

C12 - 6.6 - Double Angle Notes

$$4 \sin 6x = 8 \sin 3x \cos 3x$$

$$\sin 2x = 2 \sin x \cos x$$

Double the number in front.
Half the angle. Add a Cos

$$2 \sin x = 4 \sin \frac{1}{2}x \cos \frac{1}{2}x$$

$$\frac{1}{2} \sin 4x = 1 \sin 2x \cos 2x$$

$$2 \sin \pi = 4 \sin \left(\frac{\pi}{2}\right) \cos \left(\frac{\pi}{2}\right) = 0$$

$$8 \sin 3x \cos 3x = 4 \sin 6x$$

$$2 \sin x \cos x = \sin 2x$$

Half the number in front.
Double the angle. Cos goes away

$$4 \sin \frac{1}{2}x \cos \frac{1}{2}x = 2 \sin x$$

$$4 \sin \left(\frac{\pi}{6}\right) \cos \left(\frac{\pi}{6}\right) = 2 \sin \left(\frac{\pi}{3}\right) = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\cos 4x = \cos^2 2x - \sin^2 2x$$

$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2 \cos^2 x - 1 \\ &= 1 - 2 \sin^2 x \end{aligned}$$

Half the angle

$$\cos 4x = 2 \cos^2 2x - 1$$

$$1 - 2 \sin^2 2x = \cos 4x$$

Double the angle

$$\begin{aligned} 2 \cos^2 3x - 2 \sin^2 3x &= \\ 2 (\cos^2 3x - \sin^2 3x) &= 2 \cos 6x \end{aligned}$$

GCF

$$\begin{aligned} 4 \cos^2 5 - 2 &= \\ 2(2 \cos^2 5 - 1) &= 2 \cos 10 \end{aligned}$$

$$1 - 2 \sin^2 \pi = \cos 2\pi = 1$$

$$1 - 2 \sin^2 \left(\frac{\pi}{4}\right) = \cos \left(\frac{\pi}{2}\right) = 0$$

Simplify to sin x or cos x

$$\begin{array}{ll} 1 - \cos 2x & 1 + \cos 2x \\ 1 - (1 - 2 \sin^2 x) & 1 + (2 \cos^2 x - 1) \\ 1 - 1 + 2 \sin^2 x & 1 + 2 \cos^2 x - 1 \end{array}$$

$$2 \sin^2 x$$

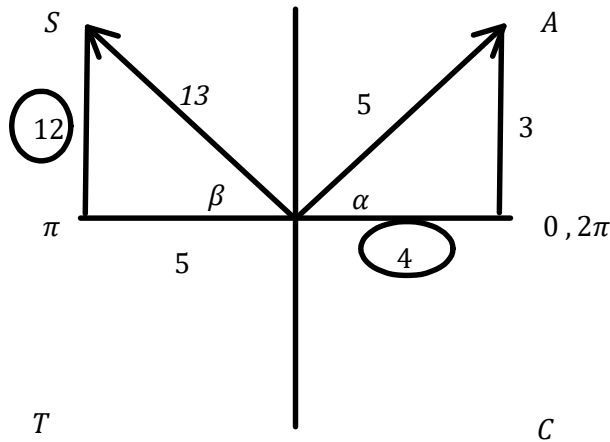
$$2 \cos^2 x$$

C12 - 6.6 - Proofs Double Angle Notes

$\tan x$	$\frac{\sin 2x}{1 + \cos 2x}$
$\frac{\sin x}{\cos x}$	$\frac{\sin 2x}{1 + (2 \cos^2 x - 1)}$ $\frac{\sin 2x}{1 + (2 \cos^2 x - 1)}$ $\frac{2 \sin x \cos x}{2 \sin x \cos x}$ $\frac{2 \cos^2 x}{2 \sin x \cos x}$ $\frac{2 \cos^2 x}{2 \sin x \cos x}$ $\frac{2 \cos^2 x}{\sin x}$ $\frac{2 \cos^2 x}{\sin x}$ $\frac{2 \cos^2 x}{\sin x}$ $\frac{\sin x}{\cos x}$

C12 - 6.6 - CosA= SinB= Sum/Double Angles Notes

Solve: $\sin\alpha = \frac{3}{5}$; QI $\cos\beta = -\frac{5}{13}$; QII $\sin(\alpha + \beta) = ?$ $\sin 2\alpha = ?$
 $\cos 2\beta = ?$



$$a^2 + b^2 = c^2$$

$$a = 4$$

$$a^2 + b^2 = c^2$$

$$b = 12$$

$$\begin{aligned} \sin(\alpha + \beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta \\ &= \frac{3}{5} \times -\frac{5}{13} + \frac{4}{5} \times \frac{12}{13} \\ &= -\frac{3}{13} + \frac{48}{65} \\ &= \frac{33}{65} \end{aligned}$$

$$\begin{aligned} \sin 2\alpha &= 2\sin\alpha\cos\alpha \\ &= 2 \times \frac{3}{5} \times \frac{4}{5} \\ &= \frac{24}{25} \end{aligned}$$

$$\begin{aligned} \cos 2\beta &= 1 - 2\sin^2\beta \\ &= 1 - 2\left(\frac{12}{13}\right)^2 \\ &= -\frac{119}{169} \end{aligned}$$