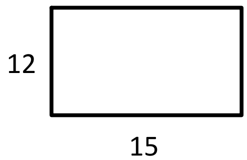
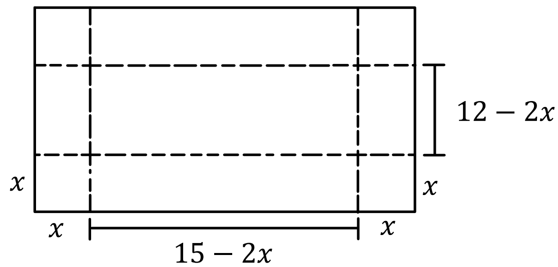
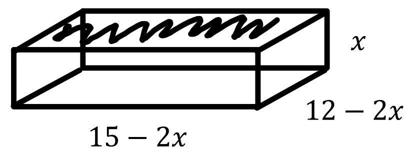
C12 - 3.5 - Open Rectangular Box Cut Side x Notes

An open rectangular box is made by cutting equal integer lengths from each corner of a 12 cm by 15 cm rectangular piece of cardboard, then folding up the sides. Find the length of the square that must be cut from each corner so the box has a volume of 162 cm^3. And find length to cut for Max Volume and find Max Volume.

$let\ x = length\ to\ cut$







$v = lwh$

$$Volume = length \times width \times height$$
$$V = (12 - 2x)(15 - 2x)x$$
$$162 = (12 - 2x)(15 - 2x)(x)$$
$$162 = 180x - 54x^2 + 4x^3$$
$$0 = 4x^3 - 54x^2 + 180x - 162$$
$$0 = 2x^3 - 27x^2 + 90x - 81$$

Potential Factors: The factors of 81: ~~± 27~~, ~~± 9~~, $\pm 3, \pm 1$

Solve by inspection: $Check: x = 1, 3$

$$f(x) = 2x^3 - 27x^2 + 90x - 81$$
$$f(3) = 2(3)^3 - 27(3)^2 + 90(3) - 81$$
$$f(3) = 54 - 243 + 270 - 81 \qquad = 0$$

$Domain$: $x > 0$, $x\ cant\ be\ negative!$
$x < 6$, $Cant\ cut\ 2\ 6's\ off\ a\ 12!$

We need to reject 6 and greater so we don't get negatives lengths.

3	2	−27	90	−81
+	↓	6	−63	81
	2	−21	27	0

$2x^2 - 21x + 27$

$\therefore x = 3\ cm$

$2x^2 - 21x + 27$

$(2x - 3)(x - 9)$

~~$x = 1.5$~~ ~~$x = 9$~~ $x < 6$

Reject non-integers

$Check\ Answer$

$l = 15 - 2x$
$l = 15 - 2(3)$
$l = 9$

$w = 12 - 2x$
$w = 12 - 2(3)$
$w = 6$

$h = x$
$h = 3$

$V = lwh$
$V = 9 \times 6 \times 3$
$V = 162\ cm^2$

$V = (12 - 2x)(15 - 2x)x$

$x = 2.2\ cm \qquad Max\ V = 177.2\ cm^2$

2nd Calc Max