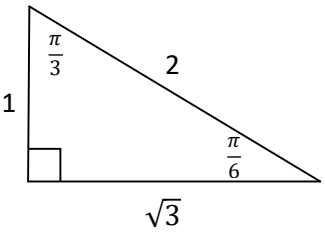
C12 - 4.5 - $\sin 2\theta$ ASTC Special Triangles Notes

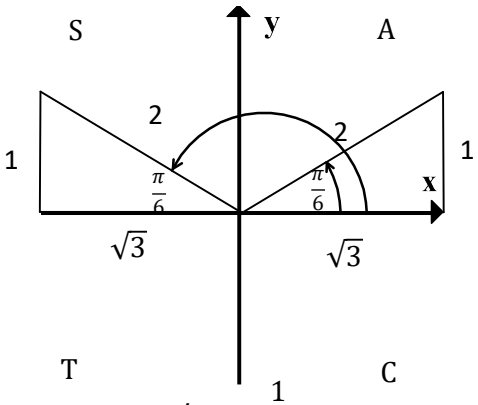
Solve for θ $0^o \le \theta < 2\pi$, *and the general solution*.

$\sin 2\theta = \frac{1}{2}$ $\sin m = \frac{1}{2}$

Let $m = 2\theta$



Draw two triangles where $\sin m$ is positive:
ASTC Quadrant I, II



Label the triangles according to special triangles and SOH CAH TOA

Label the reference angle according to special triangles.

Draw an arrow from the principal axis to the first terminal arm
Draw an arrow from the principal axis to the second terminal arm.

Solve for the arrows θ_{stp}

$sinm = \frac{1}{2}$

$m_{stp} = \frac{\pi}{6}$ $m_{stp} = \pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$

$m_{stp} = \frac{\pi}{6}, \frac{5\pi}{6}$

Check your answer: $\sin\frac{\pi}{6} = \frac{1}{2}$ $\sin\frac{5\pi}{6} = \frac{1}{2}$

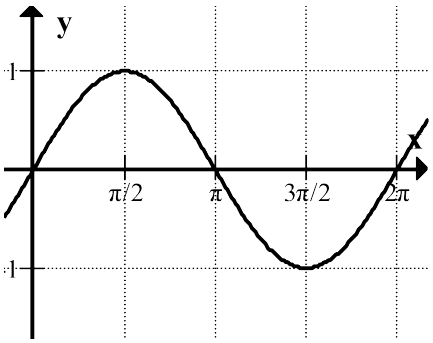
Substitute 2θ back in for m.

$m = \frac{\pi}{6}$ $m = \frac{5\pi}{6}$

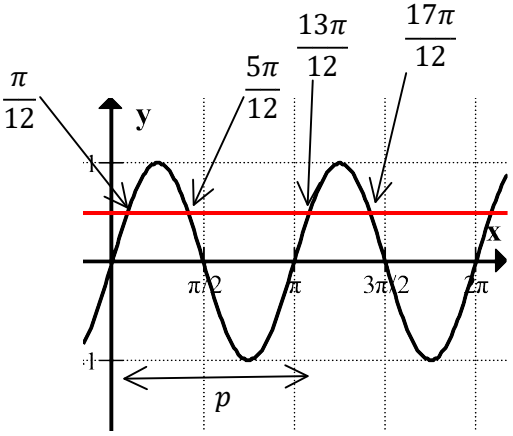
$2\theta = \frac{\pi}{6}$ $2\theta = \frac{5\pi}{6}$

$\theta = \frac{\pi}{6\times 2}$ $\theta = \frac{5\pi}{6\times 2}$

$\theta = \frac{\pi}{12}$ $\theta = \frac{5\pi}{12}$



$y = sin\theta$



$y = sin2\theta$

$y = \frac{1}{2}$

$p = \frac{2\pi}{b}$

$p = \frac{2\pi}{2}$

$= \pi$

$\theta = \theta_{stp} \pm p$

$\theta = \frac{\pi}{12} + \pi$

$\theta = \frac{13\pi}{12}$

$\theta = \theta_{stp} \pm p$

$\theta = \frac{5\pi}{12} + \pi$

$\theta = \frac{17\pi}{12}$

$\theta = \frac{13\pi}{12} + \pi$

~~$\theta = \frac{25\pi}{12} > 2\pi$~~

$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

Add/Subtract period until outside of the domain.

General Solution:

$\theta = \theta_{stp} \pm pn, n \in I$

$\theta = \frac{\pi}{12} \pm \pi n, n \in I$

$\theta = \theta_{stp} \pm pn, n \in I$

$\theta = \frac{5\pi}{12} \pm \pi n, n \in I$

The usual number of answers in the domain times b.

C12 - 4.5 - Algebra Period Equations Notes

$0 \le \theta < 2\pi$

$2sinx + 1 = 0$
$2sinx = -1$
$sinx = -\frac{1}{2}$

$5 - 3cosx = 4$
$-3cosx = -1$
$cosx = \frac{1}{3}$

$sinx = x - 1$

$x = 1.93$

$y = sinx$
$y = x - 1$

Find Intersection

$\cos\left(\frac{\pi}{2}x\right) = 0$
$cosm = 0$

$let\ m = \frac{\pi}{2}x$

...

$m = \frac{\pi}{2}$
$\frac{\pi}{2}x = \frac{\pi}{2}$

$x = 1$

$x = 1 + 4$

$x = 5$

$m = \frac{3\pi}{2}$
$\frac{\pi}{2}x = \frac{3\pi}{2}$

$x = 3$

$x = 3 + 4$

$x = 7$

Reject

$p = \frac{2\pi}{b}$
$p = \frac{2\pi}{\frac{\pi}{2}}$
$p = 2\pi \times \frac{2}{\pi}$
$p = 4$

$\tan(x - 1) = -0.2$
$tanm = -0.2$

$let\ m = x - 1$

...

$m = 2.94$
$x - 1 = 2.94$

$x = 3.94$

$x = 3.94 - \pi$

$x = 0.80$

$m = 6.09$
$x - 1 = 6.09$

$x = 7.09$

$x = 7.09 - \pi$

$x = 3.94$

Reject

$p = \frac{\pi}{b}$
$p = \frac{\pi}{1}$
$p = \pi$

$\sin\left(\frac{\pi}{4}(x - 6)\right) = \frac{1}{2}$
$sinm = \frac{1}{2}$

$let\ m = \frac{\pi}{4}(x - 6)$

Add/Subtract period until outside of the domain.

...

$m = \frac{\pi}{6}$
$\frac{\pi}{4}(x - 6) = \frac{\pi}{6}$
$x - 6 = \frac{2}{3}$
$x = \frac{20}{3}$

$x = 6.67$

$x = 6.67 - 8$

$x = -1.33$

$m = \frac{5\pi}{6}$
$\frac{\pi}{4}(x - 6) = \frac{5\pi}{6}$
$x - 6 = \frac{10}{3}$
$x = \frac{28}{3}$

$x = 9.33$

$x = 9.33 - 8$

$x = 1.33$

The usual number of answers in the domain times b.

$p = \frac{2\pi}{b}$
$p = \frac{2\pi}{\frac{\pi}{4}}$
$p = 8$