# C12 - 3.1 - Long/Synthetic Division $R = 0$ Notes

$\frac{64}{4} = ?$

*Goes Into*
*Multiply*
*Subtract*
*Bring Down*
*Repeat*

$$\begin{array}{rl} 16 & \\ 4\overline{)\,64} & \\ -\underline{\;4\downarrow} & \\ 24 & \textit{Bring down} \\ -\underline{\;24} & \\ 0 & \leftarrow \text{remainder} \end{array}$$

$$\begin{array}{rl} & \textit{quotient} \\ \textit{divisor} & \overline{)\,\textit{dividend}} \end{array}$$

$\frac{64}{4} = 16$

$64 = 4 \times 16$

$\boxed{\frac{dividend}{divisor} = quotient}$

$\boxed{dividend = (quotient)(divisor)}$

$\frac{x^2 + 5x + 6}{x + 3} = ?$

$$\begin{array}{rl} x + 2 & \\ x + 3\,\overline{)\,x^2 + 5x + 6} & \\ -\;\underline{x^2 + 3x\;\;\downarrow} & \\ 2x + 6 & \\ -\;\underline{2x + 6} & \\ 0 & \leftarrow \text{remainder} \end{array}$$

$\frac{x^2 + 5x + 6}{x + 3} = x + 2$

$x^2 + 5x + 6 = (x + 2)(x + 3)$

$\boxed{\frac{P(x)}{x - a} = Q(x)}$

$\boxed{P(x) = Q(x)(x - a)}$

Synthetic Division

$\frac{x^2 + 5x + 6}{x + 3} = ?$

$x + 3 = 0$
$x = -3$

$1x^2 + 5x + 6$

$$\begin{array}{r|rrr} -3 & 1 & 5 & 6 \\ + & \downarrow & -3 & -6 \\ \hline & 1 & 2 & 0 \end{array} \;\leftarrow \text{remainder}$$

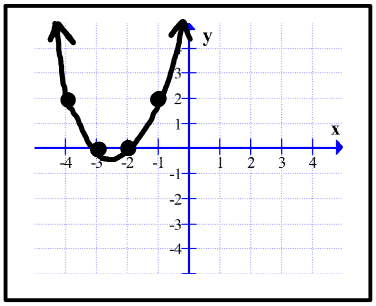
$1x + 2$

The exponents of x go down by one.

Factor Theorem

$f(x) = x^2 + 5x + 6$
$f(-3) = (-3)^2 + 5(-3) + 6$
$f(-3) = 0$

$(-3,0)$



# C12 - 3.1 - Long/Synthetic Division Notes

$\frac{65}{4} = ?$

$$\begin{array}{rl} & \boxed{16} \\ 4 & \overline{)\,65} \\ - & \underline{\;4\downarrow} \\ & \;25 \quad \textit{Bring down} \\ - & \underline{\;24} \\ & \quad 1 \leftarrow \text{remainder} \end{array}$$

$$\boxed{\begin{array}{r} \textit{quotient} \\ \textit{divisor}\ \overline{)\ \textit{dividend}} \end{array}}$$

$$\boxed{\frac{65}{4} = 16 + \frac{1}{4}} \qquad \boxed{65 = 4 \times 16 + 1}$$

$$\boxed{\frac{\textit{dividend}}{\textit{divisor}} = \textit{quotient} + \frac{\textit{remainder}}{\textit{divisor}}} \qquad \boxed{\textit{dividend} = (\textit{quotient})(\textit{divisor}) + \textit{remainder}}$$

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$\frac{x^2+5x+9}{x+3} = ?$

$$\begin{array}{rl} & \boxed{x+2} \\ x+3 & \overline{)\,x^2+5x+9} \\ - & \underline{x^2+3x\quad\downarrow} \\ & \qquad 2x+9 \\ - & \underline{\qquad 2x+6} \\ & \qquad\qquad 3 \leftarrow \text{remainder} \end{array}$$

$$\boxed{\frac{x^2+5x+9}{x+3} = x+2+\frac{3}{x+3}} \qquad \boxed{x^2+5x+9 = (x+2)(x+3)+3}$$

$$\boxed{\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}} \qquad \boxed{P(x) = Q(x)(x-a) + R}$$

## Synthetic Division

$\frac{x^2+5x+9}{x+3} = ?$

$x+3 = 0$

$x = -3$

$1x^2+5x+9$

$$\begin{array}{r|rrr} -3 & 1 & 5 & 9 \\ + & \downarrow & -3 & -6 \\ \hline & 1 & 2 & 3 \leftarrow \text{remainder} \end{array}$$

$\boxed{1x+2}\quad R{:}\,3$

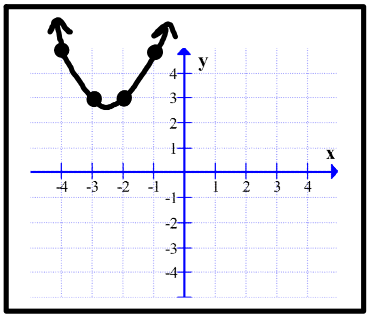
**Remainder Theorem**

$f(x) = x^2+5x+6$

$f(-3) = (-3)^2+5(-3)+9$

$f(-3) = 3$

$(-3,3)$



# C12 - 3.1 - Synthetic Division $R = 0$ Notes

$$\frac{x^3+x^2-8x+4}{x-2}$$

$x-2=0$
$x=2$

Set denominator equal to zero and solve.
$Denominator = 0$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 4 \\ + & & & & \\ \hline \end{array}$$

Place that number to the left.
Write the coefficients. $1x^3+1x^2-8x+4$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 4 \\ + & \downarrow & \nearrow 2 & 6 & -4 \\ \hline & 1 & 3 & -2 & 0 \end{array}$$

1) Bring down the first coefficient
2) $(2)\times 1 = 2$
3) $1+2=3$
4) Repeat last two steps.

$1x^2+3x-2 \qquad R=0$

$$\frac{x^3+x^2-8x+4}{x-2} = x^2+3x-2$$

$$x^3+x^2-8x+4 = (x^2+3x-2)(x-2)$$

$$\boxed{\frac{dividend}{divisor} = quotient}$$

$$\boxed{dividend = (quotient)(divisor)}$$

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$$\frac{x^3+2x^2-5x-6}{x+1}$$

$x+1=0$
$x=-1$

Set denominator equal to zero and solve.
$Denominator = 0$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ + & & & & \\ \hline \end{array}$$

Place that number to the left.
Write the coefficients. $1x^3+2x^2-5x-6$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ + & \downarrow & \nearrow -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$$\begin{array}{rl} & x^2 + x - 6 \\ \hline x+1\,\big) & x^3+2x^2-5x-6 \\ - & \underline{x^3+x^2} \downarrow \\ & \quad x^2-5x \\ - & \quad \underline{x^2+x} \downarrow \\ & \qquad -6x-6 \\ - & \qquad \underline{-6x-6} \\ & \qquad\qquad 0 \qquad R=0 \end{array}$$

$1x^2+1x-6 \qquad R=0$

$x^2+x-6$
$(x+3)(x-2)$

Factor

$$\frac{x^3+2x^2-5x-6}{x+1} = (x+3)(x-2)$$

$$x^3+2x^2-5x-6 = (x+3)(x-2)(x+1)$$

$$\boxed{\frac{P(x)}{x-a} = Q(x)}$$

$$\boxed{P(x) = Q(x)(x-a)}$$

# C12 - 3.1 - Synthetic Division Remainder/Gap Notes

$$\frac{x^3+x^2-8x+7}{x-2}$$

$$+\quad\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 7 \\ & & & & \\ \hline \end{array}$$

$$+\quad\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 7 \\ & \downarrow & \nearrow 2 & 6 & -4 \\ \hline & 1 & 3 & -2 & 3 \end{array}$$

remainder

$$1x^2+3x-2 \qquad R=3$$

$$\begin{array}{rl} & x^2+3x-2 \\ \cline{2-2} x-2\,) & x^3+x^2-8x+7 \\ - & x^3-2x^2 \\ \cline{2-2} & +3x^2-8x \\ - & 3x^2-6x \\ \cline{2-2} & -2x+7 \\ - & -2x+4 \\ \cline{2-2} & 3 \qquad R=3 \end{array}$$

*The remainder is the y value when* $x=2$

$(2,3)$

$f(2)=(2)^3+(2)^2-8(2)+7$

$f(2)=8+4-16+7$

$f(2)=3$

$$\frac{x^3+x^2-8x+6}{x-2}=x^2+3x-2+\frac{3}{x-2}$$

$$x^3+x^2-8x+6=(x^2+3x-2)(x-2)+3$$

$$\frac{dividend}{divisor}=quotient+\frac{remainder}{divisor}$$

$$dividend=(quotient)\times(divisor)+remainder$$

$$\frac{x^3+2x-12}{x-2}$$

$$1x^3+0x^2+2x-12$$

$$+\quad\begin{array}{r|rrrr} 2 & 1 & 0 & 2 & -12 \\ & & & & \\ \hline \end{array}$$

$$+\quad\begin{array}{r|rrrr} 2 & 1 & 0 & 2 & -12 \\ & \downarrow & \nearrow 2 & 4 & 12 \\ \hline & 1 & 2 & 6 & 0 \end{array}$$

$$1x^2+2x+6 \qquad R=0$$

$$\frac{x^3+2x-12}{x-2}=x^2+2x+6$$

$$x^3+2x-12=(x^2+2x+6)(x-2)$$

$$\frac{x^3+2x^2-6x-12}{x+2}$$

$$+\quad\begin{array}{r|rrrr} -2 & 1 & 2 & -6 & -12 \\ & & & & \\ \hline \end{array}$$

$$+\quad\begin{array}{r|rrrr} -2 & 1 & 2 & -6 & -12 \\ & \downarrow & \nearrow -2 & 0 & 12 \\ \hline & 1 & 0 & -6 & 0 \end{array}$$

$$1x^2+0x-6 \qquad R:0$$

$$x^2-6 \qquad R:0$$

$$\frac{x^3+2x^2-4x+8}{x+2}=x^2-6$$

$$x^3+2x^2-4x+8=(x^2-6)(x+2)$$

# C12 - 3.2 - Factor/Remainder Theorem Notes

$Factor\ \ Theorem$ If $(x - a)$ is a factor of $f(x)$, then: $f(a) = 0$

$Is$ $(x - 2)$ a factor of $f(x) = x^3 + x^2 - 8x + 4$?

$f(x) = x^3 + x^2 - 8x + 4$
$f(x) = (2)^3 + (2)^2 - 8(2) + 4$
$f(2) = 8 + 4 - 16 + 4$
$f(2) = 0$
$(2,0)$

$x - 2 = 0$
$x = 2$

$f(a) = 0$
$(x - a)$
Is a Factor

$x - intercept$

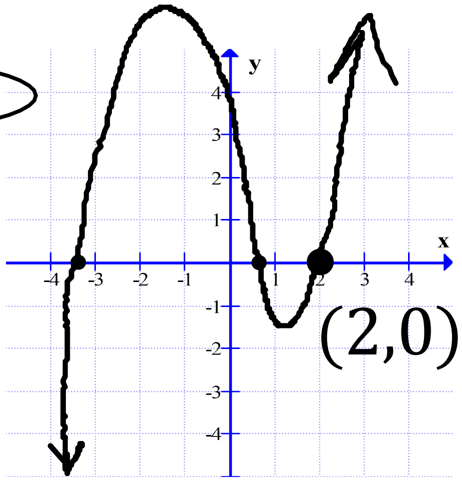
$Synthetic\ Division$

$(x - 2)$Is a Factor

$$\frac{x^3 + x^2 - 8x + 4}{x - 2}$$

Remainder = 0

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 4 \\ & \downarrow & 2 & 6 & -4 \\ \hline + & 1 & 3 & -2 & 0 \end{array}$$



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$Remainder\ Theorem$ If $(x - a)$ is not a factor of $f(x)$, then: $f(a) = remainder$

$Is$ $(x - 2)$ a factor of $f(x) = x^3 + x^2 - 8x + 5$ ?

$f(x) = x^3 + x^2 - 8x + 5$
$f(x) = (2)^3 + (2)^2 - 8(2) + 5$
$f(2) = 8 + 4 - 16 + 5$
$f(2) = 1$

$x - 2 = 0$
$x = 2$

Remainder = 1

$f(a) \neq 0 \leftarrow$ R
$(x - a)$
Is Not a Factor

$(2,1)$

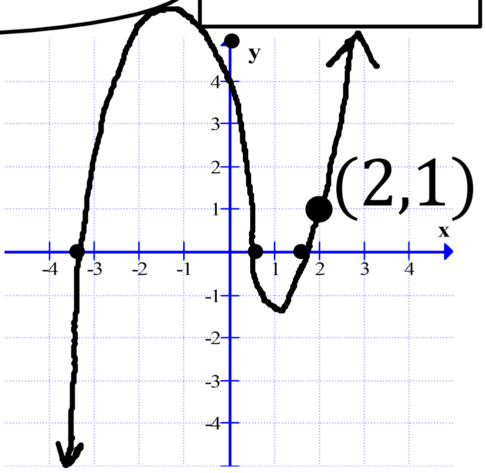
$(x - 2)$ $is$ Not a Factor!

$Synthetic\ Division$

$$\frac{x^3 + x^2 - 8x + 4}{x - 2}$$

Remainder = 1

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 5 \\ & \downarrow & 2 & 6 & -4 \\ \hline + & 1 & 3 & -2 & 1 \end{array}$$



# C12 - 3.2 - Find K Notes/HW

$Find\ k\ if\ (x+3)\ is\ a\ factor.$ $\boxed{f(-3)=0}$

$f(x)=x^3+2x^2+kx-6$

$f(-3)=(-3)^3+2(-3)^2+k(-3)-6=0$

$-15-3k=0$

$k=-5$

$f(x)=x^3+2x^2-5x-6$

$Find\ k\ if\ f(x)\ is\ divided\ by\ (x-1)$
$and\ the\ remainder\ is\ -8.$ $\boxed{f(1)=-8}$

$f(x)=x^3+2x^2-5x+k$

$f(x)=(1)^3+2(1)^2-5(1)+k=-8$

$-2+k=-8$

$k=-6$

$f(x)=x^3+2x^2-5x-6$

$Find\ k\ if\ (x-3)\ is\ a\ factor.$

$f(x)=x^3-6x^2+kx-6$ k=11

$Find\ k\ if\ f(x)\ is\ divided\ by\ (x+3)\ and\ the$
$remainder\ is\ 25.$

$f(x)=x^3+kx^2-4x-8$ k=2

$Find\ k\ if\ when\ divided\ by\ (x-5)\ the$
$remainder\ is\ 24\ if\ (x-2)\ is\ a\ factor.$

$f(x)=x^3-6x^2+11x+k$ k=-6

$Find\ k\ if\ when\ divided\ by\ (x-2)\ the$
$remainder\ is\ the\ same\ as\ if\ divided\ by\ (x-3).$

$f(x)=x^3+2x^2-4x+k$ k=-8

# C12 - 3.3 - Factoring Trinomials Notes

$\boldsymbol{f(x) = x^2 - 6x + 5}$

Potential Factors: Factors of c = $\pm 5$ and $\pm 1$

$f(x) = x^2 \ldots\ldots\ldots + 5$ $\pm 1{,}5$

$f(x) = x^2 \ldots\ldots\ldots + 5$

*Examples*:

$f(x) = (x-5)(x-1)$

$f(x) = (x+5)(x+1)$

$(x+a)(x+b) = x^2 \ldots + ab$

**Solve by inspection.**

$f(1) = 1^2 - 6(1) + 5$
$f(1) = 0$

Stop here if you want

$(x-1)$ *is a factor.*
$(1{,}0)$ $x - int$

$f(-1) = (-1)^2 - 6(-1) + 5$
$f(-1) = 12$

$(x+1)$ *is NOT a factor*
$(-1{,}12)$ $(x, y)$

$f(5) = 5^2 - 6(5) + 5$
$f(5) = 0$

$(x-5)$ *is a factor*
$(5{,}0)$ $x - int$

| *x* | *y* |
|---|---|
| *1* | *0* |
| *-1* | *12* |
| *5* | *0* |

**Do synthetic division with 1**

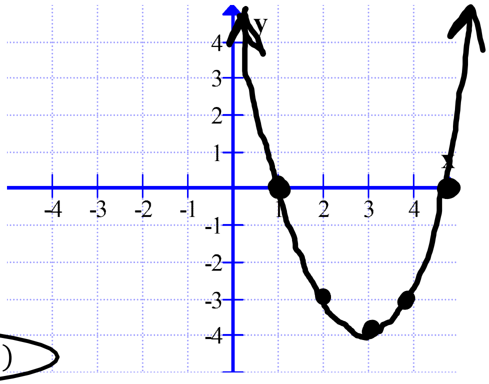
$$\begin{array}{r|rrr} 1 & 1 & -6 & 5 \\ + & \downarrow & 1 & -5 \\ \hline & 1 & -5 & 0 \end{array}$$

$x^2 - 6x + 5$

$x - 5$

$$\frac{x^2 - 6x + 5}{x - 1} = x - 5$$

$$x^2 - 6x + 5 = (x-5)(x-1)$$



**Or Do synthetic division with** 5!

$$\begin{array}{r|rrr} 5 & 1 & -6 & 5 \\ + & \downarrow & 5 & -5 \\ \hline & 1 & -1 & 0 \end{array}$$

$x - 1$

$$\frac{x^2 - 6x + 5}{x - 5} = x - 1$$

$$x^2 - 6x + 5 = (x-1)(x-5)$$

$\updownarrow$ $(x-1)$ *is a factor ?*
$f(1) = 0,$ *if you put* $+1$ *in for x it must equal zero, (or it is not a factor)*
$(+1{,}0)$ *is an x − intercept*

# C12 - 3.3 - Factoring Quadomials Notes

$f(x) = x^3 + 2x^2 - 5x - 6$

Potential Factors: Factors of c = $\pm1, \pm2, \pm3, \pm6$

$f(x) = x^3 \ldots\ldots\ldots\ldots\ldots\ldots - 6$ $\quad \pm1,2,3,6,$

$f(x) = x^3 \ldots\ldots. -6$

*Examples*:

$f(x) = (x-2)(x-3)(x-1)$

$f(x) = (x+2)(x+3)(x-1)$

$f(x) = (x+2)(x-3)(x+1)$

**Solve by inspection.**

$f(1) = (1)^3 + 2(1)^2 - 5(1) - 6$
$f(1) = 1 + 2 - 5 - 6$
$f(1) = -8$

$(x-1)$ *is NOT a factor*

$f(-1) = (-1)^3 + 2(-1)^2 - 5(-1) - 6$
$f(-1) = -1 + 2 + 5 - 6$
$f(-1) = 0$

$(x+1)$ *is a factor*

$6^3 = 216,$ *its not going to be* $6!$

$(x-a)(x+b)(x-c) = x^3 \ldots + abc$

| $x$ | $y$ |
|---|---|
| 1 | -8 |
| -1 | 0 |
| 6 | 252 |

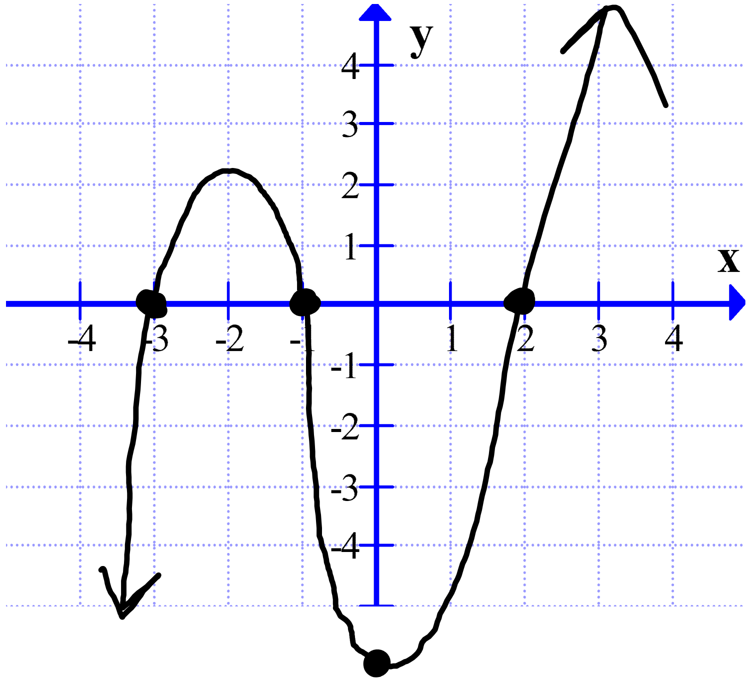
**Do synthetic division with $-1$**

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ + & \downarrow & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$1x^2 + 1x - 6$
$(x+3)(x-2)$ *Factor*

$f(x) = (x+3)(x-2)(x+1)$

$f(-3) = 0$
$f(2) = 0$
$f(-1) = 0$



y-int: $(0, -6)$

# C12 - 3.3 - Potential Factors Notes $\pm \dfrac{d}{a}$

$f(x) = x^3 + x^2 - 8x + 4$

$Potential\ Factors\colon\ \pm 1, \pm 2, \pm 4$

$factors\ of\ "d"$

**Solve by inspection**

$$\begin{aligned} f(1) &= (1)^3 + (1)^2 - 8(1) + 4 &&= -2 \\ f(-1) &= (-1)^3 + (-1)^2 - 8(-1) + 4 &&= 12 \\ f(2) &= (2)^3 + (2)^2 - 8(2) + 4 &&= 0 \end{aligned}$$

$(x-1)\ is\ NOT\ a\ factor$

$(x+1)\ is\ NOT\ a\ factor$

$(x-2)\ is\ a\ factor \quad (2{,}0)$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 4 \\ + & \downarrow & \nearrow 2 & 6 & -4 \\ \hline & 1 & 3 & -2 & 0 \end{array}$$

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$\boldsymbol{f(x) = 3x^2 + 5x - 2}$

$Potential\ Factors\colon\ \pm 2, \pm 1, \pm\frac{2}{3}, \pm\frac{1}{3}$

$factors\ of\ "c"$

$and\ \dfrac{factors\ of\ "c"}{factors\ of\ "a"}$

**Solve by inspection**

$$\begin{aligned} f(-1) &= 3(-1)^2 + 5(-1) - 2 &&= -4 \\ f(1) &= 3(1)^2 + 5(1) - 2 &&= 6 \\ f(2) &= 3(2)^2 + 5(2) - 2 &&= 20 \\ f(-2) &= 3(-2)^2 + 5(-2) - 2 &&= 0 \end{aligned}$$

$(x+1)\ is\ NOT\ a\ factor$

$(x-1)\ is\ NOT\ a\ factor$

$(x-2)\ is\ NOT\ a\ factor$

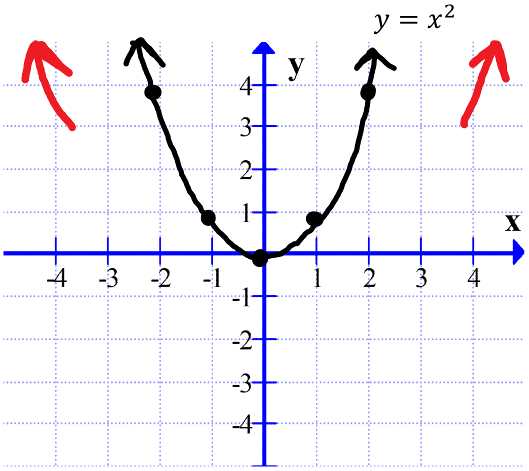
$(x+2)\ is\ a\ factor \quad (-2{,}0)$

$$\begin{array}{r|rrr} -2 & 3 & 5 & -2 \\ + & \downarrow & -6 & 2 \\ \hline & 3 & -1 & 0 \end{array}$$

# C12 - 3.4 - End Behaviour Polynomials Notes

Leading Term
Table of Values

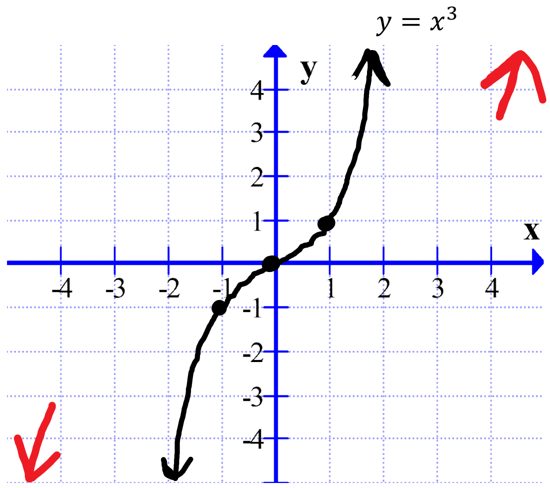
## $+\#x^{even}$



Q2,Q1

| x | y |
|---|---|
| -10 | + |
| +10 | + |

## $+\#x^{odd}$

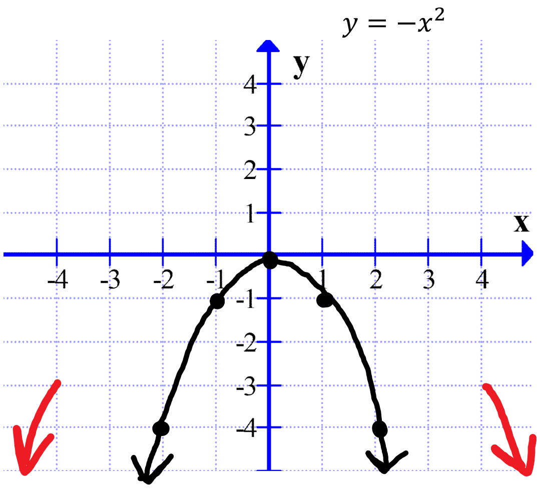


Q3,Q1

| x | y |
|---|---|
| -10 | - |
| +10 | + |

| | |
|---|---|
| $y \geq \#$ | |
| *Range* | $yER$ |
| $y \leq \#$ | |

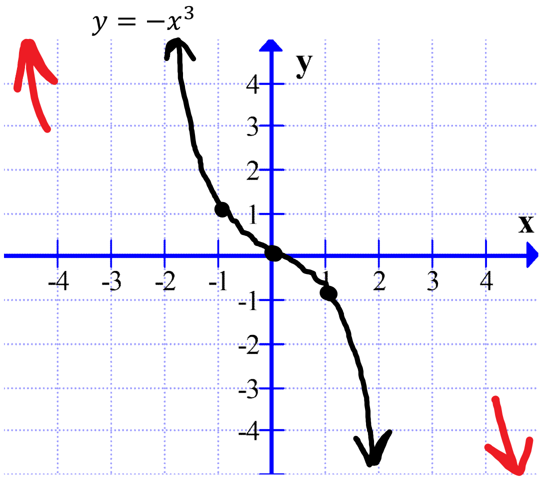
## $-\#x^{even}$



Q3,Q4

| x | y |
|---|---|
| -10 | - |
| +10 | - |

## $-\#x^{odd}$

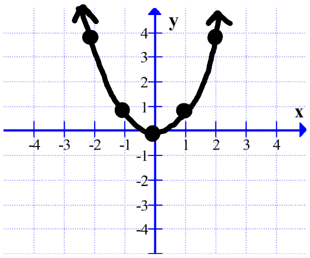


Q2,Q4

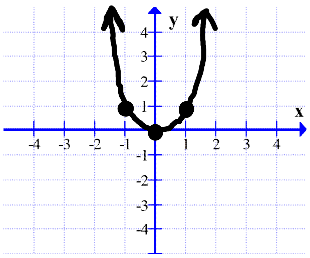
| x | y |
|---|---|
| -10 | + |
| +10 | - |

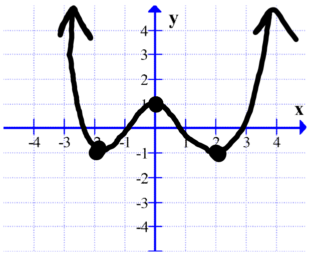
# C12 - 3.4 - End Behaviour Polynomials Notes

Leading Term
Table of Values

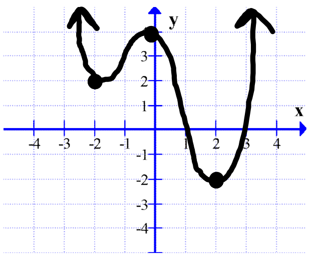


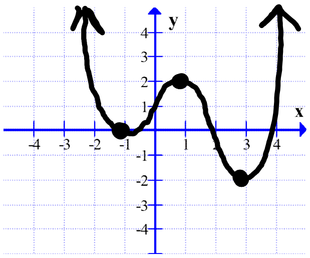
$y = x^2$

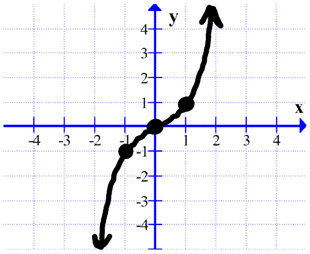




$y = x^4$ ...

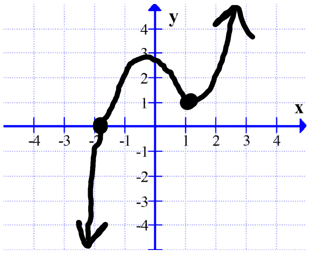


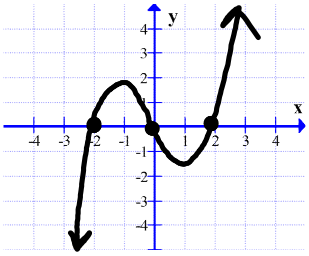


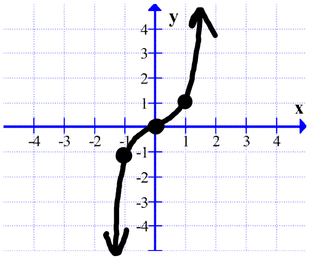


$y = x^3$

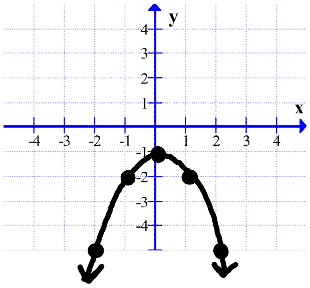
$y = x^3$ ...



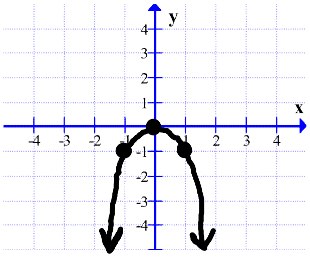




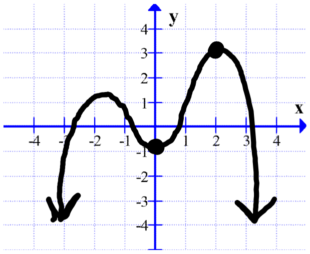
$y = x^5$



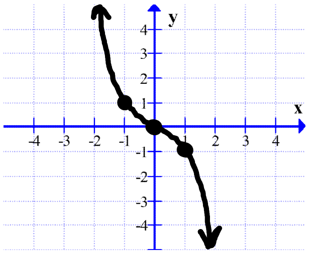
$y = -x^2$



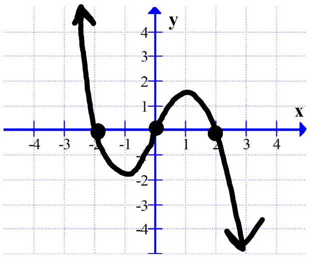
$y = -x^4$



$y = -x^4$ ...

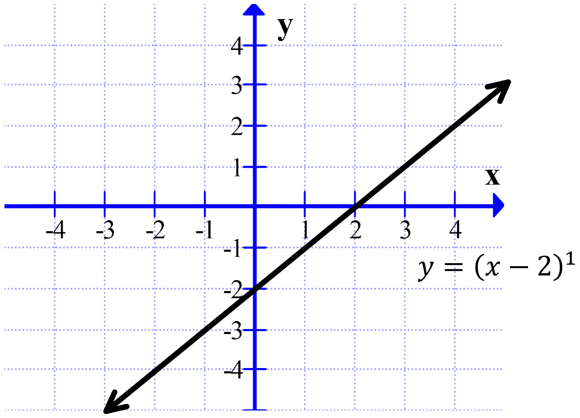


$y = -x^3$



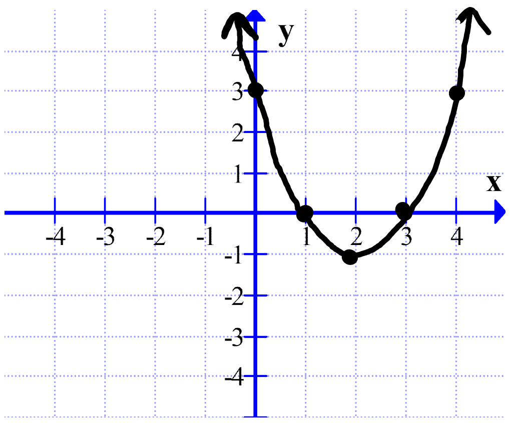
$y = -x^3$ ...

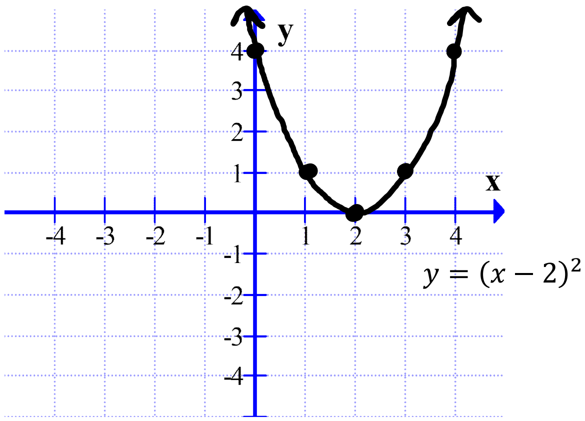
# C12 - 3.4 - Multiplicity (Factor Exponents) Graph Notes



$y = (x-2)^1$

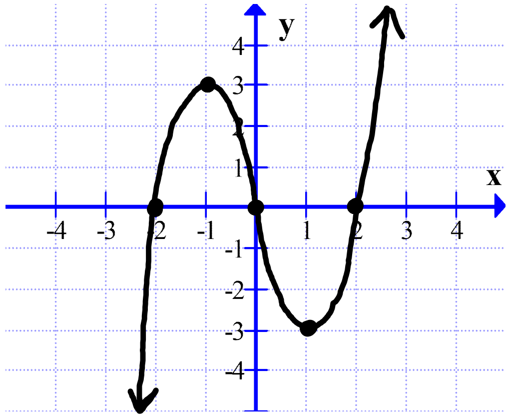
$y = (x-1)^1(x-3)^1$

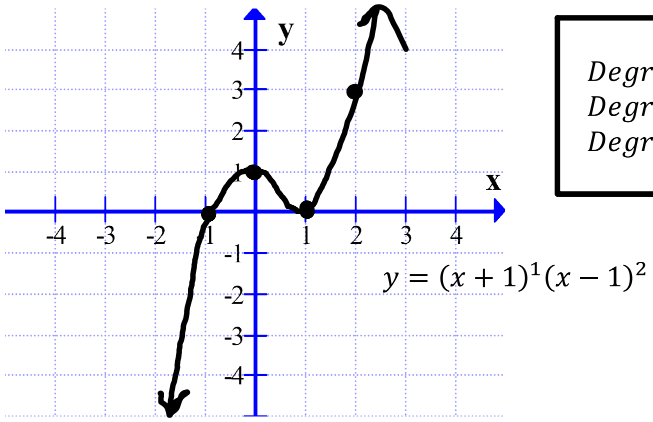




$y = (x-2)^2$

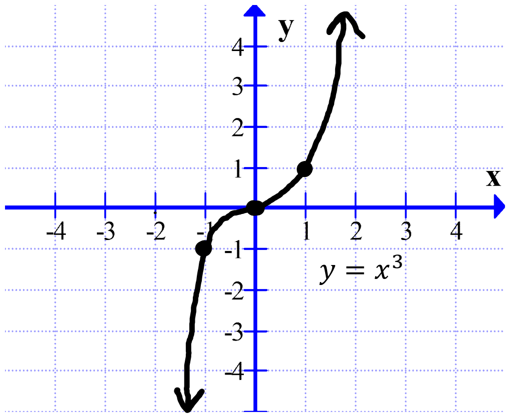
$y = x^1(x-2)^1(x+2)^1$



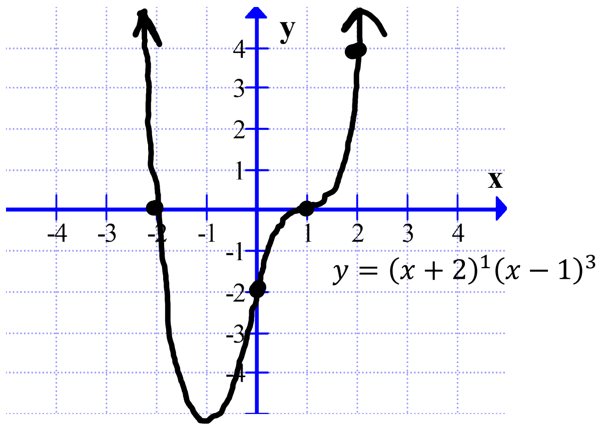


$y = (x+1)^1(x-1)^2$

$\textit{Degree 1: Straight through } x - \textit{intercept}$
$\textit{Degree 2: Bounce off } x - \textit{intercept}$
$\textit{Degree 3: Chair Shape through } x - \textit{intercept}$

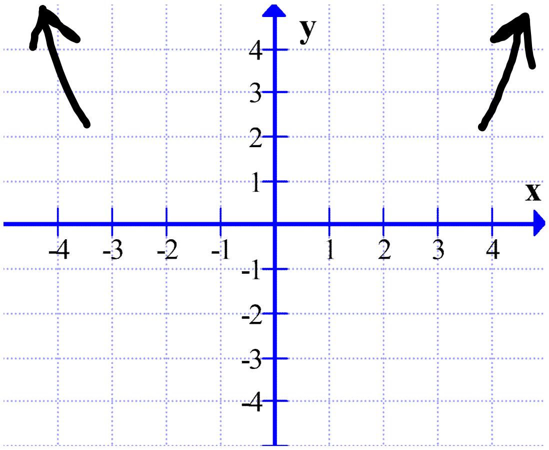


$y = x^3$



$y = (x+2)^1(x-1)^3$

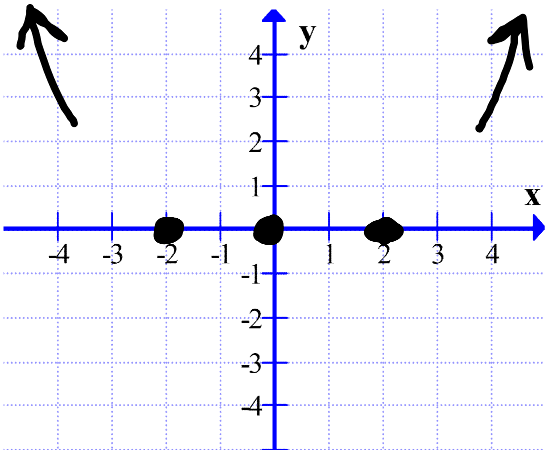
# C12 - 3.4 - Graph $y = x(x-2)^2(x+2)^3$ Notes



$\boxed{y = x(x-2)^2(x+2)^3}$

1) *End Behavior*

$y = x(x-2)^2(x+2)^3$
$y = x(x)^2(x)^3$
$y = +x^6 \xrightarrow{\qquad} y = +x^{even}$
Q3,Q1

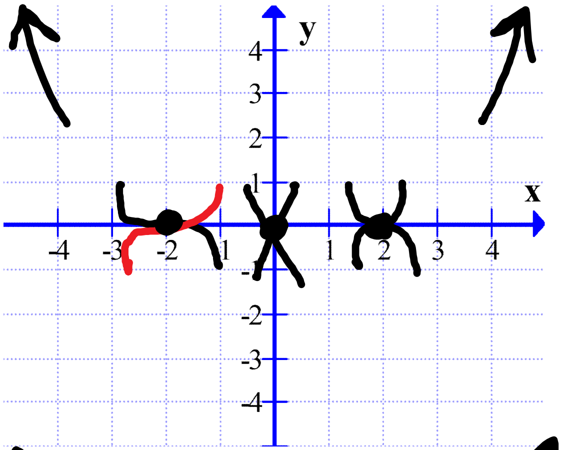


2) $x - intercepts, y\ intercept$

| | | |
|---|---|---|
| $x-2=0$<br>$x=2$ | $x=0$ | $x+2=0$<br>$x=-2$ |
| $(0,2)$ | $(0,0)$ | $(0,-2)$ |

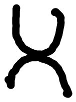
$y = x(x-2)^2(x+2)^3$
$y = 0(0-2)^2(0+2)^3$
$y = 0(-2)^2(2)^3$
$y = 0(-1)(8)$
$y = 0$

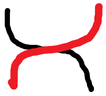
*y-int*: $(0,0)$

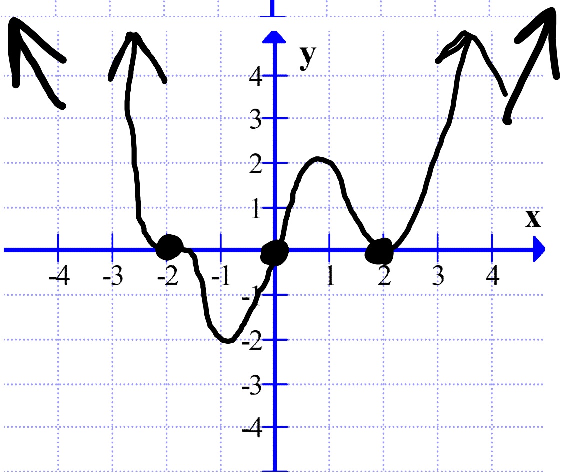


3) *Multiplicity*

| | | |
|---|---|---|
| $(x-2)^2$ | $x^1$ | $(x+2)^3$ |
| $x=2$<br>*Degree* 2 | $x=0$<br>*Degree* 1 | $x=-2$<br>*Degree* 3 |
| $U - shape$ | *Straight through* | *Chair shape* |







4) *Graph*

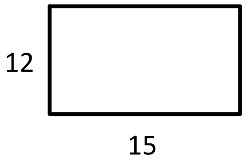
$y = x(x-2)^2(x+2)^3$

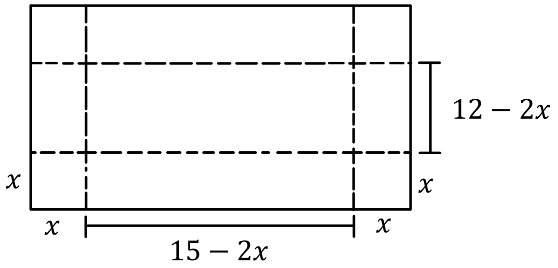
*Start from an arrow*
*Chair at* $x = -2$
*Straight through at* $x = 0$
*Bounce at* $x = 2$
*End at an arrow*

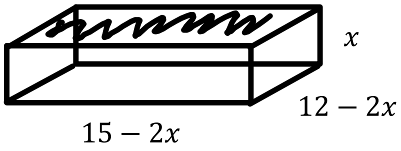
# C12 - 3.5 - Open Rectangular Box Cut Side $x$ Notes

An open rectangular box is made by cutting equal integer lengths from each corner of a 12 cm by 15 cm rectangular piece of cardboard, then folding up the sides. Find the length of the square that must be cut from each corner so the box has a volume of 162 $cm^3$. And find length to cut for Max Volume and find Max Volume.

$let\ x = length\ to\ cut$







$\boxed{v = lwh}$

$$
\begin{aligned}
Volume &= length \times width \times height \\
V &= (12 - 2x)(15 - 2x)x \\
162 &= (12 - 2x)(15 - 2x)(x) \\
162 &= 180x - 54x^2 + 4x^3 \\
0 &= 4x^3 - 54x^2 + 180x - 162 \\
0 &= 2x^3 - 27x^2 + 90x - 81
\end{aligned}
$$

**Potential Factors:** The factors of 81: $\cancel{\pm 27}, \cancel{\pm 9}, \pm 3, \pm 1$

***Solve by inspection***: $Check: x = 1, 3$

$$
\begin{aligned}
f(x) &= 2x^3 - 27x^2 + 90x - 81 \\
f(3) &= 2(3)^3 - 27(3)^2 + 90(3) - 81 \\
f(3) &= 54 - 243 + 270 - 81 \qquad = 0
\end{aligned}
$$

$Domain$: $x > 0$, $\quad x\ cant\ be\ negative$!
$x < 6$, $\quad Cant\ cut\ 2\ 6's\ off\ a\ 12$!

We need to reject 6 and greater so we don't get negatives lengths.

$$
\begin{array}{r|rrrr}
3 & 2 & -27 & 90 & -81 \\
+ & \downarrow & 6 & -63 & 81 \\
\hline
 & 2 & -21 & 27 & 0
\end{array}
$$

$2x^2 - 21x + 27$ $\quad \therefore x = 3\ cm$

$$
\begin{aligned}
&2x^2 - 21x + 27 \\
&(2x - 3)(x - 9)
\end{aligned}
$$

$\cancel{x = 1.5}$ $\quad \cancel{x = 9}$ $\quad \boxed{x < 6}$

Reject non-integers

$Check\ Answer$

$l = 15 - 2x$
$l = 15 - 2(3)$
$l = 9$

$w = 12 - 2x$
$w = 12 - 2(3)$
$w = 6$

$h = x$
$h = 3$

$V = lwh$
$V = 9 \times 6 \times 3$
$V = 162\ cm^2$

$\boxed{V = (12 - 2x)(15 - 2x)x}$

$x = 2.2\ cm \qquad Max\ V = 177.2\ cm^2$