$${}_nP_r = \frac{n!}{(n-r)!}$$
$${}_{10}P_3 = \frac{10!}{(10-3)!}$$
$${}_{10}P_3 = \frac{10!}{7!}$$
$${}_{10}P_3 = \frac{10 \times 9 \times 8 \times 7!}{7!}$$
$${}_{10}P_3 = 10 \times 9 \times 8$$
$${}_{10}P_3 = 720$$

Committee Example:

A class is voting on a committee of 3 people out of the 10 people running. How many different choices are there?

All people on a committee are equal. Order doesn't matter.

$$\frac{10 \times 9 \times 8}{3!} = \frac{720}{6} = 120$$

$${}_nC_r = \frac{n!}{r!\,(n-r)!}$$
$${}_{10}C_3 = \frac{10!}{3!\,(10-3)!}$$
$${}_{10}C_3 = \frac{10!}{3!\,(7)!}$$
$${}_{10}C_3 = \frac{10!}{3!\,7!}$$
$${}_{10}C_3 = \frac{10 \times 9 \times 8 \times 7!}{3 \times 2 \times 1!\,7!}$$
$${}_{10}C_3 = \frac{720}{6}$$
$${}_{10}C_3 = 120$$

$${}_nC_r = \frac{{}_nP_r}{r!}$$
$${}_{10}C_3 = \frac{{}_{10}P_3}{3!}$$
$${}_{10}C_3 = \frac{720}{6}$$
$${}_{10}C_3 = 120$$

The number of ways you can choose a committee is the number of ways you can choose Pres, Vice, and Sec, divided by the number of ways you can organize 3 people.

$${}_nP_r = {}_nC_r \times r!$$
$${}_{10}P_3 = {}_{10}C_3 \times 3!$$
$${}_{10}P_3 = 120 \times 6$$
$${}_{10}P_3 = 720$$

The number of ways you can choose Pres, Vice, and Sec, is the number of ways you can choose 3 people from 10 multiplied by the number of ways you can organize 3 people

C12 - 11.4 - All Minus None Notes

We have three boys and four girls. 3 b's 4 g's

How many different ways can we make a group of three, with no restrictions?

$${}_7C_3 = 35 \qquad \text{Or} \qquad \frac{(3+4)!}{3!\,4!} = 35$$

How many different ways can we make a group of three, with exactly two boys and one girl?

${}_3C_2 \times {}_4C_1$ Choose two boys from 3 boys, and 1 girl from 4 girls.

How many different ways can we make a group of three, with at least one boy?

<u>Three cases:</u>

Case 1: 1 b, 2 g **Case 2:** 2 b, 1 g **Case 3:** 3 b, 0 g

$${}_3C_1 \times {}_4C_2 \quad + \quad {}_3C_2 \times {}_4C_1 \quad + \quad {}_3C_3 \times {}_4C_0$$

$$3 \times 6 \quad + \quad 3 \times 4 \quad + \quad 1 \times 1$$

$$18 \quad + \quad 12 \quad + \quad 1 \qquad = 31$$

OR

$All - None$

(The total number of ways we can choose three people from seven minus a case with no boys)

$${}_7C_3 - {}_3C_0 \times {}_4C_3$$

$$35 - 1 \times 4$$

$$35 - 4 = 31$$

Note: ${}_7C_3 = (Case{:}\,0\ boys) + (Case{:}\,1\ boy) + (Case{:}\,2\ boys) + (Case{:}\,3\ boys)$

$35 = \quad 4 \quad + \quad 18 \quad + \quad 12 \quad + \quad 1$

$35 = 35$

We have 10 boys and 11 girls. How many different ways can we make a group of 10 with at least one boy?

$All - None$

${}_{21}C_{10} - ({}_{10}C_0 \times {}_{11}C_{10}) = 352705$

We did this instead of adding the cases 1 boys, 2 boys, 3 boys, 4 boys, 5 boys, 6 boys, 7 boys, 8 boys, 9 boys, and 10 boys.

A lot of the time it is easier to figure out the number of ways something can't be done, rather than be done, and then subtract this from the total number of possible outcomes.

A family of 5 takes a family photo. How many ways can the parents not sit together? Answer. The total number of ways the family can sit with no restrictions, Minus the number of ways they can sit Together. Think about it! Very Useful!

C12 - 11.4 - Identical Objects Notes

How many different words can we make from the letters POLE?

$4! = 24$

POLE	OLEP	EPOL	LOPE
PELO	OLPE	EPLO	LEPO
PLEO	OPLE	ELPO	LPOE
PLOE	OPEL	ELOP	LPEO
POEL	OELP	EOPL	LPEO
PEOL	OEPL	EOLP	LOEP

$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 24$

How many different words can we make from the letter POLO?

$$\frac{4!}{2!} = \frac{24}{2} = 12$$

POOL	LOOP	OLOP	OPLO
POLO	LOPO	OLPO	OOPL
PLOO	LPOO	OPOL	OOLP

$POOL = POOL$

Because these words are identical, we must divide by the number of ways we can permute the O's (i.e., 2!) so that we don't double count.

How many different words can we make from the letters PEEP?

$$\frac{4!}{2!\,2!} = \frac{24}{2 \times 2} = \frac{24}{4} = 6$$

PEEP	EPPE
PEPE	EPEP
PPEE	EEPP

A ten question multiple choice exam has solutions as follows: 5 A's, 3 B's, 1 C, 1 D. In how many different combinations could these answers be ordered?

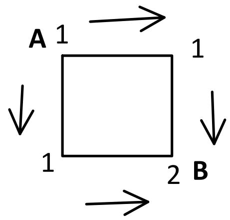
$$\frac{10!}{5!\,3!} = \frac{10 \times 9 \times 8 \times 7 \times 5!}{5!\,(3 \times 2 \times 1)} = \frac{10 \times 9 \times 8 \times 7}{6} = 840$$

$$\boxed{\frac{(\#\ of\ letters)!}{(repeating\ letter)!\,(other\ repeating\ letter)!\ \ldots}}$$

C12 - 11.4 - Paths in Squares Notes

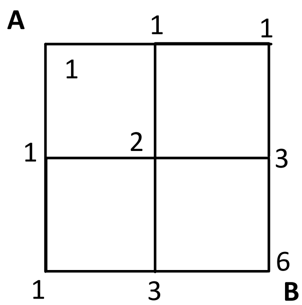
Pick a point, add points coming to it.

Paths in squares formula: $\frac{(l+w)!}{l!\,w!}$



Add numbers coming to it.

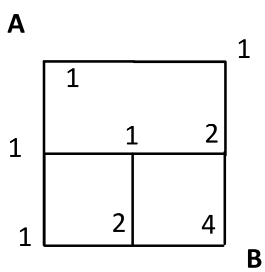
How many different paths can you follow from A to B if you only move down or to the right?



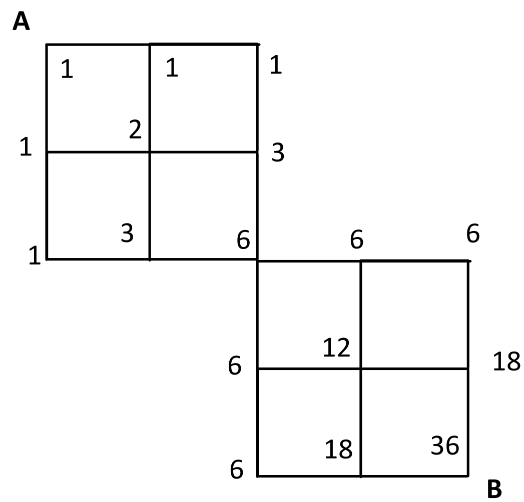
$$\frac{(2+2)!}{2!\,2!} = \frac{4!}{2!\,2!} = \frac{(4\times3\times2!)}{2!\,2!} = \frac{4\times3}{2\times1} = \frac{12}{2} = 6$$

R,R,D,D

$\frac{4!}{2!\,2!}$



You want to ask yourself, how many lines are coming towards that point from the direction they can come and add the numbers.



$$\frac{(2+2)!}{2!\,2!}\times\frac{(2+2)!}{2!\,2!} = 6\times6 = 36$$

We can only use these formulas if they are perfect rectangles or squares. No Gaps

How many ways can you get from one comer of a 3 sided rubix cube to the opposite comer if you never backtrack.

Paths in rectangular prisms formula: $\frac{(l+w+h)!}{l!\,w!\,h!}$ $= \frac{(3+3+3)!}{3!\,3!\,3!} = \frac{9!}{216} = 1680$

C12 - 11.4 - nPr nCr Algebra Notes

Solve for the missing variable

$${}_nC_2 = 10$$

$${}_nC_r = \frac{n!}{r!\,(n-r)!}$$
$${}_nC_2 = \frac{n!}{2!\,(n-2)!} = 10$$
$$\frac{n!}{2(n-2)!} = 10$$
$$\frac{n!}{(n-2)!} = 20$$
$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 20$$
$$n^2 - n = 20$$
$$n^2 - n - 20 = 0$$
$$(n-5)(n+4) = 20$$
$$n = 5 \qquad \cancel{n = -4}$$

$${}_nP_2 = 42$$

$${}_nP_r = \frac{n!}{(n-r)!}$$
$${}_nP_2 = \frac{n!}{(n-2)!} = 42$$
$$\frac{n!}{(n-2)!} = 42$$
$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 42$$
$$n^2 - n = 42$$
$$n^2 - n - 42 = 0$$
$$(n-7)(n+6) = 20$$
$$n = 7 \qquad \cancel{n = -6}$$

$${}_nC_3 = 4$$

$${}_nC_r = \frac{n!}{r!\,(n-r)!}$$
$${}_nC_3 = \frac{n!}{3!\,(n-3)!} = 4$$
$$\frac{n!}{6(n-3)!} = 4$$
$$\frac{n!}{(n-3)!} = 24$$
$$\frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 24$$
$$n(n-1)(n-2) = 24$$
$$n(n^2 - 3n + 2) = 24$$
$$n^3 - 3n^2 + 2n - 24 = 0$$

See Cubic factoring or guess and check

$${}_5C_3 = 10$$

$${}_4C_3 = 4 \qquad n = 4$$

$${}_3C_r = 3$$

$${}_nC_r = \frac{n!}{r!\,(n-r)!}$$
$${}_3C_r = \frac{3!}{r!\,(3-r)!} = 3$$
$$\frac{6}{r!\,(3-r)!} = 3$$
$$\frac{6}{3} = r!\,(3-r)!$$
$$2 = r!\,(3-r)!$$

Nope! Guess and check

$${}_3C_3 = 1$$

$${}_3C_2 = 3 \qquad r = 2$$

$${}_3C_1 = 3 \qquad r = 1$$

C12 - 11.5 - Pascal's Triangle

$;(a+b)^n$

Pascal's triangle with numbers and with ${}_nC_r$'s.

Row 1					1					$n=0$	$2^0=1$	
Row 2				1		1				$n=1$	$2^1=2$	
Row 3			1		2		1			$n=2$	$2^2=4$	
Row 4		1		3		3		1		$n=3$	$2^3=8$	
Row 5	1		4		6		4		1	$n=4$	$2^4=16$	
	$r=0$		$r=1$				${}_4C_3$					

Sum:

The sum of the row is equal to 2^n

$sum = 2^n$

$sum = 2^{row\#-1}$

$\boxed{\text{2nd \# in row is n \# = nC1}}$

$${}_0C_0$$

$${}_1C_0 \quad {}_1C_1$$

$${}_2C_0 \quad {}_2C_1 \quad {}_2C_2$$

$${}_3C_0 \quad {}_3C_1 \quad {}_3C_2 \quad {}_3C_3$$

$${}_4C_0 \quad {}_4C_1 \quad {}_4C_2 \quad {}_4C_3 \quad {}_4C_4$$

C12 - 11.5 - Binomial Expansion Notes

Binomial Expansion:

$(x+2)^2 = (x+2)(x+2) = x^2 + 4x + 4$
$(x+2)^3 = (x+2)(x+2)(x+2) = (x+2)(x^2+4x+4) = x^3 + 4x^2 + 4x + 2x^2 + 8x + 8 = x^3 + 6x^2 + 12x + 8$

$(x+2)^2 = 1x^2 + 4x + 4$
$(x+2)^3 = 1x^3 + 6x^2 + 12x + 8$

$k\ is\ always\ one\ less\ than\ the\ term\ number.$

$(a+b)^n \quad ; n+1\ terms$ $\qquad t_{k+1} = {}_nC_k a^{n-k} b^k$

$(x-5)^3$ $\quad n = 3$

$a = x$ $\quad b = -5$

$t_5 = t_{k+1}$
$5 = k + 1$
$4 = k$

Binomial	"n"	Row #	Expansion	Number of Terms
$(a+b)^0$	0	1	1	1
$(a+b)^1$	1	2	$1a + 1b$	2
$(a+b)^2$	2	3	$1a^2 + 2ab + 1b^2$	3
$(a+c)^3$	3	4	$1a^3 + 3a^2b + 3ab^2 + 1b^3$	4
$(a+b)^4$	4	5	$1a^4 + 4a^3b + 6a^2b^2 + 4xb^3 + 1b^4$	5
$(a+b)^5$	5	6	$1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$	6
$(a+b)^6$	6	7	$1a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + 1b^6$	7

Pascal's Triangle can aid in the expansion of binomials. Notice that the coefficients on each term match the numbers in Pascal's Triangle.

General Formula:

Notice that the sum of the exponents of each term is equal to n

$t_1, k = 0 \qquad t_2, k = 1$

$$(a+b)^n = {}_nC_0(a)^n(b)^0 + {}_nC_1(a)^{n-1}(b)^1 + {}_nC_2(a)^{n-2}(b)^2 + \cdots + {}_nC_{n-1}(a)^1(b)^{n-1} + {}_nC_n(a)^0(b)^n$$

What is the 5th term of expansion $(a+b)^6$.

$n = 6$
$a = a$
$b = b$

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_5 = {}_6C_4 a^{6-4} b^4$
$t_5 = 15a^2b^4$

$t_{k+1} = t_5$
$k + 1 = 5$
$k = 4$

C12 - 11.5 - Binomial Theorem Middle, x^11, x^0 Notes

FOIL $(x^2+2)^3 = (x^2+2)(x^2+2)(x^2+2) = (x^4+4x^2+4)(x^2+2) = x^6+6x^4+12x^2+8$

Which term in the binomial expansion $\left(x^2+2\right)^3$ has x^4? Find the term

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_{k+1} = {}_3C_k (x^2)^{3-k}(2)^k$
$= (x^2)^{3-k}$
$x^{6-2k} = x^4$
$6-2k=4$
$2=2k$
$k=1$

$(x^2)^{3-k} = x^4$ $\quad$ $(x^2)^{3-k}$; *is the only part that contributes to the exponent of x*

$t_{k+1} =$
$t_{1+1} = t_2$ $\quad$ The second term, t_2, will have an exponent x^4

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_2 = {}_3C_1 (x^2)^{3-1}(2)^1$
$t_2 = 3(x^2)^2 \times 2$
$t_2 = 6x^4$

The second term, $t_2 = 6x^4$

Which term in the binomial expansion $\left(x^2+2\right)^3$ is a constant? ($5 = 5x^0$) Find the term.

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_{k+1} = {}_3C_k (x^2)^{3-k}(2)^k$
$= (x^2)^{3-k}$
$x^{6-2k} = x^0$
$6-2k=0$
$6=2k$
$k=3$

$(x^2)^{3-k} = x^0$ $\quad$ ${}_nC_k$ and the negative in front of x do not contribute to finding which term it is.

$t_{k+1} =$
$t_{3+1} = t_4$

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_4 = {}_3C_3 (x^2)^{3-3}(2)^3$
$t_4 = 1(x^2)^0 \times 8$
$t_4 = 8x^0$
$t_4 = 8$

The fourth term, $t_4 = 8$

Which term in the binomial expansion $\left(x^2 - \frac{1}{x}\right)^{10}$ has x^{11}? Find the term. Note:

$$\left(x^2 - \frac{1}{x}\right)^{10} = (x^2 - x^{-1})^{10}$$

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_{k+1} = {}_{10}C_k (x^2)^{10-k}(-x^{-1})^k$
$(x^2)^{10-k}(x^{-1})^k$
$x^{20-2k}x^{-k}$
$x^{20-3k} = x^{11}$
$20-3k=11$
$9=3k$
$3=k$

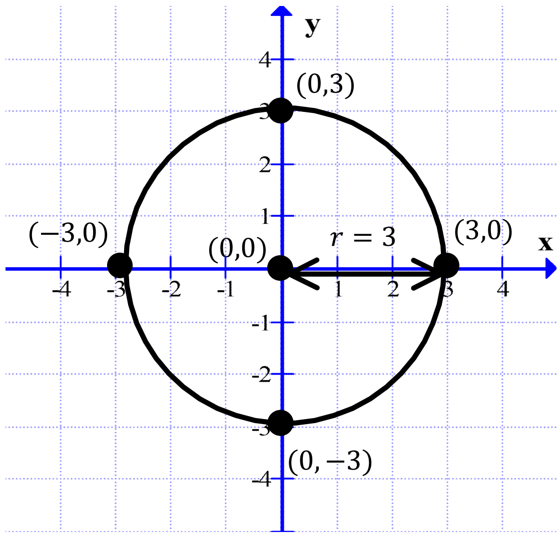
$t_{k+1} =$
$t_{3+1} = t_4$ $\quad$ The fourth term will have x^{11}.

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_{k+1} = {}_{10}C_k (x^2)^{10-k}(-x^{-1})^k$
$t_4 = {}_{10}C_3 (x^2)^{10-3}(-x^{-1})^3$
$t_4 = {}_{10}C_3 (x^2)^7(-x^{-1})^3$
$t_4 = {}_{10}C_3 x^{14}(-x^{-3})$
$t_4 = {}_{10}C_3(-x^{11})$
$t_4 = -120x^{11}$

The fourth term, $t_4 = -120x^{11}$

SAT - Conics Circles/Ellipse Notes

Circle: $x^2 + y^2 = 9$

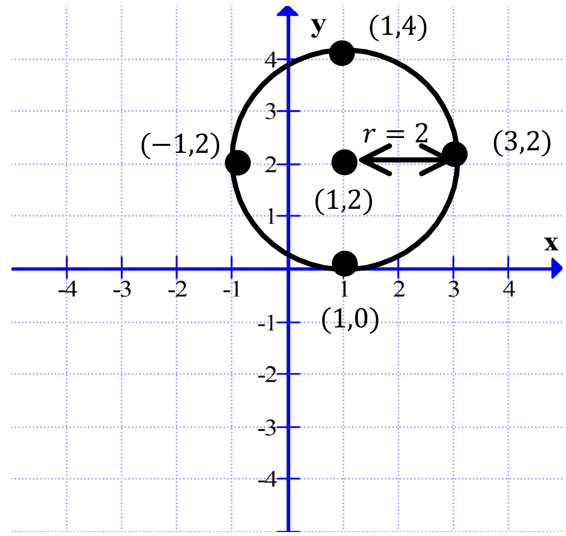


$x^2 + y^2 = r^2$ $\quad$ *Radius* $= r$

$x^2 + y^2 = 3^2$ $\quad$ $r = 3$

$(x-0)^2 + (y-0)^2 = 3^2$ $\quad$ *Center*: $(0,0)$

$(x-1)^2 + (y-2)^2 = 4$



$(x-h)^2 + (y-k)^2 = r^2$ $\quad$ *Center*: (h,k)

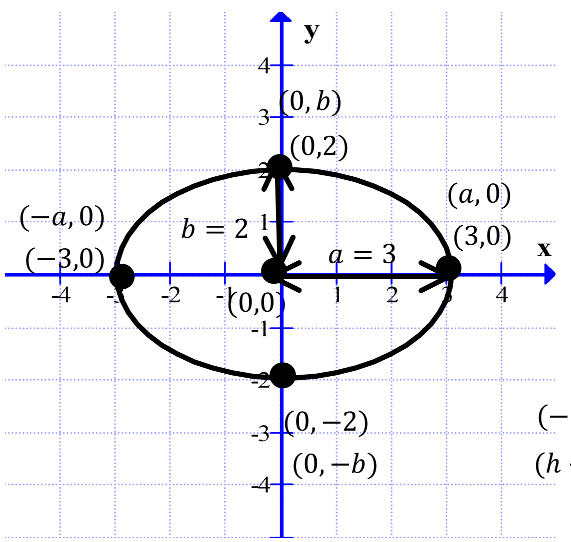
$(x-1)^2 + (y-2)^2 = 2^2$ $\quad$ $r = 2$

Center: $(1,2)$

Ellipse: $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

$\boxed{\textit{Get equal to 1.}}$

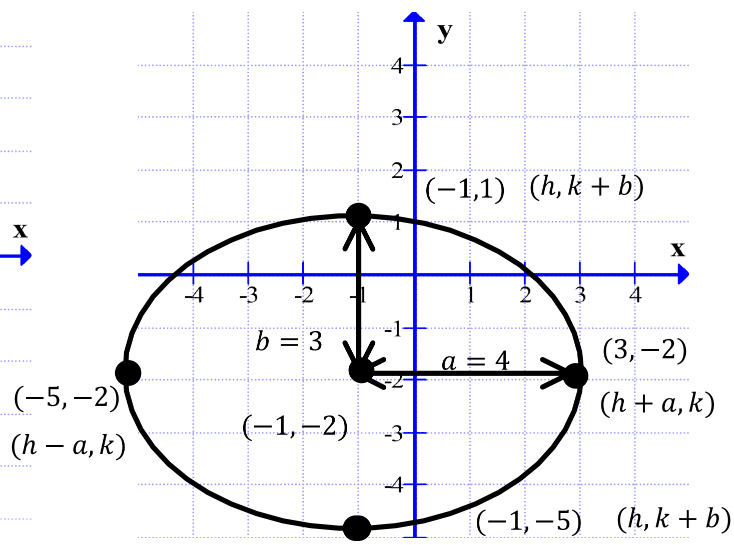
$\dfrac{(x+1)^2}{16} + \dfrac{(y+2)^2}{9} = 1$



$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $\quad$ $x - Radius = a$, $\quad y - Radius = b$

$\dfrac{x^2}{3^2} + \dfrac{y^2}{2^2} = 1$ $\quad$ $a = 3$ $\quad$ $b = 2$

Center: $(0,0)$



$\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$ $\quad$ *Center*: (h,k)

$\dfrac{(x+1)^2}{4^2} + \dfrac{(y+2)^2}{3^2} = 1$ $\quad$ $a = 4$ $\quad$ $b = 3$

Center: $(-1,-2)$

$\boxed{4x^2 + 9y^2 = 36}$; $\times$ *both sides by LCD*: 36

$$\boxed{\begin{array}{r} 9(x-1)^2 + 16(y-1)^2 = 144 \\ 9x^2 - 18x + 16y^2 - 32y - 119 = 0 \end{array}}$$

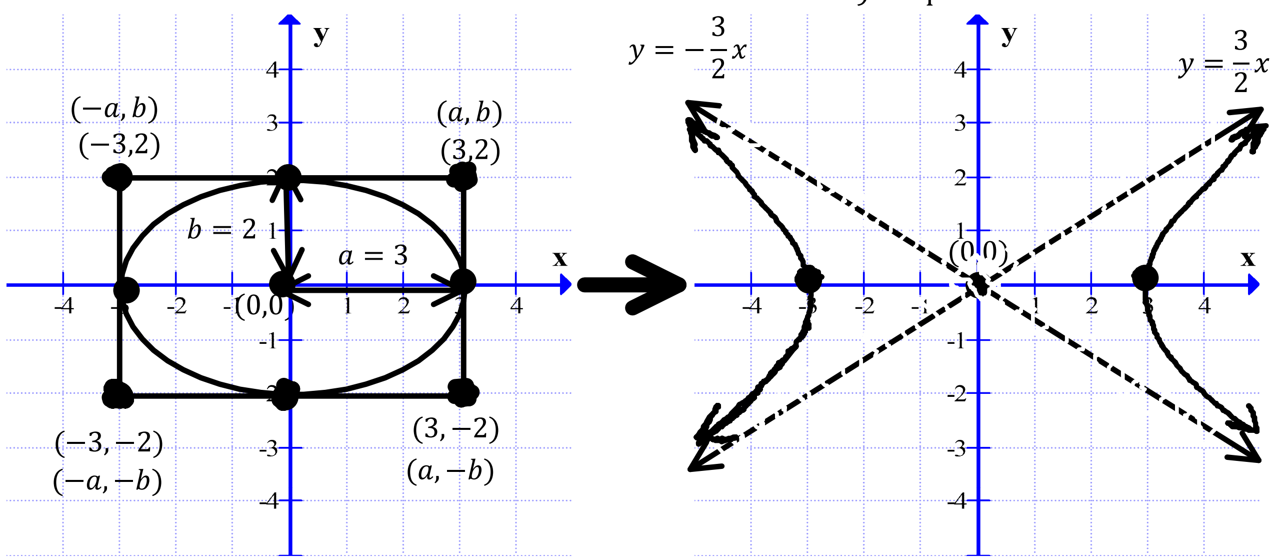
LCD
FOIL
Algebra

SAT - Conics Ellipse/Hyperbola Notes

Hyperbola:

$$\frac{x^2}{9}-\frac{y^2}{4}=1$$

$\boxed{Get\ equal\ to\ 1.}$

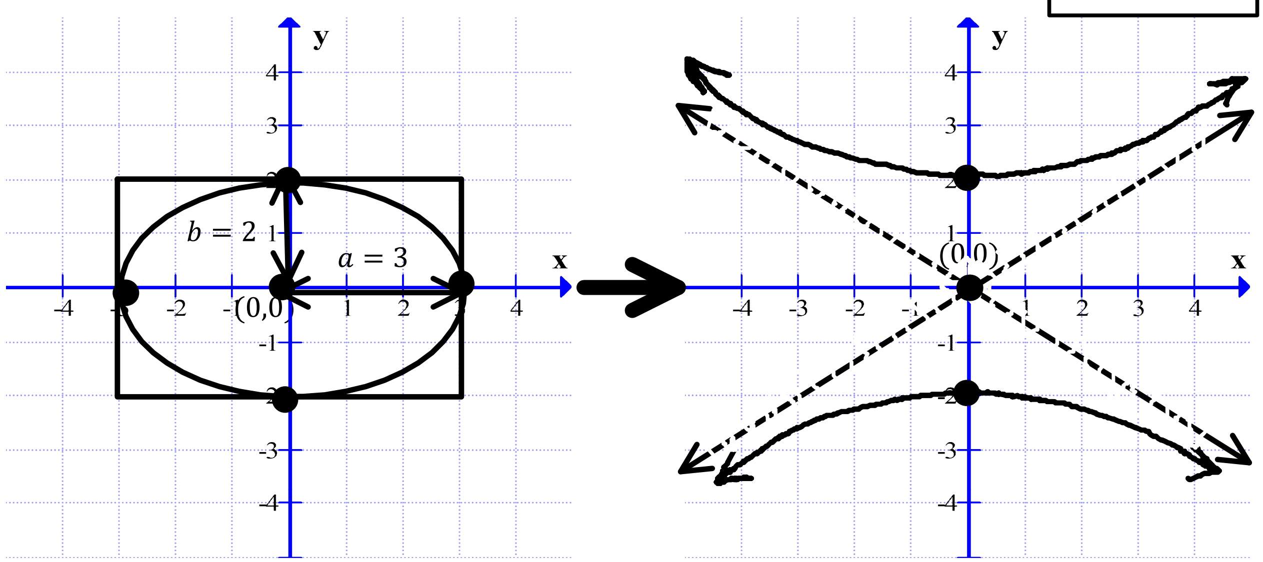


$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad Center: \quad (0,0)$$

$y=mx+b$ $\quad$ *Asymptotes*: $\quad y=\pm\frac{b}{a}x \qquad m=\pm\frac{b}{a}$

$$\frac{y^2}{9}-\frac{x^2}{4}=1$$

$$\boxed{\frac{x^2}{4}-\frac{y^2}{9}=-1}$$



$y^2-x^2; y\ first, Opens\ in\ y-direction$

$x^2-y^2; x\ first, Opens\ in\ x-direction$

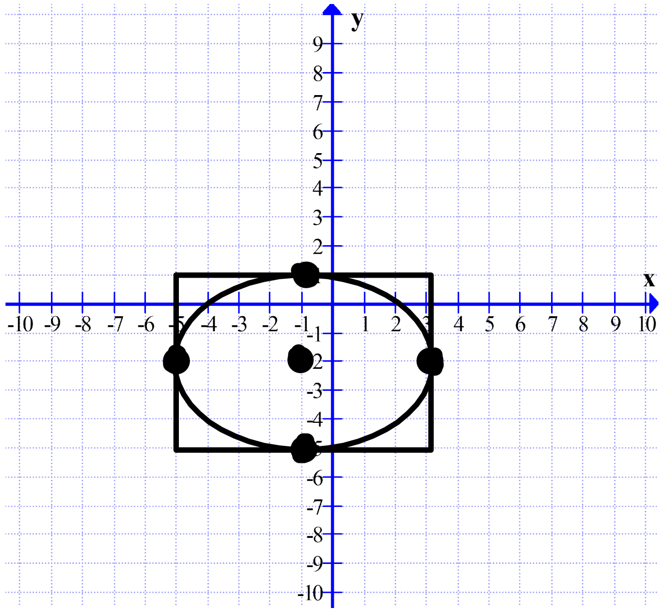
A Hyperbola and an ellipse fuse together at $(\pm a,0)$ *or* $(0,\pm b)$

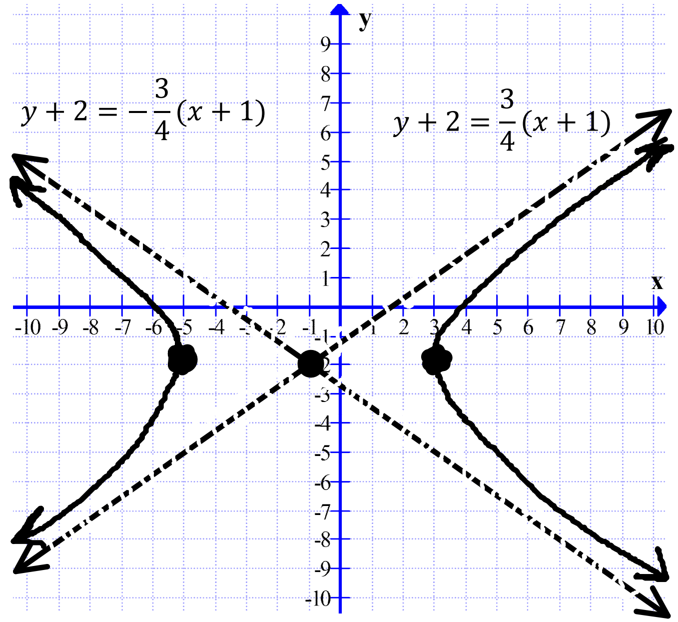
Draw a box around the Hyperbola; lines through the vertices of the rectangle are the equations of the asymptotes.

SAT - Conics Hyperbola/Conics Notes

Hyperbola:

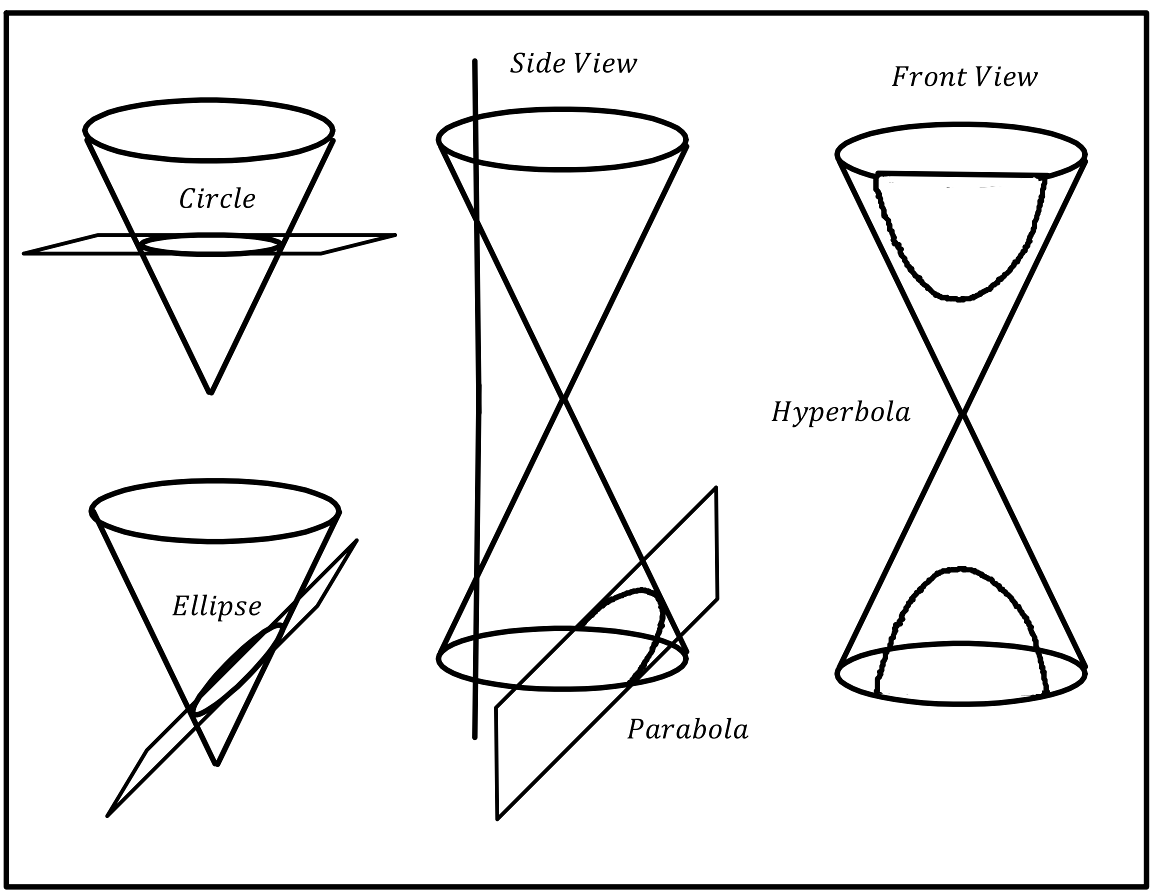
$$\frac{(x+1)^2}{16} - \frac{(y+2)^2}{9} = 1$$





$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \textit{Center}: \quad (h, k)$$

$$\textit{Asymptotes}: \quad y - k = m(x - h) \qquad m = \pm\frac{b}{a}$$



SAT - Conics Completing the Square: x and y

$x^2 - 4x + y^2 - 6y = 0$
$(x^2 - 4x) + (y^2 - 6y) = 0$
$(x^2 - 4x + 4) - 4 + (y^2 - 6y + 9) - 9 = 0$
$(x - 2)^2 + (y - 3)^2 - 13 = 0$
$(x - 2)^2 + (y - 3)^2 = 13$
$(x - 2)^2 + (y - 3)^2 = \sqrt{13}^2$

r

$x\ coord\ for\ center$

$y\ coord\ for\ center$

$3x^2 - 12x - 18y + 3y^2 = 0$ $\qquad GCF$
$3(x^2 - 4x - 6y + y^2) = 0$
$\dfrac{3(x^2 - 4x - 6y + y^2)}{3} = \dfrac{0}{3?}$
$x^2 - 4x - 6y + y^2 = 0$
$x^2 - 4x + y^2 - 6y = 0$
$(x^2 - 4x) + (y^2 - 6y) = 0$
$(x^2 - 4x + 4) - 4 + (y^2 - 6y + 9) - 9 = 0$
$(x - 2)^2 + (y - 3)^2 - 13 = 0$
$(x - 2)^2 + (y - 3)^2 = 13$
$(x - 2)^2 + (y - 3)^2 = \sqrt{13}^2$

r

$x\ coord\ for\ center$

$y\ coord\ for\ center$

Notes