

C12 - 4.6 - Equations Algebra Notes

$$\sin x + \sin x \in 2\sin x$$

$$5\cos x - 3\cos x \in 2\cos x$$

Add/Subtract Like Terms

$$3\tan x = 5 + \tan x$$

$$3m = 5 + m$$

$$2m = 5$$

$$m = 2.5$$

$$\tan x = 2.5$$

....

let $m = \tan x$

$$1 + \sin x = 4\sin x$$

$$1 + m = 4m$$

$$3m = 1$$

$$m = \frac{1}{3}$$

$$\sin x = \frac{1}{3}$$

.....

let $m = \sin x$

Algebra

$$\frac{\cos x}{\cos x + 1} = -\frac{1}{3}$$

$$\frac{m}{m + 1} = -\frac{1}{3}$$

$$3m = -m - 1$$

$$m = -\frac{1}{4}$$

$$\cos x = -\frac{1}{4}$$

....

$$m = \cos x$$

$$2\sin x = 4$$

$$\sin x = 2$$

No Solution

$\sin x = \cos x$
 $\frac{\sin x}{\cos x} = 1$
 $\tan x = 1$
 ...
 $x = \frac{\pi}{4}, \frac{5\pi}{4}$

Identities

$$5 - 2\csc x = 0$$

$$5 - 2 \times \frac{1}{\sin x} = 0$$

$$5 - \frac{2}{m} = 0$$

$$5 = \frac{2}{m}$$

$$m = \frac{2}{5}$$

$$m = 0.4$$

$$\sin x = 0.4$$

....

$\csc \theta = \frac{1}{\sin \theta}$

let $m = \sin x$

$$\sin x - \csc x = 0$$

$$\sin x - \frac{1}{\sin x} = 0$$

$$m - \frac{1}{m} = 0$$

$$\left(m - \frac{1}{m} = 0\right) \times m$$

$$m^2 - 1 = 0$$

$$(m + 1)(m - 1) = 0$$

Identities

let $m = \sin x$

Factor

$$m = 1$$

$$\sin x = 1$$

....

$$m = -1$$

$$\sin x = -1$$

....

$$\sin x \neq 0$$

C12 - 4.6 - Factoring/Distributing Notes

$$\cos x(\cos x + 1)$$

$$\cos^2 x + \cos x$$

Distribution

$$\begin{matrix} m(m+1) \\ m^2 + m \end{matrix}$$

$$\sin x - \sin^2 x$$

$$\sin x(1 - \sin x)$$

Factor

$$\begin{matrix} \sin x - \sin^2 x \\ m - m^2 \\ m(1 - m) \end{matrix}$$

$$m = \sin x$$

$$\sin x(1 - \sin x)$$

$$\sin x \cos x + \cos x$$

$$\cos x(\sin x + 1)$$

$$\begin{matrix} nm + m & n = \sin x \\ m(n + 1) & m = \cos x \end{matrix}$$

$$(\cos x + 1)(\cos x - 2)$$

$$\cos^2 x - \cos x - 2$$

$$\begin{matrix} (m+1)(m-2) \\ m^2 - m - 2 \end{matrix}$$

$$\begin{matrix} (1 + \cos x)(1 - \cos x) \\ 1 - \cancel{\cos x} + \cancel{\cos x} - \cos^2 x \end{matrix}$$

Distribution

$$\begin{matrix} (m+n)(m-n) \\ m^2 - n^2 \end{matrix}$$

$$1 - \cos^2 x$$

$$1 - \sin^2 x$$

$$(1 + \sin x)(1 - \sin x)$$

$$\begin{matrix} 1 - a^2 \\ (1 - a)(1 + a) \end{matrix}$$

$$\cos^2 x - 1$$

$$(\cos x + 1)(\cos x - 1)$$

$$\begin{matrix} a^2 - 1 \\ (a + 1)(a - 1) \end{matrix}$$

Differences of squares

$$\sin^2 x - \cos^2 x$$

$$(\sin x + \cos x)(\sin x - \cos x)$$

$$\begin{matrix} m^2 - n^2 \\ (m + n)(m - n) \end{matrix}$$

$$\begin{matrix} \cos^4 x - \sin^4 x \\ (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) \\ (\cos 2x)(1) \end{matrix}$$

$$\begin{matrix} m^4 - n^4 \\ (m^2 + n^2)(m^2 - n^2) \end{matrix}$$

$$\cos 2x$$

Identities

$$\begin{matrix} \sin^2 \theta + \cos^2 \theta = 1 \\ \cos 2\theta = \cos^2 \theta - \sin^2 \theta \end{matrix}$$

$$\sin^2 x + \sin x - 2$$

$$(\sin x + 2)(\sin x - 1)$$

Factor

$$\begin{matrix} \sin^2 x + \sin x - 2 \\ m^2 + m - 2 \\ (m + 2)(m - 1) \end{matrix}$$

$$\text{let } m = \sin x$$

$$(\sin x + 2)(\sin x - 1)$$

$$\begin{matrix} \sin^2 \theta + 2\sin \theta \cos \theta + \cos^2 \theta \\ (\sin \theta + \cos \theta)(\sin \theta + \cos \theta) \end{matrix}$$

$$(\cos x + \sin x)^2$$

$$\begin{matrix} m^2 + 2mn + n^2 \\ (m + n)(m + n) \end{matrix}$$

$$-\sin x + 1 = 1 - \sin x$$

$$2 + \sin x + \sin^2 x = \sin^2 x + \sin x + 2$$

Rearrange order of Terms

$$\sin^2 x + \tan x + \cos^2 x = \sin^2 x + \cos^2 x + \tan x = 1 + \tan x$$

C12 - 4.6 - Solving Equations Notes

$$\begin{aligned} \cos^2 x + \cos x &= 0 \\ \cos x(\cos x + 1) &= 0 \quad \text{Factor} \\ \cos x &= 0 & \cos x + 1 &= 0 \\ & & \cos x &= -1 \end{aligned}$$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \pi$

$$\begin{aligned} \cos^2 x + \cos x &= 0 \\ m^2 + m &= 0 & \text{let } m = \cos x \\ m(m + 1) &= 0 & \text{Factor} \\ m = 0 & & m = -1 \\ \cos x = 0 & & \cos x = -1 \end{aligned}$$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \pi$

$$\begin{aligned} \sin^2 x + \sin x - 2 &= 0 \\ m^2 + m - 2 &= 0 & \text{let } m = \sin x \\ (m + 2)(m - 1) &= 0 \end{aligned}$$

$m = -2$
 $\sin x = -2$
 Reject

$m = 1$
 $\sin x = 1$

$x = \frac{\pi}{2}$

$-1 \leq \sin x \leq 1$
 $-1 \leq \cos x \leq 1$

$$\begin{aligned} 2 \sin^2 x + \sin x - 1 &= 0 \\ 2m^2 + m - 1 &= 0 & \text{let } m = \sin x \\ \dots & & \\ (2m - 1)(m + 1) &= 0 & \text{Factor} \end{aligned}$$

$m = \frac{1}{2}$
 $\sin x = \frac{1}{2}$

$m = -1$
 $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{3\pi}{2}$

$$\begin{aligned} 3 \cos^2 x - 8 \cos x - 5 &= 0 \\ 3m^2 - 8m - 5 &= 0 & \text{let } m = \cos x \end{aligned}$$

$m = 3.18$ $m = -0.52$

Quadform

$\cos x = -0.52$

$x = 2.12$ $x = 4.16$

C12 - 4.6 - Identities Chapter 6 Notes

Identities

$$\begin{aligned} \sin 2x + \cos x &= 0 \\ 2\sin x \cos x + \cos x &= 0 \\ \cos x(2\sin x - 1) &= 0 \end{aligned}$$

$$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$$

$$\begin{aligned} \cos x = 0 & \quad 2\sin x - 1 = 0 \\ \text{---} & \quad \sin x = \frac{1}{2} \\ \text{---} & \quad \text{---} \end{aligned}$$

$$\begin{aligned} \sin x - \cos^2 x - 1 &= 0 \\ \sin x - (1 - \sin^2 x) - 1 &= 0 \\ \sin x - 1 + \sin^2 x - 1 &= 0 \\ \sin^2 x + \sin x - 2 &= 0 \end{aligned}$$

Identities

$$\boxed{\cos^2 x = 1 - \sin^2 x}$$

Identities

$$\boxed{\cos 2x = 1 - 2\sin^2 x}$$

$$\begin{aligned} \sin x + \cos 2x &= 0 \\ \sin x - (1 - 2\sin^2 x) &= 0 \\ 2\sin^2 x + \sin x - 1 &= 0 \end{aligned}$$

$$\begin{aligned} \frac{\cos x}{\cos x + 1} &= -\frac{1}{3} \\ \frac{m}{m+1} &= -\frac{1}{3} \\ 3m &= -m - 1 \\ m &= -\frac{1}{4} \\ \cos x &= -\frac{1}{4} \\ \text{---} \end{aligned}$$

$$\begin{aligned} 1 + \cos x &= \sin x \\ (1 + \cos x)^2 &= (\sin x)^2 \\ 1 + 2\cos x + \cos^2 x &= \sin^2 x \\ 1 + 2\cos x + \cos^2 x &= 1 - \cos^2 x \\ 2\cos^2 x + 2\cos x &= 0 \\ 2\cos x(\cos x + 1) &= 0 \end{aligned}$$

$$\begin{aligned} \cos x \cos 2x - \sin x \sin 2x &= -1 \\ \cos x \cos 2x - \sin x \sin 2x &= -1 \\ \cos(2x + x) &= -1 \\ \cos 3x &= 1 \\ \text{---} \end{aligned}$$

$$\cos x = 0 \quad \cos x = -1$$

$$\boxed{\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

Identities