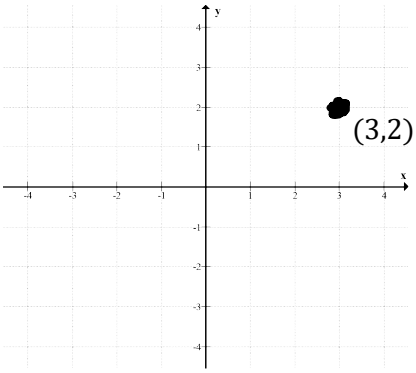
C12 - 1.1 - VHT Point Notes

Find new point.

$y = f(x)$



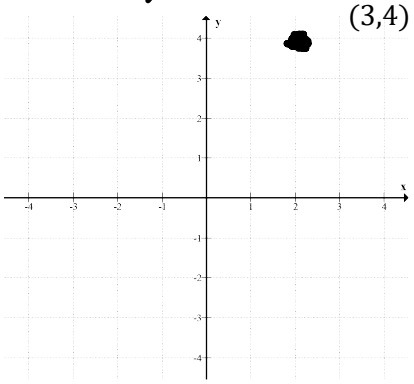
Point

$(3,2)$ *is on* $f(x)$

Function Notation

$y = f(x)$

$y - 2 = f(x) + 2$



Operation

$(3,2)$

$VT = +2$ $(3,4)$ Add 2 to y-value

UP TWO

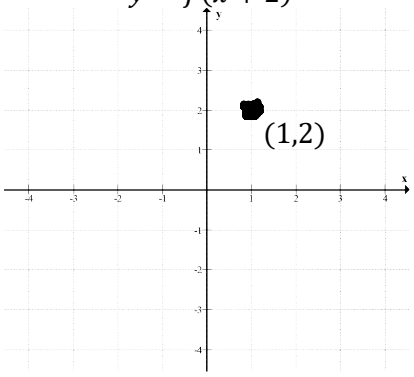
A Vertical Translation up 2

$y + 2$

Mapping Notation

$(x, y + 2)$

$y = f(x + 2)$



$(3,2)$

$HT = -2$ $(1,2)$ Subtract 2 from x-value

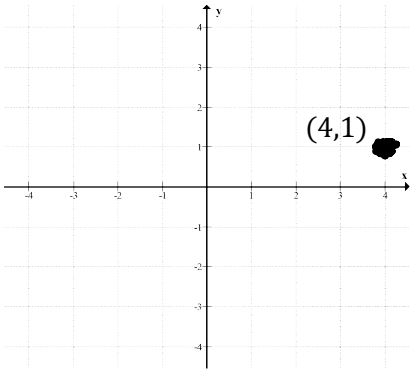
LEFT 2

A Horizontal Translation left 2

$x - 2$

$(x - 2, y)$

$y + 1 = f(x - 1)$



$(3,2)$

$HT = +1$ $(4,2)$ Add 1 to x-value

$VT = -1$ $(4,1)$ Subtract 1 from y-value

RIGHT 1
DOWN 1

A Horizontal Translation right 1
A Vertical Translation down 1

$x + 1$ $y - 1$

$(x + 1, y - 1)$

Do exactly what you see outside of the brackets on the right-hand side to the **y-value**
Do the **Opposite** of what you see inside the brackets to the **x-value**. Attached to the variable.
Do the **Opposite** of what you see on the left hand side to the **y-value**. Attached to the variable.

C12 - 1.1 - VHT Function Notation f(x) Notes

$y = f(x)$

$f(x) = x^2$ *Given*

$f(3) = ?$ $(3, y)$ *What is y when x is 3.*

$f(x) = x^2$
$f(x) = (x)^2$
$f(3) = (3)^2$

Put 3 in for x.

Put whatever is inside the brackets in for x. Substitute with Brackets

$f(3) = 9$ (3,9)

$y = x^2$
$y = (3)^2$
$y = 9$

x	y
3	9

$f(x) = x^2$

$f(x + 2) = ?$

$f(x) = x^2$

$f(x + 2) = (x + 2)^2$

Put $(x + 2)$ in for x.

Function Notation

$g(x) = ?$

$g(x) = f(x + 2)$

$g(x) = (x + 2)^2$

$HT = -2$

Let's call it $g(x)$

$f(x) + 1 = ?$

$f(x) = x^2$

$f(x) + 1 = x^2 + 1$

$f(x) + 1$

$m(x) = ?$

$m(x) = f(x) + 1$

$m(x) = x^2 + 1$

$VT = +1$

Let's call it $m(x)$

$f(x)$ does not mean $f \times x$
$f(x)$ is one thing
We dont divide by any part of $f(x)$ or $f(\#)$
Cant Distribute into/Factor out of a function $f(x)$

y is a variable
f is a function

$y = f(x)$ $y = m(x)$ $y = g(x)$

$g(x) \neq f(x) \neq m(x)$

Unless they do

C12 - 1.1 - VHT Graph y= Notes

Vertical Translation Up One

$$VT = +1$$

$$y = x^2$$
$$y - 1 = x^2 \qquad y \to y - 1$$
$$y = x^2 + 1$$

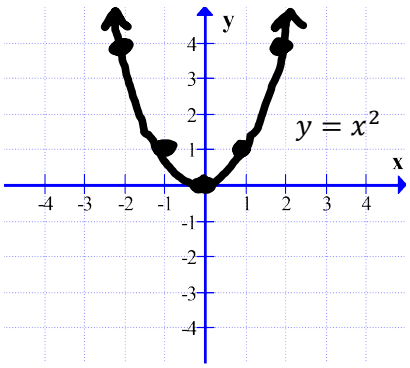
$Put\ y - 1\ in\ for\ y$

Substitute the Opposite Operation for the Variable

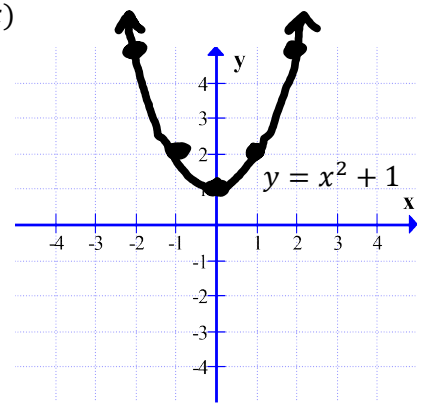
$g(x) = x^2 + 1$

Let's call it $g(x)$

$y = f(x)$



$g(x) = x^2 + 1$



x	y
-2	4
-1	1
0	0
1	1
2	4

x	y
-2	5
-1	2
0	1
1	2
2	5

Add 1 to the y-value

$Up\ 1$

Horizontal Translation Left One

$$HT = -1$$

$$y = \sqrt{x}$$
$$y = \sqrt{x + 1} \qquad x \to x + 1$$

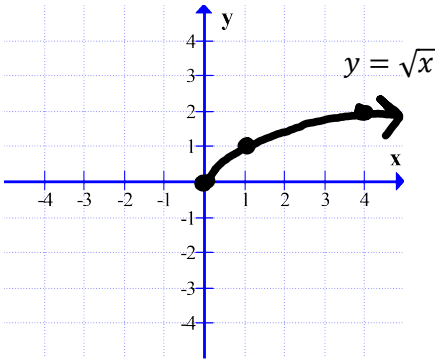
$Put\ x + 1\ in\ for\ x$

Substitute the Opposite Operation for the Variable

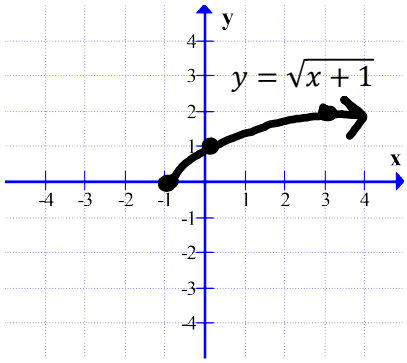
$m(x) = \sqrt{x + 1}$

Let's call it $m(x)$

$y = f(x)$



$m(x) = f(x + 1)$



x	y
-1	und
0	0
1	1
4	2

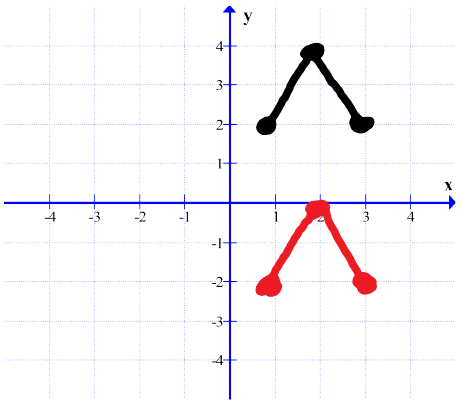
x	y
-2	und
-1	0
0	1
3	2

Subtract 1 from the x-value

$Left\ 1$

C12 - 1.1 - VHT Graphs f(x) Notes

Find the transformed equation of $f(x)$ in all forms.



$y = f(x)$

$y = f(x) + k$

$y = f(x) + 4$

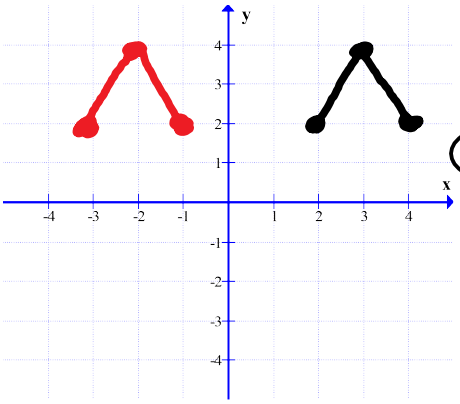
$y - k = f(x)$

$y - 4 = f(x)$

$y = f(x)$
$y - 4 = f(x)$
$y = f(x) + 4$

$VT = 4 \quad y \to y - 4$

$y = f(x)$



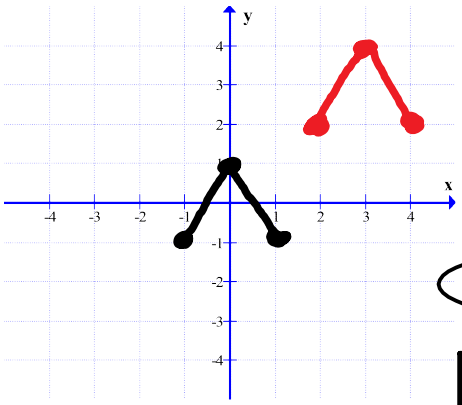
$y = f(x - h)$

$y = f(x - 5)$

$y = f(x)$
$y = f(x - 5)$

$HT = +5 \quad x \to x - 5$

$y = f(x)$



$y = f(x - h) + k$

$y = f(x + 3) - 3$

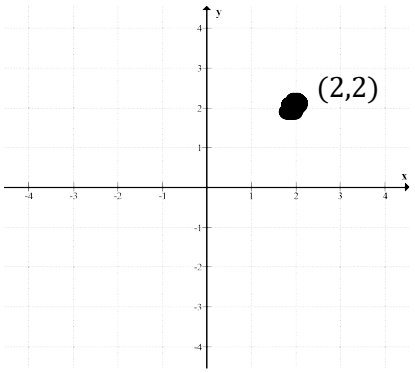
$y = f(x)$
$y = f(x + 3)$
$y + 3 = f(x + 3)$
$y = f(x + 3) - 3$

$HT = -3 \quad x \to x + 3$
$VT = -3 \quad y \to y + 3$

C12 - 1.2 - VHCE Point Notes

Find new point

$y = f(x)$



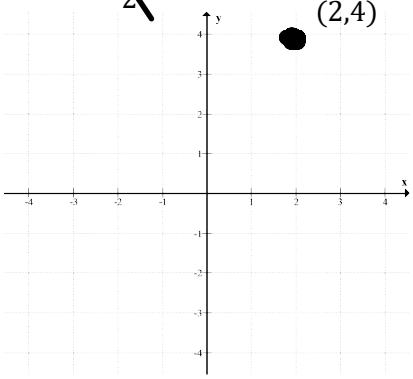
Point

$(2,2)$ *is on* $f(x)$

Function Notation

$y = f(x)$

$\frac{1}{2}y = 2f(x)$



Operation

$(2,2)$

$VE = 2$ $(2,4)$

Multiply y-value by 2

y times 2

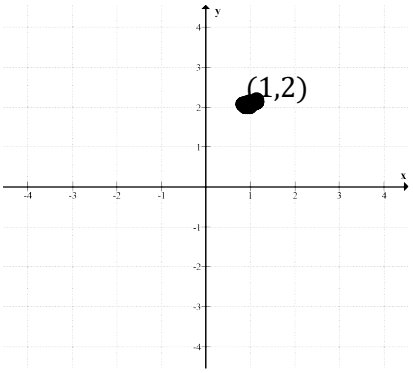
A Vertical Expansion by a Factor of 2

$2y$

Mapping Notation

$(x, 2y)$

$g(x) = f(2x)$



$(2,2)$

$HC = \frac{1}{2}$ $(1,2)$

Multiply x-value by $\frac{1}{2}$

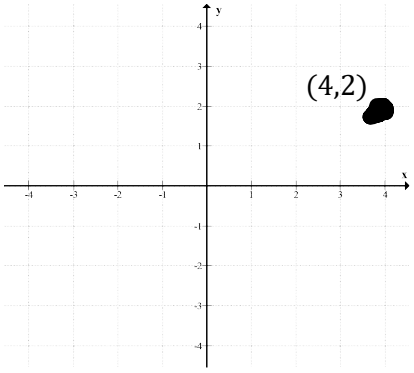
$(\frac{1}{2}x, y)$

x times $\frac{1}{2}$

A Horizontal Compression by a Half

$\frac{1}{2}x$

$h(x) = f(\frac{1}{2}x)$



$(2,2)$

$HE = 2$ $(4,2)$

Multiply x-value by 2

$(2x, y)$

x times 2

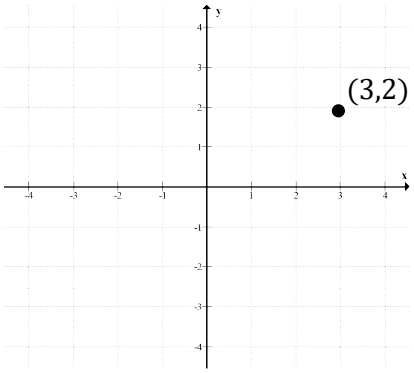
A Horizontal Expansion by 2

$2x$

Do exactly what you see outside of the brackets on the right-hand side to the **y-value**
Do the **Opposite** of what you see inside the brackets to the **x-value**. Attached to the variable.
Do the **Opposite** of what you see on the left hand side to the **y-value**. Attached to the variable.

C12 - 1.2 - VHR Point Notes

$Find\ g(x)$

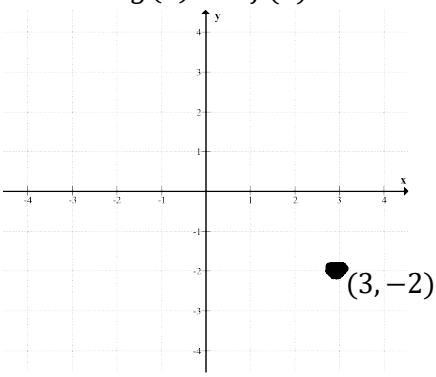


Point

$(3,2) is\ on\ f(x)$

Function Notation

$g(x) = -f(x)$



Operation

(3,2)

VR $(3, -2)$

y times -1

$-y$

Multiply y-value by -1

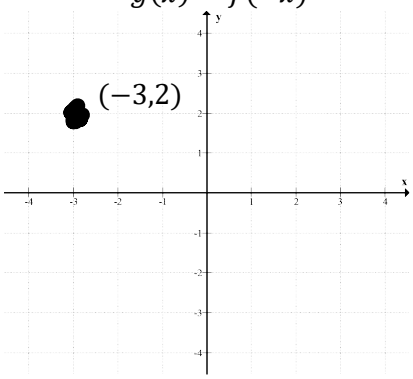
A Vertical Reflection

Mapping Notation

$(x, -y)$

Reflection

$g(x) = f(-x)$



(3,2)

HR $(-3,2)$

x times -1

$-x$

Multiply x-value by -1

A Horizontal Reflection

$(-x, y)$

Reflection

Remember: beDMAS. Function Operations 1st. Inside Out.

C12 - 1.2 - VHCER Function Notation f(x) Notes

$y = f(x)$

$f(x) = x^2$ *Given*

$f(3) = ?$ $(3, y)$ *What is y when x is 3.*

$f(x) = x^2$
$f(x) = (x)^2$
$f(3) = (3)^2$

Put 3 in for x.

Put whatever is inside the brackets in for x. Substitute with Brackets

$y = x$
$y = (3)^2$
$y = 9$

x	y
3	9

$f(3) = 9$ (3,9)

$f(2x) = ?$

$f(x) = x^2$

$f(2x) = (2x)^2$

Let's call it y

Put 2x in for x

Function Notation

$y = ?$

$y = f(2x)$

$y = (2x)^2$

$HC = \frac{1}{2}$

$2f(x) = ?$

$f(x) = x^2$

$2f(x) = 2x^2$

Let's call it $k(x)$

$2 \times f(x)$

$k(x) = ?$

$k(x) = 2f(x)$

$k(x) = 2x^2$

$VE = 2$

$-f(x) = ?$

$f(x) = x^2$

$-f(x) = -x^2$

Let's call it $n(x)$

$-ve\ f(x)$

$n(x) = ?$

$n(x) = -f(x)$

$n(x) = -x^2$

VR

Vertical Reflection

C12 - 1.2 - VHCE Graph y= Notes

Vertical Expansion by a factor of 2 $VE = 2$

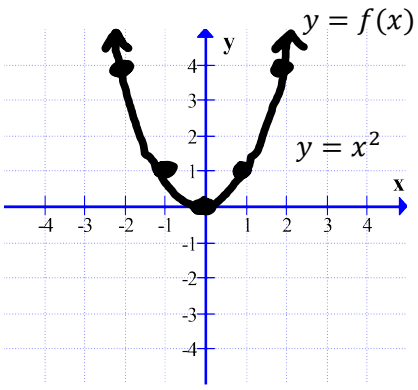
$y = x^2$

$\frac{1}{2}y = x^2$ $\quad y \to \frac{1}{2}y$

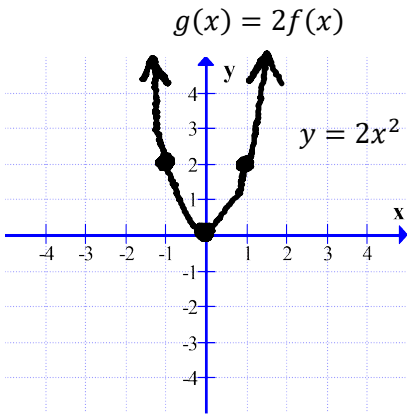
$y = 2x^2$

$Put \frac{1}{2}y \; in \; for \; y$

Substitute the Opposite Operation for the Variable



x	y
-2	4
-1	1
0	0
1	1
2	4



x	y
-2	8
-1	2
0	0
1	2
2	8

Multiply y values by 2

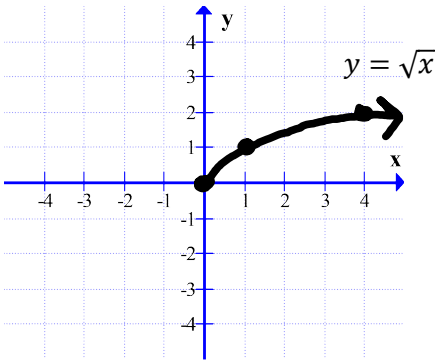
Horizontal Compression by a factor of $\frac{1}{2}$ $HC = \frac{1}{2}$

$y = \sqrt{x}$

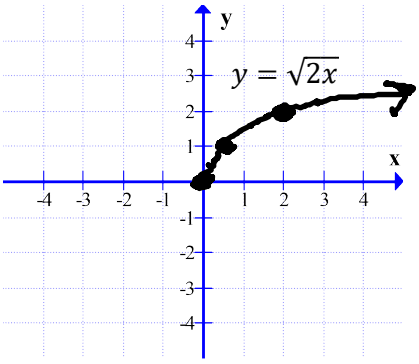
$y = \sqrt{2x}$ $\quad x \to 2x$

$Put \; 2x \; in \; for \; x$

Substitute the Opposite Operation for the Variable



x	y
-1	und
0	0
1	1
4	2



x	y
-1	und
0	0
$\frac{1}{2}$	1
2	2

Multiply x values by $\frac{1}{2}$

C12 - 1.2 - VHR Graph y= Notes

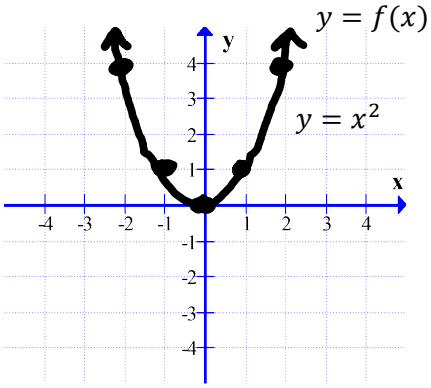
Vertical Reflection VR

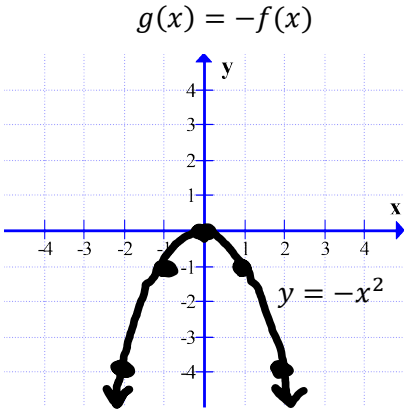
$y = x^2$
$-y = x^2$
$y = -x^2$

$y \to -y$

$Put\ -y\ in\ for\ y$

Substitute the Opposite Operation for the Variable





x	y
-2	4
-1	1
0	0
1	1
2	4

x	y
-2	-4
-1	-1
0	0
1	-1
2	-4

Over the x-axis

Multiplying y by negative 1

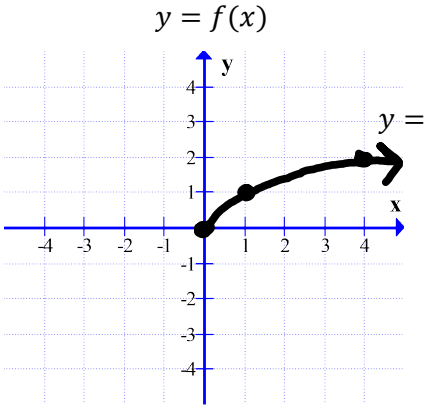
Horizontal Reflection

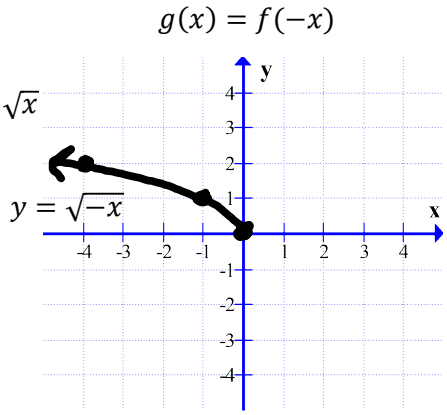
$y = \sqrt{x}$
$y = \sqrt{-x}$

HR $x \to -x$

$Put\ -x\ in\ for\ x$

Substitute the Opposite Operation for the Variable





x	y
-1	und
0	0
1	1
4	2

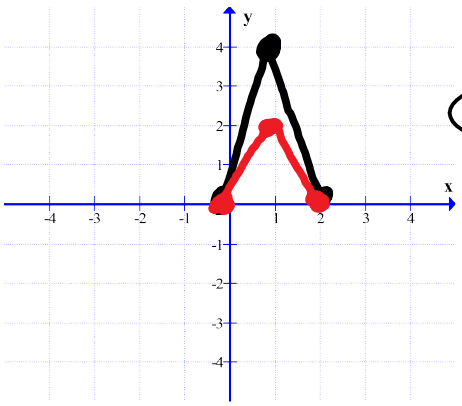
x	y
1	und
0	0
-1	1
-4	2

Over the y-axis

Multiplying x by negative 1

C12 - 1.2 - VHCER Graphs f(x) Notes

Find the transformed equation of $f(x)$ in all forms.



$y = af(x)$

$y = 2f(x)$

$ay = f(x)$

$\frac{1}{2}y = f(x)$

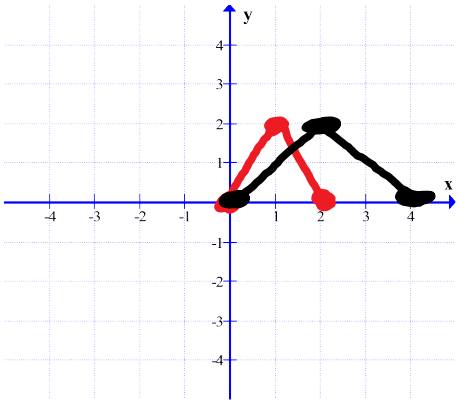
$y = f(x)$

$\frac{1}{2}y = f(x)$

$y = 2f(x)$

$VE = 2 \quad y \to \frac{1}{2}y$

$y = f(x)$



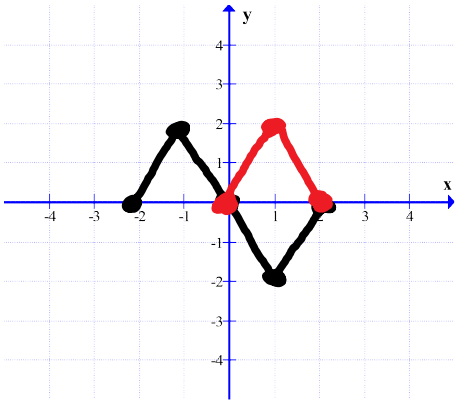
$y = f(bx)$

$y = f(\frac{1}{2}x)$

$HE = 2 \quad x \to \frac{1}{2}x$

$y = f(x)$

$y = f(bx)$



$y = af(x)$

$HR \quad x \to -x$

$y = f(x)$

$y = f(-x)$

$y = f(x)$

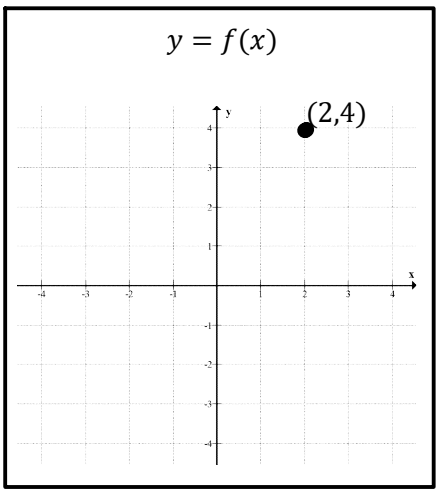
$-y = f(x)$

$VR \quad y \to -y$

$y = -f(x)$

C12 - 1.3 - VHTCER Point/s/Algebra/Factor/Order Notes

$(2,4)$ *is on* $f(x)$. *Find the point on* $g(x)$ *if*: $g(x) = f(x-2)+3$



	(2,4)	
$HT = +2$	(4,4)	Add 2 to x-value
$VT = +3$	(4,7)	Add 3 to y-value

$(2,4)$ *is on* $f(x)$. *Find the point on* $g(x)$ *if*: $g(x) = -2f(x+1)-1$

	(2,4)	
VR	$(2,-4)$	Multiply y-value by -1
$VE = 2$	$(2,-8)$	Multiply y-value by 2
$HT = -1$	$(1,-8)$	Subtract 1 from x-value
$VT = -1$	$(1,-9)$	Subtract 1 from y-value

$(2,4)$ *is on* $f(x)$. *Find the point on* $g(x)$ *if*: $g(x) = f\left(-\frac{1}{2}x\right)$

	(2,4)	
HR		
$HE = 2$	$(-2,4)$	Multiply x-value by -1
	$(-4,4)$	Multiply x-value by 2

$(2,4)$ *and* $(4,6)$ *are on* $f(x)$. *Find the point on* $g(x)$ *if*: $g(x) = f(2(x-2))$

	(2,4)	(4,6)	
$HC = \frac{1}{2}$	(1,4)	(2,6)	Multiply x-value by a half
$HT = +2$	(3,4)	(4,6)	Add 2 to x-value

Two Points

$g(x) = f(2x-4)$
$g(x) = f(2(x-2))$

$HC = \frac{1}{2}$
$HT = +2$

$y = f(1-x)$
$y = f(-(-1+x))$
$y = f(-(x-1)$

HR
$HT = +1$

Factor Brackets

; so x has a coefficient of 1

$2g(x) - 4 = f(x)$
$2g(x) = f(x) + 4$
$g(x) = \frac{1}{2}f(x) + 2$

Algebra
$VC = \frac{1}{2}$
$VT = +2$

$(2,4)$ *is on* $f(x)$. *Find the point on* $g(x)$ *if*: $g(x) = f^{-1}(x+2)$

1.4

	(2,4)	
f^{-1}	(4,2)	Function operations 1st
$HT = -2$	(2,2)	Subtract 2 from x

C12 - 1.3 - VHTCER Function Notation f(x) Notes

$y = f(x)$

$f(x) = x^2$

Function Notation

$3f(-x) + 2 = ?$

$f(x) = x^2$

$3f(-x) + 2 = 3(-x)^2 + 2$

Let's call it $d(x)$

$3 \times f(-x) + 2$

$d(x) = ?$

$d(x) = 3f(-x) + 2$

$d(x) = 3(-x)^2 + 2$

$2f(x-1) + 5 = ?$

$f(x) = x^2$

$2f(x-1) + 5 = 2(x-1)^2 + 5$

Let's call it $n(x)$

$Put\ x-1\ in\ for\ x$
$+5\ to\ 2f(x-1)$

$n(x) = ?$

$n(x) = 2f(x-1) + 5$

$n(x) = 2(x-1)^2 + 5$

C12 - 1.3 - VHTCER y= Notes

Find the new equation.

$y = x^2 + x$

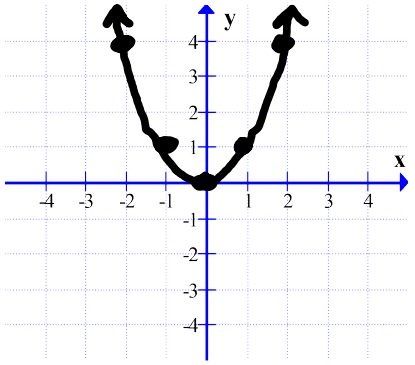
A Horizontal Reflection
A vertical expansion by a factor of 2
A vertical translation up 1
A horizontal translation left 5

$y = x^2 + x$
$y = (-x)^2 + (-x)$ → HR → $x \to -x$
$y = x^2 - x$
Algebra
$\frac{1}{2}y = x^2 - x$ → $VE = 2$ → $y \to \frac{1}{2}y$
$y = 2x^2 - 2x$
Algebra
$y - 1 = 2x^2 - 2x$ → $VT = +1$ → $y \to y - 1$
$y = 2x^2 - 2x + 1$

$y = 2(x + 5)^2 - 2(x + 5) + 1$ → $HT = -5$ → $x \to x + 5$

Foil?

C12 - 1.3 - VHTCER Graph y= Notes



$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

Vertical Expansion by a factor of 2 **AND A** Vertical Translation Up One

$y = x^2$

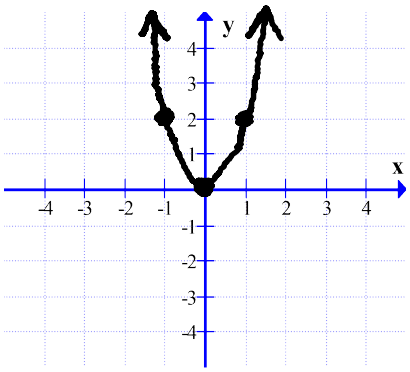
$\frac{1}{2}y = x^2$ $\quad VE = 2$ $\quad y \to \frac{1}{2}y$ $\quad$ *Put* $\frac{1}{2}y$ *in for* y $\quad$ **Substitute the Opposite Operation for the Variable**

$y = 2x^2$

$VE = 2$

$y \times 2$

Multiply y *values by* 2



$y = 2x^2$

x	y
-2	8
-1	2
0	0
1	2
2	8

$y = 2x^2$

$y - 1 = 2x^2$ $\quad VT = +1$ $\quad y \to y - 1$ $\quad$ *Put* $y - 1$ *in for* y $\quad$ **Substitute the Opposite Operation for the Variable**

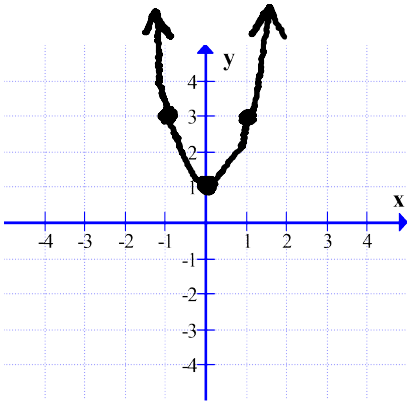
$y = 2x^2 + 1$

$VT = +1$

$y + 1$

Up 1

Add 1 *to the* $y -$ *values*

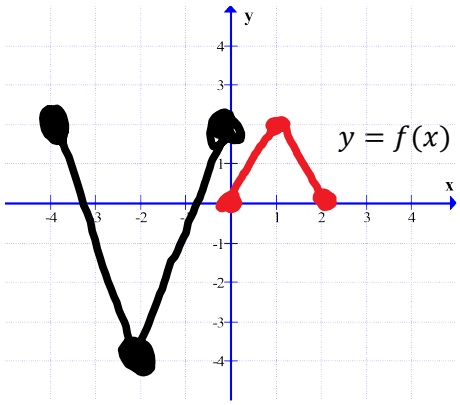


$y = 2x^2 + 1$

x	y
-2	9
-1	3
0	1
1	3
2	9

C12 - 1.3 - VHTCER Graph f(x) Notes

Find the transformed equation.



$y = af(b(x - h)) + k$

How wide is it?

2 units

How wide is it now?

4 units

What happened?

HE=2

$x \to \frac{1}{2}x$

How tall is it?

2 units

How tall is it now?

6 units

What happened?

VE=3

$y \to \frac{1}{3}y$

Any reflections?

VR

$y \to -y$

Or do multiple intercepts to make sure.

Pick a point, not an intercept, do expansions, compressions, and reflections.

$HE = 2$ $(1,2)$ $(2,2)$

$VE = 3$ $(2,6)$

VR $(2,-6)$

Has it moved?

$HT = -4$ $(2,-6)$ $(-2,-6)$

$VT = +2$ $(-2,-4)$

$x \to x + 4$

$y \to y - 2$

$y = f(x)$

$y = f\left(\frac{1}{2}x\right)$

$\frac{1}{3}y = f\left(\frac{1}{2}x\right)$

$y = 3f\left(\frac{1}{2}x\right)$

$-y = 3f\left(\frac{1}{2}x\right)$

$y = -3f\left(\frac{1}{2}x\right)$

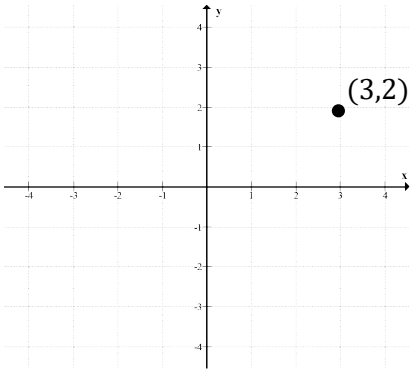
$y = -3f\left(\frac{1}{2}(x + 4)\right)$

$y - 2 = -3f\left(\frac{1}{2}(x + 4)\right)$

$y = -3f\left(\frac{1}{2}(x + 4)\right) + 2$

C12 - 1.4 - Point $f^{-1}(x)$ Inverse Notes

$Find\ g(x)$

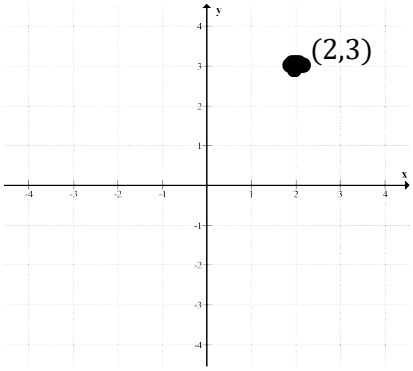


Point

$(3,2) is\ on\ f(x)$

Function Notation

$g(x) = f^{-1}(x)$



Operation

(3,2)

$f^{-1}(x)$ (2,3)

$x <-> y$

(y, x)

$Switch\ x\ and\ y$

Inverse

Mapping Notation

(y, x)

Inverse 1st. Function Operations 1st. Inside Out.

C12 - 1.4 - Graph/Algebra $f^{-1}(x)$ Inverse Notes

$f(x) = 2x + 2$

$y = 2x + 2$

$x = 2y + 2$ — *Switch* x and y

$x - 2 = 2y$

$\frac{x}{2} - 1 = y$

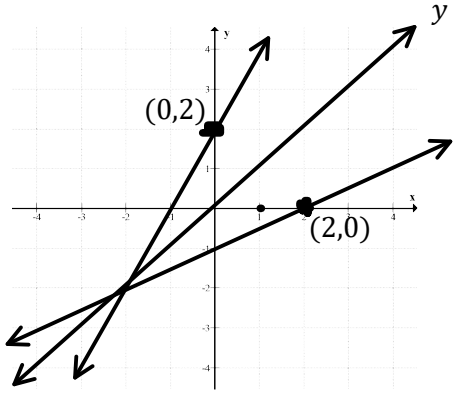
$y = \frac{1}{2}x - 1$ — Solve for y

$f^{-1}(x) = \frac{1}{2}x - 1$ — Write in Function Notation

$y = f(x)$

$f(x) = 2x + 2$

x	2x+2
-2	-2
-1	0
0	2
1	4
2	6



$y = x$

$f^{-1}(x) = \frac{1}{2}x - 1$

x	2x-4
-2	-2
0	-1
2	0
4	1
6	2

Switch x and y

Remember: The inverse is a diagonal reflection over the line $y = x$

Check your answer $f^{-1}(f(x)) = ?$

$f^{-1}(x) = \frac{1}{2}x - 1$

$f^{-1}(2x - 4) = \frac{1}{2}(2x + 2) - 1$

$f^{-1}(2x - 4) = x$ $\quad f^{-1}(f(x)) = x$ ✓

$f(f^{-1}(x)) = ?$

$f(x) = 2x + 2$

$f\left(\frac{1}{2}x + 2\right) = 2\left(\frac{1}{2}x - 1\right) + 2$

$f\left(\frac{1}{2}x + 2\right) = x$ $\quad f(f^{-1}(x)) = x$ ✓

$f(x) = \frac{x}{x+1}$

$y = \frac{x}{x+1}$

$x = \frac{y}{y+1}$ — Switch x and y

$x(y+1) = y$ — Multiply

$xy + x = y$ — Distribute

$x = y - xy$ — Combine like terms (y's on one side)

$x = y(1-x)$ $\quad GCF = y$ — Factor

$\frac{x}{1-x} = y$ — Divide

$y = \frac{x}{1-x}$

$f^{-1}(x) = \frac{x}{1-x}$

A function has an inverse function if it is One-to-One, Or if you restrict the domain.

C12 - 1.5 - Order Matters Point/Functions Notes

$y = f(x)$

Find the new point.

$(x, f(x)) = (2,4)$

x	y
2	4

A vertical expansion by a factor of 2
A vertical translation up 2

$VE = 2$ $(2,4)$
$VT = +2$ $(2,8)$
$(2,10)$

x	y
2	10

A vertical translation up 2
A vertical expansion by a factor of 2

$VT = +2$ $(2,4)$
$VE = 2$ $(2,6)$
$(2,12)$

Find the new equation.

$f(x) = x^2$

x	y
2	4

A vertical expansion by a factor of 2
A vertical translation up 2

$f(x) = x^2$
$y = x^2$
$\frac{1}{2}y = x^2$ $Put \frac{1}{2}y \; in \; for \; y$
$y = 2x^2$
$y - 2 = 2x^2$ $Put \; "y - 2" \; in \; for \; y$
$y = 2x^2 + 2$

x	y
2	10

A vertical translation up 2
A vertical expansion by a factor of 2

$f(x) = x^2$
$y = x^2$
$y - 2 = x^2$ $Put \; "y - 2" \; in \; for \; y$
$y = x^2 + 2$
$\frac{1}{2}y = x^2 + 2$ $Put \frac{1}{2}y \; in \; for \; y$
$y = 2x^2 + 4$

Remember: We always substitute the opposite operation for the variable.
Remember: Order matters. An addition then a multiplication is far different from the same multiplication and then the same addition. **Think about it!**
Remember: Do the operations in the order you are asked or follow DMAS