C12 - 1.0 -Trans Notes

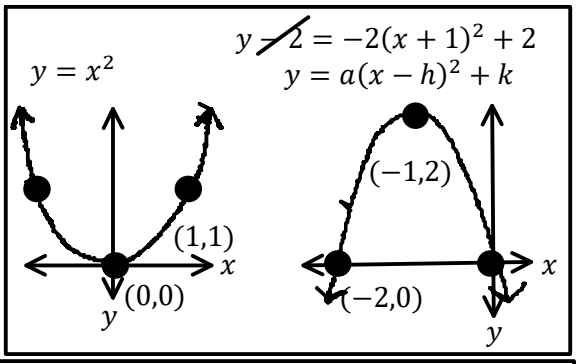
$y^* = f(x)$

$a : VR;\ a < 0, VE\ or\ VC = a.$ $\quad k : VT = k\ (Up, Down)$

$y - k = af(b(x-h)) + k$

$b : HR;\ b < 0, HE\ or\ HC = \frac{1}{b}.$ $\quad h : HT = h^*\ (Left, Right)$

$y = x^2$ $\quad y - 2 = -2(x+1)^2 + 2$ $\quad y = a(x-h)^2 + k$



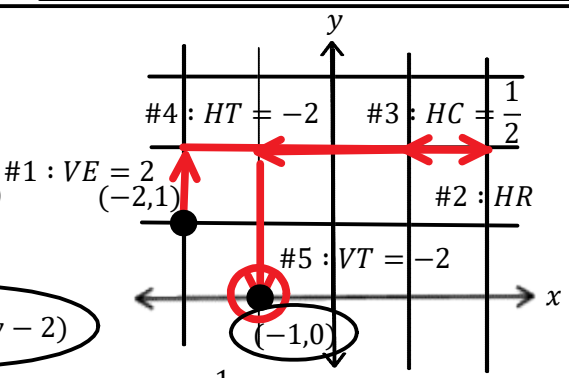
Transform Points :

$2y + 4 = 4f(-4-2x)$

$2y = 4f(-4-2x) - 4$ Subtract Both Sides

$y = 2f(-4-2x) - 2$ Divide Both Sides

$y = 2f(-2(x+2)) - 2$ Factor Brackets #($1x$ …)



	(−2,1)	(4,−1)	
$VE = 2$	(−2,2)	(4,−2)	$y \times 2$
HR	(+2,2)	(−4,−2)	$x \times -1$
$-HC = \frac{1}{2}$	(1,2)	(−2,−2)	$x \times \frac{1}{2}$
$HT - 2$	(−1,2)	(−4,−2)	$x - 2$
$VT = -2$	(−1,0)	(−4,−4)	$y - 2$

Mapping

$(-\frac{1}{2}x - 2, 2y - 2)$

$(-\frac{1}{2}(-2) - 2, 2(1) - 2)$

$(-1,0)$

$(-\frac{1}{2}(4) - 2, 2(-1) - 2)$

$(-4,-4)$

$y = |f^{-1}(x-1)| - 2$ Inverse 1st! DMAS, Inside Out!

	(−2,3)	
$f^{-1}(x)$	(3,−2)	Switch x & y
$HT = +1$	(4,−2)	$x + 1$
$\lvert y \rvert$	(4,+2)	Make y positive
$VT = -2$	(4,0)	$y - 2$

Backwards Steps :

(2,6) is on $2f(x+1)$.

What point is on $f(x)$?

(3,3)

(3,6) $VE = 2$

(2,6) $HT = -1$

$f(x) = x^2$ **Function Notation :**

$f(2) = (2)^2 = 4$ Sub 2 for x

$f(x) = x^2$

$f(x-2) = (x-2)^2$ Sub $x-2$ for x

$g(x) = (x-2)^2$ $HT = +2$

Two Aliens on earth named "y" & "x" are the same age. $y = x$. If y time-travels 2 years ahead and lands back on earth the same age as when they (y) left, where x is two years older, what must you substitute for x for the equation to hold true?

$let\ y = y's\ age$ Now $y = x$

$let\ x = x's\ age$

x	y
5	5
8	8

Arbitrary #'s!

Then $y = x - 2$ $\quad x \to x - 2$

x	y
7	5
10	8

x	y
3	5
6	8

$y = x + 2$

$x \to x + 2$

Not what you'd think!

Transform Functions/Equations :

$f(x) = \sqrt{x}$

$y = \sqrt{x}$

$\frac{1}{2}y = \sqrt{x}$

$y = 2\sqrt{x}$

$y = 2\sqrt{4x}$

$y + 1 = 2\sqrt{4x} - 1$

$y = 2\sqrt{4(x+4)} - 1$

$y = 4\sqrt{x+4} - 1$

Substitute the Opposite Operation for the Variable

$VE = 2$ Algebra $\quad y \to \frac{1}{2}y$

$HC = \frac{1}{4}$ $\quad x \to 4x$

$VT = -1$

$HT = -4$ $\quad y \to y + 1$

Radical Laws $\quad x \to x + 4$

HR & VR Reflections

$y = -x^3 - x^2 + x$

$y = -(-x)^3 - (-x)^2 + (-x)$ $\quad HR\ \ x \to -x$

$y = +x^3 - x^2 - x$ $\quad$ BEDMAS

$-y = x^3 - x^2 - x$ $\quad VR\ \ y \to -y$

$y = -x^3 + x^2 + x$ $\quad Divide\ by\ -1$

$g(x) = -x^3 + x^2 + x$ Call it whatever you want*

Invariant Points:

HR/HE/HC: y-int (0,y)

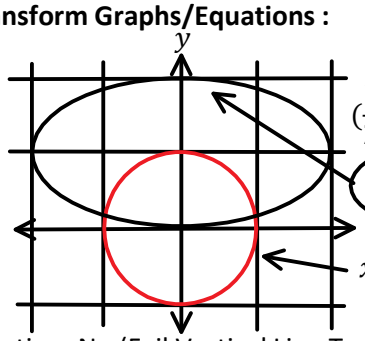
VR/VE/VC: x-int (x,0)

Inverses: (a,a)

Roots: (x,0) (x,1)

C12 - 1.0 -Trans Graphs Notes

Transform Graphs/Equations :



$x^2+y^2=1$

$x^2+(y-1)^2=1$

$(\frac{1}{2}x)^2+(y-1)^2=1$

$\frac{x^2}{4}+(y-1)^2=1$

$VT=+1$ $\quad y \to y-1$

$HE=2$ $\quad x \to \frac{1}{2}x$

Exponent/Fraction Laws

Do it in the order they told you to do it! (or DMAS*)

Function: No (Fail Vertical Line Test)

$g(x)=f\left(\frac{1}{2}x\right)+1$

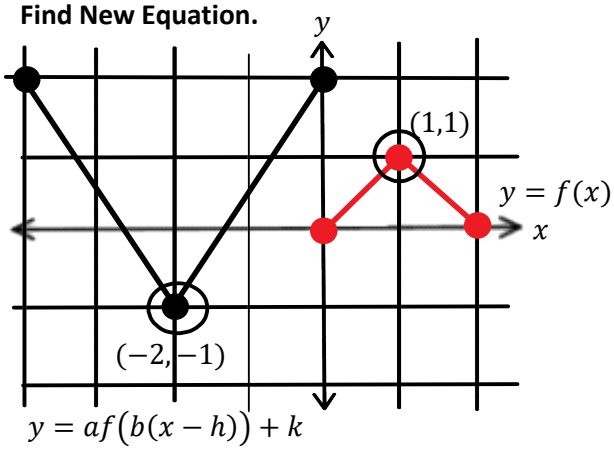
Call it whatever you want*
(Unless they name it for you!)

$x^2+y^2=1$

$y=+\sqrt{1-x^2}$

$y=-\sqrt{1-x^2}$

Find New Equation.



$y=af\big(b(x-h)\big)+k$

How Wide is it/now? $(2 \to 4)$
What Happened?
How Tall is it/now? $(1 \to 3)$
What Happened?
Any Reflections?
Pick a Point (Not an Intercept!)
Do what you said so far!
Has it Moved?

$y=f(x)$

$HE=2$ $\quad \frac{(1,1)}{(2,1)}$ $\quad y=f\left(\frac{1}{2}x\right)$ $\quad x \to \frac{1}{2}x$

$VE=3$ $\quad (2,3)$ $\quad \frac{1}{3}y=3f\left(\frac{1}{2}x\right)$ $\quad y \to \frac{1}{3}y$

VR $\quad (2,-3)$ $\quad -y=-3f\left(\frac{1}{2}x\right)$ $\quad y \to -y$

$HT=-4$ $\quad (-2,-3)$

$VT=+2$ $\quad (-2,-1)$ ✓ $\quad y=-3f\left(\frac{1}{2}(x+4)\right)$ $\quad x \to x+4$

$y-2=-3f\left(\frac{1}{2}(x+4)\right)+2$ $\quad y \to y-2$

C12 - 1.0 - Trans Inverse Notes $y = f(x)$

Find the Inverse : Check by taking the Inverse!

$f(x) = 2x - 1$
$y = 2x - 1$
$x = 2y - 1$
$2y = x + 1$
$y = \frac{1}{2}x + \frac{1}{2}$
$f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$

Switch x and y

Algebra

Function Notation

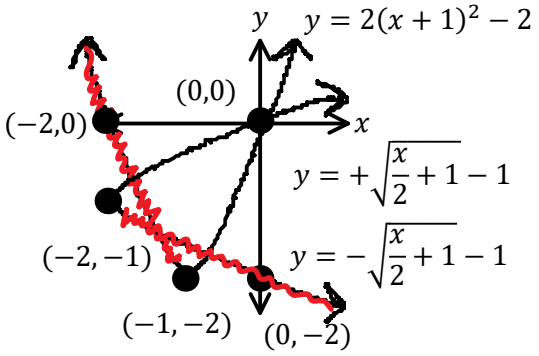
$f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$
$y = \frac{1}{2}x + \frac{1}{2}$
$x = \frac{1}{2}y + \frac{1}{2}$
$2x = y + 1$
$y = 2x - 1$ ✓

Diagonal Reflection Switch Domain and Range*

$f(x) = \frac{x}{x+1}$
$y = \frac{x}{x+1}$
$x = \frac{y}{y+1}$ Switch x and y
$x(y+1) = y$ Multiply
$xy + x = y$ Distribute
$x = y - xy$ Combine like terms(y's on one side)
$x = y(1-x)$ Factor
$\frac{x}{1-x} = y$ Divide

$y = \frac{x}{1-x}$ $f^{-1}(x) = \frac{x}{1-x}$

$f(x) = 2(x+1)^2 - 2$
$x = 2(y+1)^2 - 2$ Switch x & y
$x + 2 = 2(y+1)^2$ Add
$\frac{x}{2} + 1 = (y+1)^2$ Divide
$\pm\sqrt{\frac{x}{2} + 1} = y + 1$ Square Root
$y = \pm\sqrt{\frac{x}{2} + 1} - 1$ Algebra/Mirror
$f^{-1}(x) = \pm\sqrt{\frac{x}{2} + 1} - 1$



Function: No (Fail Vertical Line Test)
Function: Yes (Pass Vertical Line Test)
; if $f(x), x \geq -1$ or if $f(x), x \leq -1$.

Alternate Check :

$f(f^{-1}(x)) = ?$

$f(x) = \frac{x}{x+1}$

$f\left(\frac{x}{1-x}\right) = \left(\frac{\frac{x}{1-x}}{\frac{x}{1-x}+1}\right) \times \frac{LCD}{\frac{1-x}{1-x}}$

$= \frac{x}{x + (1-x)}$

$= x$ ✓

$f^{-1}(f(x)) = ?$

$f(x) = \frac{x}{1-x}$

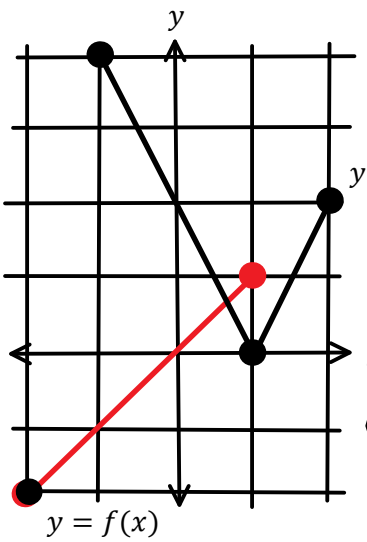
$f\left(\frac{x}{x+1}\right) = \left(\frac{\frac{x}{x+1}}{1-\frac{x}{x+1}}\right) \times \frac{LCD}{\frac{x+1}{x+1}}$

$= \frac{x}{x+1-(x)}$

$= x$ ✓

Transform/Find Points/Graphs/ Equations/Functions
Domain & Range
Inverse/Algebra
Substitution

Domain & Range :



$y = 2|f(x-1)|$

$f(x)$
Domain : $-2 \leq x \leq 1$
Range: $-2 \leq y \leq 1$
$2|f(x-1)|$
Domain : $-1 \leq x \leq 2$
Range : $0 \leq y \leq 4$

(Order Matters)

$y = x^2$
$\frac{1}{2}y = x^2$
$y = 2x^2$
$y - 2 = 2x^2$
$y = 2x^2 + 2$

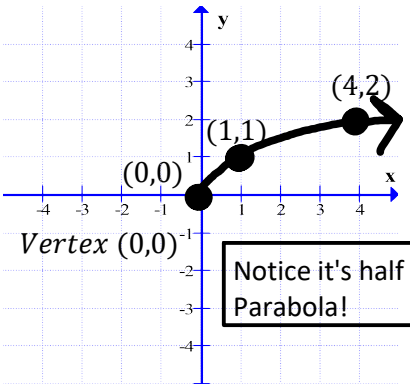
$VE = 2$ $y \to \frac{1}{2}y$ Algebra
$VT = +2$ $y \to y - 2$ Algebra

$y = x^2$
$y - 2 = x^2$
$y = x^2 + 2$
$\frac{1}{2}y = x^2 + 2$
$y = 2x^2 + 4$

We don't distribute into a function!
$\frac{1}{2}y = 3(2f(2x-1) - 1) + 1$
$\frac{1}{2}y = 6f(2x-1) - 3 + 1$
$y = 12f(2x-1) - 4$

*C12 - 2.0 - Trans Root Notes

Graph : $y = \sqrt{x}$



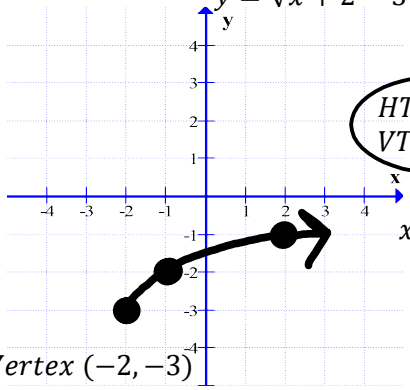
Notice it's half a Parabola!

x	y
-1	und
0	0
1	1
4	2
9	3

$D: x \geq 0$

$R: y \geq 1$

$y = \sqrt{x+2} - 3$



$HT = -2$
$VT = -3$

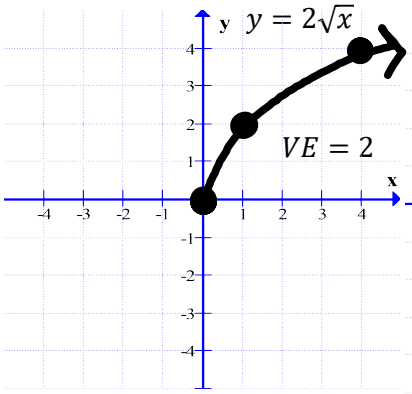
$x + 2 \geq 0$

$x \geq -2$

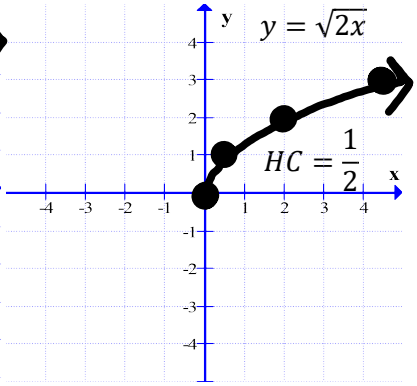
$y \geq -3$

Remember: Choose increments of x in your table of values that square root easily.

$y = 2\sqrt{x}$



$y = \sqrt{2x}$



$y = a\sqrt{b(x-h)} + k$

$Vertex\ (h,k)$

Domain: Set Underneath root ≥ zero and solve

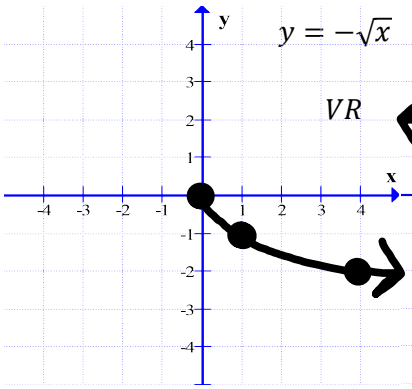
$D: b(x-h) \geq 0$

$R: y \geq k, a > 0$

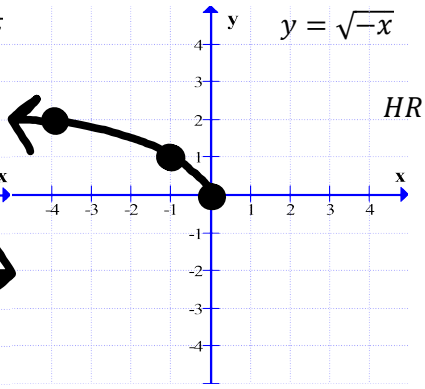
$y \leq k, a < 0$

TOV

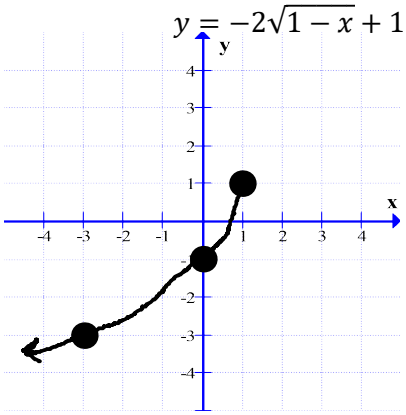
$y = -\sqrt{x}$



$y = \sqrt{-x}$



$y = -2\sqrt{1-x} + 1$



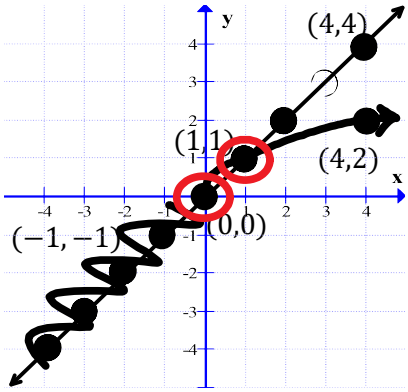
$1 - x \geq 0$

$-x \geq -1$

$x \leq 1$

$y \leq 1$

Draw the graph of $\sqrt{x}$ from the graph of $f(x)$ and label the invariant points and state the domain and range.



$y = f(x) = x$

$y = \sqrt{f(x)} = \sqrt{x}$

x	$f(x)$	$\sqrt{f(x)}$
-1	-1	und
0	0	0
1	1	1
4	4	2

Pick an x value on f(x).
Put your pencil there!
Square root the y − value
Draw the new point.

Remember:
Cant square root a negatives!
Choose x-values whose y values can square root evenly if possible.
Invariant points are on the line $y = 1\ (x, 1), and\ y = 0\ (x, 0)$