C12 - 11.0 - FCP/Blanks/Factorials/Trials Notes

FCP - Fundamental Counting Principle

___ , ___ , ___

\# Options ___ , ___ , ___ Options

Blanks

A person has 3 shirts and 2 pairs of pants. How many different outfits can they wear?

$3 \times 2 = \boxed{6}$

A woman has 4 pairs of shoes, 3 dresses and 5 hats. How many different outfits can she wear?

$4 \times 3 \times 5 = \boxed{60}$

How many 5 digit numbers are there?

$9 \times 10 \times 10 \times 10 \times 10 = \boxed{90{,}000}$

A number can't start with a 0
i.e. 02345 = 2345, a 4 digit number.

How many ways can you organize 3 people in a row? $3 \times 2 \times 1 = \boxed{3!}$

Factorial

$3! = 3 \times 2 \times 1 = \boxed{6}$

$5! = 5(4)(3)(2)(1) = \boxed{120}$

$99! = \boxed{99 \times 98 \times 97!}$ **Close the factorial!**

MATH PRB !

$-3! = -3 \times 2 \times 1 = \boxed{-6}$

$(-3)! = \boxed{\textit{no solution}}$

$(0.5)! = \boxed{\textit{no solution}}$

$$\frac{7!}{4!} = \frac{7(6)(5)\cancel{(4)(3)(2)(1)}}{\cancel{4(3)(2)(1)}} = \frac{7(6)(5)}{1} = 7(6)(5) = \boxed{210}$$

$$\frac{7!}{4!} = \frac{7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!}} = 7 \times 6 \times 5 = \boxed{210}$$

$n! = n(n-1)(n-2)\ldots(3)(2)(1)$

$n! = \boxed{n(n-1)!}$

$(n-1)! = \boxed{(n-1)(n-2)(n-3)!}$

$(n+2)! = \boxed{(n+2)(n+1)(n)!}$

$$\frac{n!}{(n-2)!} = \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = \boxed{n(n-1) = n^2 - n}$$

Expand the bigger one

$n! + (n+1)! =$

$n! + (n+1)n! =$ GCF

$n!\big(1 + (n+1)\big) =$

$\boxed{n!\,(n+2)}$

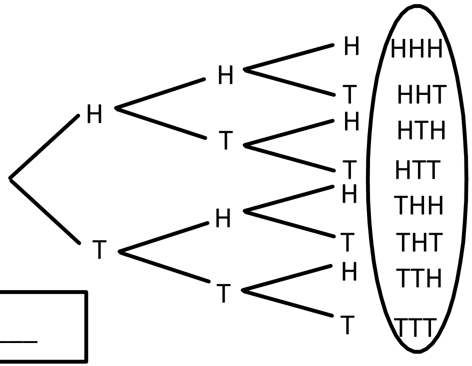
$\frac{10! - 9!}{8!} =$ Separate Fractions

$\frac{10!}{8!} - \frac{9!}{8!} =$

$10 \times 9 - 9 = \boxed{90}$

If you flip a coin three times what is the total number of outcomes?

$2^3 = \boxed{8}$



___ , ___ , ___

$\underset{HT}{2}$, $\underset{HT}{2}$, $\underset{HT}{2}$ $= 2 \times 2 \times 2 = \boxed{8}$

A six question test has A, B, C, D, multiple-choice answers. How many answer keys are there possible?

$4^6 = \boxed{4096}$

If a family has 8 children what is the number of combinations of boys and girls?

$2^8 = \boxed{256}$

A license plate has 3 letters followed by 3 digits.

Repeats allowed ___ , ___ , ___

$26 \times 26 \times 26 \times 10 \times 10 \times 10 = 1757600$

No Repeats

$26 \times 25 \times 24 \times 10 \times 9 \times 8 = 11232000$

${}_{26}P_3 \times {}_{10}P_3 = 11232000$

C12 - 11.0 - Choosing ABC nPr nCr Notes

Arranging Three of the Letters of ABC

No restrictions (repeats allowed)

$$\underset{\text{Eg.(A, B or C)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg.(A, B or C)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg. (A, B or C)}}{\underline{\quad 3 \quad}} = 3^3 = \boxed{27}$$

$\boxed{27}$

AAA	AAB	ABA	BAA	ABC
BBB	AAC	ACA	CAA	ACB
CCC	BBA	BAB	ABB	BAC
	BBC	BCB	CBB	BCA
	CCA	CAC	ACC	CAB
	CCB	CBC	BCC	CBA

$\mathit{Case}\ 1\!:\ 3\ \mathit{same} \quad + \quad \mathit{Case}\ 2\!:\ 2\ \mathit{same}, 1\ \mathit{different} \quad + \quad \mathit{Case}\ 3\!:\ 3\ \mathit{different}$

$${}_3C_1 + {}_3C_2 \times 2 \times 1 \times 1 + {}_3C_2 \times 2 \times 1 \times 1 + {}_3C_2 \times 2 \times 1 \times 1 + {}_3C_3 \times 3!$$

$$3 + 3 \times 2 + 3 \times 2 + 3 \times 2 + 1 \times 3!$$

$$= 3 + 6 + 6 + 6$$

$$= \boxed{27}$$

No repeats

Order matters

$ABC \quad ACB \quad BAC \quad BCA \quad CBA \quad CAB$ $\boxed{6}$

$$\underset{\text{Eg.(A or B or C)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg.(A or C)}}{\underline{\quad 2 \quad}} \times \underset{\text{Eg. (C)}}{\underline{\quad 1 \quad}} = 3! = \boxed{6}$$

$${}_3P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1} = 3! = \boxed{6}$$

$\boxed{\text{MATH}}$ $\boxed{\text{PRB}}$ $\boxed{\text{nPr}}$ 1st # 1st

Order doesn't matter

$ABC = ACB = BAC = BCA = CBA = CAB$ $\boxed{1}$

$${}_3C_3 = \frac{3!}{3!\,(3-3)!} = \frac{3!}{3! \times 0!} = \frac{3!}{3!} = \frac{3 \times 2 \times 1}{3 \times 2 \times 1} = \boxed{1}$$

$\boxed{\text{MATH}}$ $\boxed{\text{PRB}}$ $\boxed{\text{nCr}}$ 1st # 1st

Arranging Two of the Letters of ABC

No restrictions (repeats allowed)

$\boxed{9}$

$$\underset{\text{(A, B or C)}}{\underline{\quad 3 \quad}} \times \underset{\text{(A, B or C)}}{\underline{\quad 3 \quad}} = \boxed{9}$$

AA	AB	AC	CB
BB	BA	CA	BC
CC			

$\mathit{Case}\ 1\!:\ 2\ \mathit{same} \quad + \quad \mathit{Case}\ 2\!:\ 2\ \mathit{different} \quad =$

$${}_3C_1 + {}_3C_2 \times 2!$$

$$3 + 3 \times 2!$$

$$3 + 6$$

$$\boxed{= 9}$$

No repeats

Order matters

$$\underset{\text{(A or B or C)}}{\underline{\quad 3 \quad}} \times \underset{\text{(B or C*)}}{\underline{\quad 2 \quad}} = \boxed{6}$$

~~AA~~	AB	AC	CB
~~BB~~	BA	CA	BC
~~CC~~			

$\boxed{6}$

$${}_3P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = 3! = \boxed{6}$$

Order doesn't matter

$AB = BA \quad AC = CA \quad BC = CB$ $\boxed{3}$

$${}_3C_2 = \frac{3!}{2!\,(3-2)!} = \frac{3!}{2! \times 1!} = \frac{3!}{2!} = \boxed{3}$$

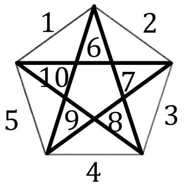
C12 - 11.0 - Choosing Notes

5 people, how many handshakes?

$${}_nC_r = \frac{n!}{r!\,(n-r)!}$$

$${}_5C_2 = \frac{5!}{2!\,(5-2)!}$$

$$\boxed{{}_5C_2 = 10}$$



C12 - 11.0 - Cases Notes

How many four digit numbers can we make from the numbers 0,1,2,3 with no restrictions?

$$\underset{\substack{\text{Eg. (1,2,3)}\\ \text{NOT '0'}}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg. (0,1 ,2,3)}}{\underline{\quad 4 \quad}} \times \underset{\text{Eg. (0,1,2,3)}}{\underline{\quad 4 \quad}} \times \underset{\text{Eg. (0,1,2,3)}}{\underline{\quad 4 \quad}} = 3 \times 4^3 = \boxed{192}$$

How many 4 digit numbers can we make from the numbers 0,1,2,3 without repeating numbers?

1230
1203
1320
1302
1023
1032

2130
2103
2310
2301
2013
2031

3210
3201
3120
3102
3021
3012

$$\boxed{18} \quad \underset{\substack{\text{Eg. (1,2,3)}\\ \text{NOT '0'}}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg. (0,2,3)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg. (0,3)}}{\underline{\quad 2 \quad}} \times \underset{\text{Eg. (3)}}{\underline{\quad 1 \quad}} = \boxed{18}$$

$${}_3P_1 \times {}_3P_3 = \boxed{18}$$

How many 4 digit **EVEN** numbers can we make from the numbers 0,1,2,3 with no repeats?

1230
1320
2130
2310
3120
3210

$+$

1302
1032
3012
3102

$= \boxed{10}$

An even number must have a 0 or 2 last.
If the 0 is last, we can use the 2 first. Case #1.
But, if we use the 2 last, the 0 cannot come first. Case #2.
Therefore, 2 cases.

Case 1: $\underset{\text{Eg. (1,2,3)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg. (2,3)}}{\underline{\quad 2 \quad}} \times \underset{\text{Eg. (3)}}{\underline{\quad 1 \quad}} \times \underset{\text{Eg. (0 or }\cancel{2}\text{)}}{\underline{\quad 1 \quad}} = 6$

Case 2: $\underset{\text{Eg. (1,3)}}{\underline{\quad 2 \quad}} \times \underset{\text{Eg. (0,3)}}{\underline{\quad 2 \quad}} \times \underset{\text{Eg. (0)}}{\underline{\quad 1 \quad}} \times \underset{2}{\underline{\quad 1 \quad}} = 4$

$= \boxed{10}$

Add cases.

If the last number* affects the first numbers you can choose from: multiple cases.

C12 - 11.0 - President/Committee Notes

How many ways can you organize 3 people?

$$\underset{1,2,3}{\underline{\quad 3 \quad}} \times \underset{1,2}{\underline{\quad 2 \quad}} \times \underset{1}{\underline{\quad 1 \quad}} = 3! = \boxed{6}$$

$${}_3P_3 = \frac{3!}{(3-3)!}$$
$${}_3P_3 = \frac{3!}{0!}$$
$$\boxed{{}_3P_3 = 6}$$

A president, secretary and treasurer are chosen from 10 people. How many different choices are there?

$$\underset{1\text{-}10}{\underline{\quad 10 \quad}} \times \underset{1\text{-}9}{\underline{\quad 9 \quad}} \times \underset{1\text{-}8}{\underline{\quad 8 \quad}} = \boxed{720}$$

$${}_{10}P_3 = \frac{10!}{(10-3)!}$$
$${}_{10}P_3 = \frac{10!}{7!}$$
$${}_{10}P_3 = 10 \times 9 \times 8$$
$$\boxed{{}_{10}P_3 = 720}$$

A committee of 3 people is chosen from 10 people. How many different choices are there?

$${}_{10}C_3 = \frac{10!}{3!\,(10-3)!}$$
$${}_{10}C_3 = \frac{10!}{3!\,(7)!}$$
$${}_{10}C_3 = \frac{10 \times 9 \times 8 \times 7!}{3 \times 2 \times 1!\,7!}$$
$${}_{10}C_3 = \frac{720}{6}$$
$$\boxed{{}_{10}C_3 = 120}$$

$$\frac{10 \times 9 \times 8}{3!} = \frac{720}{6} = \boxed{120}$$

$${}_nC_r = \frac{{}_nP_r}{r!}$$
$${}_{10}C_3 = \frac{{}_{10}P_3}{3!}$$
$${}_{10}C_3 = \frac{720}{6}$$
$$\boxed{{}_{10}C_3 = 120}$$

$${}_nP_r = {}_nC_r \times r!$$
$${}_{10}P_3 = {}_{10}C_3 \times 3!$$
$${}_{10}P_3 = 120 \times 6$$
$$\boxed{{}_{10}P_3 = 720}$$

The number of ways you can choose a committee is :
The number of ways you can choose Pres, Vice, and Sec, divided by the number of ways you can organize 3 people.

C12 - 11.0 - All Minus None/Not Notes

We have three boys and four girls. 3 b's 4 g's

How many different ways can we make a group of three, with no restrictions? $\boxed{{}_7C_3 = 35}$ **OR** $\frac{(3+4)!}{3!\,4!} = \boxed{35}$

How many different ways can we make a group of three, with exactly two boys and one girl? ${}_3C_2 \times {}_4C_1 = 3 \times 4 = \boxed{12}$

Choose 2 boys from 3 boys, and 1 girl from 4 girls.

$p(2b, 1g) = \boxed{\frac{12}{35}}$

How many different ways can we make a group of three, with at least one boy?

<u>**Three cases:**</u> **Case 1:** 1 b, 2 g **Case 2:** 2 b, 1 g **Case 3:** 3 b, 0 g

$$\begin{array}{ccccc} {}_3C_1 \times {}_4C_2 & + & {}_3C_2 \times {}_4C_1 & + & {}_3C_3 \times {}_4C_0 \\ 3 \times 6 & + & 3 \times 4 & + & 1 \times 1 \\ 18 & + & 12 & + & 1 \end{array} \boxed{= 31} \qquad p(b \geq 1) = \boxed{\frac{31}{35}}$$

OR

$All - None$

${}_7C_3 - {}_3C_0 \times {}_4C_3$

$35 - 1 \times 4$

$35 - 4 = \boxed{31}$

$All - 0\,b, 3g\ (No\ boys)$

(The total number of ways we can choose three people from seven minus a case with no boys)

$$\begin{array}{l} \text{Note: } {}_7C_3 = (0\ boys) + (1\ boy) + (2\ boys) + (3\ boys) \\ \qquad 35 = \quad 4 \quad + \quad 18 \quad + \quad 12 \quad + \quad 1 \\ \qquad 35 = 35 \end{array}$$

We have 10 boys and 11 girls. How many different ways can we make a group of 10 with at least one boy?

$All - None$

${}_{21}C_{10} - ({}_{10}C_0 \times {}_{11}C_{10}) = 352705$

We did this instead of adding the cases 1 boys, 2 boys, 3 boys, 4 boys, 5 boys, 6 boys, 7 boys, 8 boys, 9 boys, and 10 boys.

A lot of the time it is easier to figure out the number of ways something can't be done, rather than be done, and then subtract this from the total number of possible outcomes.

We have 10 boys and 11 girls. How many different ways can we make a group of 10 with at least two boy?

$All - 1\ boy - 0\ boy$

${}_{21}C_{10} - ({}_{10}C_1 \times {}_{11}C_9) - ({}_{10}C_0 \times {}_{11}C_{10}) = 352155$

A family of 5 takes a family photo. How many ways can the parents not sit together? Answer. The total number of ways the family can sit with no restrictions, Minus the number of ways the parents can sit Together. Think about it! Very Useful!

A-Parent
B-Parent
C-Child
D-Child
E-Child

ABCDE*

${}_5P_5 = 5! = \boxed{120}$

CDE*

${}_3P_3 = 3! = \boxed{6}$

AB*

${}_2P_2 = 2! = \boxed{2}$

___ , ___ , ___ , ___

4 Locations

(AB)CDE C(AB)DE CD(AB)E CDE(AB)

$3! \times 2! \times 4 = \boxed{48}$

OR $4!\,2! = \boxed{48}$

Treat parents like one thing.

$120 - 48 = \boxed{72}$

C12 - 1.0 - Identical Objects/Pathways Notes

How many different words can we make from the letters POLE?

$\underline{4}\times\underline{3}\times\underline{2}\times\underline{1} = 4! = 24$

POLE
PELO
PLEO
PLOE
POEL
PEOL

OLEP
OLPE
OPLE
OPEL
OELP
OEPL

EPOL
EPLO
ELPO
ELOP
EOPL
EOLP

LOPE
LEPO
LPOE
LPEO
LPEO
LOEP

How many different words can we make from the letter POLO?

$$\frac{4!}{2!} = \frac{24}{2} = 12$$

POOL
POLO
PLOO

LOOP
LOPO
LPOO

OLOP
OLPO
OPOL

OPLO
OOPL
OOLP

How many different words can we make from the letters PEEP?

$$\frac{4!}{2!\,2!} = \frac{24}{2\times 2} = \frac{24}{4} = 6$$

Divide by 2! Twice.
(2 E's and 2 P's)

PEEP
PEPE
PPEE

EPPE
EPEP
EEPP

$POOL = POOL$

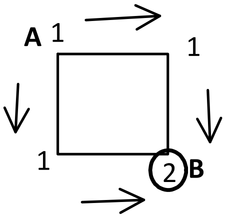
Because these words are identical, we must divide by the number of ways we can permute the O's (ie., 2!) so that we don't double count.

A ten question multiple choice exam has solutions as follows: 5 A's, 3 B's, 1 C, 1 D. In how many different ways could these answers be ordered?

$$\frac{10!}{5!\,3!} = \frac{10\times9\times8\times7\times5!}{5!\,(3\times2\times1)} = \frac{10\times9\times8\times7}{6} = 840$$

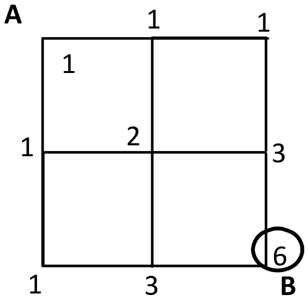
Paths in squares formula:

Add numbers coming to it.



You want to ask yourself, how many lines are coming towards that point from the direction they can come and add the numbers.

How many different paths can you follow from A to B if you only move down or to the right?

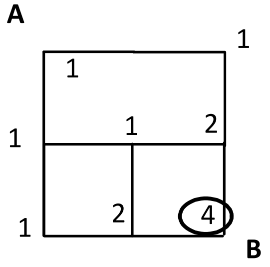


$$\boxed{\frac{(l+w)!}{l!\,w!}}$$

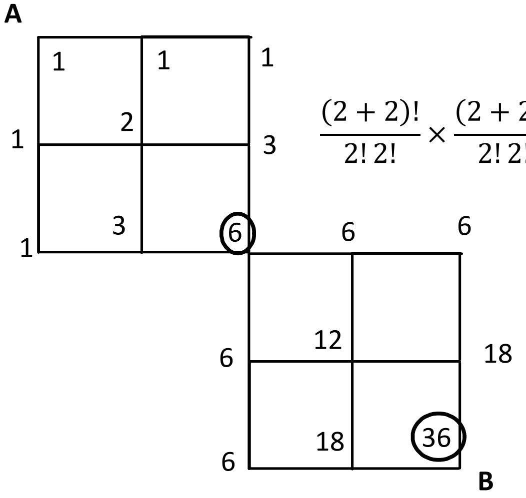
$$\frac{(2+2)!}{2!\,2!} = \frac{4!}{2!\,2!} = \frac{4\times 2}{2} = 6$$

R,R,D,D
$\frac{4!}{2!\,2!}$

No Formula if broken



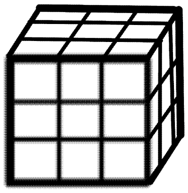
We can only use these formulas if they are perfect rectangles or squares or cubes. No Gaps



$$\frac{(2+2)!}{2!\,2!}\times\frac{(2+2)!}{2!\,2!} = 6\times 6 = 36$$

How many ways can you get from one comer of a 3 sided Rubix cube to the opposite comer if you never backtrack.

Paths in rectangular prisms formula:



$$\frac{(l+w+h)!}{l!\,w!\,h!} = \frac{(3+3+3)!}{3!\,3!\,3!} = \frac{9!}{216} = 1680$$

C12 - 11.0 - nPr nCr Algebra Notes

Solve for the missing variable

$_nC_2 = 10$

$_nC_r = \frac{n!}{r!\,(n-r)!}$

$_nC_2 = \frac{n!}{2!\,(n-2)!} = 10$

$\frac{n!}{2(n-2)!} = 10$

$\frac{n!}{(n-2)!} = 20$

$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 20$

$n^2 - n = 20$

$n^2 - n - 20 = 0$

$(n-5)(n+4) = 20$

$n = 5$ $\quad \cancel{n = -4}$

$_5C_2 = 10$ Check

$_nP_2 = 42$

$_nP_r = \frac{n!}{(n-r)!}$

$_nP_2 = \frac{n!}{(n-2)!} = 42$

$\frac{n!}{(n-2)!} = 42$

$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 42$

$n^2 - n = 42$

$n^2 - n - 42 = 0$

$(n-7)(n+6) = 20$

$n = 7$ $\quad \cancel{n = -6}$

$_7P_2 = 42$ Check

$_nC_3 = 4$

$_nC_r = \frac{n!}{r!\,(n-r)!}$

$_nC_3 = \frac{n!}{3!\,(n-3)!}$

$4 = \frac{n!}{6(n-3)!}$

$24 = \frac{n!}{(n-3)!}$

$24 = \frac{n(n-1)(n-2)(n-3)!}{(n-3)!}$

$n(n-1)(n-2) = 24$

$n(n^2 - 3n + 2) = 24$

$n^3 - 3n^2 + 2n - 24 = 0$

Cubic factoring/guess and check

$_5C_3 = 10$

$_4C_3 = 4$ $\quad n = 4$

Be careful, may be two values!

$_3C_r = 3$

$_nC_r = \frac{n!}{r!\,(n-r)!}$

$_3C_r = \frac{3!}{r!\,(3-r)!} = 3$

$3 = \frac{6}{r!\,(3-r)!}$

$\frac{6}{3} = r!\,(3-r)!$

$2 = r!\,(3-r)!$

$_3C_3 = 1$

$_3C_2 = 3$ $\quad r = 2$

$_3C_1 = 3$ $\quad r = 1$

Nope! Guess and check.

What is the 5th term of expansion $(a+b)^6$.

$t_{k+1} = {}_nC_k a^{n-k} b^k$

$t_5 = {}_6C_4 a^{6-4} b^4$

$t_5 = 15a^2b^4$

$\boxed{(a+b)^n}$

$n = 6$

$a = a$

$b = b$

$t_{k+1} = t_5$

$k + 1 = 5$

$k = 4$

Check

$1a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + 1b^6$

Which term in the binomial expansion $(x^2+2)^3$ has x^4? Find the term.

$t_{k+1} = {}_nC_k a^{n-k} b^k$

$t_{k+1} = {}_3C_k (x^2)^{3-k} (2)^k$

$t_2 = {}_3C_1 (x^2)^{3-1} (2)^1$

$t_2 = 3(x^2)^2 \times 2$

$t_2 = 6x^4$

The second term, $t_2 = 6x^4$

$(x^2)^{3-k} = x^4$

$x^{6-2k} = x^4$

$6 - 2k = 4$

$2 = 2k$

$k = 1$

$(x^2)^{3-k}$; *is the only part that contributes to the exponent of x.*

$t_{k+1} =$

$t_{1+1} = t_2$

The second term, t_2, will have an exponent x^4

$(x^2+2)^3 = (x^2+2)(x^2+2)(x^2+2) = (x^4 + 4x^2 + 4)(x^2+2) = x^6 + 6x^4 + 12x^2 + 8$

Which term in the binomial expansion $(x^2+2)^3$ is a constant? (eg. $5 = 5x^0$) Find the term.

$t_{k+1} = {}_nC_k a^{n-k} b^k$

$t_{k+1} = {}_3C_k (x^2)^{3-k} (2)^k$

$t_4 = {}_3C_3 (x^2)^{3-3} (2)^3$

$t_4 = 1(x^2)^0 \times 8$

$t_4 = 8x^0$

$t_4 = 8$

The fourth term, $t_4 = 8$

$(x^2)^{3-k} = x^0$

$x^{6-2k} = x^0$

$6 - 2k = 0$

$6 = 2k$

$k = 3$

$t_{k+1} =$

$t_{3+1} = t_4$

Which term in the binomial expansion $\left(x^2 - \frac{1}{x}\right)^{10} = (x^2 - x^{-1})^{10}$ has x^{11}? Find the term.

$t_{k+1} = {}_nC_k a^{n-k} b^k$

$t_{k+1} = {}_{10}C_k (x^2)^{10-k} (-x^{-1})^k$

$t_4 = {}_{10}C_3 (x^2)^{10-3} (-x^{-1})^3$

$t_4 = {}_{10}C_3 (x^2)^7 (-x^{-1})^3$

$t_4 = {}_{10}C_3 x^{14} (-x^{-3})$

$t_4 = {}_{10}C_3 (-x^{11})$

$t_4 = -120x^{11}$

$(x^2)^{10-k}(x^{-1})^k = x^{11}$

$x^{20-2k} x^{-k} = x^{11}$

$x^{20-3k} = x^{11}$

$20 - 3k = 11$

$9 = 3k$

$3 = k$

The fourth term, $t_4 = -120x^{11}$

$t_{k+1} =$

$t_{3+1} = t_4$

C12 - 11.0 - Table of Cards

SAMPLE SPACE

Hearts ♥	**Diamonds ♦**	**Spades ♠**	**Clubs ♣**
Ace ♥	Ace ♦	Ace ♠	Ace ♣
2 ♥	2 ♦	2 ♠	2 ♣
3 ♥	3 ♦	3 ♠	3 ♣
4 ♥	4 ♦	4 ♠	4 ♣
5 ♥	5 ♦	5 ♠	5 ♣
6 ♥	6 ♦	6 ♠	6 ♣
7 ♥	7 ♦	7 ♠	7 ♣
8 ♥	8 ♦	8 ♠	8 ♣
9 ♥	9 ♦	9 ♠	9 ♣
10 ♥	10 ♦	10 ♠	10 ♣
Jack ♥	Jack ♦	Jack ♠	Jack ♣
Queen ♥	Queen ♦	Queen ♠	Queen ♣
King ♥	King ♦	King ♠	King ♣

Ace is both high and low*

52 card deck
4 suits
13 cards in each suit
4 of each rank

5 card poker hands $_{52}C_5 = 2598960\ Hands$ $P(hand) = \frac{\#\ of}{_{52}C_5}$

Hand							$_nC_r$	# of
Royal Flush	Ace ♥	King ♥	Queen ♥	Jack ♥	10 ♥	10-Ace same suit	$_4C_1 \times 1 = 4$	4
Straight Flush	5 ♠	6 ♠	7 ♠	8 ♠	10 ♠	5 card run same suit	$_4C_1 \times 10 - 4 = 36$	36
4 of a Kind	7 ♥	7 ♦	7 ♠	7 ♣	3 ♦	4 same rank, 1 other	$_{13}C_1\ _4C_4\ _{48}C_1$	624
Full House	2 ♥	2 ♦	2 ♠	4 ♦	4 ♣	3 same rank, 1 pair	$_{13}C_1\ _4C_3\ _{12}C_1\ _4C_2$	3744
Flush	4 ♠	8 ♠	Jack ♠	2 ♠	6 ♠	All same suit, no straight	$_4C_1 \times\ _{13}C_5 - 40$	5108
Straight	3 ♥	4 ♣	5 ♦	6 ♠	7 ♣	5 card run, not all same suit	$(\ _4C_1)^5 \times 10 - 40$	10200
3 of a kind	9 ♥	9 ♦	9 ♠	2 ♦	5 ♣	3 kind, 2 others not a pair	$_{13}C_1\ _4C_3\ _{12}C_2\ (_4C_1)^2$	54912
2 pair	4 ♥	4 ♠	5 ♦	5 ♠	Queen ♠	2 different pairs, 1 other	$_{13}C_2(\ _4C_2)^2\ _{44}C_1$	*123552*
Pair	King ♦	King ♠	6 ♥	9 ♠	2 ♦	1 pair 3 others	$_{13}C_1\ _4C_2\ _{12}C_3(\ _4C_1)^3$	1098240
High Card	Jack ♠	8 ♦	4 ♠	2 ♦	7 ♠	None of the above	$_{52}C_5 - above\ sum$	1302540

$3\ Kind\ \neq$

$_{13}C_1\ _4C_3\ _{12}C_1\ _4C_1\ _{11}C_1\ _4C_1$

$Pair\ \neq$

$_{13}C_1\ _4C_2\ _{12}C_1\ _4C_1\ _{11}C_1\ _4C_1\ _{10}C_1\ _4C_1$

$2\ Pair\ \neq$

$_{13}C_1\ _4C_2\ _{12}C_1\ _4C_2\ _{11}C_1\ _4C_1$

Note:

$_{48}C_1 =\ _{12}C_1\ _4C_1$

$_{44}C_1 =\ _{11}C_1\ _4C_1$