C12 - 7.2 - Solving Exponential Equations Notes

Solve for x

$$2^x = 4^2$$
$$2^x = (2^2)^2$$
$$2^x = 2^4$$
$$x = 4$$

$4 = 2^2$

Same Base: Make exponents equal to each other

Check Answer:

$$2^4 = 4^2$$
$$16 = 16$$ ✓

$$2^x 2^1 = 2^5$$
$$2^{x+1} = 2^5$$
$$x + 1 = 5$$
$$x = 4$$

Add Exponents

$$2^x 2^1 = 2^5$$
$$2^4 2^1 = 2^5$$
$$2^5 = 2^5$$ ✓

$$4^{x+1} = 8^{2x-2}$$
$$(2^2)^{x+1} = (2^3)^{2x-2}$$
$$2^{2x+2} = 2^{6x-6}$$
$$2x + 2 = 6x - 6$$
$$8 = 4x$$
$$x = 2$$

$4 = 2^2$

$8 = 2^3$

Change of Base
Multiply Exponents

Solve

$$4^{x+1} = 8^{2x-2}$$
$$4^{2+1} = 8^{2(2)-2}$$
$$4^3 = 8^2$$
$$64 = 64$$ ✓

$$2^{x^2-x} = 1$$
$$2^{x^2-x} = 2^0$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$x = 0 \quad x = 1$$

$2^0 = 1$

Change of Base

Factor

Solve

$$2^{x^2-x} = 1$$
$$2^{0^2-0} = 1$$
$$2^0 = 1$$
$$1 = 1$$ ✓

$$2^{x^2-x} = 1$$
$$2^{1^2-1} = 1$$
$$2^0 = 1$$
$$1 = 1$$ ✓

$$2^{x^2-3x} = \frac{1}{4}$$
$$2^{x^2-3x} = 2^{-2}$$
$$x^2 - 3x = -2$$
$$x^2 - 3x + 2 = 0$$
$$(x-2)(x-1) = 0$$
$$x = 2 \quad x = 1$$

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

Change of Base

Factor

Solve

$$2^{x^2-3x} = \frac{1}{4}$$
$$2^{2^2-3(2)} = \frac{1}{4}$$
$$2^{-2} = \frac{1}{4}$$
$$\frac{1}{4} = \frac{1}{4}$$ ✓

$$2^{x^2-3x} = \frac{1}{4}$$
$$2^{1^2-3(1)} = \frac{1}{4}$$
$$2^{-2} = \frac{1}{4}$$
$$\frac{1}{4} = \frac{1}{4}$$ ✓

$$x^{\frac{2}{5}} = 3$$
$$\left(x^{\frac{2}{5}}\right)^{\frac{5}{2}} = (3)^{\frac{5}{2}}$$
$$x = 3^{\frac{5}{2}}$$

$$\frac{2}{5} \times \frac{5}{2} = 1$$

$$x = 15.5885$$

Take both/sides to reciprocal exponent of variable

Brackets around the left side
Brackets around the right side

$$x^{\frac{2}{5}} = 3$$
$$\left(3^{\frac{5}{2}}\right)^{\left(\frac{2}{5}\right)} = 3$$
$$3 = 3$$ ✓

$$(x+1)^{\frac{2}{3}} = 16$$
$$\left((x+1)^{\frac{2}{3}}\right)^{\left(\frac{3}{2}\right)} = (16)^{\frac{3}{2}}$$
$$x + 1 = \sqrt[2]{16^3}$$
$$x + 1 = 4^3$$
$$x + 1 = 64$$
$$x = 63$$

$x^2 = 9$ $(x^2)^{\frac{1}{2}} = 9^{\frac{1}{2}}$ $x = \pm 3$	Square root both sides $\sqrt{x} = x^{\frac{1}{2}}$

$$x^{\frac{m}{n}} = \sqrt[x]{x^m}$$

$$(x+1)^{\frac{2}{3}} = 16$$
$$(63+1)^{\frac{2}{3}} = 16$$
$$64^{\frac{2}{3}} = 16$$
$$\sqrt[3]{64^2} = 16$$
$$4^2 = 16$$
$$16 = 16$$ ✓