# C12 - 6.1 - Ratios $cscx\ secx\ cotx$ Notes

$\frac{sinx}{sinx} = 1$ $\quad \frac{\sin^2 x}{sinx} = sinx$ $\quad \frac{\sin^3 x}{sinx} = \sin^2 x$

$\frac{\cos x}{cosx} = 1$ $\quad \frac{\cos^2 x}{cosx} = cosx$ $\quad \frac{\cos^3 x}{\cos^2 x} = \cos x$

$\sin^2 x = (sinx)(sinx) \neq sinx^2$

$\cos^2 x = (cosx)(cosx) \neq cosx^2$

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$\frac{sinx}{1} \times \frac{1}{cosx}$

$\frac{sinx}{cosx}$

$= tanx$

$tanx = \frac{sinx}{cosx}$

$sinxtanx$

$sinx \times \frac{sinx}{cosx}$

$= \frac{\sin^2 x}{cosx}$

$cosxtanx$

$cosx \times \frac{sinx}{cosx}$

$= sinx$

$\frac{sinxcosx}{sinx} = cosx$

$\frac{cosxsinx}{cosx} = sinx$

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$\frac{sinx}{tanx}$

$\frac{sinx}{\left(\frac{sinx}{cosx}\right)}$

$sinx \div \frac{sinx}{cosx}$

$sinx \times \frac{cosx}{sinx}$

$= cosx$

$\frac{cosx}{tanx}$

$\frac{cosx}{\left(\frac{sinx}{cosx}\right)}$

$cosx \div \frac{sinx}{cosx}$

$cosx \times \frac{cosx}{sinx}$

$= \frac{\cos^2 x}{sinx}$

$\frac{tanx}{cosx}$

$\frac{\left(\frac{sinx}{cosx}\right)}{\frac{cosx}{1}}$

$\frac{sinx}{cosx} \div \frac{cosx}{1}$

$\frac{sinx}{cosx} \times \frac{1}{cosx}$

$= \frac{sinx}{\cos^2 x}$

$\frac{tanx}{sinx}$

$\frac{\left(\frac{sinx}{cosx}\right)}{\frac{sinx}{1}}$

$\frac{sinx}{cosx} \div \frac{sinx}{1}$

$\frac{sinx}{cosx} \times \frac{1}{sinx}$

$\frac{1}{cosx}$

$= secx$

$secx = \frac{1}{cosx}$

*Flip and Multiply*

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$secxcosx =$

$\frac{1}{cosx} \times cosx =$

$\frac{cosx}{cosx} = 1$

$secx = \frac{1}{cosx}$

$secxsinx =$

$\frac{1}{cosx} \times sinx =$

$\frac{sinx}{cosx} = tanx$

$\frac{sinx}{cosx} = tanx$

$secxtanx =$

$\frac{1}{cosx} \times \frac{sinx}{cosx} = \frac{sinx}{\cos^2 x}$

$cscxsinx = \frac{1}{sinx} \times sinx = \frac{sinx}{sinx} = 1$

$cscx = \frac{1}{sinx}$

$csxcosx =$

$\frac{1}{sinx} \times cosx =$

$\frac{cosx}{sinx} = cotx$

$\frac{cosx}{sinx} = cotx$

$cscxtanx =$

$\frac{1}{sinx} \times \frac{sinx}{cosx} =$

$\frac{1}{\cos x} = secx$

# C12 - 6.2 - Add Subtract Fractions Notes

$$\frac{1}{cosx} - \frac{\sin x}{cosx} = \frac{1 - sinx}{cosx}$$

*Add Fractions: LCD*

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$$\frac{1}{sinx} - sinx$$
$$\frac{1}{sinx} - sinx \times \frac{sinx}{sinx}$$
$$\frac{1}{sinx} - \frac{\sin^2 x}{sinx}$$
$$\frac{1 - \sin^2 x}{sinx}$$
$$\frac{\cos^2 x}{sinx}$$

$$\sin^2 x + \cos^2 x = 1$$
$$\cos^2 x = 1 - \sin^2 x$$

$$\frac{1}{sinxcosx} - \frac{sinx}{cosx}$$
$$\frac{1}{sinxcosx} - \frac{sinx}{cosx} \times \frac{sinx}{sinx}$$
$$\frac{1}{sinxcosx} - \frac{\sin^2 x}{sinxcosx}$$
$$\frac{1 - \sin^2 x}{sinxcosx}$$
$$\frac{\cos^2 x}{sinxcosx}$$
$$\frac{cosx}{sinx}$$
$$cotx$$

$$\frac{sinx + cosx}{cosx}$$
$$\frac{sinx}{cosx} + \frac{cosx}{cosx}$$
$$tanx + 1$$

*Separate Fractions*

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$$\frac{\frac{1}{cosx} - cosx}{1 - \frac{sinx}{cosx}}$$
$$\frac{\frac{1}{cosx} - \frac{\cos^2 x}{cosx}}{\frac{cosx}{cosx} - \frac{sinx}{cosx}}$$
$$\frac{\frac{1 - \cos^2 x}{cosx}}{\frac{cosx - sinx}{cosx}}$$
$$\frac{1 - \cos^2 x}{cosx} \times \frac{cosx}{cosx - siinx}$$
$$\frac{1 - \cos^2 x}{cosx - sinx}$$
$$\frac{\sin^2 x}{cosx - sinx}$$

Add Fractions: LCD
Flip and Multiply

$$\frac{\frac{1}{cosx} - cosx}{1 - \frac{sinx}{cosx}} \qquad LDC = cosx$$
$$\left(\frac{\frac{1}{cosx} - cosx}{1 - \frac{sinx}{cosx}}\right) \times \frac{cosx}{cosx}$$
$$\frac{1 - \cos^2 x}{cosx - sinx}$$
$$\frac{\sin^2 x}{cosx - sinx}$$

Multiply top and bottom by LCD

$$\frac{\frac{1}{cosx} - cosx}{1 - \frac{sinx}{cosx}}$$
$$\frac{1 - \cos^2 x}{cosx - sinx}$$

# C12 - 6.3 - Proofs Pythag Reciprocal Fractions Notes

| $\tan x \csc x$ | $=$ | $\sec x$ |
|---|---|---|
| $\left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x}\right)$ | | |
| $\frac{1}{\cos x}$ | | |
| $\sec x$ | | ✓ |

| $\frac{\cot x}{\csc x}$ | $=$ | $\cos x$ |
|---|---|---|
| $\frac{\left(\frac{\cos x}{\sin x}\right)}{\left(\frac{1}{\sin x}\right)}$ | | |
| $\frac{\cos x}{\sin x} \times \frac{\sin x}{1}$ | | |
| $\cos x$ | | ✓ |

| $1 + \tan^2 x$ | $=$ | $\sec^2 x$ |
|---|---|---|
| $1 + \frac{\sin^2 x}{\cos^2 x}$ | | $\frac{1}{\cos^2 x}$ |
| $\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$ | | |
| $\frac{\cos^2 x + \sin^2 x}{\cos^2 x}$ | | |
| $\frac{1}{\cos^2 x}$ | | ✓ |

| $\csc x \cos^2 x + \sin x$ | $=$ | $\csc x$ |
|---|---|---|
| $\frac{1}{\sin x} \times \cos^2 x + \sin x$ | | $\frac{1}{\sin x}$ |
| $\frac{\cos^2 x}{\sin x} + \sin x \times \frac{\sin x}{\sin x}$ | | |
| $\frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x}$ | | |
| $\frac{\cos^2 x + \sin^2 x}{\sin x}$ | | |
| $\frac{1}{\sin x}$ | | ✓ |

| $\cot x + \tan x$ | $=$ | $\csc x \sec x$ |
|---|---|---|
| $\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$ | | $\csc x \sec x$ |
| $\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$ | | |
| $\frac{1}{\sin x \cos x}$ | | |
| $\left(\frac{1}{\sin x}\right)\left(\frac{1}{\cos x}\right)$ | | |
| $\csc x \sec x$ | | ✓ |

| $\frac{1 + \cos x}{1 + \sec x}$ | $=$ | $\cos x$ |
|---|---|---|
| $\left(\frac{1 + \cos x}{1 + \frac{1}{\cos x}}\right)$ | | $\cos x$ |
| $\frac{(1 + \cos x)}{\left(\frac{\cos x + 1}{\cos x}\right)}$ | | |
| $(1 + \cos x) \times \frac{\cos x}{\cos x + 1}$ | | |
| $\frac{\cos x(1 + \cos x)}{\cos x + 1}$ | | |
| $\cos x$ | | ✓ |

# C12 - 6.4 - Proofs Conjugate Notes

Conjugate:

$a+b \longleftrightarrow a-b$

$a-b \longleftrightarrow a+b$

Conjugate:

$1-sinx \longleftrightarrow 1+sinx$

$1+sinx \longleftrightarrow 1-sinx$

Conjugate:

$1+cosx \longleftrightarrow 1-cosx$

$1-cosx \longleftrightarrow 1+cosx$

$\frac{\square}{1+cosx} \times \frac{1-cosx}{1-cosx}$

$\frac{\square}{1+sinx} \times \frac{1-sinx}{1-sinx}$

$\frac{\square}{1-cosx} \times \frac{1+cosx}{1+cosx}$

$\frac{\square}{1-sinx} \times \frac{1+sinx}{1+sinx}$

**Prove that the two sides are equal.**

$\frac{sinx}{1+cosx} = \frac{1-cosx}{sinx}$

The conjugate

$\times \frac{1-cosx}{1-cosx}$

$\frac{sinx}{1+cosx} \times \frac{1-cosx}{1-cosx}$

$\frac{(1-cosx)}{\sin x}$

1) Multiply the top and bottom by the conjugate of the denominator
2) FOIL the bottom
3) Pythagorean Identity
4) Simplify

$\frac{sinx\,(1-cosx)}{(1+cosx)(1-cosx)}$

$(1+cosx)(1-cosx)$

$1 - cosx + cosx - \cos^2 x$

$1-\cos^2 x$

FOIL (FL)

$(a+b)(a-b)$

$a^2 - ab + ab + b^2$

$a^2 - b^2$

$\frac{sinx(1-cosx)}{1-cosx+cosx-\cos^2 x}$

$\frac{sinx(1-cosx)}{1-\cos^2 x}$

$\sin^2 x - \cos^2 x = 1$

$\sin^2 x = 1-\cos^2 x$

Now we have the Pythagorean identity

$\frac{sinx(1-cosx)}{\sin^2 x}$

$\frac{(1-cosx)}{\sin x}$

$RHS$ ✓

Conj
FL
Pythag
Simp

# C12 - 6.4 - Proofs Foil Conjugate Fact Frac Notes

$$\frac{1-cosx}{sinx} = \frac{sinx}{1+cosx}$$

| | |
|---|---|
| $\frac{(1-cosx)}{sin\,x}$ | $\frac{sinx}{1+cosx} \times \frac{1-cosx}{1-cosx}$ **Conjugate!** |
| ✓ | $\frac{sinx(1-cosx)}{1-\cos^2 x}$ |
| | $\frac{sinx(1-cosx)}{\sin^2 x}$ |
| | $\frac{(1-cosx)}{sin\,x}$ |

*Foil*

$$(sinx-1)(sinx+1) = -\cos^2 x$$

| | |
|---|---|
| $sin^2\,x\ -1$ | ✓ |
| $-\cos^2 x$ | |

*Factor*

$$\frac{1+cosx}{\sin^2 x} = \frac{1}{1-cos\,x}$$

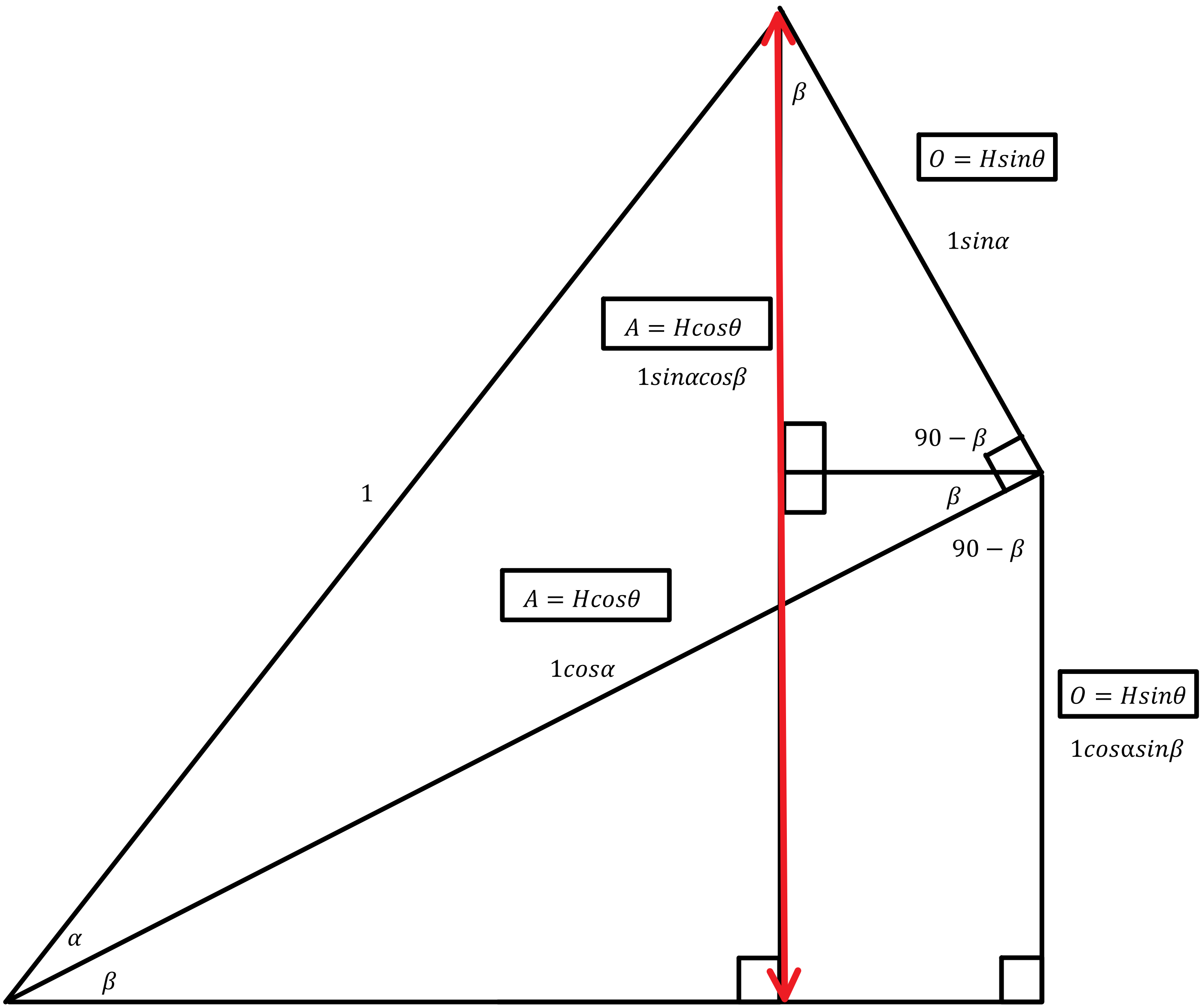
| | |
|---|---|
| $\frac{1+cosx}{1-cos^2\,x}$ | $\frac{1}{1-cos\,x}$ |
| $\frac{1+cosx}{(1-cos\,x)(1+cosx)}$ | ✓ |
| $\frac{1}{1-cos\,x}$ | |

Add and Subtract Fractions

$$\frac{1}{1+cosx} + \frac{1}{1-cosx} = 2\csc^2 x$$

| | |
|---|---|
| $\frac{1}{1+cosx} \times \frac{1-cosx}{1-cosx} + \frac{1}{1-cosx} \times \frac{1+cosx}{1+cosx}$ | $2 \times \frac{1}{\sin^2 x}$ |
| $\frac{(1-cosx)+(1+cosx)}{(1-cosx)(1+cosx)}$ | $\frac{2}{\sin^2 x}$ |
| $\frac{1-cosx+1+cosx}{1-\cos^2 x}$ | ✓ |
| $\frac{2}{\sin^2 x}$ | |

# C12 - 6.5 - Sum and Differences Angle Theory




$$\sin(\alpha+\beta) = sin\alpha cos\beta + cos\alpha sin\beta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

# C12 - 6.5 - Simplify/Expand Sum Difference Notes

$$\sin(a+b) = sinacosb + sinbcosa$$

$$\sin(x+\pi) = sinxcos\pi + sin\pi cosx$$
$$= sinx \times -1 + 0 \times cosx$$
$$= -sinx$$

$$\cos(a+b) = cosacosb - sinasinb$$

$$cos45cos30 + sin45sin30 = \cos(45^o - 30^o)$$
$$= cos15$$

$$\sin\left(\frac{\pi}{12}\right) =$$
$$sin15^0 =$$

$$\frac{\pi}{12} = 15^0$$

$$15 = 45 - 30$$ Or $$\frac{\pi}{12} = \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{4} - \frac{\pi}{3}$$

$$\sin(45^0 - 30^0) = sin45cos30 - sin30cos45$$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

*Rationalize!*

$$\sin(a-b) = sinacosb - sinbcosa$$

Sin is the same sign sincos:cossin
Cos is the opposite sign coscos:sinsin

$$\cos(-75) = \cos(-45-30)$$
$$= \cos(-45)\cos(30) + \sin(-45)\cos(30)$$

$$\cos(-x) = cosx \qquad \sin(-x) = -sinx$$

OR

$$cos75^o =$$
$$\cos(45^o + 30^o) = cos45cos30 - sin45sin30$$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\cos(a-b) = cosacosb + sinasinb$$

$$sec15^o =$$
$$\frac{1}{cos15^o} =$$
$$\frac{1}{\cos(45^o - 30^o)} = \frac{1}{(cos45cos30 + sin45sin30)}$$
$$= \frac{1}{\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}}$$
$$= \frac{1}{\left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)}$$
$$= \frac{1}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$
$$= 1 \times \frac{2\sqrt{2}}{\sqrt{3}+1}$$
$$= \frac{2\sqrt{2}}{\sqrt{3}+1}$$

$$\cos\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right) + \sin\left(\frac{\pi}{6}+x\right)\sin\left(\frac{\pi}{6}-x\right) =$$
$$\cos\left(\frac{\pi}{6} + x - (\frac{\pi}{6} - x)\right) = \cos(2x)$$

$$sin255 = -sin105$$
$$\sin255 = \sin(360-105) = \sin 360cos105 - sin105cos360 = 0 - sin105(1) = -sin105 = -\sin(45+60)$$

| A combination of special and quadrantal angles…^* | $255 = 180 + 75$ | $285 = 180 + 105$ | $195 = 180 + 15$ |
|---|---|---|---|
| | $255 = 180 + (45+30)$ | $285 = 180 + (60+45)$ | $195 = 90 + 105$ |

# C12 - 6.6 - Double Angle Notes

$4\sin 6x = 8\sin 3x\cos 3x$

$2\sin x = 4\sin\frac{1}{2}x\cos\frac{1}{2}x$

$\frac{1}{2}\sin 4x = 1\sin 2x\cos 2x$

$2sin\pi = 4\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right) = 0$

$sin2x = 2sinxcosx$

Double the number in front.
Half the angle. Add a Cos

---

$8\sin 3x\cos 3x = 4\sin 6x$

$4\sin\frac{1}{2}x\cos\frac{1}{2}x = 2\sin x$

$4\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right) = 2\sin\left(\frac{\pi}{3}\right) = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$2sinxcosx = sin2x$

Half the number in front.
Double the angle. Cos goes away

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$cos4x = \cos^2 2x - \sin^2 2x$

$cos4x = 2\cos^2 2x - 1$

$$\begin{aligned} cos2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}$$

Half the angle

$1 - 2\sin^2 2x = cos4x$

Double the angle

$2\cos^2 3x - 2\sin^2 3x =$
$2(\cos^2 3x - \sin^2 3x) = 2cos6x$

GCF

$4\cos^2 5 - 2 =$
$2(2\cos^2 5 - 1) = 2cos10$

$1 - 2\sin^2\pi = cos2\pi = 1$

$1 - 2\sin^2\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

$Simplify\ to\ sinx\ or\ cosx$

$1 - cos2x$
$1 - (1 - 2\sin^2 x)$
$1 - 1 + 2\sin^2 x$
$2\sin^2 x$

$1 + cos2x$
$1 + (2\cos^2 x - 1)$
$1 + 2\cos^2 x - 1$
$2\cos^2 x$

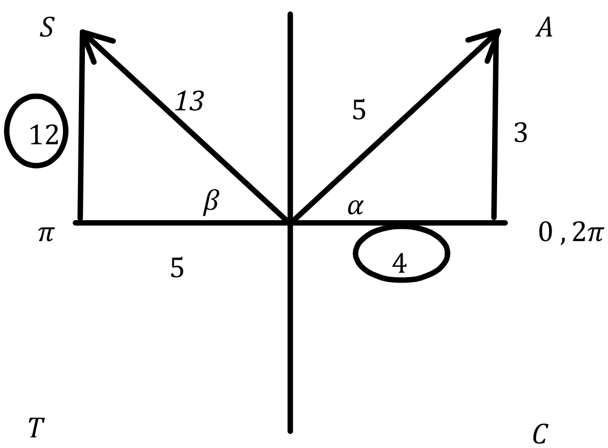
# C12 - 6.6 - Proofs Double Angle Notes

| $tanx$ | $\frac{sin2x}{1+cos2x}$ |
|---|---|
| $\frac{sinx}{cosx}$ | $\frac{sin2x}{1+(2\cos^2 x-1)}$ |
| | $\frac{sin2x}{1+(2\cos^2 x-1)}$ |
| | $\frac{2sinxcosx}{2\cos^2 x}$ |
| | $\frac{2sinxcosx}{2\cos^2 x}$ |
| | $\frac{2sinxcosx}{2\cos^2 x}$ |
| | $\frac{sinx}{\cos x}$ |

# C12 - 6.6 - CosA= SinB= Sum/Double Angles Notes

*Solve*: $sin\alpha = \frac{3}{5}\ ;QI$ $cos\beta = -\frac{5}{13}\ ;QII$ $\sin(\alpha+\beta) = ?$ $sin2\alpha = ?$

$cos2\beta = ?$




$a^2 + b^2 = c^2$
$a = 4$

$a^2 + b^2 = c^2$
$b = 12$

$$\sin(\alpha+\beta) = sin\alpha cos\beta + cos\alpha sin\beta$$
$$= \frac{3}{5} \times -\frac{5}{13} + \frac{4}{5} \times \frac{12}{13}$$
$$= -\frac{3}{13} + \frac{48}{65}$$
$$= \frac{33}{65}$$

$$sin2\alpha = 2sin\alpha cos\alpha$$
$$= 2 \times \frac{3}{5} \times \frac{4}{5}$$
$$= \frac{24}{25}$$

$$cos2\beta = 1 - 2\sin^2\beta$$
$$= 1 - 2\left(\frac{12}{13}\right)^2$$
$$= -\frac{119}{169}$$