C12 - 3.1 - Long/Synthetic Division $R = 0$ Notes

$\frac{64}{4} = ?$

Goes Into
Multiply
Subtract
Bring Down
Repeat

$$\begin{array}{r} 16 \\ 4\overline{)\,64} \\ -\,4\downarrow \\ \hline 24 \\ -\,24 \\ \hline 0 \end{array}$$

Bring down

remainder

$$\frac{64}{4} = 16$$

$$\frac{dividend}{divisor} = quotient$$

$$64 = 4 \times 16$$

$$dividend = (quotient)(divisor)$$

$\frac{x^2 + 5x + 6}{x + 3} = ?$

$$\begin{array}{r} x + 2 \\ x + 3\overline{)\,x^2 + 5x + 6} \\ -\,(x^2 + 3x)\downarrow \\ \hline 2x + 6 \\ -\,(2x + 6) \\ \hline 0 \end{array}$$

remainder

$$\frac{x^2 + 5x + 6}{x + 3} = x + 2$$

$$\frac{P(x)}{x - a} = Q(x)$$

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

$$P(x) = Q(x)(x - a)$$

Synthetic Division

$\frac{x^2 + 5x + 6}{x + 3} = ?$

$1x^2 + 5x + 6$

$x + 3 = 0$
$x = -3$

$$\begin{array}{r|ccc} -3 & 1 & 5 & 6 \\ + & & -3\downarrow & -6 \\ \hline & 1 & 2 & 0 \end{array}$$

remainder

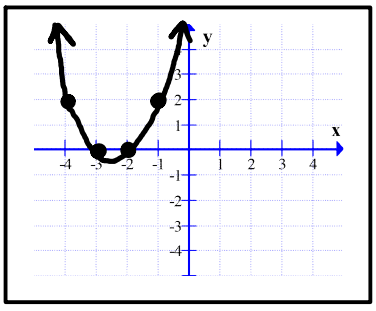
$1x + 2$

The exponents of x go down by one.

Factor Theorem

$f(x) = x^2 + 5x + 6$
$f(-3) = (-3)^2 + 5(-3) + 6$
$f(-3) = 0$

$(-3,0)$



C12 - 3.1 - Long/Synthetic Division Notes

$\frac{65}{4} = ?$

$$\begin{array}{r} 16 \\ 4\overline{)\,65} \\ -\,4\downarrow \\ \hline 25 \\ -\,24 \\ \hline 1 \end{array}$$

Bring down

remainder

$$\frac{65}{4} = 16 + \frac{1}{4}$$

$$65 = 4 \times 16 + 1$$

$$\frac{dividend}{divisor} = quotient + \frac{remainder}{divisor}$$

$$dividend = (quotient)(divisor) + remainder$$

$\frac{x^2 + 5x + 9}{x + 3} = ?$

$$\begin{array}{r} x + 2 \\ x + 3\overline{)\,x^2 + 5x + 9} \\ -\,(x^2 + 3x)\downarrow \\ \hline 2x + 9 \\ -\,(2x + 6) \\ \hline 3 \end{array}$$

remainder

$$\frac{x^2 + 5x + 9}{x + 3} = x + 2 + \frac{3}{x + 3}$$

$$x^2 + 5x + 9 = (x + 2)(x + 3) + 3$$

$$\frac{P(x)}{x - a} = Q(x) + \frac{R}{x - a}$$

$$P(x) = Q(x)(x - a) + R$$

Synthetic Division

$\frac{x^2 + 5x + 9}{x + 3} = ?$

$1x^2 + 5x + 9$

$x + 3 = 0$

$x = -3$

$$\begin{array}{r|ccc} -3 & 1 & 5 & 9 \\ + & & \downarrow -3 & -6 \\ \hline & 1 & 2 & 3 \end{array}$$

remainder

$1x + 2$ $R: 3$

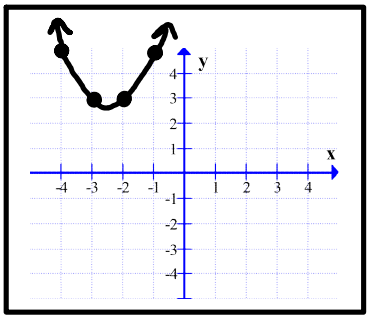
Remainder Theorem

$f(x) = x^2 + 5x + 6$

$f(-3) = (-3)^2 + 5(-3) + 9$

$f(-3) = 3$

$(-3,3)$



C12 - 3.1 - Synthetic Division $R = 0$ Notes

$$\frac{x^3 + x^2 - 8x + 4}{x - 2}$$

$x - 2 = 0$
$x = 2$

Set denominator equal to zero and solve.
$Denominator = 0$

$$\begin{array}{r|cccc} 2 & 1 & 1 & -8 & 4 \\ + & & & & \\ \hline & & & & \end{array}$$

Place that number to the left.
Write the coefficients. $1x^3 + 1x^2 - 8x + 4$

$$\begin{array}{r|cccc} 2 & 1 & 1 & -8 & 4 \\ + & & \downarrow & & \\ & & 2 & 6 & -4 \\ \hline & 1 & 3 & -2 & 0 \end{array}$$

1) Bring down the first coefficient
2) $(2) \times 1 = 2$
3) $1 + 2 = 3$
4) Repeat last two steps.

$1x^2 + 3x - 2 \quad R = 0$

$$\frac{x^3 + x^2 - 8x + 4}{x - 2} = x^2 + 3x - 2$$

$$x^3 + x^2 - 8x + 4 = (x^2 + 3x - 2)(x - 2)$$

$$\frac{dividend}{divisor} = quotient$$

$$dividend = (quotient)(divisor)$$

$$\frac{x^3 + 2x^2 - 5x - 6}{x + 1}$$

$x + 1 = 0$
$x = -1$

Set denominator equal to zero and solve.
$Denominator = 0$

$$\begin{array}{r|cccc} -1 & 1 & 2 & -5 & -6 \\ + & & & & \\ \hline & & & & \end{array}$$

Place that number to the left.
Write the coefficients. $1x^3 + 2x^2 - 5x - 6$

$$\begin{array}{r|cccc} -1 & 1 & 2 & -5 & -6 \\ + & & \downarrow & & \\ & & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$$\begin{array}{r} x^2 + x - 6 \\ x + 1 \overline{)\, x^3 + 2x^2 - 5x - 6} \\ -\,(x^3 + x^2) \\ \hline x^2 - 5x \\ -\,(x^2 + x) \\ \hline -6x - 6 \\ -\,(-6x - 6) \\ \hline 0 \end{array} \qquad R = 0$$

$1x^2 + 1x - 6 \quad R = 0$

$x^2 + x - 6$
$(x + 3)(x - 2)$

Factor

$$\frac{x^3 + 2x^2 - 5x - 6}{x + 1} = (x + 3)(x - 2)$$

$$x^3 + 2x^2 - 5x - 6 = (x + 3)(x - 2)(x + 1)$$

$$\frac{P(x)}{x - a} = Q(x)$$

$$P(x) = Q(x)(x - a)$$

C12 - 3.1 - Synthetic Division Remainder/Gap Notes

$$\frac{x^3+x^2-8x+7}{x-2}$$

$$\begin{array}{r|cccc} 2 & 1 & 1 & -8 & 7 \\ + & & & & \\ \hline & & & & \end{array}$$

$$\begin{array}{r|cccc} 2 & 1 & 1 & -8 & 7 \\ + & & 2 & 6 & -4 \\ \hline & 1 & 3 & -2 & 3 \end{array}$$

remainder

$1x^2+3x-2 \quad R=3$

$$\begin{array}{r} x^2+3x-2 \\ x-2 \;\big)\; x^3+x^2-8x+7 \\ -\;(x^3-2x^2) \\ \hline +3x^2-8x \\ -\;(3x^2-6x) \\ \hline -2x+7 \\ -\;(-2x+4) \\ \hline 3 \end{array} \qquad R=3$$

The remainder is the y value when $x=2$ $\quad f(2)=(2)^3+(2)^2-8(2)+7$

$f(2)=8+4-16+7$

$f(2)=3$

(2,3)

$$\frac{x^3+x^2-8x+6}{x-2}=x^2+3x-2+\frac{3}{x-2}$$

$$x^3+x^2-8x+6=(x^2+3x-2)(x-2)+3$$

$$\frac{dividend}{divisor}=quotient+\frac{remainder}{divisor}$$

$$dividend=(quotient)\times(divisor)+remainder$$

$$\frac{x^3+2x-12}{x-2}$$

$1x^3+0x^2+2x-12$

$$\begin{array}{r|cccc} 2 & 1 & 0 & 2 & -12 \\ + & & & & \\ \hline & & & & \end{array}$$

$$\begin{array}{r|cccc} 2 & 1 & 0 & 2 & -12 \\ + & & 2 & 4 & 12 \\ \hline & 1 & 2 & 6 & 0 \end{array}$$

$1x^2+2x+6 \quad R=0$

$$\frac{x^3+2x-12}{x-2}=x^2+2x+6$$

$$x^3+2x-12=(x^2+2x+6)(x-2)$$

$$\frac{x^3+2x^2-6x-12}{x+2}$$

$$\begin{array}{r|cccc} -2 & 1 & 2 & -6 & -12 \\ + & & & & \\ \hline & & & & \end{array}$$

$$\begin{array}{r|cccc} -2 & 1 & 2 & -6 & -12 \\ + & & -2 & 0 & 12 \\ \hline & 1 & 0 & -6 & 0 \end{array}$$

$1x^2+0x-6 \quad R:0$

$x^2-6 \quad R:0$

$$\frac{x^3+2x^2-4x+8}{x+2}=x^2-6$$

$$x^3+2x^2-4x+8=(x^2-6)(x+2)$$