

C12 - 3.2 - Factor/Remainder Theorem Notes

Factor Theorem If $(x - a)$ is a factor of $f(x)$, then: $f(a) = 0$

Is $(x - 2)$ a factor of $f(x) = x^3 + x^2 - 8x + 4$?

$$\begin{aligned} f(x) &= x^3 + x^2 - 8x + 4 \\ f(x) &= (2)^3 + (2)^2 - 8(2) + 4 \\ f(2) &= 8 + 4 - 16 + 4 \\ f(2) &= 0 \\ (2,0) \end{aligned}$$

$$\begin{aligned} x - 2 &= 0 \\ x &= 2 \end{aligned}$$

$f(a) = 0$
 $(x - a)$
Is a Factor

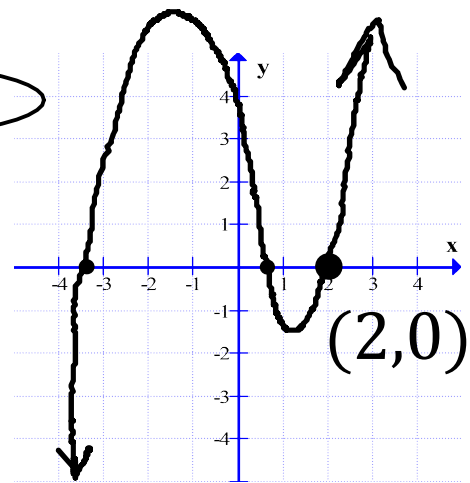
x - intercept
Synthetic Division

$$\begin{array}{r} x^3 + x^2 - 8x + 4 \\ x - 2 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 4 \\ & \downarrow & \nearrow & & \\ + & 1 & 3 & -2 & 0 \end{array}$$

$(x - 2)$ Is a Factor

Remainder = 0



Remainder Theorem If $(x - a)$ is not a factor of $f(x)$, then: $f(a) = \text{remainder}$

Is $(x - 2)$ a factor of $f(x) = x^3 + x^2 - 8x + 5$?

$$\begin{aligned} f(x) &= x^3 + x^2 - 8x + 5 \\ f(x) &= (2)^3 + (2)^2 - 8(2) + 5 \\ f(2) &= 8 + 4 - 16 + 5 \\ f(2) &= 1 \end{aligned}$$

$$\begin{aligned} x - 2 &= 0 \\ x &= 2 \end{aligned}$$

$f(a) \neq 0 \leftarrow R$
 $(x - a)$
Is Not a Factor

$(2,1)$

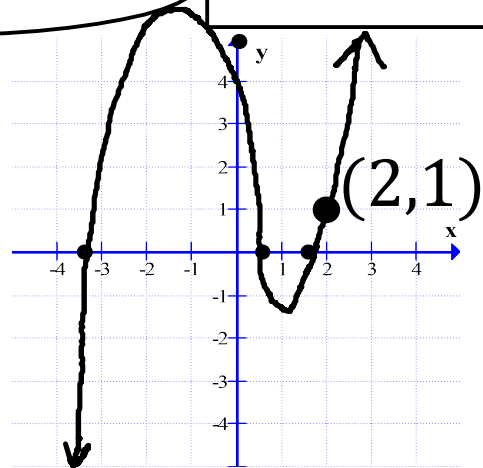
Synthetic Division

$$\begin{array}{r} x^3 + x^2 - 8x + 5 \\ x - 2 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 5 \\ & \downarrow & \nearrow & & \\ + & 1 & 3 & -2 & 1 \end{array}$$

$(x - 2)$ is Not a Factor!

Remainder = 1



C12 - 3.2 - Find K Notes/HW

Find k if $(x + 3)$ is a factor.

$$f(-3) = 0$$

$$f(x) = x^3 + 2x^2 + kx - 6$$

$$f(-3) = (-3)^3 + 2(-3)^2 + k(-3) - 6 = 0$$

$$-15 - 3k = 0$$

$$k = -5$$

$$f(x) = x^3 + 2x^2 - 5x - 6$$

Find k if $f(x)$ is divided by $(x - 1)$ and the remainder is -8 .

$$f(1) = -8$$

$$f(x) = x^3 + 2x^2 - 5x + k$$

$$f(1) = (1)^3 + 2(1)^2 - 5(1) + k = -8$$

$$-2 + k = -8$$

$$k = -6$$

$$f(x) = x^3 + 2x^2 - 5x - 6$$

Find k if $(x - 3)$ is a factor.

$$f(x) = x^3 - 6x^2 + kx - 6 \quad k=11$$

Find k if $f(x)$ is divided by $(x + 3)$ and the remainder is 25.

$$f(x) = x^3 + kx^2 - 4x - 8 \quad k=2$$

Find k if when divided by $(x - 5)$ the remainder is 24 if $(x - 2)$ is a factor.

$$f(x) = x^3 - 6x^2 + 11x + k \quad k=-6$$

Find k if when divided by $(x - 2)$ the remainder is the same as if divided by $(x - 3)$.

$$f(x) = x^3 + 2x^2 - 4x + k \quad k=-8$$