One radian is equal to the length of the arc of a circle with radius = 1.

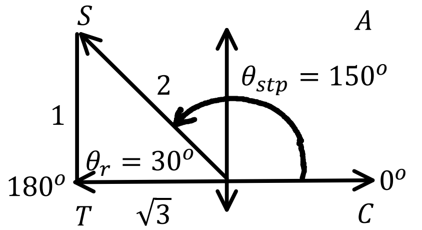
C12 - 4.0 - Trig Notes

$sin\theta \neq 2$

$\theta = \theta_{stp}$; Unless otherwise stated.

Rad <-> Deg : $30^o \times \frac{\pi}{180^o} = \frac{\pi}{6} = 0.52$

$\frac{5\pi}{6} \times \frac{180^o}{\pi} = 150^o$

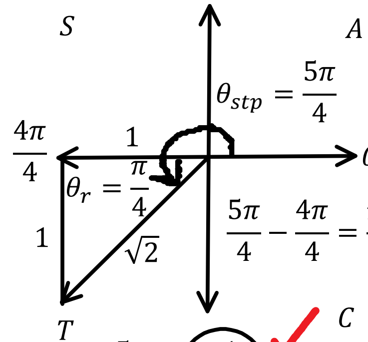


$180 - 150 = 30$

$cos150^o = -\frac{\sqrt{3}}{2} = 0.866$

$\theta_{cot} = -210^0, 150^o, 510^o$ ...

$\theta_{gen} = 150^o + 360n, n \in I$



$\sin\frac{5\pi}{4} = -\frac{1}{\sqrt{2}} = -0.707$

$\theta_{cot} = -\frac{3\pi}{4}; \frac{5\pi}{4}, \frac{13\pi}{4}$ ...

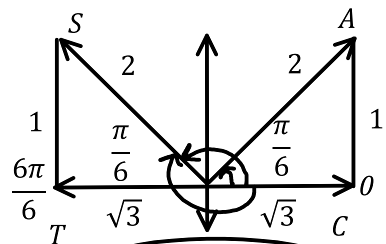
$\theta_{gen} = \frac{5\pi}{4} + 2\pi n, n \in I$

$p^* = 360^o, 2\pi$

Degrees are for children unless you're taking Physics

$2sin\theta - 1 = 0$ **Domain**

$sin\theta = \frac{1}{2}$; $-\frac{3\pi}{2} \leq \theta < 2\pi$



$\theta_{stp} = -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

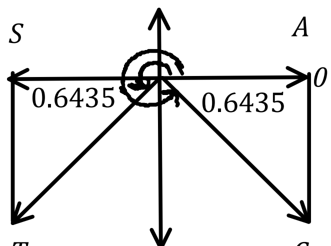
Check :

$\sin\frac{-7\pi}{6} = \sin\frac{\pi}{6} = \sin\frac{5\pi}{6} = \frac{1}{2}$

$\theta_{gen} = \frac{\pi}{6} + 2\pi n, n \in I$

$\theta_{gen} = \frac{5\pi}{6} + 2\pi n, n \in I$

$sin\theta = -\frac{0.6}{1}$; $0 \leq \theta < 2\pi$



$\theta_r = \sin^{-1}\left(+\frac{0.6}{1}\right)$

$\theta_r = 0.644$

$\pi + 0.6435$

$2\pi - 0.6435$

$\theta_{stp} = 3.785, 5.640$

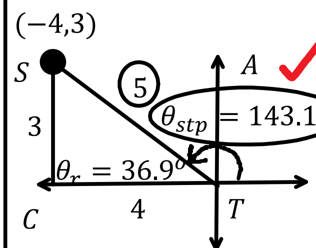
$sin3.785 = -0.6$

$sin3.6652 = -0.6$

$\theta_{gen} = 3.785 + 2\pi n, n \in I$

$\theta_{gen} = 5.640 + 2\pi n, n \in I$

SOH-CAH-TOA is a magical fairy land to teach grade 10's trig.



$a^2 + b^2 = c^2$

$c = 5$

$180 - 36.9 = 143.1$

$sin143.1 = 0.6 = \frac{3}{5}$

SOH-CAH-TOA
CHO-SHA-CAO

Reciprocal
Flip It!

$sin\theta = +\frac{3}{5}$ | $cscx = +\frac{5}{3}$

$cos\theta = -\frac{4}{5}$ | $secx = -\frac{5}{4}$

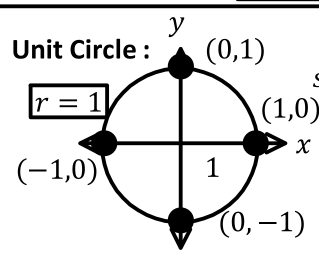
$tan\theta = -\frac{3}{4}$ | $cotx = -\frac{4}{3}$

$\theta_r = \tan^{-1}\left(+\frac{3}{4}\right)$

$\theta_r = 36.9^o$

θ_r : Only inverse positives.

Unit Circle :



$r = 1$

$sin90^o = 1$

$sin\theta = y$

$\tan\frac{3\pi}{2} = und$

$tan\theta = \frac{y}{x}$

$cos\theta = 0$; $0 < \theta < 360$

$cos\theta = x$

$\cos\frac{\pi}{2} = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos\frac{3\pi}{2} = 0$

$\theta_{gen} = \frac{\pi}{2} + \pi n, n \in I$

$\frac{3\pi}{2} - \frac{\pi}{2} = \pi$

$\csc\frac{5\pi}{6} = +\frac{2}{1} = \frac{H}{O}$ $\quad csc\theta = \frac{1}{sin\theta}$

$\sin\frac{5\pi}{6} = +\frac{1}{2} = \frac{O}{H}$ $\quad \frac{1}{\sin\frac{5\pi}{6}} = 2$

We don't flip the angle!

$csc\theta = \frac{2}{1} = \frac{H}{O}$

$sin\theta = \frac{1}{2} = \frac{O}{H}$

...

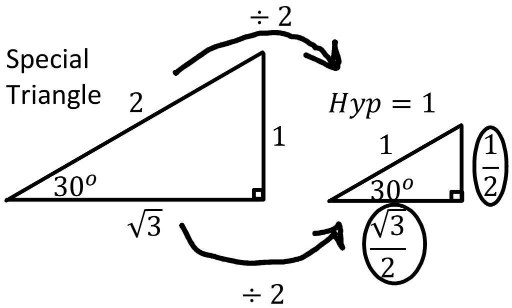
$cot\theta = und = \frac{\#}{0}$

$tan\theta = \frac{0}{\#} = 0$

...

C12 - 4.0 - Co/Sine Unit Circle Definition Notes

Half a Special Triangle so $HYP = 1$.



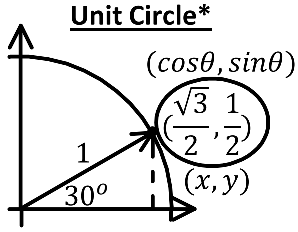
$sin\theta = \frac{O}{H}$

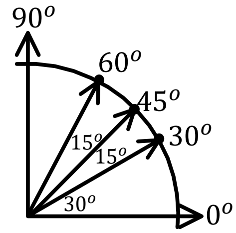
$sin\theta = \frac{1}{2}$

$sin\theta = \frac{O}{H}$

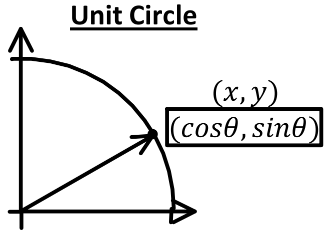
$sin\theta = \frac{\frac{1}{2}}{1} = \frac{1}{2}$

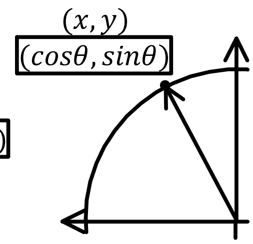
Unit Circle*



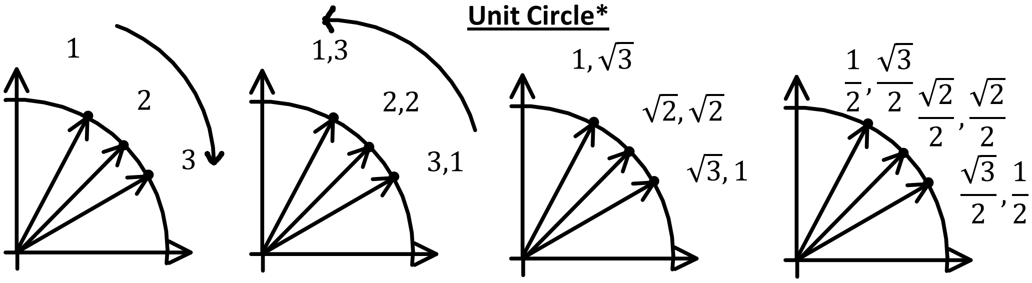


Unit Circle





Unit Circle*

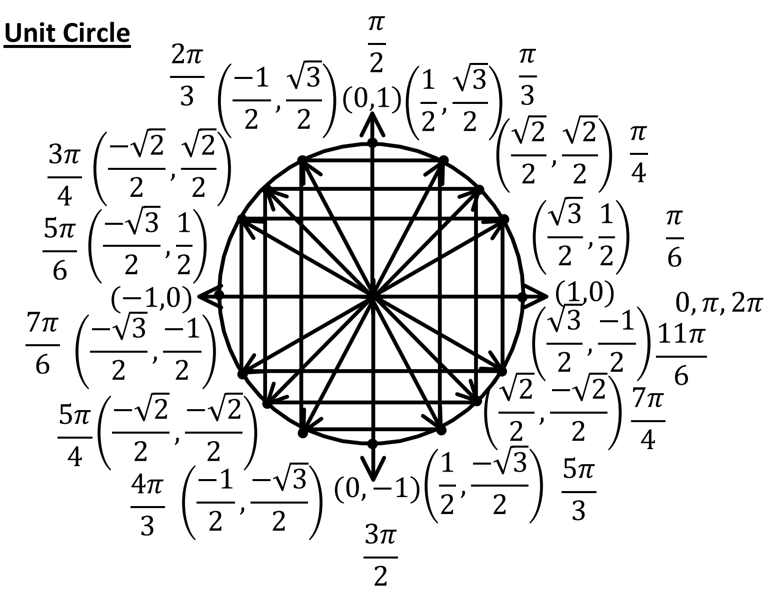


$-ve\ x\ QII$
$-ve\ x, y\ QIII$
$-ve\ y\ QIV$

This is obviously $\frac{1}{2}$

Steps : 123 → 123 → *Root the tops** → *All over 2* → *(Brackets)*

Unit Circle



$cos\theta = x$ $sin\theta = y$

$tan\theta = \frac{y}{x} = \frac{rise}{run} = \frac{sin\theta}{cos\theta}$

$$sin\theta = \sum_{k=0}^{\infty} \frac{(-1)^k \theta^{2k+1}}{(2k+1)!} = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} \dots$$

$$cos\theta = \sum_{k=0}^{\infty} \frac{(-1)^k \theta^{2k}}{(2k)!} = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} \dots$$

$$tan\theta = \cdots = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \frac{17\theta^7}{315} \dots$$

Rationalize the numerator/denominator.

$\frac{\sqrt{2}}{2} =$

$\frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{\sqrt{2}} =$

$\frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\frac{3}{2\sqrt{3}} =$

$\frac{3}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} =$

$\frac{3\sqrt{3}}{2 \times 3} = \frac{\sqrt{3}}{2}$

Rationalize to recognize Special Triangles/Unit Circle more easily.

C12 - 4.0 - Trig Notes

NPV's : $\frac{1}{tan\theta}$ Denominator $\neq 0$

$\frac{1}{\frac{sin\theta}{cos\theta}}$ $cos\theta \neq 0$ $sin\theta \neq 0$

... ...

$\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}$ $\theta \neq 0, \pi$

$\theta \neq \frac{\pi}{2} + \pi n, n \in I$ $\theta \neq \pi n, n \in I$

$\frac{3\pi}{2} - \frac{\pi}{2} = \pi$ $\pi - 0 = \pi$

$\frac{1}{cos\theta + 1}$

$cos\theta + 1 \neq 0$

$cos\theta \neq -1$

...

$\frac{1}{sin\theta - \frac{1}{2}}$

$sin\theta - \frac{1}{2} \neq 0$

$sin\theta \neq \frac{1}{2}$

...

$\frac{1}{\cos^2 x - 1}$

$\cos^2 x - 1 \neq 0$

$\cos^2 x \neq 1$

$cosx \neq \pm 1$

...

$\frac{1}{\sin^2 x + 1}$

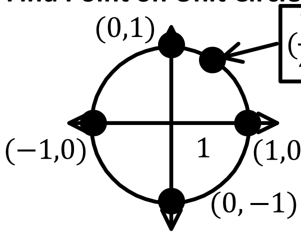
$\sin^2 x + 1 \neq 0$

$\sin^2 x \neq -1$

$sinx \neq \sqrt{-1}$

No Restrictions

Find Point on Unit Circle :



$x^2 + y^2 = 1$

$\left(\frac{1}{4}\right)^2 + y^2 = 1$

$\frac{1}{16} + y^2 = \frac{16}{16}$

$y^2 = \frac{15}{16}$

$y = \pm\frac{\sqrt{15}}{4}$

Is the Point on the Unit Circle :

$(-\frac{3}{4}, \frac{1}{4})$

$x^2 + y^2 = 1$

$\left(-\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 \neq 1$

$\frac{9}{16} + \frac{1}{16} \neq 1$

$\frac{10}{16} \neq 1$

Not on Unit Circle

$(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$

$x^2 + y^2 = 1$

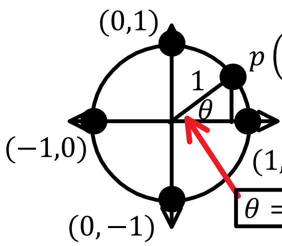
$\left(-\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 = 1$

$\frac{3}{4} + \frac{1}{4} = 1$

$1 = 1$ ✓

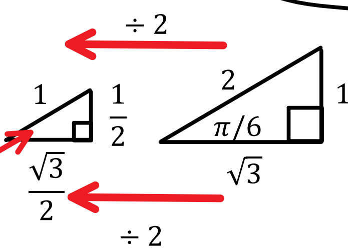
On Unit Circle

Solve the Point on the Unit Circle :

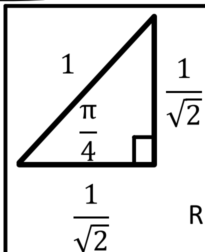


$p\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\theta = \pi/6$



Similar Triangles



$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

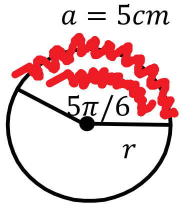
Rationalize!

Arc Length/Sector Area :

Find the Sector Area and Radius of the circle if arc-length subtended by θ.

$let\ a = arc - length$

$a = 5cm$



$a = \theta r$

$5 = \frac{5\pi}{6} r$

$\frac{5 \times 6}{5\pi} = r$

$r = \frac{6}{\pi} = 1.91cm$ ✓

$C = 2\pi r$

$C = 2\pi(1.91)$

$C = 12cm$

Logic Check

$A = \frac{ar}{2}$

$A = \frac{5 \times 1.91}{2}$

$A = 4.78cm^2$ ✓

$A = \pi r^2$

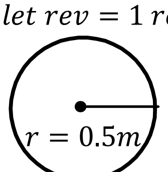
$A = \pi(1.91)^2$

$A = 11.46cm^2$

Find the angular velocity of a wheel travelling 25 $\frac{m}{s}$ if the radius 0.5 m. Find the arc in 0.1 s.

$let\ w = angular\ velocity$

$let\ rev = 1\ revolution$



$C = 2\pi r$

$C = 2\pi(0.5)$

$C = 3.14\ m$

Number of Turns :

25m 1s

3.14 m

$\frac{25}{3.14} = 7.96\ Revs$

$w = \frac{\theta}{t}$

$w = \frac{7.96\ revs}{1\ s}$

$w = \frac{7.96\ (2\pi)}{1\ s}$ ✓

$w = \frac{50\ radians}{s}$

$a = \theta r$

$a = 50(0.5)$

$a = 25m$

Length of tire that touches the road.

$1\ Rev = 2\pi$

$6.28 = 1\ rev$

$\frac{50}{6.28} = 7.96\ revs$

If you turn about 8 times with a circumference of about 3m you will be about 25m.

C12 - 4.0 - Trig Notes

... See Above

Algebra :

$sin\theta + sin\theta - 1 = 0$
$2sin\theta = 1$
$sin\theta = \frac{1}{2}$
...

$\cos^2\theta = 1$
$cos\theta = \pm 1$
$cos\theta = 1 \quad cos\theta = -1$
...

$\sin^2\theta = \frac{1}{2} \quad ; \quad 0 \le \theta < 2\pi$
$sin\theta = \pm\frac{1}{\sqrt{2}}$
...
4 triangles!

$\theta = \frac{\pi}{4} \quad \theta = \frac{3\pi}{4} \quad \frac{3\pi}{4} - \frac{\pi}{4} = \frac{\pi}{2}$

$\theta = \frac{5\pi}{4} \quad \theta = \frac{7\pi}{4} \quad \frac{5\pi}{4} - \frac{3\pi}{4} = \frac{\pi}{2}$

$\theta_{gen} = \frac{\pi}{4} + \frac{\pi}{2}n, n \in \mathrm{I}$

$2\sin^2\theta + sin\theta - 1 = 0$ Factoring
$2m^2 + m - 1 = 0$ let $m = sin\theta$
$(2m-1)(m+1) = 0$

$2m - 1 = 0 \quad m + 1 = 0$
$m = \frac{1}{2} \quad m = -1$
$sin\theta = \frac{1}{2} \quad sin\theta = -1$
... ...

$\frac{cosx}{cosx+1} = -\frac{1}{3}$
$\frac{m}{m+1} = -\frac{1}{3}$ let $m = cosx$
$3m = -m - 1$
$m = -\frac{1}{4}$
$cosx = -\frac{1}{4}$

$2sin\theta cos\theta + cos\theta = 0$
$cos\theta(2sin\theta + 1) = 0$
$cos\theta = 0 \quad 2sin\theta + 1 = 0$
... $sin\theta = \frac{1}{2}$
...

$\tan^2\theta + tan\theta = 3$
$m^2 + m - 3 = 0$ let $m = tan\theta$
...
Quadform!

$m = 1.3 \quad m = -2.3$
$tan\theta = 1.3 \quad tan\theta = -2.3$
$\theta = \tan^{-1}(+1.3) \quad \theta = \tan^{-1}(+2.3)$
$\theta_r = 0.915 \quad \theta_r = 1.161$
... ...

Period Change : $y = sinbx$ The usual number of answers in the domain times b.

$sin2\theta = \frac{1}{2} \quad ; \quad 0 \le \theta < 2\pi$
$sinm = \frac{1}{2}$ let $m = 2\theta$
...

$m = \frac{\pi}{6} \quad m = \frac{5\pi}{6}$
$2\theta = \frac{\pi}{6} \quad 2\theta = \frac{5\pi}{6}$
$\theta = \frac{\pi}{12} \quad \theta = \frac{5\pi}{12}$

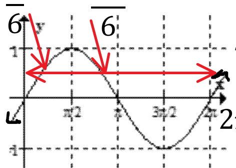
$\theta = \theta + p \quad \theta = \theta + p$
$\theta = \frac{\pi}{12} + \pi \quad \theta = \frac{5\pi}{12} + \pi$
$\theta = \frac{13\pi}{12} \quad \theta = \frac{17\pi}{12}$

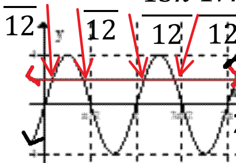
Period
$p = \frac{2\pi}{b}$
$p = \frac{2\pi}{2}$
$p = \pi$

~~$\theta = \frac{13\pi}{12} + \pi$~~
~~$\theta = \frac{25\pi}{12} > 2\pi$~~

Add/Subtract period until outside of the domain.

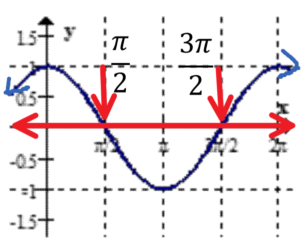
Calc $y_1 = y_2$
$y_1 = LHS$
$y_2 = RHS$





$\theta_{gen} = \frac{\pi}{12} + \pi n, n \in \mathrm{I}$
$\theta_{gen} = \frac{5\pi}{12} + \pi n, n \in \mathrm{I}$

$\cos\frac{1}{2}\theta = 0 \quad ; \quad 0 \le \theta < 2\pi$
$cosm = 0$ let $m = \frac{1}{2}\theta$
$cos\theta = y$
...
$m = \frac{\pi}{2} \quad m = \frac{3\pi}{2}$
$\frac{1}{2}\theta = \frac{\pi}{2} \quad \frac{1}{2}\theta = \frac{3\pi}{2}$
$\theta = \pi$ ~~$\theta = 3\pi$~~



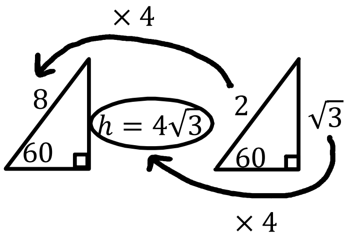
$p = 4\pi$

$\theta_{gen} = \pi + 4\pi n, n \in \mathrm{I}$

$\sin\left(\frac{\pi}{4}(x-6)\right) = \frac{1}{2} \quad ; \quad 0 \le x < 2\pi$
$sinm = \frac{1}{2}$ let $m = \frac{\pi}{4}(x-6)$
...
$x = 1.33$, ~~6.67~~, ~~9.33~~

C12 - 4.0 - Trig Notes

Solve for h.



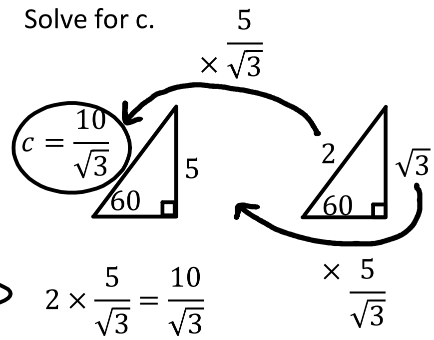
$$\sin\theta = \frac{opp}{hyp}$$

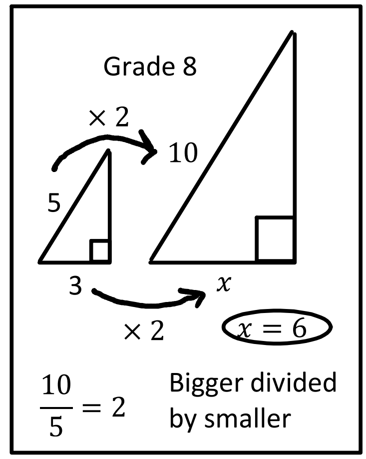
$$\sin 60 = \frac{h}{8}$$

$$8 \times \frac{\sqrt{3}}{2} = \frac{h}{8} \times 8$$

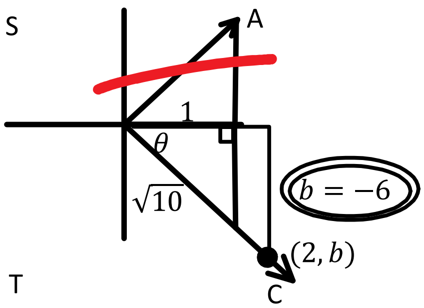
$$h = 4\sqrt{3}$$

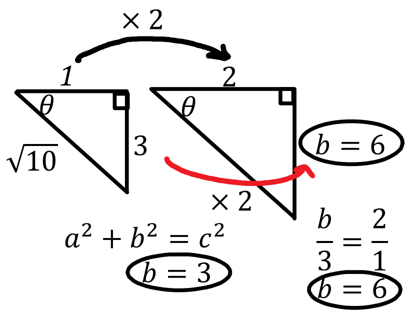
Solve for c.



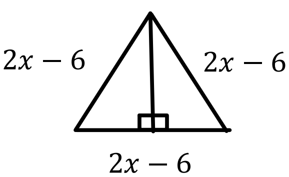


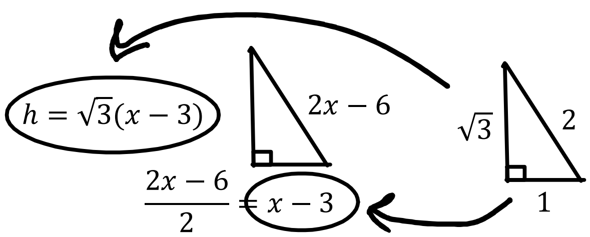
$$\cos\theta = \frac{1}{\sqrt{10}} \qquad \tan\theta < 0 \qquad \text{Find } b\text{ ; } (2, b)$$





Find Area (Hard)





$$A = \frac{bh}{2}$$

$$A = \frac{(2x-6)\sqrt{3}(x-3)}{2}$$

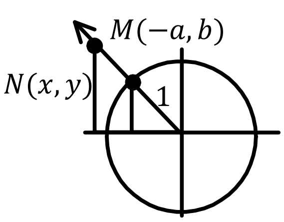
$$A = \sqrt{3}(x-3)^2$$

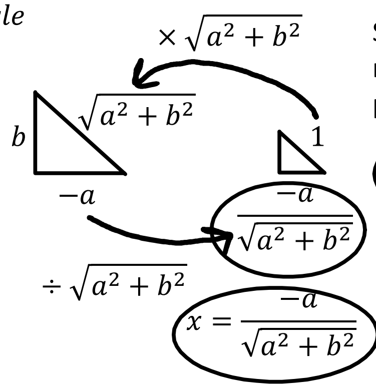
~~$(2x-6)^2 - (x-3)^2 = b^2$~~

~~$4x^2 - 24x + 36 - x^2 + 6x - 9 = b^2$~~

~~$3x^2 - 18x + 25 = b^2$~~

Find N(x, y) on unit circle





Squaring negatives gives positive.

$$y = \frac{b}{\sqrt{a^2 + b^2}}$$

$$x = \frac{-a}{\sqrt{a^2 + b^2}}$$

C12 - 4/6.0 - Trig Notes

Identities :

$$\begin{aligned} sinx - \cos^2 x - 1 &= 0 \\ sinx - (1 - \sin^2 x) - 1 &= 0 \\ sinx - 1 + \sin^2 x - 1 &= 0 \\ \sin^2 x + sinx - 2 &= 0 \end{aligned}$$

$$\boxed{\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ \cos^2 x &= 1 - \sin^2 x \end{aligned}}$$

$$\begin{aligned} sinx - cscx &= 0 \\ sinx - \frac{1}{sinx} &= 0 \\ m - \frac{1}{m} &= 0 \end{aligned}$$

$$\left(m - \frac{1}{m} = 0\right) \times m$$

$$m^2 - 1 = 0$$

...

$$\boxed{csc\theta = \frac{1}{sin\theta}}$$

$$\boxed{let\ m = sin\theta}$$

$$\begin{aligned} sin2x &= cosx \\ 2sinxcosx &= cosx \\ 2sinxcosx - cosx &= 0 \\ cosx(2sinx - 1) &= 0 \end{aligned}$$

$$\boxed{sin2x = 2sinxcosx}$$

$cosx = 0$ … $sinx = \frac{1}{2}$ …

$$\begin{gathered} \cos^4 x - \sin^4 x \\ (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) \\ (cos2x)(1) \\ cos2x \end{gathered}$$

…

$$\boxed{\begin{gathered} m^4 - n^4 \\ (m^2 + n^2)(m^2 - n^2) \end{gathered}}$$

$$\begin{aligned} cosxcos2x - sinxsin2x &= -1 \\ cosxcos2x - sinxsin2x &= -1 \\ \cos(2x + x) &= -1 \\ cos3x &= 1 \end{aligned}$$

....

$$\begin{aligned} \cos 2x &= -1 \\ 2\sin^2 x - 1 &= -1 \\ 2\sin^2 x &= 0 \\ \sin^2 x &= 0 \\ \sin x &= 0 \end{aligned}$$

…

$$\boxed{cos2x = 2\sin^2 x - 1}$$

$$\boxed{\cos(\alpha + \beta) = cos\alpha cos\beta - sin\alpha sin\beta}$$

$$\begin{aligned} cos\theta - cos2\theta &= 0 \\ cos\theta - (2\cos^2 \theta - 1) &= 0 \\ -2\cos^2 \theta + cos\theta + 1 &= 0 \\ 2\cos^2 \theta - cos\theta - 1 &= 0 \end{aligned}$$

…

Rad<->Deg
ASTC/0/π
SOHCAHTOA Ratios
CHOSHACAO
Inverse (+ve)= θ_r
Special $\Delta's/Point$
Unit Circle ; $x^2 + y^2 = 1$
Rationalize
Algebra/Factoring/Quadform
$let\ m = sin\theta$
Expressions vs Equations
Domain
Period Change $m^* = 2x$
NPV's ; Denominator $\neq 0$
Point on Unit Circle
Similar Triangles
Arc-Length/Sector Area
Angular Velocity $1\ rev = 2\pi$
Trig Identities

$0 < \theta_r < 90$
θ_{stp} : ± Arrows/s
$0 < \theta_{pri} < 360^o$
$\theta_{cot} = \theta_{stp} \pm p^* n$ ...
$\theta_{gen} = \theta_{stp} + p^* n, n \in \mathrm{I}$

$sin\theta \neq 2$
Range :
$-1 \leq sin\theta \leq 1$
$-1 \leq cos\theta \leq 1$