C12 - 11.11 - Binomial Expansion Notes

Binomial Expansion:

$(x+2)^2 = (x+2)(x+2) = x^2 + 4x + 4$
$(x+2)^3 = (x+2)(x+2)(x+2) = (x+2)(x^2+4x+4) = x^3 + 4x^2 + 4x + 2x^2 + 8x + 8 = x^3 + 6x^2 + 12x + 8$

$(x+2)^2 = 1x^2 + 4x + 4$
$(x+2)^3 = 1x^3 + 6x^2 + 12x + 8$

k *is always one less than the term number.*

$(a+b)^n \quad ; n+1$ *terms*

$t_{k+1} = {}_nC_k a^{n-k} b^k$

$(x-5)^3$

$n = 3$

$b = -5$

$a = x$

$t_5 = t_{k+1}$
$5 = k+1$
$4 = k$

Binomial	"n"	Row #	Expansion	Number of Terms
$(a+b)^0$	0	1	1	1
$(a+b)^1$	1	2	$1a + 1b$	2
$(a+b)^2$	2	3	$1a^2+2ab+1b^2$	3
$(a+c)^3$	3	4	$1a^3+3a^2b+3ab^2+1b^3$	4
$(a+b)^4$	4	5	$1a^4+4a^3b+6a^2b^2+4xb^3+1b^4$	5
$(a+b)^5$	5	6	$1a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+1b^5$	6
$(a+b)^6$	6	7	$1a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+1b^6$	7

Pascal's Triangle can aid in the expansion of binomials. Notice that the coefficients on each term match the numbers in Pascal's Triangle.

General Formula:

Notice that the sum of the exponents of each term is equal to n

$t_1, k=0 \qquad t_2, k=1$

$$(a+b)^n = {}_nC_0(a)^n(b)^0 + {}_nC_1(a)^{n-1}(b)^1 + {}_nC_2(a)^{n-2}(b)^2 + \cdots + {}_nC_{n-1}(a)^1(b)^{n-1} + {}_nC_n(a)^0(b)^n$$

What is the 5th term of expansion $(a+b)^6$.

$n = 6$
$a = a$
$b = b$

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_5 = {}_6C_4 a^{6-4} b^4$
$t_5 = 15a^2b^4$

$t_{k+1} = t_5$
$k+1 = 5$
$k = 4$

C12 - 11.11 - Binomial Theorem Middle, x^11, x^0 Notes

FOIL $(x^2+2)^3 = (x^2+2)(x^2+2)(x^2+2) = (x^4+4x^2+4)(x^2+2) = x^6+6x^4+12x^2+8$

Which term in the binomial expansion $\left(x^2+2\right)^3$ has x^4? Find the term

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_{k+1} = {}_3C_k (x^2)^{3-k}(2)^k$
$= (x^2)^{3-k}$
$x^{6-2k} = x^4$
$6-2k = 4$
$2 = 2k$
$k = 1$

$(x^2)^{3-k} = x^4$

$(x^2)^{3-k}$; *is the only part that contributes to the exponent of* x

$t_{k+1} =$
$t_{1+1} = t_2$

The second term, t_2, will have an exponent x^4

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_2 = {}_3C_1 (x^2)^{3-1}(2)^1$
$t_2 = 3(x^2)^2 \times 2$
$t_2 = 6x^4$

The second term, $t_2 = 6x^4$

Which term in the binomial expansion $\left(x^2+2\right)^3$ is a constant? ($5 = 5x^0$) Find the term.

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_{k+1} = {}_3C_k (x^2)^{3-k}(2)^k$
$= (x^2)^{3-k}$
$x^{6-2k} = x^0$
$6-2k = 0$
$6 = 2k$
$k = 3$

$(x^2)^{3-k} = x^0$

${}_nC_k$ and the negative in front of x do not contribute to finding which term it is.

$t_{k+1} =$
$t_{3+1} = t_4$

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_4 = {}_3C_3 (x^2)^{3-3}(2)^3$
$t_4 = 1(x^2)^0 \times 8$
$t_4 = 8x^0$
$t_4 = 8$

The fourth term, $t_4 = 8$

Which term in the binomial expansion $\left(x^2 - \frac{1}{x}\right)^{10}$ has x^{11}? Find the term. Note:

$$\left(x^2 - \frac{1}{x}\right)^{10} = (x^2 - x^{-1})^{10}$$

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_{k+1} = {}_{10}C_k (x^2)^{10-k}(-x^{-1})^k$
$(x^2)^{10-k}(x^{-1})^k$
$x^{20-2k}x^{-k}$
$x^{20-3k} = x^{11}$
$20-3k = 11$
$9 = 3k$
$3 = k$

$t_{k+1} =$
$t_{3+1} = t_4$

The fourth term will have x^{11}.

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_{k+1} = {}_{10}C_k (x^2)^{10-k}(-x^{-1})^k$
$t_4 = {}_{10}C_3 (x^2)^{10-3}(-x^{-1})^3$
$t_4 = {}_{10}C_3 (x^2)^7(-x^{-1})^3$
$t_4 = {}_{10}C_3 x^{14}(-x^{-3})$
$t_4 = {}_{10}C_3 (-x^{11})$
$t_4 = -120x^{11}$

The fourth term, $t_4 = -120x^{11}$