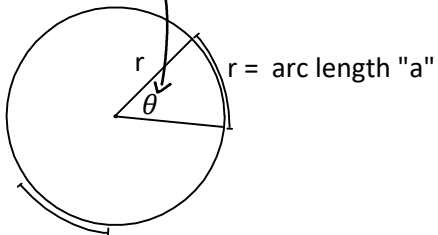


C12 - 4.1 - Degree/Radian Notes

"One radian is equal to the length of the arc of a circle with radius = 1.

1 Radian is the central angle whose arc is equal to the radius

$$1 \text{ rad} = 57.3^\circ$$



$$1 \text{ rad} = 57.3^\circ$$

$$1 \text{ inch} = 2.54 \text{ cm}$$

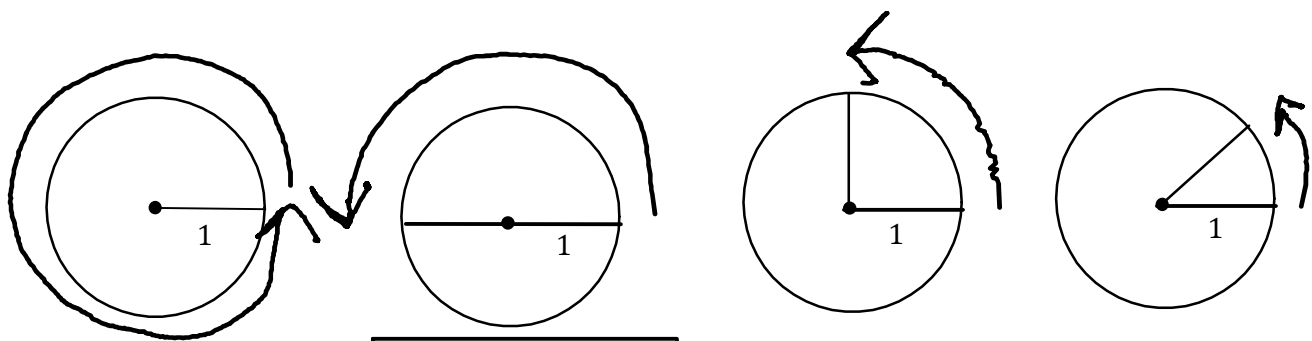
$$\theta_{\text{rad}} = \frac{a}{r}$$

$$\theta_{\text{rad}} = \frac{r}{r}$$

$$\theta_{\text{rad}} = 1 \text{ rad}$$

One Radian equals 57.3°

Arc length



$$\theta = 360^\circ = 2\pi_{\text{rad}}$$

$$C = 2\pi r$$

$$C = 2\pi(1)$$

$$C = 2\pi$$

$$C = 6.28$$

$$\theta = 180^\circ = \pi_{\text{rad}}$$

$$C = 2\pi r$$

$$C = 2\pi(1)$$

$$C = 2\pi$$

$$\frac{C}{2} = \frac{2\pi}{2}$$

$$C = \pi$$

$$\theta = 90^\circ = \frac{\pi}{2}_{\text{rad}}$$

$$C = 2\pi r$$

$$C = 2\pi(1)$$

$$C = 2\pi$$

$$\frac{C}{4} = \frac{2\pi}{4}$$

$$C = \frac{\pi}{2}$$

$$\theta = 45^\circ = \frac{\pi}{4}_{\text{rad}}$$

$$C = 2\pi r$$

$$C = 2\pi(1)$$

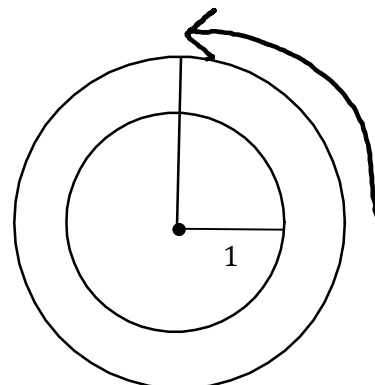
$$C = 2\pi$$

$$\frac{C}{8} = \frac{2\pi}{8}$$

$$C = \frac{\pi}{4}$$

Notice the size of the circle does not matter.

$$90^\circ = \frac{\pi}{2}$$



$$\theta = 90^\circ = \frac{\pi}{2}_{\text{rad}}$$

C12 - 4.1 - Degree/Radian Conversion Notes

Degrees to Radians:

Radians to Degrees:

$$\frac{180^\circ}{\pi_{rad}}$$

$$\frac{\pi_{rad}}{180^\circ}$$

$$\times \frac{\pi}{180^\circ}$$

$$\times \frac{180^\circ}{\pi}$$

π and 180° are the same thing, just in different units

Find θ in radians

$$30^\circ =? \quad 30^\circ \times \frac{\pi}{180^\circ} = \frac{30\pi}{180} = \frac{\pi}{6} = 0.52$$

$$120^\circ =? \quad 120^\circ \times \frac{\pi}{180^\circ} = \frac{120\pi}{180} = \frac{2\pi}{3}$$

$$99^\circ =? \quad 99^\circ \times \frac{\pi}{180^\circ} = \frac{99\pi}{180} = \frac{11\pi}{20}$$

Find θ in degrees

$$\frac{\pi}{3_{rad}} =? \quad \frac{\pi}{3_{rad}} \times \frac{180^\circ}{\pi} = \frac{180\pi}{3\pi} = 60^\circ$$

$$\frac{2\pi}{5_{rad}} =? \quad \frac{2\pi}{5_{rad}} \times \frac{180^\circ}{\pi} = \frac{360\pi}{5\pi} = 72^\circ$$

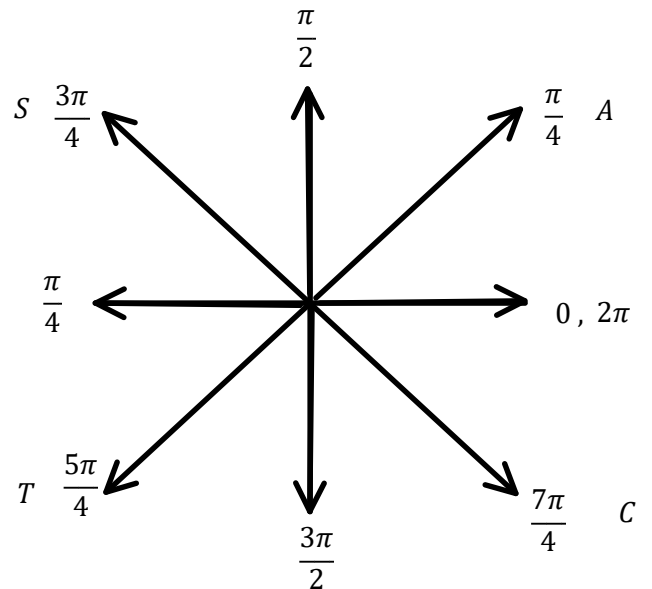
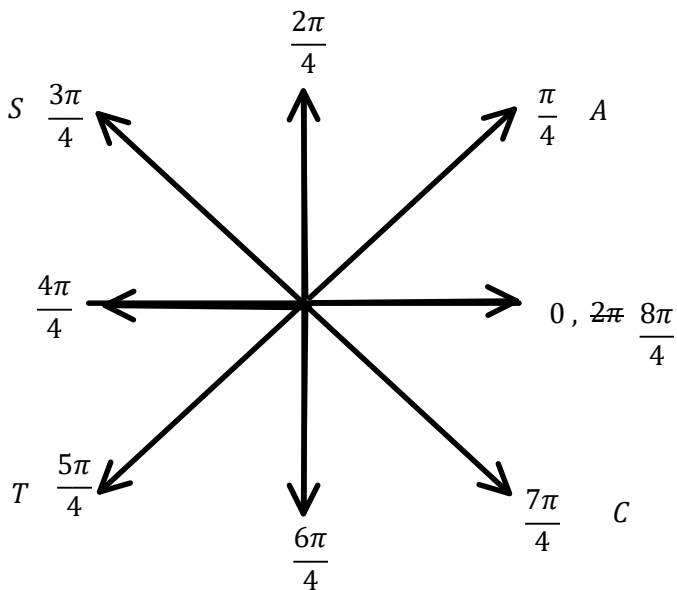
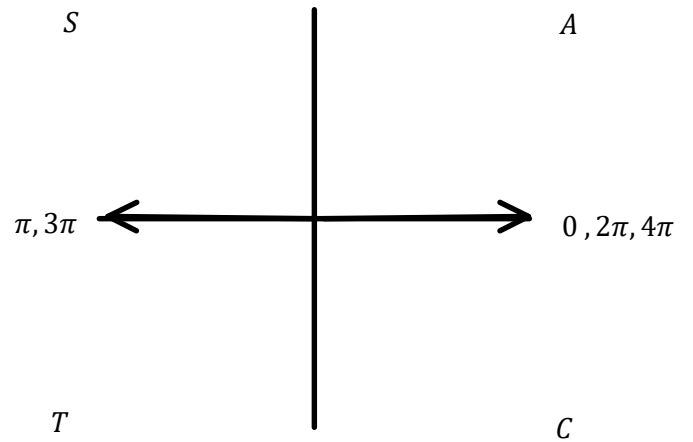
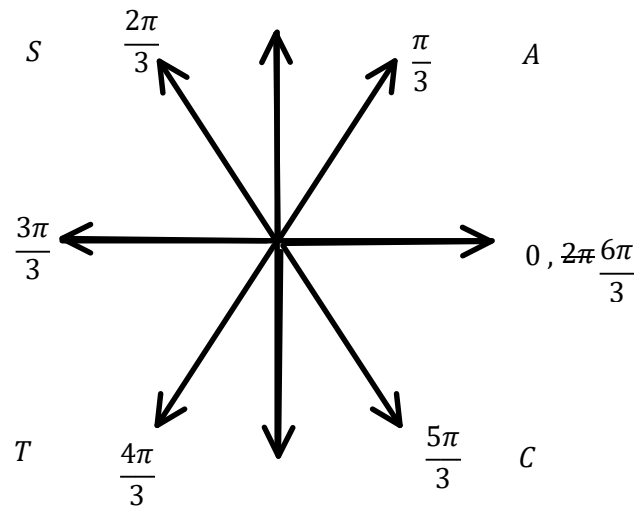
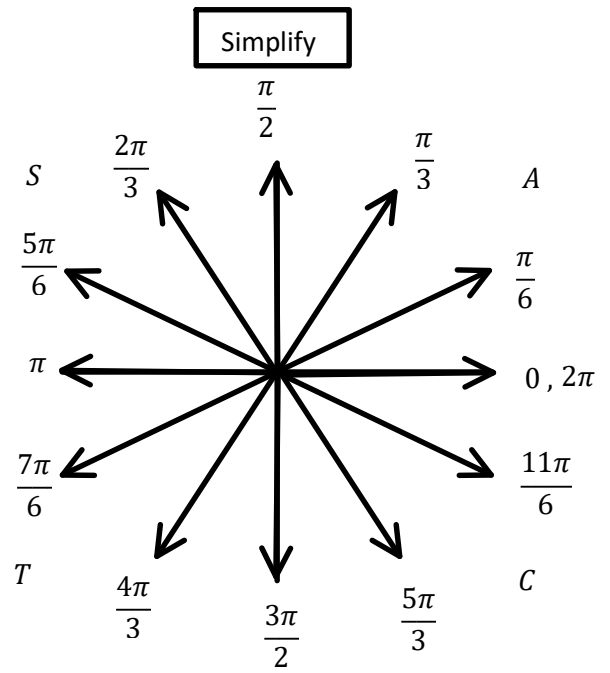
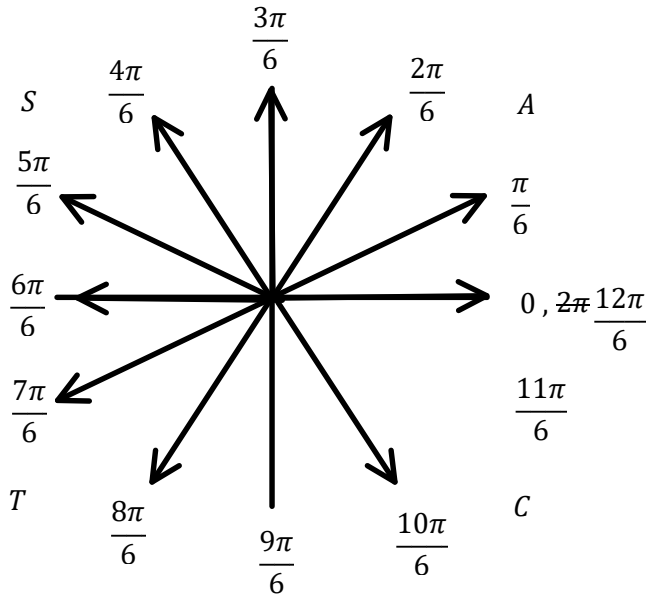
$$1.57_{rad} =? \quad 1.57_{rad} \times \frac{180^\circ}{\pi} = 90^\circ$$

$$3 =? \quad 3_{rad} \times \frac{180^\circ}{\pi} = \frac{540}{\pi} = 171.89^\circ$$

Degrees	Radians	Radians	Radians
0°	0_{rad}	0_{rad}	0_{rad}
15°	$\frac{\pi}{12_{rad}}$	$\frac{\pi}{12_{rad}}$	0.26_{rad}
30°	$\frac{2\pi}{12_{rad}}$	$\frac{\pi}{6_{rad}}$	0.52_{rad}
45°	$\frac{3\pi}{12_{rad}}$	$\frac{\pi}{4_{rad}}$	0.79_{rad}
60°	$\frac{4\pi}{12_{rad}}$	$\frac{\pi}{3_{rad}}$	1.05_{rad}
75°	$\frac{5\pi}{12_{rad}}$	$\frac{5\pi}{12_{rad}}$	1.31_{rad}
90°	$\frac{6\pi}{12_{rad}}$	$\frac{\pi}{2_{rad}}$	1.57_{rad}
180°	$\frac{12\pi}{12} = \pi_{rad}$	π_{rad}	3.14_{rad}
270°	$\frac{3\pi}{2_{rad}}$	$\frac{3\pi}{2_{rad}}$	4.71_{rad}
360°	$2\pi_{rad}$	$2\pi_{rad}$	6.28_{rad}
720°	$4\pi_{rad}$	$4\pi_{rad}$	12.56_{rad}

If there are no units it is in radians.

C12 - 4.1 - $\frac{\# \pi}{\#}$ Notes

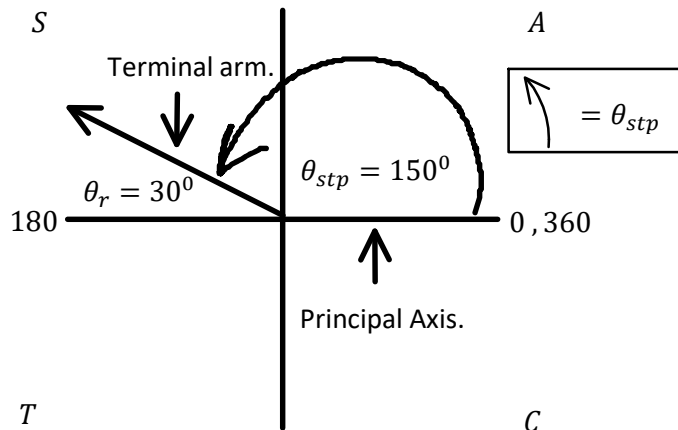


C12 - 4.2 - θ_r, θ_{stp} Notes

(always positive, between 0 and $\pi/2$)

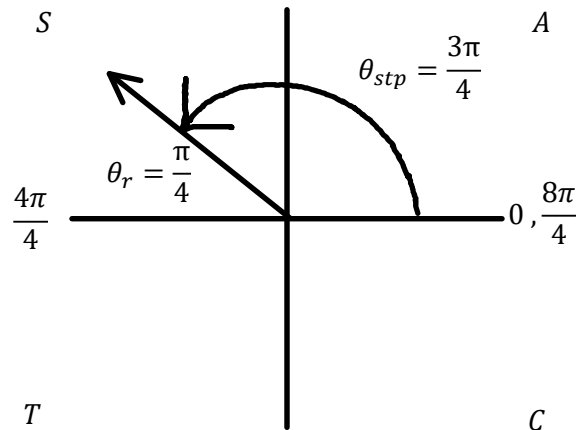
θ_r : the "reference angle" is the angle between the terminal arm and the x -axis.

θ_{stp} : the "angle in standard position" from the principal axis (+ x -axis) to the terminal arm.



$$\theta_r = 180 - 150 \quad \theta_{stp} = 180 - 30$$

$$\theta_r = 30^\circ \quad \theta_{stp} = 150^\circ$$



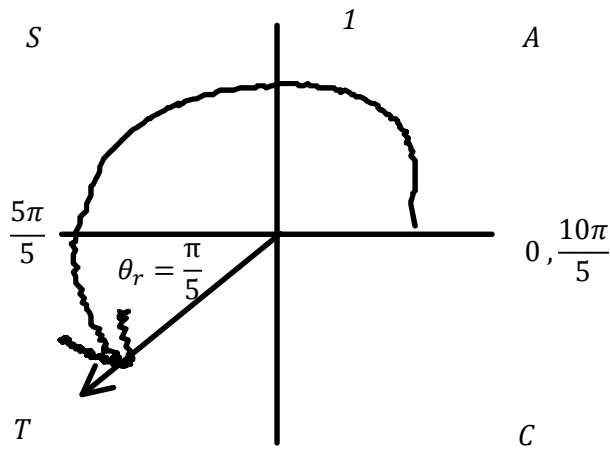
LCD

$$\theta_r = \pi - \theta_{stp} \quad \theta_{stp} = \pi - \theta_r$$

$$\theta_r = \pi - \frac{3\pi}{4} \quad \theta_{stp} = \pi - \frac{\pi}{4}$$

$$\theta_r = \frac{4\pi}{4} - \frac{3\pi}{4} \quad \theta_{stp} = \frac{4\pi}{4} - \frac{\pi}{4}$$

$$\theta_r = \frac{\pi}{4} \quad \theta_{stp} = \frac{3\pi}{4}$$

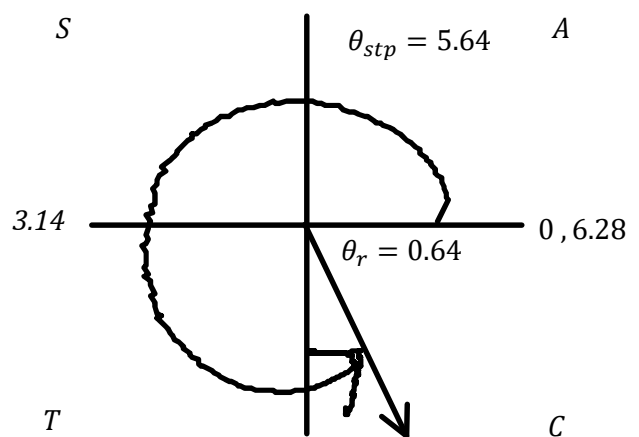


$$\theta_r = \pi + \theta_{stp} \quad \theta_{stp} = \pi + \theta_r$$

$$\theta_r = \pi + \frac{6\pi}{5} \quad \theta_{stp} = \pi + \frac{\pi}{5}$$

$$\theta_r = \frac{5\pi}{5} + \frac{6\pi}{5} \quad \theta_{stp} = \frac{5\pi}{5} + \frac{\pi}{5}$$

$$\theta_r = \frac{\pi}{5} \quad \theta_{stp} = \frac{6\pi}{5}$$



$$\theta_r = 2\pi - \theta_{stp} \quad \theta_{stp} = 2\pi - \theta_r$$

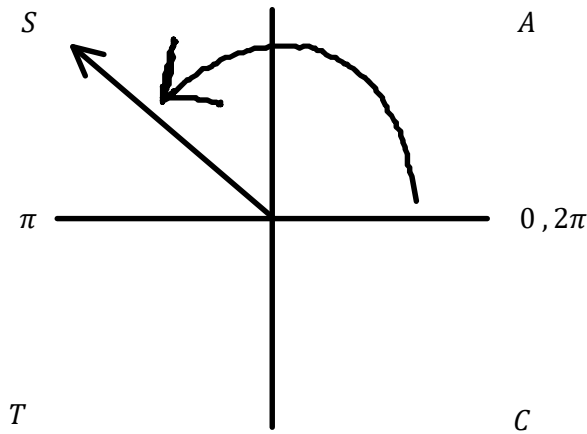
$$\theta_r = 2\pi - 5.64 \quad \theta_{stp} = 2\pi - 0.64$$

$$\theta_r = 0.64 \quad \theta_{stp} = 5.64$$

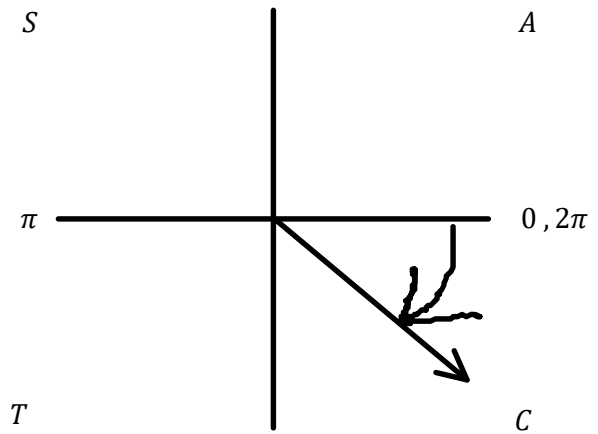
$$\theta_{cot} = \theta_{stp} \pm 2\pi n, nEI$$

C12 - 4.2 - $\pm \theta_{stp}, \theta_{cot}, \theta_{gen}$ Notes

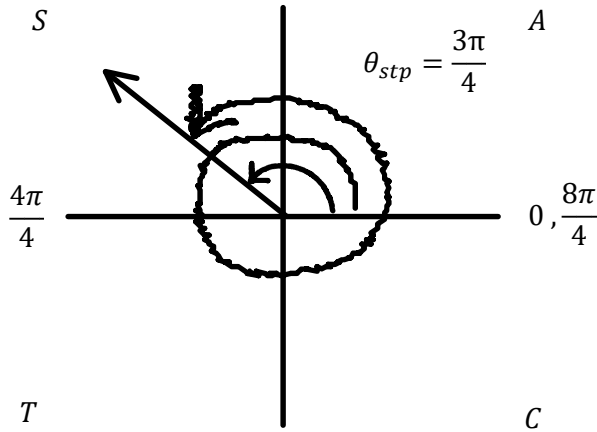
Counter-clockwise rotation is a positive θ_{stp}



Clockwise rotation is a negative θ_{stp}



θ_{cot} : the "co-terminal angle" is any angle with the same terminal arm.



$$\theta_{cot} = \theta_{stp} \pm 2\pi$$

$$\theta_{cot} = \frac{3\pi}{4} + 2\pi$$

$$\theta_{cot} = \frac{3\pi}{4} + \frac{8\pi}{4}$$

$$\theta_{cot} = \frac{11\pi}{4}$$

$$\theta_{cot} = \theta_{stp} \pm 2\pi$$

$$\theta_{cot} = \frac{3\pi}{4} - 2\pi$$

$$\theta_{cot} = \frac{3\pi}{4} - \frac{8\pi}{4}$$

$$\theta_{cot} = -\frac{5\pi}{4}$$

θ_{gen} : the "general solution" is all angles with the same terminal arm.

$$\theta_{gen} = \theta_{stp} \pm 2\pi n, nEI$$

$$\theta_{gen} = \frac{3\pi}{4} \pm 2\pi n, nEI$$

Basic logic will calculate θ_{stp} and θ_r much more easily than using these formulas.

$$\frac{9\pi}{2}$$

$$\frac{9\pi}{2} - 2\pi$$

$$\frac{9\pi}{2} - \frac{4\pi}{2}$$

$$\frac{5\pi}{2}$$

$$\frac{5\pi}{2} - 2\pi$$

$$\frac{5\pi}{2} - \frac{4\pi}{2}$$

$$\frac{\pi}{2}$$

OR

$$\frac{9\pi}{2} \div 2\pi$$

$$\frac{9\pi}{2} \times \frac{1}{2\pi}$$

$$\frac{9}{4} = 2.25$$

$$\frac{9\pi}{2} - 2(2\pi)$$

$$\frac{9\pi}{2} - 4\pi$$

$$\frac{9\pi}{2} - \frac{8\pi}{2}$$

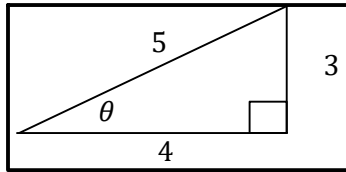
$$\frac{\pi}{2}$$

OR

$$0.25 \times 2\pi = \frac{\pi}{2}$$

You may need to add or subtract 2π more than once.

C12 - 4.3 - Find Ratio/Type in Calc Notes



Degrees are for children, unless you are taking physics.

$\sin\theta = \frac{O}{H}$ $\csc\theta = \frac{H}{O}$ $\sin\theta = \frac{3}{5}$ $\csc\theta = \frac{5}{3}$ The first letters switch s<->c	$\cos\theta = \frac{A}{H}$ $\sec\theta = \frac{H}{A}$ $\cos\theta = \frac{4}{5}$ $\sec\theta = \frac{5}{4}$ The first letters switch c<->s No one would do this! $\sec\theta = \frac{5}{4}$	$\sec\theta = \frac{1}{\cos\theta}$ $\sec\theta = \frac{1}{\left(\frac{4}{5}\right)}$ $\sec\theta = 1 \times \frac{5}{4}$ $\sec\theta = \frac{5}{4}$	$\tan\theta = \frac{O}{A}$ $\cot\theta = \frac{A}{O}$ $\tan\theta = \frac{3}{4}$ $\cot\theta = \frac{4}{3}$ The ones with the t's
--	--	---	---

Type in Calculator (Degrees or Radians)

$\sin 25^\circ = 0.42$ $\cos 180^\circ = -1$ $\sin 30^\circ = \frac{1}{2}$ $\tan(-980^\circ) = -5.67$ $\sin 2.5 = 0.60$ $\cos \frac{\pi}{3} = \frac{1}{2}$ $\tan(5\pi) = 0$ $\cos \pi = -1$	$\csc 140^\circ =$ $\csc\theta = \frac{1}{\sin\theta}$ $\csc 140^\circ = \frac{1}{\sin 140^\circ}$ $\csc 140^\circ = -1.56$ $\csc 3.4 =$ $\csc\theta = \frac{1}{\sin\theta}$ $\csc 3.4 = \frac{1}{\sin 3.4}$ $\csc 3.4 = -3.91$	$\sec 65^\circ =$ $\sec\theta = \frac{1}{\cos\theta}$ $\sec 65^\circ = \frac{1}{\cos 65^\circ}$ $\sec\theta = 2.37$ $\sec\left(\frac{3}{5}\right) =$ $\sec\theta = \frac{1}{\cos\theta}$ $\sec\left(\frac{3}{5}\right) = \frac{1}{\cos\left(\frac{3}{5}\right)}$ $\sec\left(\frac{3}{5}\right) = 1.21$	$\cot 25^\circ =$ $\cot\theta = \frac{1}{\tan\theta}$ $\cot 25^\circ = \frac{1}{\tan 25^\circ}$ $\cot 25^\circ = 2.14$ $\cot 250 =$ $\cot\theta = \frac{1}{\tan\theta}$ $\cot 250 = \frac{1}{\tan 250}$ $\cot 250 = -0.25$
--	--	---	---

Find θ in Degrees

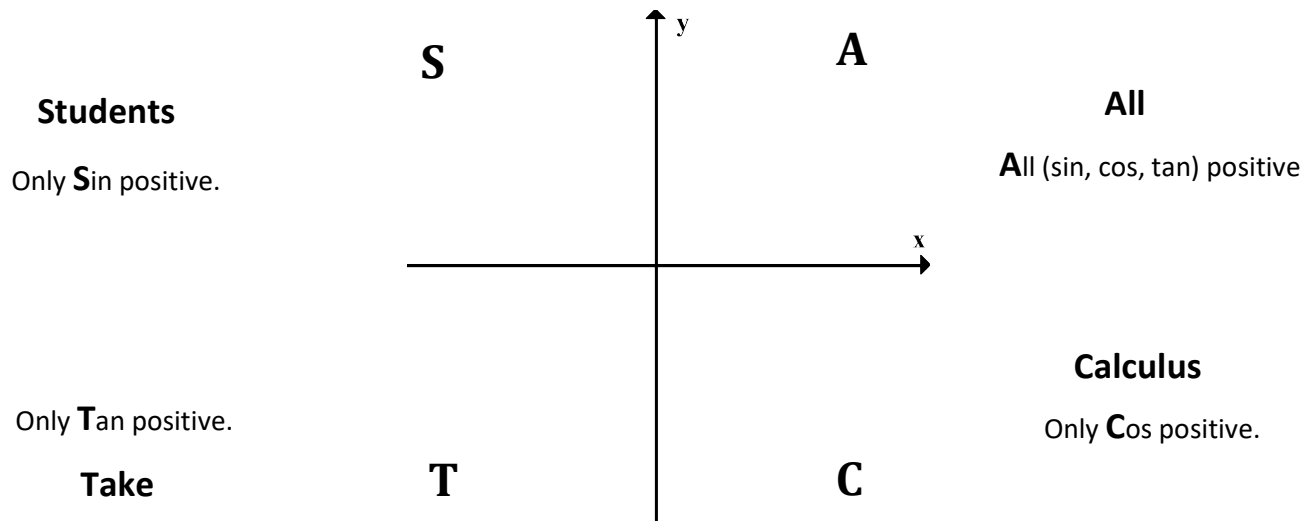
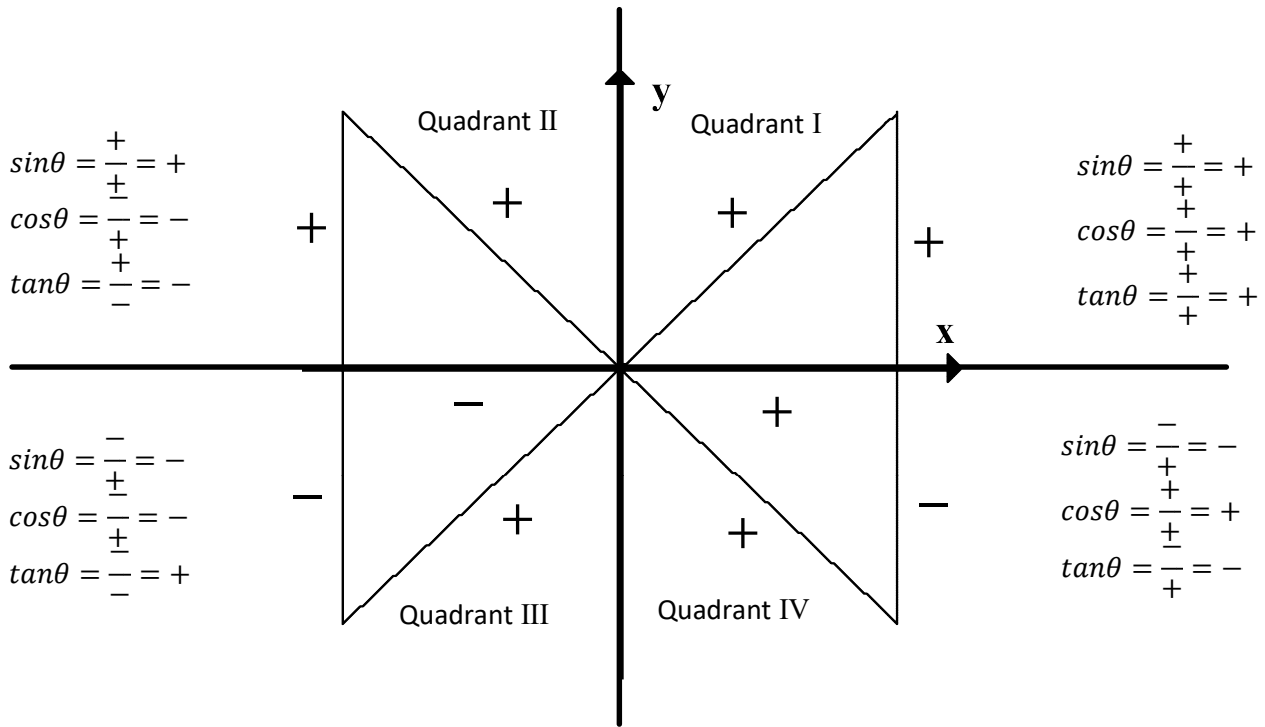
$\sin\theta = \frac{3}{5}$ $\theta = \sin^{-1}\left(\frac{3}{5}\right)$ $\theta = 36.9^\circ$	$\sec\theta = \frac{2}{1}$ $\cos\theta = \frac{1}{2}$ $\theta = \cos^{-1}\left(\frac{1}{2}\right)$ $\theta = 60^\circ$	$\sec\theta = \frac{H}{A}$ $\cos\theta = \frac{A}{H}$
---	---	--

Find θ in Radians

$\cos\theta = \frac{3}{5}$ $\theta = \cos^{-1}\left(\frac{3}{5}\right)$ $\theta = 0.93$	$\cot\theta = 3$ $\cot\theta = \frac{3}{1}$ $\tan\theta = \frac{1}{3}$ $\theta = \tan^{-1}\left(\frac{1}{3}\right)$ $\theta = 0.32$	$\cot\theta = \frac{A}{O}$ $\tan\theta = \frac{O}{A}$
---	---	--

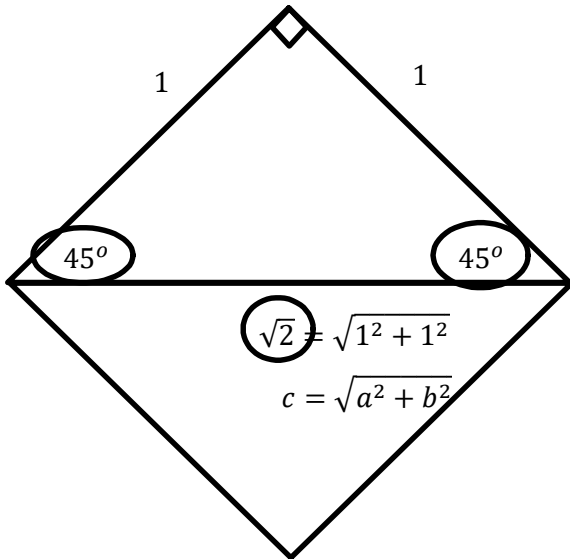
C12 - 4.3 - ASTC Notes

$(+)^2 + (-)^2 = +$	Remember: the hypotenuse is always positive.	$(+)^2 + (+)^2 = +$
$\sqrt{+} = +$		$\sqrt{+} = +$

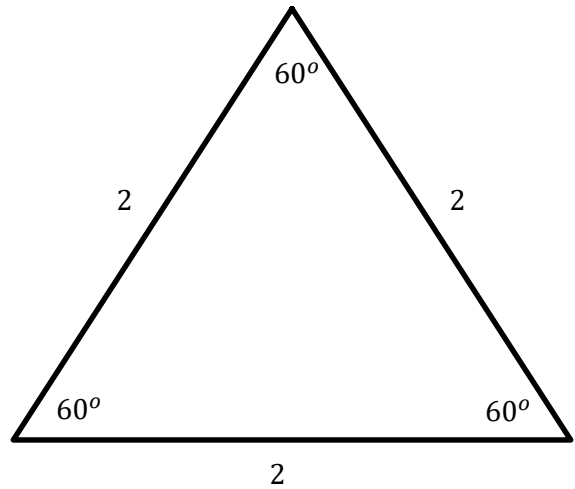


C12 - 4.3 - Special Triangles 30,45,60 sin/cos/tan Notes

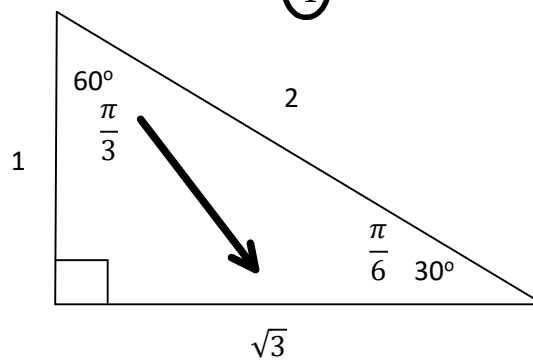
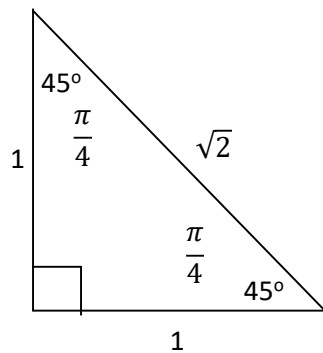
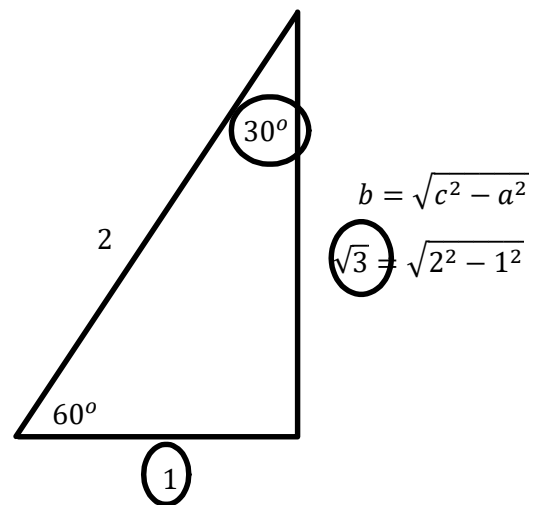
Right Isosceles, with sides =1



Half an equilateral with sides 2



Diagonal of a square with sides lengths of 1



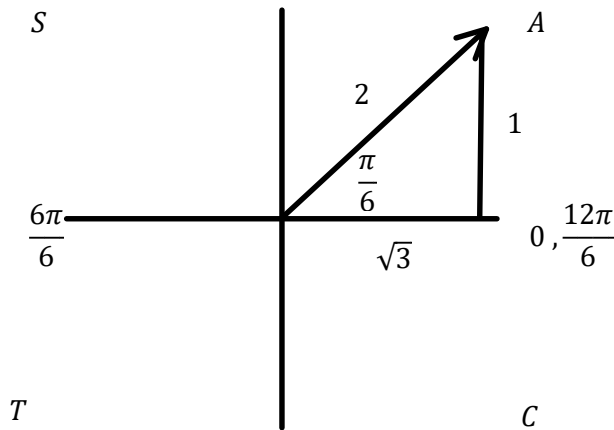
60 > 30
 $\sqrt{3} > 1$
 60 must open up to the root 3.
 And Vice Versa

$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$	$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$	$\sin \frac{\pi}{6} = \frac{1}{2}$
$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$	$\cos \frac{\pi}{3} = \frac{1}{2}$	$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$
$\tan \frac{\pi}{4} = \frac{1}{1}$	$\tan \frac{\pi}{3} = \frac{\sqrt{3}}{1}$	$\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$

C12 - 4.3 - $\sin\theta = ?$ Notes

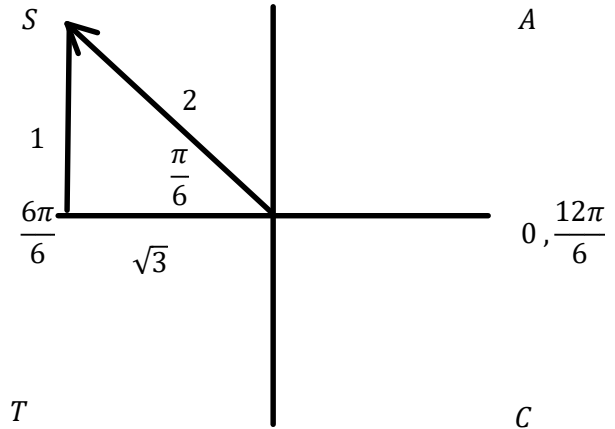
$$\sin\frac{\pi}{6} = ?$$

$$\sin\frac{\pi}{6} = \frac{1}{2}$$



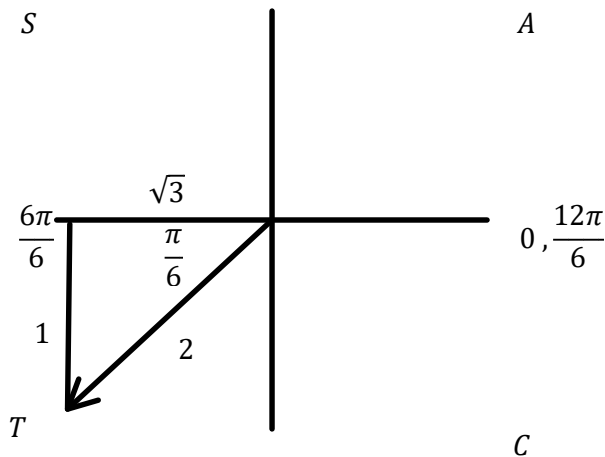
$$\sin\frac{5\pi}{6} = ?$$

$$\sin\frac{5\pi}{6} = \frac{1}{2}$$



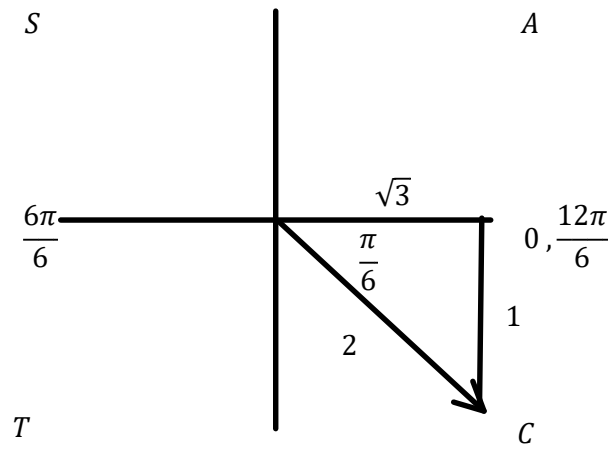
$$\sin\frac{7\pi}{6} = ?$$

$$\sin\frac{7\pi}{6} = -\frac{1}{2}$$



$$\sin\frac{11\pi}{6} = ?$$

$$\sin\frac{11\pi}{6} = -\frac{1}{2}$$



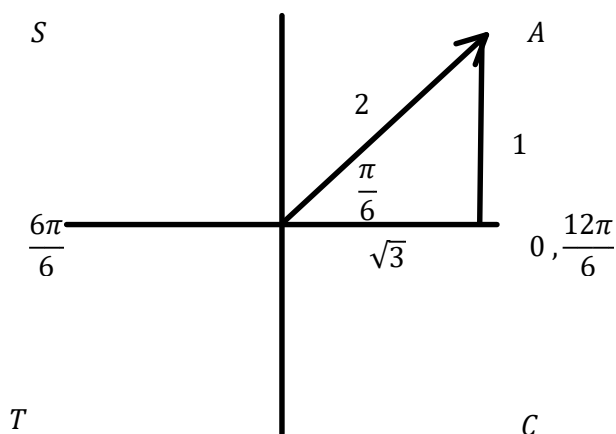
SOH - CAH - TOA

$$\begin{array}{ccc} \sin\theta = \frac{O}{H} & \cos\theta = \frac{A}{H} & \tan\theta = \frac{O}{A} \\ \csc\theta = \frac{1}{\sin\theta} = \frac{H}{O} & \sec\theta = \frac{1}{\cos\theta} = \frac{H}{A} & \cot\theta = \frac{1}{\tan\theta} = \frac{A}{O} \end{array}$$

C12 - 4.3 - $\sin\theta, \cos\theta, \tan\theta = ?$ Notes

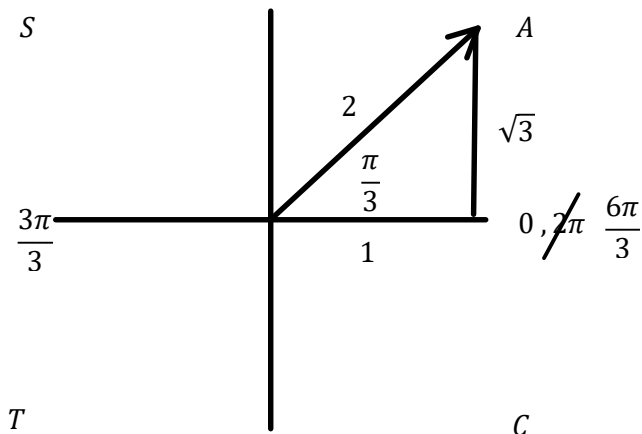
$$\cos \frac{\pi}{6} = ?$$

$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$



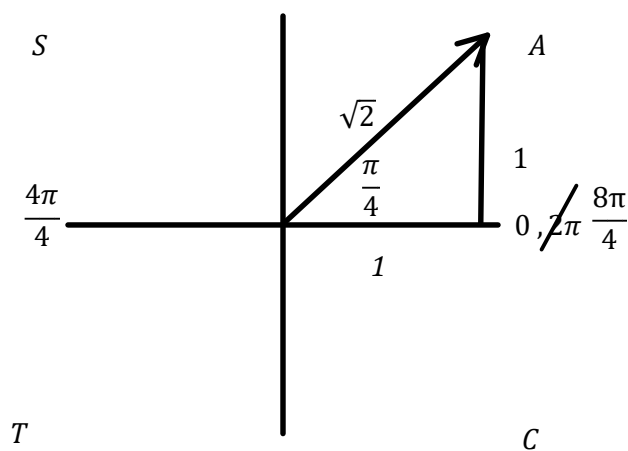
$$\sin \frac{\pi}{3} = ?$$

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$



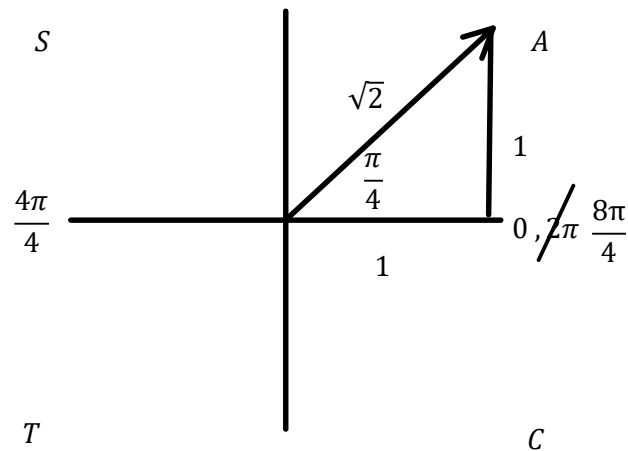
$$\sin \frac{\pi}{4} = ?$$

$$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$



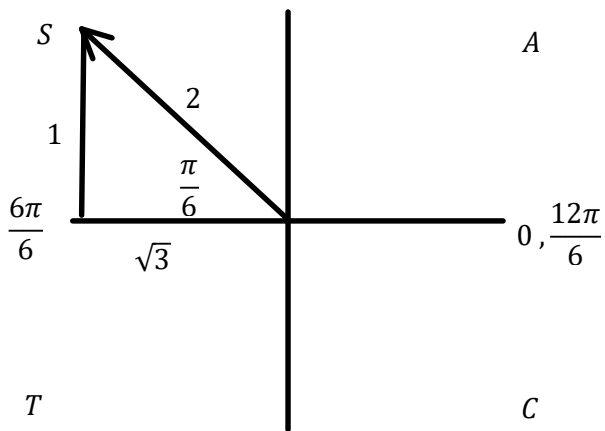
$$\tan \frac{\pi}{4} = ?$$

$$\tan \frac{\pi}{4} = 1$$



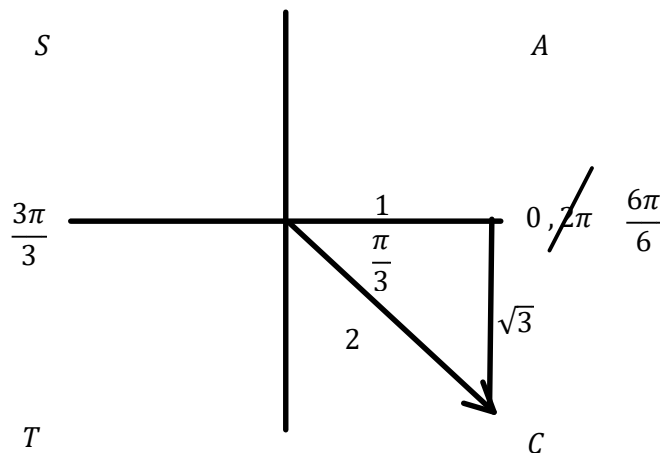
$$\cos \frac{5\pi}{6} = ?$$

$$\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$$



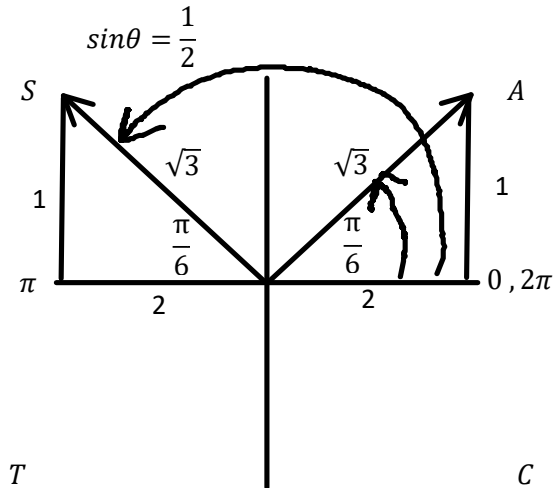
$$\tan \frac{5\pi}{3} = ?$$

$$\tan \frac{5\pi}{3} = -\sqrt{3}$$



C12 - 4.3 - $\sin\theta = \frac{1}{2}$ Notes

Solve for $\theta, 0^\circ \leq \theta < 2\pi$.



$$\theta_{stp} = \frac{\pi}{6} \quad \theta_{stp} = \pi - \frac{\pi}{6}$$

$$= \frac{6\pi}{6} - \frac{\pi}{6}$$

$$= \frac{5\pi}{6}$$

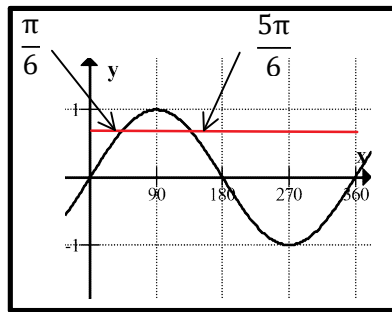
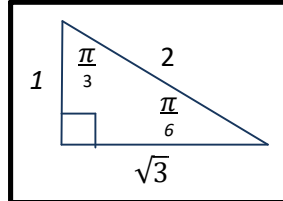
$$\theta_{stp} = \frac{\pi}{6}, \frac{5\pi}{6}$$

Solve for the arrows θ_{stp}

Draw two triangles where $\sin\theta$ is positive: ASTC Quadrant I, II

Label triangles based on special triangles/SOH CAH TOA
Label the reference angle according to special triangles.

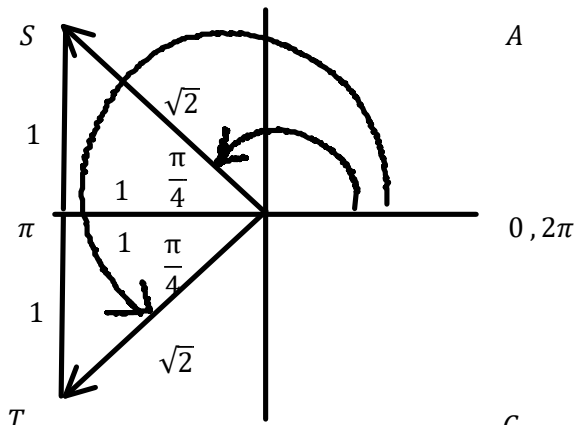
Draw an arrow from the principal axis to:
The first and second terminal arm.



Check your answer: $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \quad \sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$

Solve for $\theta, 0^\circ \leq \theta < 2\pi$ and state the **General Solution**.

$$\cos x = -\frac{1}{\sqrt{2}}$$



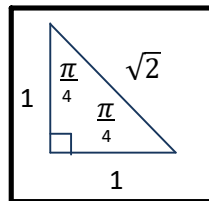
$$\theta_{stp} = \pi - \frac{\pi}{4} \quad \theta_{stp} = \pi + \frac{\pi}{4}$$

$$\theta_{stp} = \frac{4\pi}{4} - \frac{\pi}{4}$$

$$\theta_{stp} = \frac{3\pi}{4} \quad \theta_{stp} = \frac{4\pi}{4} + \frac{\pi}{4}$$

$$\theta_{stp} = \frac{5\pi}{4}$$

$$\theta_{stp} = \frac{3\pi}{4}, \frac{5\pi}{4}$$



General Solution: $\theta = \theta_{stp} \pm pn, n \in I$

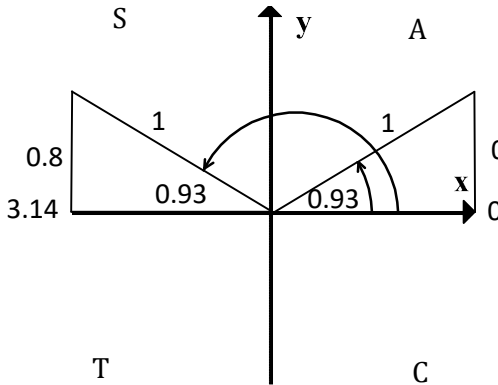
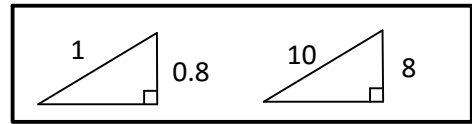
$$\theta = \frac{3\pi}{4} \pm 2\pi n, n \in I \quad \theta = \frac{5\pi}{4} \pm 2\pi n, n \in I$$

C12 - 4.3 - $\sin\theta = .8$ & Point Notes

Solve for $\theta, 0^\circ \leq \theta < 2\pi$ and general solution

$$\sin\theta = 0.8$$

$$\sin\theta = \frac{0.8}{1} = \frac{8}{10}$$



Draw two triangles where $\sin\theta$ is positive:
ASTC Quadrant I, II

0.8 Label the triangles according to SOH CAH TOA

Solve for θ_r :

$$\theta_r = \sin^{-1}\left(+\frac{O}{H}\right)$$

$$\sin\theta = \frac{0.8}{1}$$

$$\theta_r = \sin^{-1}\left(+\frac{0.8}{1}\right)$$

$$\theta_r = 0.93$$

Draw an arrow from the principal axis to the first terminal arm,
draw an arrow from the principal axis to the second terminal arm.

Only inverse positives = θ_r

Solve for the arrows θ_{stp}

$$\theta_{stp} = 0.93 \quad \theta_{stp} = \pi - 0.93$$

$$= 2.21$$

Check your answer:

$$\sin 0.93 = 0.8$$

$$\sin 2.21 = 0.8$$

$$\theta_{stp} = 0.93, 2.21$$

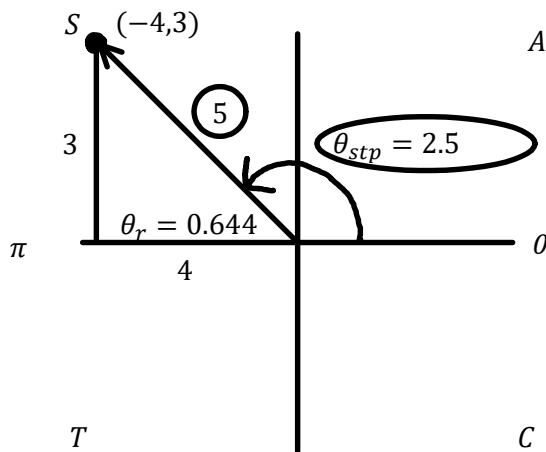
$$\text{General Solution: } \theta = \theta_{stp} \pm pn, n \in I$$

$$\theta = 0.93 \pm 2\pi n, n \in I$$

$$\theta = \theta_{stp} \pm pn, n \in I$$

$$\theta = 2.21 \pm 2\pi n, n \in I$$

Find $\sin x, \cos x, \tan x, \csc x, \sec x,$ and $\cot x$ for the following point. Find θ_{stp}



$$\sin\theta = +\frac{3}{5}$$

$$\csc x = +\frac{5}{3}$$

$$\cos\theta = -\frac{4}{5}$$

$$\sec x = -\frac{5}{4}$$

$$\tan\theta = -\frac{3}{4}$$

$$\cot x = -\frac{4}{3}$$

$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = c^2$$

$$9 + 16 = c^2$$

$$25 = c^2$$

$$\sqrt{25} = \sqrt{c^2}$$

$$5 = c$$

$$\tan\theta = \frac{O}{A}$$

$$\tan\theta = \frac{-3}{4}$$

$$\tan\theta = -0.75$$

$$\theta = \tan^{-1}(+0.75)$$

$$\theta = 0.644$$

$$\pi - 0.644 = 2.50$$

$$\theta_{stp} = 2.50$$

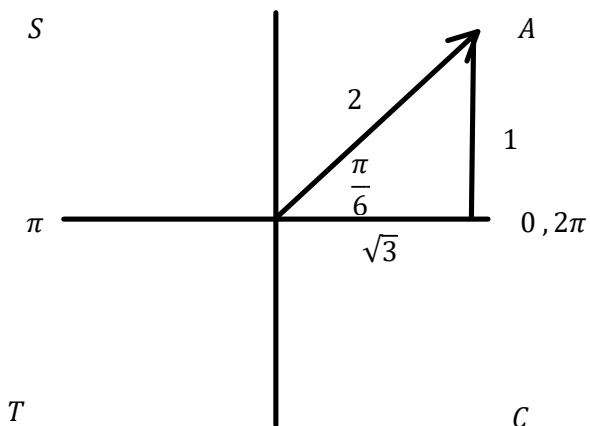
Only inverse positives = θ_r

C12 - 4.3 - $\csc\theta, \sec\theta, \cot\theta = ?$ Notes

$$\sec \frac{\pi}{6} = ?$$

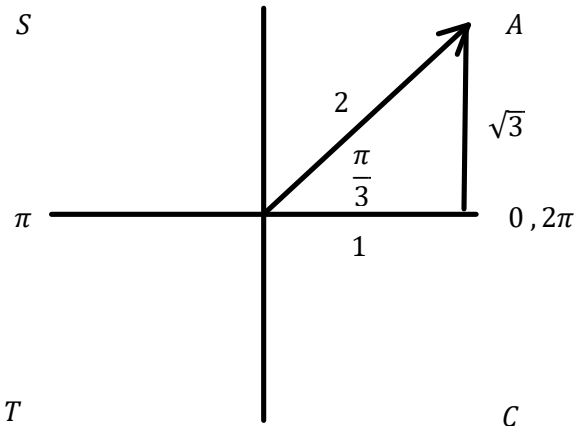
$$\sec \frac{\pi}{6} = \frac{2}{\sqrt{3}}$$

$$\sec \frac{\pi}{6} \neq \cos \left(\frac{6}{\pi} \right)$$



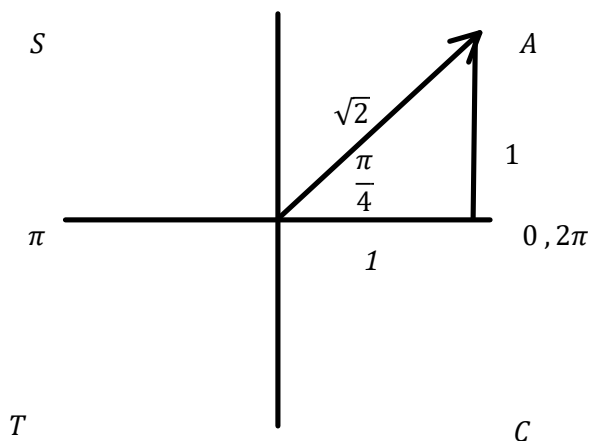
$$\csc \frac{\pi}{3} = ?$$

$$\csc \frac{\pi}{3} = \frac{2}{\sqrt{3}}$$



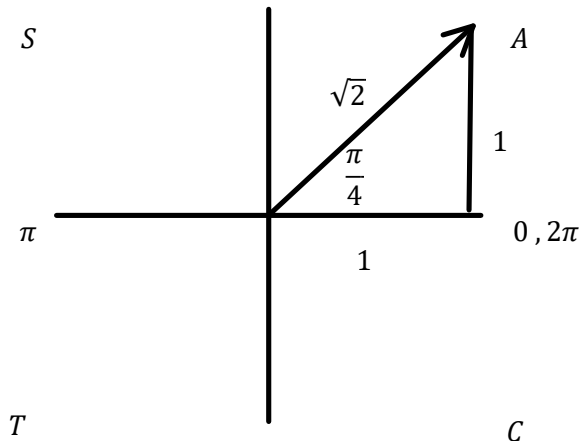
$$\csc \frac{\pi}{4} = ?$$

$$\csc \frac{\pi}{4} = \sqrt{2}$$



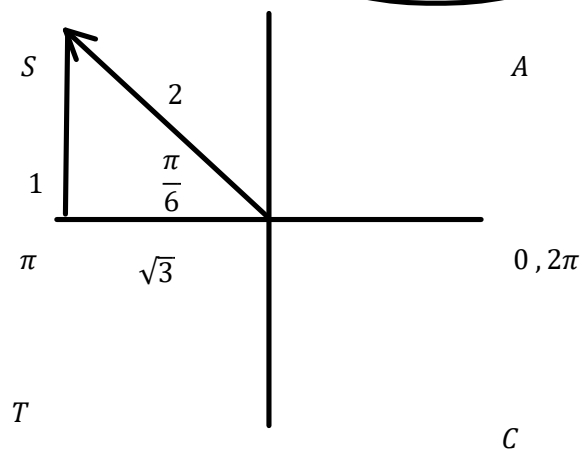
$$\cot \frac{\pi}{4} = ?$$

$$\cot \frac{\pi}{4} = 1$$



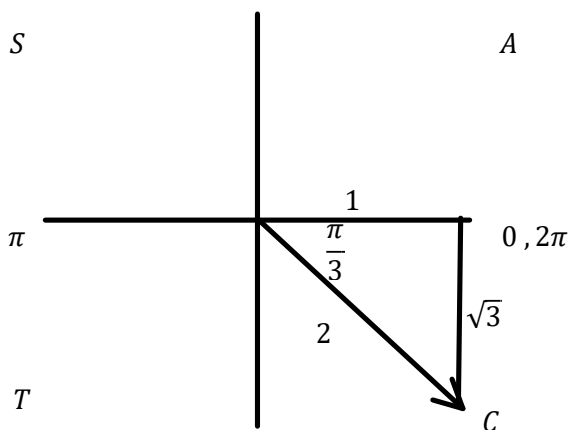
$$\sec \frac{5\pi}{6} = ?$$

$$\sec \frac{5\pi}{6} = -\frac{2}{\sqrt{3}}$$



$$\cot \frac{5\pi}{3} = ?$$

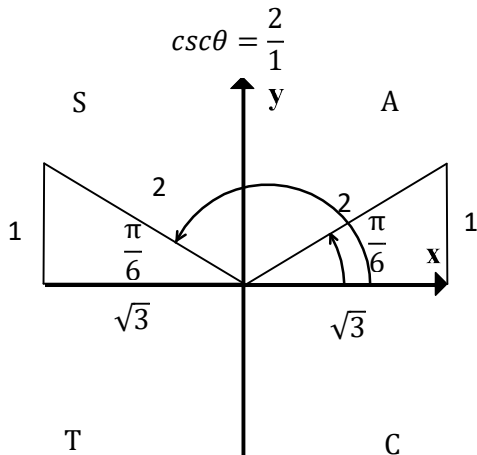
$$\cot \frac{5\pi}{3} = -\frac{1}{\sqrt{3}}$$



C12 - 4.3 - $\csc\theta = 2$ Notes

Solve for $\theta, 0^\circ \leq \theta < 2\pi$.

OR $\sin\theta = \frac{1}{2}$



Draw two triangles where $\csc\theta$ is positive:
ASTC Quadrant I, II

Label the triangles according to special triangles/SOH CAH TOA

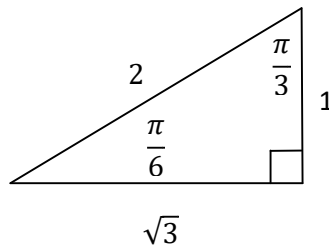
Label the reference angle according to special triangles.

Draw an arrow from the principal axis to the first terminal arm
Draw an arrow from the principal axis to the second terminal arm.

$$\theta_{stp} = \frac{\pi}{6}$$

$$\begin{aligned} \theta_{stp} &= \pi - \frac{\pi}{6} \\ &= \frac{6\pi}{6} - \frac{\pi}{6} \\ &= \frac{5\pi}{6} \end{aligned}$$

$$\theta_{stp} = \frac{\pi}{6}, \frac{5\pi}{6}$$



Solve for the arrows θ_{stp}

Check your answer:

$$\begin{aligned} \cot\theta &= 0.1 \\ \cot\theta &= \frac{0.1}{1} = \frac{1}{10} \\ \tan\theta &= \frac{1}{0.1} = \frac{10}{1} \\ \theta &= \tan^{-1}(10) \\ \theta &= 84.24^\circ \\ \dots \\ \theta &= 84.24, 264.29^\circ \end{aligned}$$

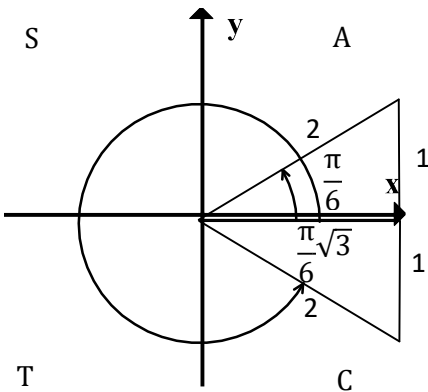
Solve for $\theta, 0^\circ \leq \theta < 2\pi$ and state the General Solution.

$$\sec\theta = \frac{2}{\sqrt{3}}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} \theta_{stp} &= \frac{\pi}{6} & \theta_{stp} &= 2\pi - \frac{\pi}{6} \\ & & &= \frac{11\pi}{6} \end{aligned}$$

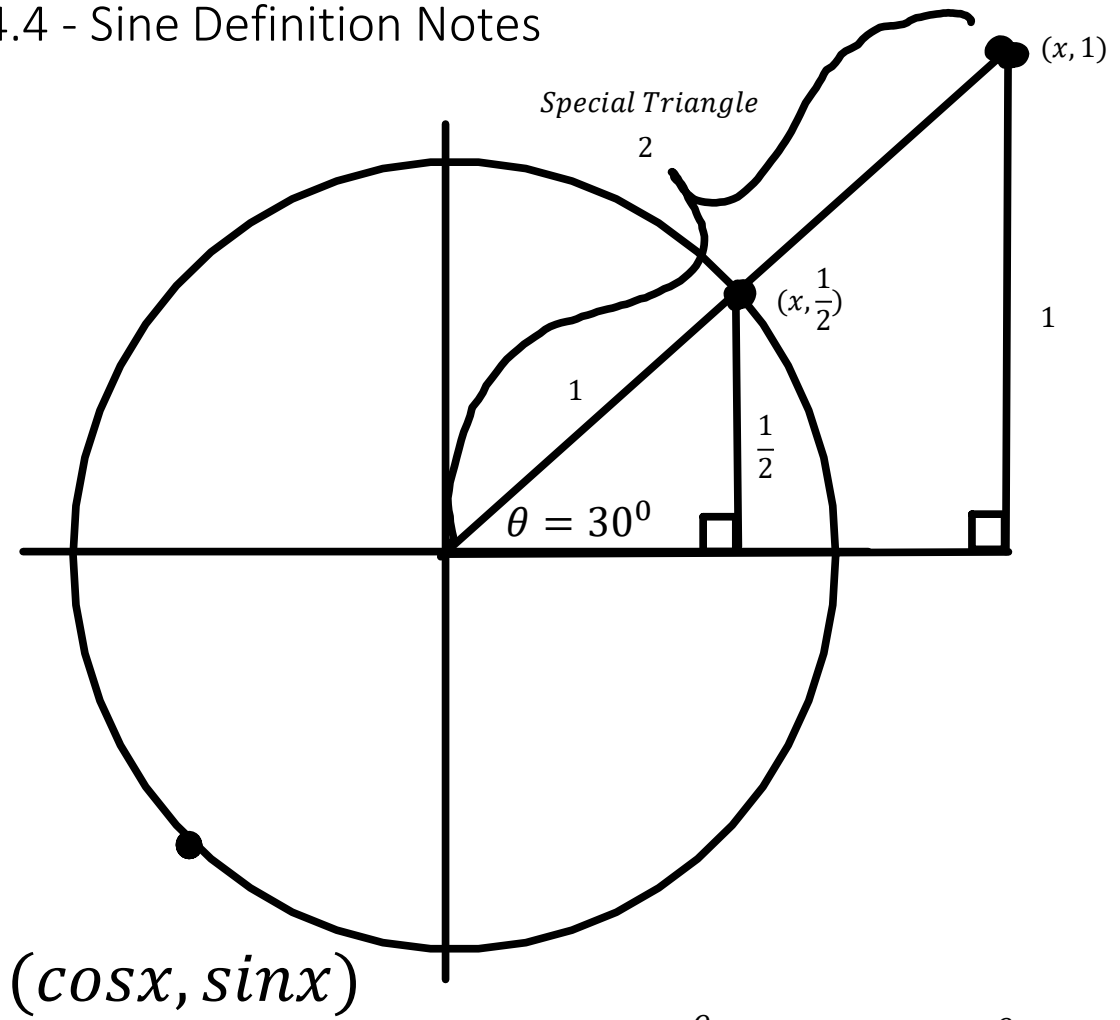
$$\theta_{stp} = \frac{\pi}{6}, \frac{11\pi}{6}$$



General Solution: $\theta = \theta_{stp} \pm pn, n \in I$

$$\theta = \frac{\pi}{6} \pm 2\pi n, n \in I \quad \theta = \frac{11\pi}{6} \pm 2\pi n, n \in I$$

C12 - 4.4 - Sine Definition Notes



$$\sin \theta = \frac{O}{H}$$

$$\sin \theta = \frac{1}{2}$$

$$\sin \theta = \frac{O}{H}$$

$$\sin \theta = \frac{\frac{1}{2}}{1}$$

$$\sin \theta = \frac{1}{2}$$

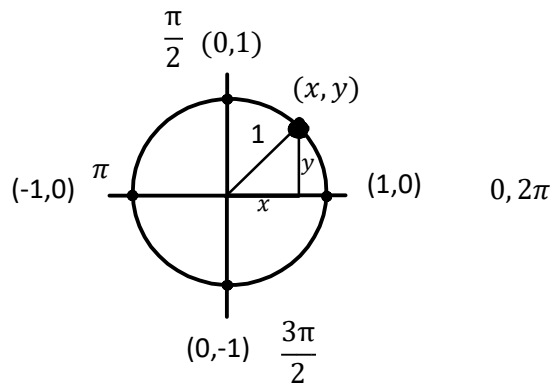
$$(x, y)$$

$$(\cos x, \sin x)$$

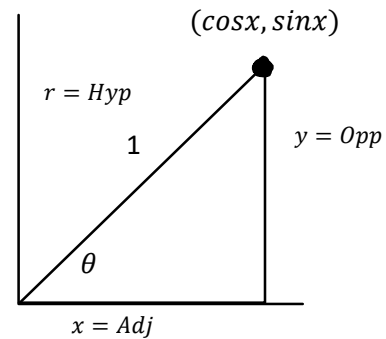
$$\tan x = m$$

$$\sin \theta = y$$

C11 - 4.4 - Unit Circle Quadrantal Angle Notes



Radius of unit circle = 1
Hyp = 1



$$\sin\theta = \frac{Opp}{Hyp}$$

$$\sin\theta = \frac{y}{1}$$

$$\boxed{\sin\theta = y}$$

$$\cos\theta = \frac{Adj}{Hyp}$$

$$\cos\theta = \frac{x}{1}$$

$$\boxed{\cos\theta = x}$$

$$\tan\theta = \frac{Opp}{Adj}$$

$$\boxed{\tan\theta = \frac{y}{x}}$$

$$\sin 0 = \frac{0}{1}$$

$$\sin 0 = 0$$

$$\cos\left(\frac{3\pi}{2}\right) = \frac{0}{1}$$

$$\cos\left(\frac{3\pi}{2}\right) = 0$$

$$\tan 0 = \frac{0}{1}$$

$$\tan 0 = 0$$

$$\sin\left(\frac{3\pi}{2}\right) = \frac{-1}{1}$$

$$\sin\left(\frac{3\pi}{2}\right) = -1$$

$$\cos 2\pi = \frac{1}{1}$$

$$\cos 2\pi = 1$$

$$\tan\left(\frac{3\pi}{2}\right) = \frac{-1}{0}$$

$$\tan\left(\frac{3\pi}{2}\right) = \text{UND}$$

$$\csc\theta = \frac{Hyp}{Opp}$$

$$\boxed{\csc\theta = \frac{1}{\sin\theta}}$$

$$\sec\theta = \frac{Hyp}{Adj}$$

$$\boxed{\sec\theta = \frac{1}{\cos\theta}}$$

$$\cot\theta = \frac{Adj}{Opp}$$

$$\boxed{\csc\theta = \frac{1}{y}}$$

$$\csc 0 = \frac{1}{\sin 0}$$

$$\boxed{\sec\theta = \frac{1}{x}}$$

$$\sec\left(\frac{\pi}{2}\right) = \frac{1}{\cos\left(\frac{\pi}{2}\right)}$$

$$\boxed{\cot\theta = \frac{x}{y}}$$

$$\csc\theta = \frac{1}{0}$$

$$\csc 0 = \frac{1}{0}$$

$$\sec\left(\frac{\pi}{2}\right) = \frac{1}{0}$$

$$\sec\left(\frac{\pi}{2}\right) = \frac{1}{0}$$

$$\cot 0 = \frac{0}{1}$$

$$\csc 0 = \text{und}$$

$$\csc\theta = \text{und}$$

$$\sec\left(\frac{\pi}{2}\right) = \text{und}$$

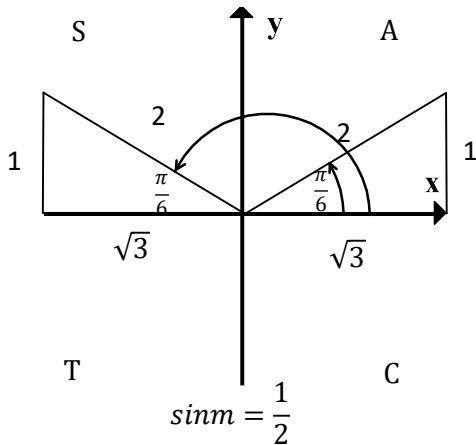
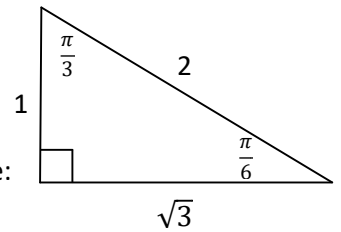
$$\cot 0 = 0$$

C12 - 4.5 - sin 2θ ASTC Special Triangles Notes

Solve for θ $0^\circ \leq \theta < 2\pi$, and the general solution.

$$\sin 2\theta = \frac{1}{2} \quad \sin m = \frac{1}{2}$$

Let $m = 2\theta$



Draw two triangles where $\sin m$ is positive:
ASTC Quadrant I, II

Label the triangles according to special triangles
and SOH CAH TOA

Label the reference angle according to
special triangles.

Draw an arrow from the principal axis to the first terminal arm
Draw an arrow from the principal axis to the second terminal arm.

Solve for the arrows θ_{stp}

$$m_{stp} = \frac{\pi}{6} \quad m_{stp} = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$m_{stp} = \frac{\pi}{6}, \frac{5\pi}{6}$$

Check your answer:

$$\sin \frac{\pi}{6} = \frac{1}{2}$$

$$\sin \frac{5\pi}{6} = \frac{1}{2}$$

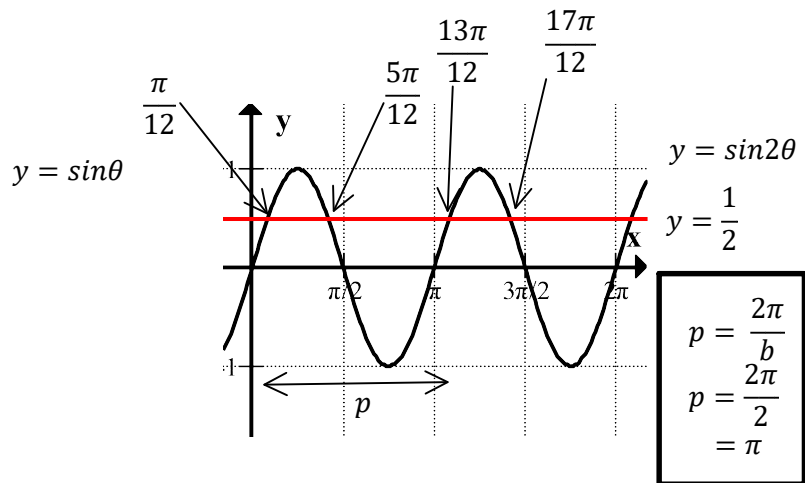
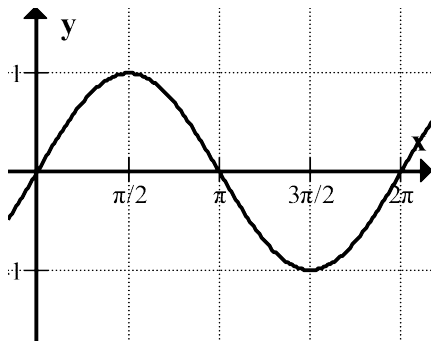
$$m = \frac{\pi}{6} \quad m = \frac{5\pi}{6}$$

$$2\theta = \frac{\pi}{6} \quad 2\theta = \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{6 \times 2} \quad \theta = \frac{5\pi}{6 \times 2}$$

$$\theta = \frac{\pi}{12} \quad \theta = \frac{5\pi}{12}$$

Substitute 2θ back in for m .



$$\theta = \theta_{stp} \pm p$$

$$\theta = \frac{\pi}{12} + \pi$$

$$\theta = \frac{13\pi}{12}$$

$$\theta = \theta_{stp} \pm p$$

$$\theta = \frac{5\pi}{12} + \pi$$

$$\theta = \frac{17\pi}{12}$$

~~$$\theta = \frac{13\pi}{12} + \pi$$~~
~~$$\theta = \frac{25\pi}{12} > 2\pi$$~~

$$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

Add/Subtract
period until
outside of the
domain.

General Solution: $\theta = \theta_{stp} \pm pn, n \in I$ $\theta = \theta_{stp} \pm pn, n \in I$

$\theta = \frac{\pi}{12} \pm \pi n, n \in I$ $\theta = \frac{5\pi}{12} \pm \pi n, n \in I$

The usual number of
answers in the
domain times b.

C12 - 4.5 - Algebra Period Equations Notes

$$0 \leq \theta < 2\pi$$

$2\sin x + 1 = 0$ $2\sin x = -1$ $\sin x = -\frac{1}{2}$ \dots	$5 - 3\cos x = 4$ $-3\cos x = -1$ $\cos x = \frac{1}{3}$ \dots
---	---

$$\sin x = x - 1$$

$$x = 1.93$$

$$y = \sin x$$

$$y = x - 1$$

Find Intersection

$$\cos\left(\frac{\pi}{2}x\right) = 0$$

$$\cos m = 0$$

$$\text{let } m = \frac{\pi}{2}x$$

$$\dots$$

$$m = \frac{\pi}{2}$$

$$\frac{\pi}{2}x = \frac{\pi}{2}$$

$$x = 1$$

$$m = \frac{3\pi}{2}$$

$$\frac{\pi}{2}x = \frac{3\pi}{2}$$

$$x = 3$$

$$p = \frac{2\pi}{b}$$

$$p = \frac{2\pi}{2}$$

$$p = 2\pi \times \frac{2}{\pi}$$

$$p = 4$$

$$x = 1 + 4$$

$$x = 5$$

$$x = 3 + 4$$

$$x = 7$$

Reject

$$\tan(x - 1) = -0.2$$

$$\tan m = -0.2$$

$$\text{let } m = x - 1$$

$$\dots$$

$$m = 2.94$$

$$x - 1 = 2.94$$

$$x = 3.94$$

$$m = 6.09$$

$$x - 1 = 6.09$$

$$x = 7.09$$

Reject

$$x = 3.94 - \pi$$

$$x = 0.80$$

$$x = 7.09 - \pi$$

$$x = 3.94$$

$$p = \frac{\pi}{b}$$

$$p = \frac{\pi}{1}$$

$$p = \pi$$

$$\sin\left(\frac{\pi}{4}(x - 6)\right) = \frac{1}{2}$$

$$\sin m = \frac{1}{2}$$

$$\text{let } m = \frac{\pi}{4}(x - 6)$$

$$\dots$$

$$m = \frac{\pi}{6}$$

$$\frac{\pi}{4}(x - 6) = \frac{\pi}{6}$$

$$x - 6 = \frac{2}{3}$$

$$x = \frac{20}{3}$$

$$x = 6.67$$

$$x = 6.67 - 8$$

$$x = -1.33$$

$$m = \frac{5\pi}{6}$$

$$\frac{\pi}{4}(x - 6) = \frac{5\pi}{6}$$

$$x - 6 = \frac{10}{3}$$

$$x = \frac{28}{3}$$

$$x = 9.33$$

$$x = 9.33 - 8$$

$$x = 1.33$$

$$p = \frac{2\pi}{b}$$

$$p = \frac{2\pi}{4}$$

$$p = \frac{\pi}{2}$$

Add/Subtract period until outside of the domain.

The usual number of answers in the domain times b.

C12 - 4.6 - Equations Algebra Notes

$$\sin x + \sin x \in 2\sin x$$

$$5\cos x - 3\cos x \in 2\cos x$$

Add/Subtract Like Terms

$$3\tan x = 5 + \tan x$$

$$3m = 5 + m$$

$$2m = 5$$

$$m = 2.5$$

$$\tan x = 2.5$$

....

let $m = \tan x$

$$1 + \sin x = 4\sin x$$

$$1 + m = 4m$$

$$3m = 1$$

$$m = \frac{1}{3}$$

$$\sin x = \frac{1}{3}$$

.....

let $m = \sin x$

Algebra

$$\frac{\cos x}{\cos x + 1} = -\frac{1}{3}$$

$$\frac{m}{m + 1} = -\frac{1}{3}$$

$$3m = -m - 1$$

$$m = -\frac{1}{4}$$

$$\cos x = -\frac{1}{4}$$

....

$$m = \cos x$$

$$2\sin x = 4$$

$$\sin x = 2$$

No Solution

$\sin x = \cos x$
 $\frac{\sin x}{\cos x} = 1$
 $\tan x = 1$
 ...
 $x = \frac{\pi}{4}, \frac{5\pi}{4}$

Identities

$$5 - 2\csc x = 0$$

$$5 - 2 \times \frac{1}{\sin x} = 0$$

$$5 - \frac{2}{m} = 0$$

$$5 = \frac{2}{m}$$

$$m = \frac{2}{5}$$

$$m = 0.4$$

$$\sin x = 0.4$$

....

$\csc \theta = \frac{1}{\sin \theta}$

let $m = \sin x$

$$\sin x - \csc x = 0$$

$$\sin x - \frac{1}{\sin x} = 0$$

$$m - \frac{1}{m} = 0$$

$$\left(m - \frac{1}{m} = 0\right) \times m$$

$$m^2 - 1 = 0$$

$$(m + 1)(m - 1) = 0$$

Identities

let $m = \sin x$

Factor

$$m = 1$$

$$\sin x = 1$$

....

$$m = -1$$

$$\sin x = -1$$

....

$$\sin x \neq 0$$

C12 - 4.6 - Factoring/Distributing Notes

$$\cos x(\cos x + 1)$$

$$\cos^2 x + \cos x$$

Distribution

$$\begin{matrix} m(m+1) \\ m^2 + m \end{matrix}$$

$$\sin x - \sin^2 x$$

$$\sin x(1 - \sin x)$$

Factor

$$\begin{matrix} \sin x - \sin^2 x \\ m - m^2 \\ m(1 - m) \end{matrix}$$

$$m = \sin x$$

$$\sin x(1 - \sin x)$$

$$\sin x \cos x + \cos x$$

$$\cos x(\sin x + 1)$$

$$\begin{matrix} nm + m & n = \sin x \\ m(n + 1) & m = \cos x \end{matrix}$$

$$(\cos x + 1)(\cos x - 2)$$

$$\cos^2 x - \cos x - 2$$

$$\begin{matrix} (m+1)(m-2) \\ m^2 - m - 2 \end{matrix}$$

$$\begin{matrix} (1 + \cos x)(1 - \cos x) \\ 1 - \cancel{\cos x} + \cancel{\cos x} - \cos^2 x \end{matrix}$$

Distribution

$$\begin{matrix} (m+n)(m-n) \\ m^2 - n^2 \end{matrix}$$

$$1 - \cos^2 x$$

$$1 - \sin^2 x$$

$$(1 + \sin x)(1 - \sin x)$$

$$\begin{matrix} 1 - a^2 \\ (1 - a)(1 + a) \end{matrix}$$

$$\cos^2 x - 1$$

$$(\cos x + 1)(\cos x - 1)$$

$$\begin{matrix} a^2 - 1 \\ (a + 1)(a - 1) \end{matrix}$$

Differences of squares

$$\sin^2 x - \cos^2 x$$

$$(\sin x + \cos x)(\sin x - \cos x)$$

$$\begin{matrix} m^2 - n^2 \\ (m + n)(m - n) \end{matrix}$$

$$\begin{matrix} \cos^4 x - \sin^4 x \\ (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) \\ (\cos 2x)(1) \end{matrix}$$

$$\begin{matrix} m^4 - n^4 \\ (m^2 + n^2)(m^2 - n^2) \end{matrix}$$

$$\cos 2x$$

Identities

$$\begin{matrix} \sin^2 \theta + \cos^2 \theta = 1 \\ \cos 2\theta = \cos^2 \theta - \sin^2 \theta \end{matrix}$$

$$\sin^2 x + \sin x - 2$$

$$(\sin x + 2)(\sin x - 1)$$

Factor

$$\begin{matrix} \sin^2 x + \sin x - 2 \\ m^2 + m - 2 \\ (m + 2)(m - 1) \end{matrix}$$

$$\text{let } m = \sin x$$

$$(\sin x + 2)(\sin x - 1)$$

$$\begin{matrix} \sin^2 \theta + 2\sin \theta \cos \theta + \cos^2 \theta \\ (\sin \theta + \cos \theta)(\sin \theta + \cos \theta) \end{matrix}$$

$$(\cos x + \sin x)^2$$

$$\begin{matrix} m^2 + 2mn + n^2 \\ (m + n)(m + n) \end{matrix}$$

$$-\sin x + 1 = 1 - \sin x$$

$$2 + \sin x + \sin^2 x = \sin^2 x + \sin x + 2$$

Rearrange order of Terms

$$\sin^2 x + \tan x + \cos^2 x = \sin^2 x + \cos^2 x + \tan x = 1 + \tan x$$

C12 - 4.6 - Solving Equations Notes

$$\begin{aligned} \cos^2 x + \cos x &= 0 \\ \cos x(\cos x + 1) &= 0 \quad \text{Factor} \\ \cos x &= 0 & \cos x + 1 &= 0 \\ & & \cos x &= -1 \end{aligned}$$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \pi$

$$\begin{aligned} \cos^2 x + \cos x &= 0 \\ m^2 + m &= 0 & \text{let } m = \cos x \\ m(m + 1) &= 0 & \text{Factor} \\ m = 0 & & m = -1 \\ \cos x = 0 & & \cos x = -1 \end{aligned}$$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \pi$

$$\begin{aligned} \sin^2 x + \sin x - 2 &= 0 \\ m^2 + m - 2 &= 0 & \text{let } m = \sin x \\ (m + 2)(m - 1) &= 0 \end{aligned}$$

$m = -2$
 $\sin x = -2$
 Reject

$m = 1$
 $\sin x = 1$

$x = \frac{\pi}{2}$

$-1 \leq \sin x \leq 1$
 $-1 \leq \cos x \leq 1$

$$\begin{aligned} 2 \sin^2 x + \sin x - 1 &= 0 \\ 2m^2 + m - 1 &= 0 & \text{let } m = \sin x \\ \dots & & \\ (2m - 1)(m + 1) &= 0 & \text{Factor} \end{aligned}$$

$m = \frac{1}{2}$
 $\sin x = \frac{1}{2}$

$m = -1$
 $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{3\pi}{2}$

$$\begin{aligned} 3 \cos^2 x - 8 \cos x - 5 &= 0 \\ 3m^2 - 8m - 5 &= 0 & \text{let } m = \cos x \end{aligned}$$

$m = 3.18$ $m = -0.52$

Quadform

$\cos x = -0.52$

$x = 2.12$ $x = 4.16$

C12 - 4.6 - Identities Chapter 6 Notes

Identities

$$\begin{aligned} \sin 2x + \cos x &= 0 \\ 2\sin x \cos x + \cos x &= 0 \\ \cos x(2\sin x - 1) &= 0 \end{aligned}$$

$$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$$

$$\begin{aligned} \cos x = 0 & \quad 2\sin x - 1 = 0 \\ \text{---} & \quad \sin x = \frac{1}{2} \\ \text{---} & \quad \text{---} \end{aligned}$$

$$\begin{aligned} \sin x - \cos^2 x - 1 &= 0 \\ \sin x - (1 - \sin^2 x) - 1 &= 0 \\ \sin x - 1 + \sin^2 x - 1 &= 0 \\ \sin^2 x + \sin x - 2 &= 0 \end{aligned}$$

Identities

$$\boxed{\cos^2 x = 1 - \sin^2 x}$$

Identities

$$\boxed{\cos 2x = 1 - 2\sin^2 x}$$

$$\begin{aligned} \sin x + \cos 2x &= 0 \\ \sin x - (1 - 2\sin^2 x) &= 0 \\ 2\sin^2 x + \sin x - 1 &= 0 \end{aligned}$$

$$\begin{aligned} \frac{\cos x}{\cos x + 1} &= -\frac{1}{3} \\ \frac{m}{m+1} &= -\frac{1}{3} \\ 3m &= -m - 1 \\ m &= -\frac{1}{4} \\ \cos x &= -\frac{1}{4} \end{aligned}$$

$$\begin{aligned} 1 + \cos x &= \sin x \\ (1 + \cos x)^2 &= (\sin x)^2 \\ 1 + 2\cos x + \cos^2 x &= \sin^2 x \\ 1 + 2\cos x + \cos^2 x &= 1 - \cos^2 x \\ 2\cos^2 x + 2\cos x &= 0 \\ 2\cos x(\cos x + 1) &= 0 \end{aligned}$$

$$\begin{aligned} \cos x \cos 2x - \sin x \sin 2x &= -1 \\ \cos x \cos 2x - \sin x \sin 2x &= -1 \\ \cos(2x + x) &= -1 \\ \cos 3x &= 1 \\ \text{---} & \end{aligned}$$

$$\boxed{\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

Identities

$$\cos x = 0 \quad \cos x = -1$$

C12 - 4.7 - NPV Trig Notes

$$\frac{1}{\cos\theta}$$

$$\frac{\tan\theta}{\cos\theta}$$

$$\frac{\sec\theta}{\cos\theta}$$

$$\cos\theta \neq 0$$

$$0 \leq \theta < 2\pi$$

General Solution

$$\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\theta \neq \frac{\pi}{2} + \pi n, n \in \mathbb{I}$$

$$\cos\theta = y$$

$$p = \frac{3\pi}{2} - \frac{\pi}{2} = \pi$$

$$\frac{1}{\sin\theta}$$

$$\frac{\cot\theta}{\sin\theta}$$

$$\frac{\csc\theta}{\sin\theta}$$

$$\sin\theta \neq 0$$

$$\theta \neq 0, \pi$$

$$\theta \neq \pi n, n \in \mathbb{I}$$

$$\sin\theta = y$$

$$\frac{1}{\frac{\tan\theta}{\cos\theta}}$$

$$\sin\theta \neq 0$$

...

$$\frac{1}{\frac{\cot\theta}{\sin\theta}}$$

$$\sin\theta \neq 0$$

...

Any denominator or any part of a fraction that will make a denominator zero.

$$\frac{1}{\cos\theta + 1}$$

$$\cos\theta + 1 \neq 0$$

$$\cos\theta \neq -1$$

$$\theta \neq \pi$$

$$\theta \neq \pi + 2\pi n, n \in \mathbb{I}$$

$$\frac{1}{\sin\theta - \frac{1}{2}}$$

$$\sin\theta - \frac{1}{2} \neq 0$$

$$\sin\theta \neq \frac{1}{2}$$

...

$$\frac{1}{\cos^2 x - 1}$$

$$\cos^2 x - 1 \neq 0$$

$$\sin^2 x \neq 1$$

$$\sin x \neq \pm 1$$

...

$$\frac{1}{\sin^2 x + 1}$$

$$\sin^2 x + 1 \neq 0$$

$$\sin^2 x \neq -1$$

$$\sin x \neq \sqrt{-1}$$

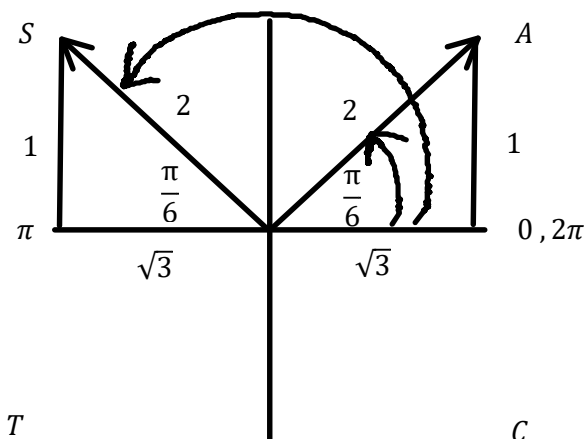
No Restrictions

C12 - 4.7 - ASTC General Solutions

$$\theta = \theta_{stp} \pm pn, n \in I$$

Solve for $\theta, 0 \leq \theta < 2\pi$, and find general solution.

$$\sin\theta = \frac{1}{2}$$

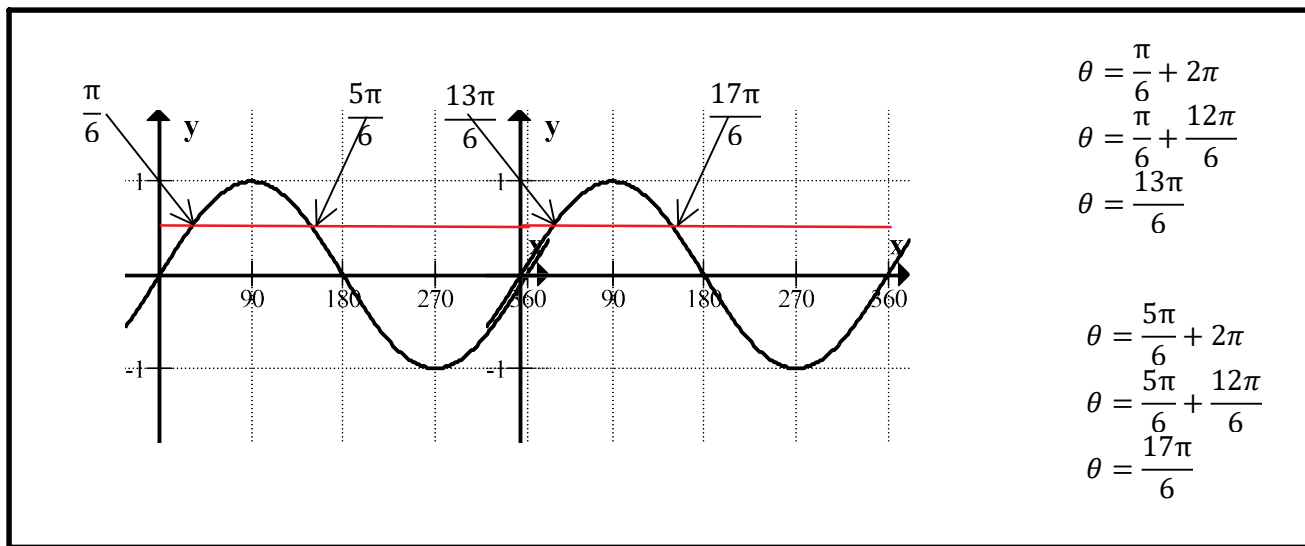


$$\theta = \frac{\pi}{6}$$

$$\theta = \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{6} + 2\pi n, n \in I$$

$$\theta = \frac{5\pi}{6} + 2\pi n, n \in I$$



$$\theta = \frac{\pi}{6} + 2\pi$$

$$\theta = \frac{\pi}{6} + \frac{12\pi}{6}$$

$$\theta = \frac{13\pi}{6}$$

$$\theta = \frac{5\pi}{6} + 2\pi$$

$$\theta = \frac{5\pi}{6} + \frac{12\pi}{6}$$

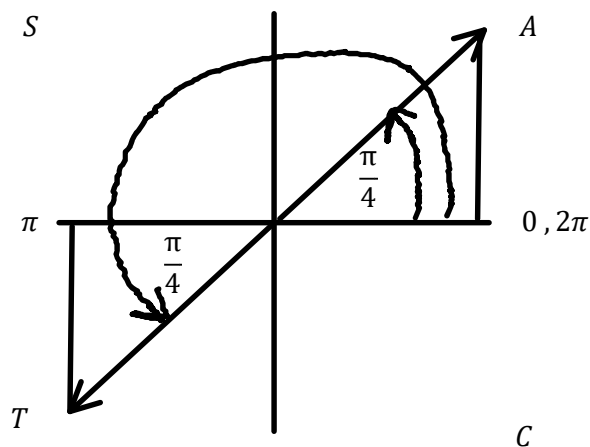
$$\theta = \frac{17\pi}{6}$$

C12 - 4.7 - ASTC Reject General Solutions

$$\theta = \theta_{stp} \pm pn, n \in I$$

Solve for $\theta, 0 \leq \theta < 2\pi$, and find general solution.

$\tan\theta = 1$



$$\theta = \frac{\pi}{4}$$

$$\theta = \frac{5\pi}{4}$$

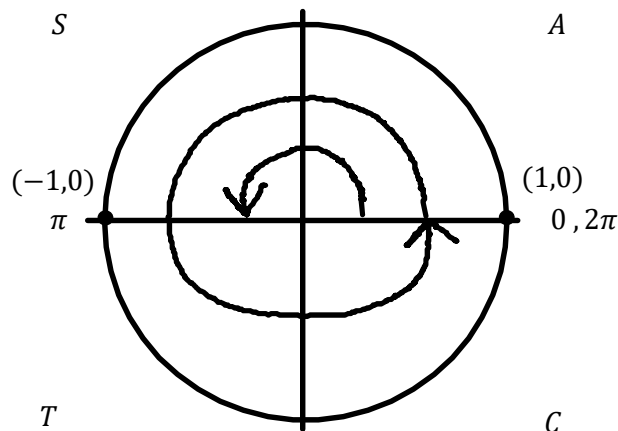
$$\theta = \frac{\pi}{4} + \pi n, n \in I$$

~~$$\theta = \frac{5\pi}{4} + \pi n, n \in I$$~~

$$\frac{\pi}{4}, \frac{5\pi}{4}, \dots$$

$$\frac{\pi}{4} + p + p + p \dots$$

$\sin\theta = 0$



$$\theta = 0$$

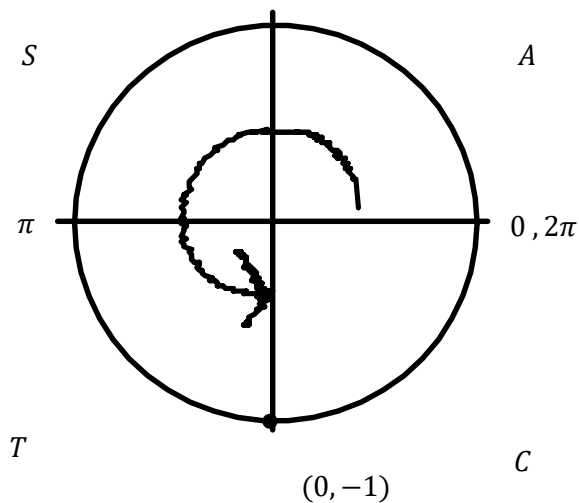
$$\theta = \pi$$

$$\theta = 0 + 2\pi n, n \in I$$

$$\theta = \pi + 2\pi n, n \in I$$

$$\theta = \pi n, n \in I$$

$\sin\theta = -1$



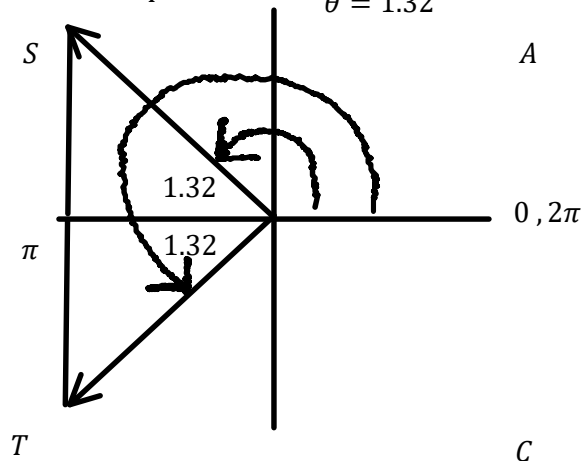
$$\theta = \frac{3\pi}{2}$$

$$\theta = \frac{3\pi}{2} + 2\pi n, n \in I$$

$$\cos\theta = -\frac{1}{4}$$

$$\theta = \cos^{-1}\left(\frac{1}{4}\right)$$

$$\theta = 1.32$$



$$\theta = 1.82$$

$$\theta = 4.46$$

$$\theta = 1.82 + 2\pi n, n \in I$$

$$\theta = 4.46 + 2\pi n, n \in I$$

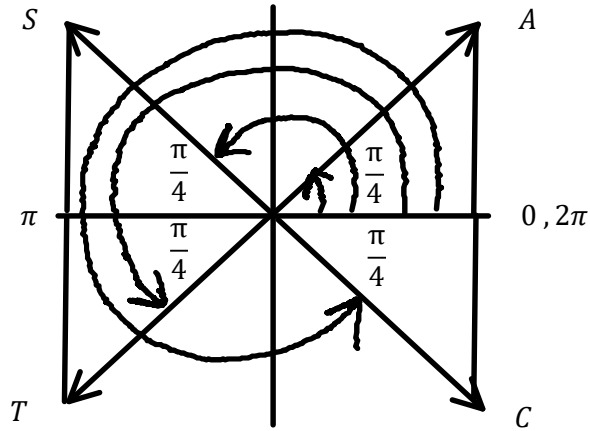
C12 - 4.7 - Square Root General Solutions

$$\theta = \theta_{stp} \pm pn, n \in I$$

Solve for $\theta, 0 \leq \theta < 2\pi$, and find general solution.

$$\sin^2 \theta = \frac{1}{2}$$

$$\sin \theta = \pm \frac{1}{\sqrt{2}}$$



$$\theta = \frac{\pi}{4}$$

$$\theta = \frac{3\pi}{4}$$

$$\theta = \frac{5\pi}{4}$$

$$\theta = \frac{7\pi}{4}$$

$$\theta = \frac{\pi}{4} + \frac{\pi}{2}n, n \in I$$

$$\tan^2 \theta = 1$$

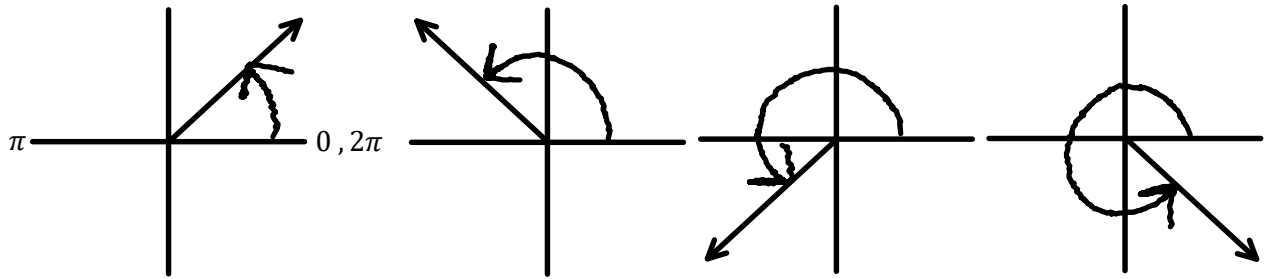
$$\frac{\sin^2 \theta}{\cos^2 \theta} = 1$$

$$\sin^2 \theta = \cos^2 \theta$$

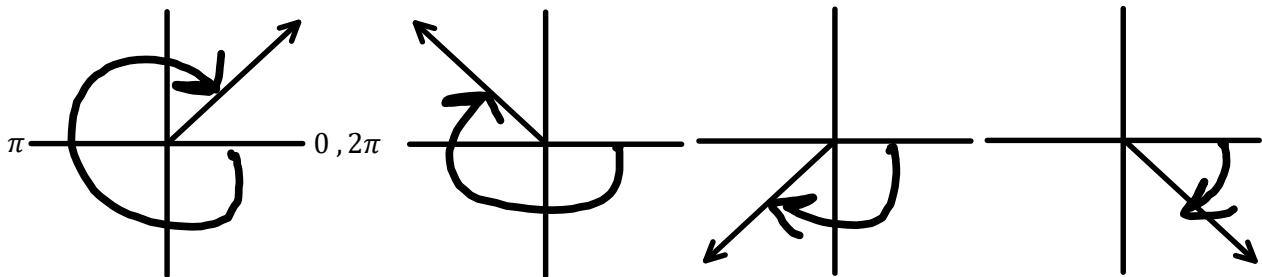
$$\theta = \frac{\pi}{4} + \frac{\pi}{2}n, n \in I$$

C12 - 4.7 - Domain Change Notes

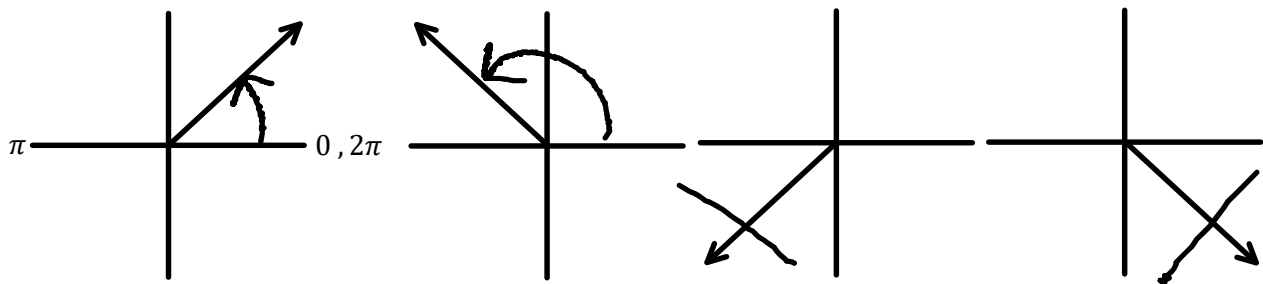
$$0 \leq \theta < 2\pi$$



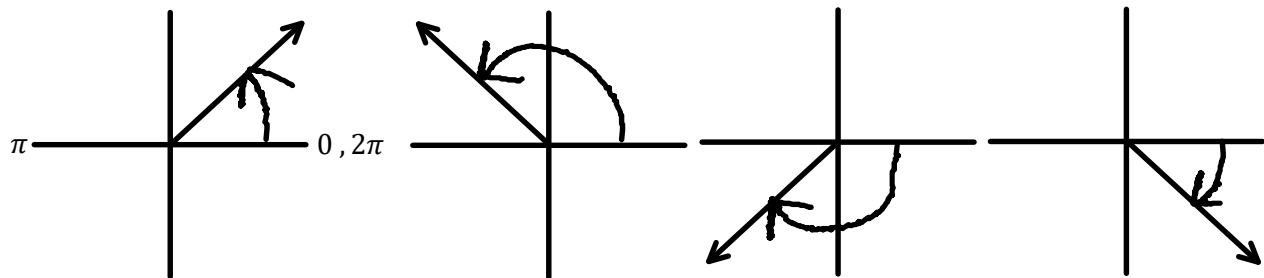
$$-2\pi \leq \theta < 0$$



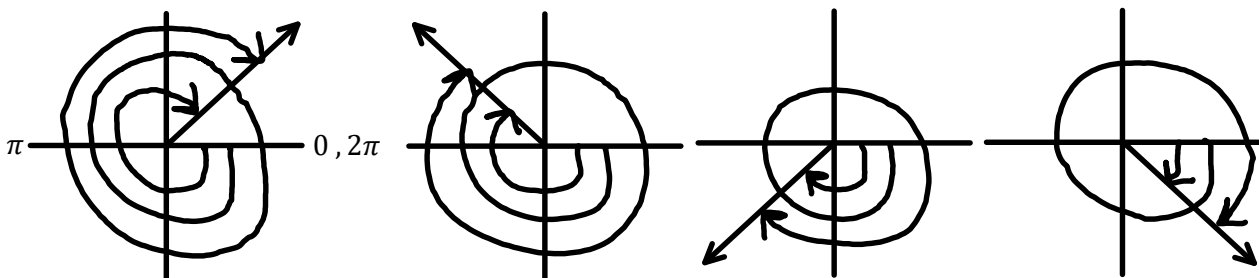
$$0 \leq \theta < \pi$$



$$-\pi \leq \theta < \pi$$

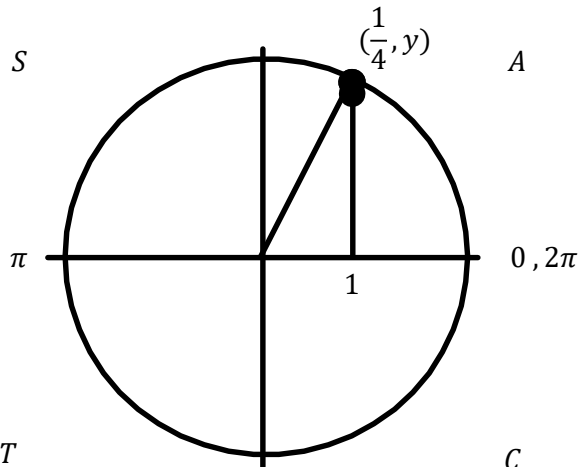


$$-4\pi \leq \theta < 0$$



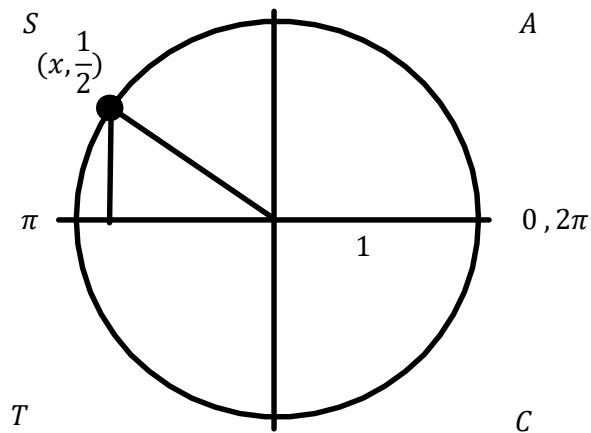
C12 - 4.8 - Solve (x,y) Unit Circle Notes

Solve the point on the unit circle



$$\begin{aligned}
 x^2 + y^2 &= 1 \\
 \left(\frac{1}{4}\right)^2 + y^2 &= 1 \\
 \frac{1}{16} + y^2 &= \frac{16}{16} \\
 y^2 &= \frac{15}{16}
 \end{aligned}$$

$$y = \pm \frac{\sqrt{15}}{4} \quad \left(\frac{1}{4}, \frac{\sqrt{15}}{4}\right)$$



$$\begin{aligned}
 x^2 + y^2 &= 1 \\
 x^2 + \left(\frac{1}{2}\right)^2 &= 1 \\
 x^2 + \frac{1}{4} &= \frac{4}{4} \\
 x^2 &= \frac{3}{4}
 \end{aligned}$$

$$x = \pm \frac{\sqrt{3}}{2} \quad \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

Is the point on the unit circle

$$\left(-\frac{3}{4}, \frac{1}{4}\right)$$

$$\begin{aligned}
 x^2 + y^2 &= 1 \\
 \left(-\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 &\neq 1 \\
 \frac{9}{16} + \frac{1}{16} &\neq 1 \\
 \frac{10}{16} &\neq 1
 \end{aligned}$$

Not on Unit Circle

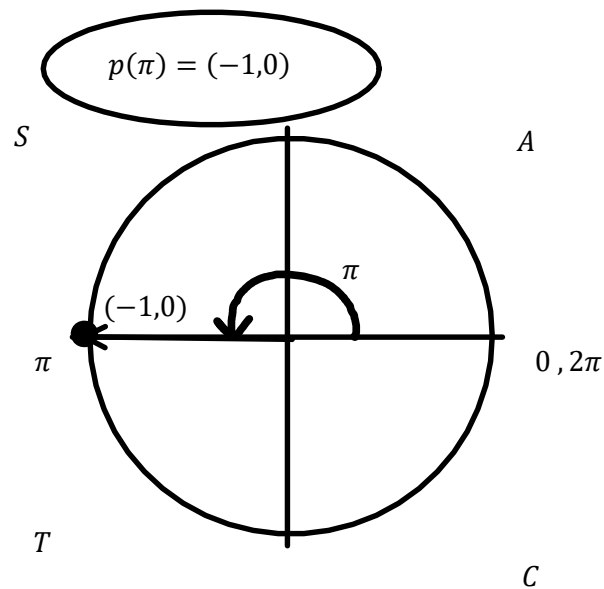
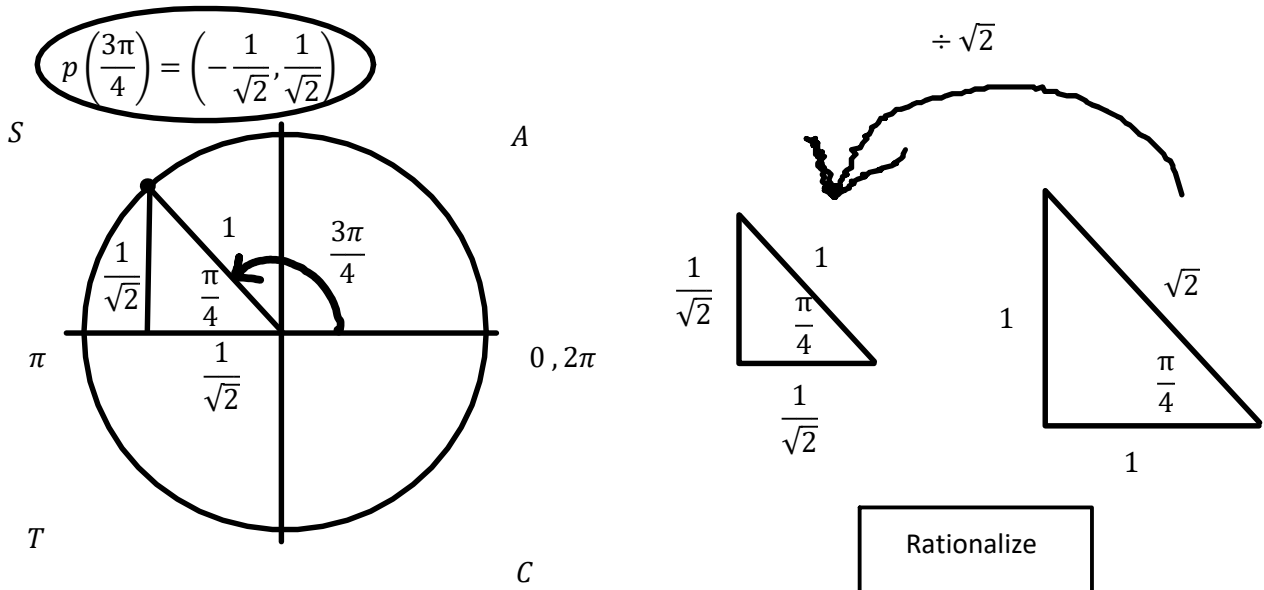
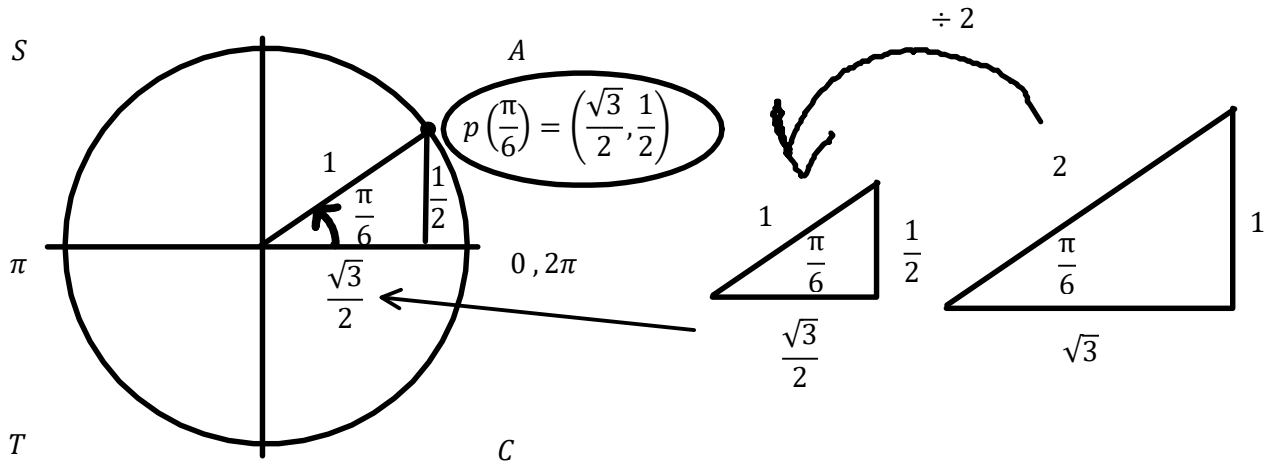
$$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

$$\begin{aligned}
 x^2 + y^2 &= 1 \\
 \left(-\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 &= 1 \\
 \frac{3}{4} + \frac{1}{4} &= 1 \\
 1 &= 1
 \end{aligned}$$

On Unit Circle

C12 - 4.8 - Solve $p(\theta)$ Unit Circle Notes

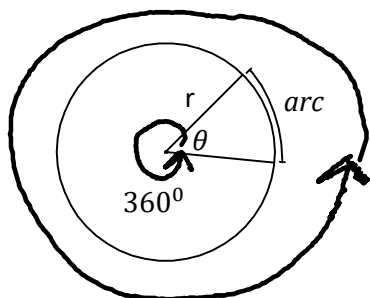
Solve the point on the unit circle



<p>Rationalize</p> $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

C12 - 4.9 - Arc Length, Sector Area Notes

θ in radians



Circumference

$$\frac{\text{arc length}}{\text{Circumference}} = \frac{\theta}{360^\circ}$$

Grade 8-11

$$\frac{\text{arc length}}{\text{Circumference}} = \frac{\theta}{2\pi}$$

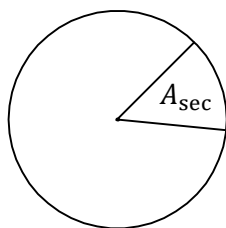
$$\begin{aligned} \frac{a}{2\pi r} &= \frac{\theta}{2\pi} \\ 2\pi \times \frac{a}{2\pi r} &= \frac{\theta}{2\pi} \times 2\pi \\ \frac{a}{r} &= \theta \\ r \times \frac{a}{r} &= \theta \times r \end{aligned}$$

$$a = \theta r$$

$$a = \theta r$$

θ must be in radians

Sector Area



$$\frac{\text{Area}_{\text{sector}}}{\text{Area}_{\text{total}}} = \frac{\text{arc length}}{\text{Circumference}}$$

$$\frac{A_{\text{sec}}}{\pi r^2} = \frac{a}{2\pi r}$$

$$A_{\text{sec}} = \frac{ar}{2}$$

$$A = \frac{\theta r^2}{2}$$

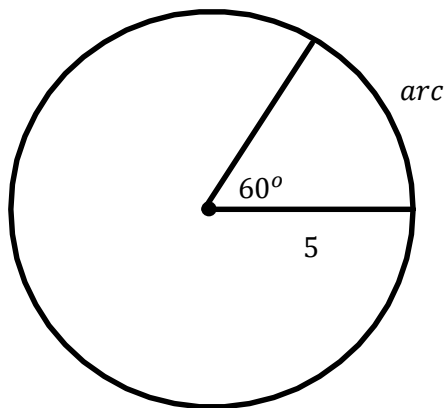
$$A = \frac{ar}{2}$$

$$\frac{A_{\text{sec}}}{\pi r^2} = \frac{a}{2\pi r} = \frac{\theta}{360^\circ} = \frac{\theta}{2\pi}$$

They are all equal to each other.

C12 - 4.9 - Arc Length Notes

Find the arc length



$$\frac{a}{C} = \frac{\theta}{360}$$

$$\frac{a}{2\pi r} = \frac{360}{360}$$

$$a = \frac{\pi(5)}{3}$$

$$a = 5.24$$

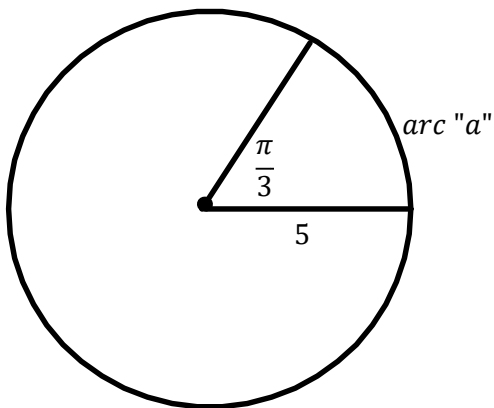
$$C = 2\pi r$$

$$C = 2\pi \times 5$$

$$C = 31.4$$

$$\frac{60^\circ}{360^\circ} = \frac{1}{6} \text{ of circle}$$

$$\frac{1}{6} \times 31.4 = 5.2$$

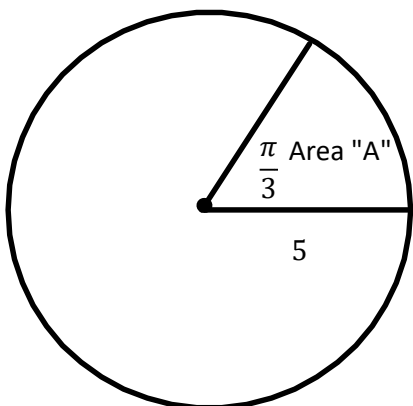


$$a = \theta r$$

$$a = \frac{\pi}{3} \times 5$$

$$a = 5.24$$

Find the Area of the Sector



$$\frac{Area_{sector}}{Area_{Total}} = \frac{\theta}{2\pi}$$

$$\frac{A_{sec}}{\pi r^2} = \frac{\theta}{2\pi}$$

$$A_{sec} = \frac{\theta r^2}{2}$$

$$A_{sec} = \frac{\left(\frac{\pi}{3} \times 5^2\right)}{2}$$

$$A_{sec} = 13.09$$

$$A = \pi r^2$$

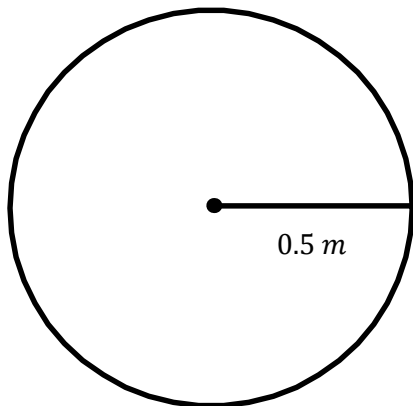
$$A = \pi(5)^2$$

$$A = 78.54$$

$$\frac{1}{6} \times 78.54 = 13.09$$

C12 - 4.9 - Angular Velocity Notes

Find the angular velocity of a wheel travelling 25 meters per second if the radius 0.5 meters. Find the arc in 0.1 seconds.



$$w = \frac{\theta}{t}$$

Angular Velocity "w"

$$C = 2\pi r$$

$$C = 2\pi(0.5)$$

$$C = 3.14 \text{ m}$$

$$1 \text{ Rev} = 2\pi$$

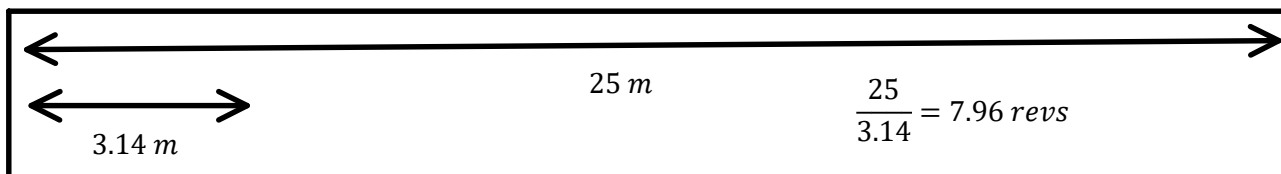
$$w = \frac{v}{C} \times 2\pi$$

$$w = \frac{v}{r}$$

$$\frac{25 \text{ m}}{\text{s}} \times \frac{1 \text{ revolution}}{3.14 \text{ m}} = 7.96 \frac{\text{Rev}}{\text{s}}$$

$$7.96 \times \frac{2\pi}{1} = 15.92 \frac{\text{Rad}}{\text{s}}$$

$$w = 15.92\pi \frac{\text{Rad}}{\text{s}}$$



$$25 \frac{\text{m}}{\text{s}} \Rightarrow 0.04 \frac{\text{s}}{\text{m}}$$

$$\frac{25}{1} \Rightarrow \frac{1}{25} = 0.04$$

$$15.92\pi \frac{\text{Rad}}{\text{s}} \times 0.1 \text{ s} = 5 \text{ Rad}$$

$$w = \frac{\theta}{t}$$

$$a = \theta r$$

$$a = 5(0.5)$$

$$a = 2.5 \text{ m}$$

$$\frac{25 \text{ m}}{\text{s}} \times \frac{1 \text{ revolution}}{3.14 \text{ m}} = 7.96 \frac{\text{Rev}}{\text{s}}$$

$$1 \text{ Rev} = 360^\circ$$

$$7.96 \times \frac{360^\circ}{1} = 2865.6 \frac{\circ}{\text{s}}$$

$$w = 2865.6 \frac{\circ}{\text{s}}$$

$$w = \frac{v}{C} \times 360^\circ$$

$$25 \frac{\text{m}}{\text{s}} \times 0.1 \text{ s} = 2.5 \text{ m}$$

$$d = vt$$