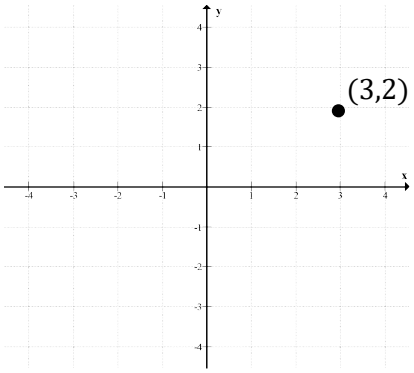
C12 - 1.4 - Point $f^{-1}(x)$ Inverse Notes

$Find\ g(x)$

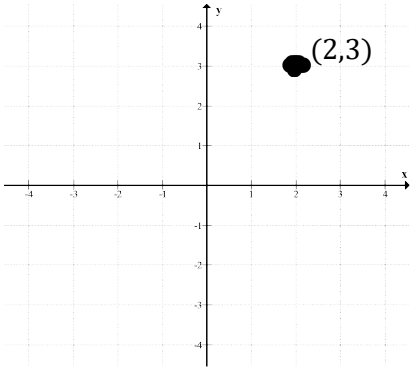


Point

$(3,2) is\ on\ f(x)$

Function Notation

$g(x) = f^{-1}(x)$



Operation

(3,2)

$f^{-1}(x)$ (2,3)

$x <-> y$

(y, x)

$Switch\ x\ and\ y$

Inverse

Mapping Notation

(y, x)

Inverse 1st. Function Operations 1st. Inside Out.

C12 - 1.4 - Graph/Algebra $f^{-1}(x)$ Inverse Notes

$f(x) = 2x + 2$

$$y = 2x + 2$$
$$x = 2y + 2$$
$$x - 2 = 2y$$
$$\frac{x}{2} - 1 = y$$
$$y = \frac{1}{2}x - 1$$
$$f^{-1}(x) = \frac{1}{2}x - 1$$

Switch x and y

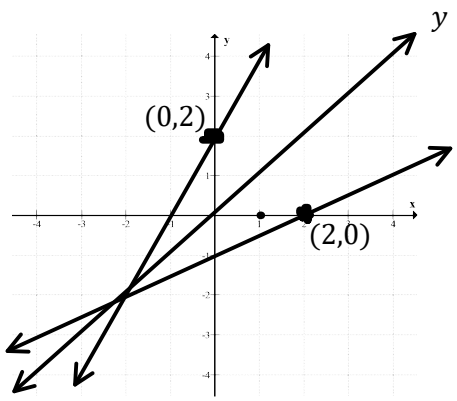
$y = f(x)$

Solve for y

Write in Function Notation

$f(x) = 2x + 2$

x	2x+2
-2	-2
-1	0
0	2
1	4
2	6



$f^{-1}(x) = \frac{1}{2}x - 1$

x	2x-4
-2	-2
0	-1
2	0
4	1
6	2

Switch x and y

Remember: The inverse is a diagonal reflection over the line $y = x$

Check your answer $f^{-1}(f(x)) = ?$

$$f^{-1}(x) = \frac{1}{2}x - 1$$
$$f^{-1}(2x - 4) = \frac{1}{2}(2x + 2) - 1$$
$$f^{-1}(2x - 4) = x \qquad f^{-1}(f(x)) = x$$

✓

$f(f^{-1}(x)) = ?$

$$f(x) = 2x + 2$$
$$f\left(\frac{1}{2}x + 2\right) = 2\left(\frac{1}{2}x - 1\right) + 2$$
$$f\left(\frac{1}{2}x + 2\right) = x \qquad f(f^{-1}(x)) = x$$

✓

$$f(x) = \frac{x}{x+1}$$
$$y = \frac{x}{x+1}$$
$$x = \frac{y}{y+1}$$
$$x(y+1) = y$$
$$xy + x = y$$
$$x = y - xy$$
$$x = y(1-x) \qquad GCF = y$$
$$\frac{x}{1-x} = y$$
$$y = \frac{x}{1-x}$$
$$f^{-1}(x) = \frac{x}{1-x}$$

Switch x and y
Multiply
Distribute
Combine like terms (y's on one side)
Factor
Divide

A function has an inverse function if it is One-to-One, Or if you restrict the domain.