C12 - 7.0 - Exponentials Notes

Simplify : $5^2 \times 5^3 = 5^5$ Add Exponents $\frac{3^5}{3^2} = 3^3$ Subtract Exponents Check on Calculator

$(2^2)^3 = 2^6$ $(2x)^3 = 2^3x^3 = 8x^3$ Check Answer $x^* = 3$

$(2x)^3 = 8x^3$

$(2(3))^3 = 8(3)^3$

$216 = 216$

$\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2}$ Multiply/ Distribute Exponents

$(3 \times 4)^2 = 3^2 \times 4^2$

$3a^{-2} = \frac{3}{a^2}$ $3^{-3}a^{-2} = \frac{1}{3^3a^2}$ $(2x)^{-3} = \frac{1}{(2x)^3}$ $\left(\frac{5}{3}\right)^{-2} = \frac{3^2}{5^2}$ Negative Exponents

$4^2 = (2^2)^2 = 2^4$ $27^4 = (3^3)^4 = 3^{12}$ Change Base $\frac{1}{25} = \frac{1}{5^2} = 5^{-2}$

$8^{\frac{2}{3}} =$

$\sqrt[3]{8^2} =$

$2^2 = 4$

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$

$\frac{1}{\sqrt{2}} =$

$\frac{1}{2^{\frac{1}{2}}} = 2^{-\frac{1}{2}}$

$\sqrt[4]{\frac{1}{16}} =$

$\frac{\sqrt[4]{1}}{\sqrt[4]{16}} = \frac{1}{2}$

Radicals

$\frac{3^4 \times 3^{-3}}{9} =$

$\frac{3^1}{3^2} =$

$3^{-1} =$

$\frac{1}{3^1} = \frac{1}{3}$

$\frac{4^2 \times 16^3}{128^2} =$

$\frac{((2^2)^2 \times (2^4)^3)}{(2^7)^2} =$

$\frac{2^4 \times 2^{12}}{2^{14}} =$

$\frac{2^{16}}{2^{14}} = 2^{(16-14)} = 4$

$\frac{2^{7x+5} \times 2^{3x+3}}{2^{2x-4}} =$

$\frac{2^{10x+8}}{2^{2x-4}} =$

2^{8x+12}

$3^x \times 3 =$

$3^x \times 3^1 = 3^{x+1}$

$6^{x+1} = 6^x(6^1) = 6(6^x)$

$\frac{6^x}{6} =$

$\frac{6^x}{6^1} = 6^{x-1}$

$7^{x-1} = 7^x \times 7^{-1} = \frac{7^x}{7^1}$

$4^{1-x} = 4^1(4^{-x})$

$= \frac{4}{4^x}$

$(5^2)^x = 5^{2x}$ $5^{2x} = (5^x)^2 = (5^2)^x$

$3^{2x+1} = 3^{2x}3^1$

$= (3^x)^2 3^1$

$= 3(3^x)^2$

$6^x = (2 \times 3)^x$

$= 2^x \times 3^x$

Solve for x : STO x Calculator

$2^x = 4^2$

$2^x = (2^2)^2$

$2^x = 2^4$

$x = 4$

Check Answer:

$2^4 = 4^2$

$16 = 16$

Same Base: Make Exponents Equal to each other.

$2^x2^1 = 2^5$

$2^{x+1} = 2^5$

$x + 1 = 5$

$x = 4$

$2^x2^1 = 2^5$

$2^42^1 = 2^5$

$2^5 = 2^5$

$4^{x+1} = 8^{2x-2}$

$(2^2)^{x+1} = (2^3)^{2x-2}$

$2^{2x+2} = 2^{6x-6}$

$2x + 2 = 6x - 6$

$8 = 4x$

$x = 2$

$4^{x+1} = 8^{2x-2}$

$4^{2+1} = 8^{2(2)-2}$

$4^3 = 8^2$

$64 = 64$

$2^{x^2-x} = 1$

$2^{x^2-x} = 2^0$

$x^2 - x = 0$

$x(x-1) = 0$ Factor

$x = 0$ $x = 1$

$2^{x^2-x} = 1$

$2^{0^2-0} = 1$

$2^0 = 1$

$1 = 1$

$2^{x^2-x} = 1$

$2^{1^2-1} = 1$

$2^0 = 1$

$1 = 1$

$2^{x^2-3x} = \frac{1}{4}$

$2^{x^2-3x} = 2^{-2}$

$x^2 - 3x = -2$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 2$ $x = 1$

$2^{x^2-3x} = \frac{1}{4}$

$2^{2^2-3(2)} = \frac{1}{4}$

$2^{-2} = \frac{1}{4}$

$\frac{1}{4} = \frac{1}{4}$

$2^{x^2-3x} = \frac{1}{4}$

$2^{1^2-3(1)} = \frac{1}{4}$

$2^{-2} = \frac{1}{4}$

$\frac{1}{4} = \frac{1}{4}$

$x^{\frac{2}{5}} = 3$

$\left(x^{\frac{2}{5}}\right)^{\frac{5}{2}} = (3)^{\frac{5}{2}}$

$x = 3^{\frac{5}{2}}$

Take both/sides to reciprocal exponent of variable

$x^{\frac{2}{5}} = 3$

$\left(3^{\frac{5}{2}}\right)^{\left(\frac{2}{5}\right)} = 3$

$3 = 3$

$x^2 = 9$

$(x^2)^{\frac{1}{2}} = 9^{\frac{1}{2}}$

$x = \pm 3$

Square root both sides

$\sqrt{x} = x^{\frac{1}{2}}$

$(x+1)^{\frac{2}{3}} = 16$

$\left((x+1)^{\frac{2}{3}}\right)^{\left(\frac{3}{2}\right)} = (16)^{\frac{3}{2}}$

$x + 1 = \pm\sqrt[2]{16^3}$

$x + 1 = \pm 4^3$

$x + 1 = \pm 64$

$x = 63, -65$

$(x+1)^{\frac{2}{3}} = 16$

$(63+1)^{\frac{2}{3}} = 16$

$64^{\frac{2}{3}} = 16$

$\sqrt[3]{64^2} = 16$

$4^2 = 16$

$16 = 16$

C12 - 7.0 - Exponentials Notes

Solve for x :

$2(3^x) + 3^x = 243$ — let $m = 3^x$

$$2m + m = 243$$
$$3m = 243$$
$$m = 81$$
$$3^x = 81$$
$$3^x = 3^4$$
$$x = 4$$ ✓

Check:
$$2(3^x) + 3^x = 243$$
$$2(3^4) + 3^4 = 243$$
$$2(81) + 81 = 243$$
$$243 = 243$$

$7^x + 7^{x+1} = 392$ — Let $m = 7^x$

$$7^x + 7^x 7^1 = 392$$
$$m + 7m = 392$$
$$8m = 392$$
$$m = 49$$
$$7^x = 49$$
$$7^x = 7^2$$
$$x = 2$$ ✓

Check:
$$7^x + 7^{x+1} = 392$$
$$7^2 + 7^{2+1} = 392$$
$$49 + 343 = 392$$
$$392 = 392$$

$9^{2x} - 2(9^x) = 3$ — let $m = 9^x$

$$(9^x)^2 - 2(9^x) - 3 = 0$$
$$m^2 - 2m - 3 = 0$$
$$(m-3)(m+1) = 0$$

$$m - 3 = 0 \quad m = 3 \quad 9^x = 3 \quad (3^2)^x = 3^1 \quad 3^{2x} = 3^1 \quad 2x = 1 \quad x = \frac{1}{2}$$ ✓

$$m + 1 = 0 \quad m = -1 \quad 9^x = -1$$ No Solution ✓

Check:
$$9^{2x} - 2(9^x) - 3 = 0$$
$$9^{2\left(\frac{1}{2}\right)} - 2\left(9^{\left(\frac{1}{2}\right)}\right) - 3 = 0$$
$$9^1 - 2(3) - 3 = 0$$
$$9 - 6 - 3 = 0$$
$$0 = 0$$

$3^x - 3 = 4(3^{-x})$ — let $m = 3^x$

$$3^x - 3 = \frac{4}{3^x}$$
$$m - 3 = \frac{4}{m}$$
$$\left(m - 3 = \frac{4}{m}\right) \times m$$
$$m^2 - 3m = 4$$
$$m^2 - 3m - 4 = 0$$
$$(m-4)(m+1) = 0$$

$$m - 4 = 0 \quad m = 4 \quad 3^x = 4 \quad x = 1.2619$$ ✓

Calc $y_1 = y_2$
$y_1 = LHS$
$y_2 = RHS$
$y_1 = 3^x - 3$
$y_2 = 4(3^{-x})$

$$m + 1 = 0 \quad m = -1 \quad 3^x = -1$$ No Sol ✓

Check:
$$3^{1.2619} - 3 = 4(3^{-1.2619})$$
$$1 = 1$$

$3^{2x+1} - 4(3^{x+1}) + 9 = 0$ — let $m = 3^x$

$$3^{2x}3^1 - 4(3^x 3) + 9 = 0$$
$$(3^x)^2 3 - 4(3^x)3 + 9 = 0$$
$$3(3^x)^2 - 12(3^x) + 9 = 0$$
$$3m^2 - 12m + 9 = 0$$
$$m^2 - 4m + 3 = 0$$
$$(m-1)(m-3) = 0$$

$$m - 1 = 0 \quad m = 1 \quad 3^x = 1 \quad 3^x = 3^0 \quad x = 0$$ ✓

$$m - 3 = 0 \quad m = 3 \quad 3^x = 3 \quad 3^x = 3^1 \quad x = 1$$ ✓

Check:
$$3^{2x+1} - 4(3^{x+1}) + 9 = 0$$
$$3^{2(0)+1} - 4\left(3^{(0)+1}\right) + 9 = 0$$
$$3^1 - 4(3) + 9 = 0$$
$$3 - 12 + 9 = 0$$
$$0 = 0$$

$$3^{2x+1} - 4(3^{x+1}) + 9 = 0$$
$$3^{2(1)+1} - 4\left(3^{(1)+1}\right) + 9 = 0$$
$$3^3 - 4(3^2) + 9 = 0$$
$$27 - 36 + 9 = 0$$
$$0 = 0$$

$2(2^x)^2 - 3(2^x) + 1 = 0$ — let $m = 2^x$

$$2m^2 - 3m + 1 = 0$$
$$(2m-1)(m-1) = 0$$

$$2m - 1 = 0 \quad m = \frac{1}{2} \quad 2^x = \frac{1}{2} \quad 2^x = 2^{-1} \quad x = -1$$ ✓

$$m - 1 = 0 \quad m = 1 \quad 2^x = 1 \quad 2^x = 2^0 \quad x = 0$$ ✓

$10^x - 4(5^x) - 5(2^x) + 20 = 0$ — let $m = 2^x$ — let $n = 5^x$

$$2^x \times 5^x - 4(5^x) - 5(2^x) + 20 = 0$$
$$mn - 4n - 5m + 20 = 0$$
$$(mn - 4n)(-5m + 20) = 0$$
$$n(m-4) - 5(m-4) = 0$$
$$(n-5)(m-4) = 0$$

$$n - 5 = 0 \quad n = 5 \quad 5^x = 5 \quad 5^x = 5^1 \quad x = 1$$ ✓

$$m - 4 = 0 \quad m = 4 \quad 2^x = 4 \quad 2^x = 2^2 \quad x = 2$$ ✓

Check:
$$10^x - 4(5^x) - 5(2^x) + 20 = 0$$
$$10^1 - 4(5^1) - 5(2^1) + 20 = 0$$
$$10 - 20 - 10 + 20 = 0$$
$$0 = 0$$

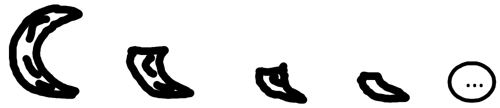
$$10^x - 4(5^x) - 5(2^x) + 20 = 0$$
$$10^2 - 4(5^2) - 5(2^2) + 20 = 0$$
$$100 - 100 - 20 + 20 = 0$$
$$0 = 0$$

Check:
$$2(2^x)^2 - 3(2^x) + 1 = 0$$
$$2(2^{-1})^2 - 3(2^{-1}) + 1 = 0$$
$$2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 1 = 0$$
$$2\left(\frac{1}{4}\right) - \frac{3}{2} + 1 = 0$$
$$0 = 0$$

$$2(2^x)^2 - 3(2^x) + 1 = 0$$
$$2(2^0)^2 - 3(2^0) + 1 = 0$$
$$2(1)^2 - 3(1) + 1 = 0$$
$$2 - 3 + 1 = 0$$
$$0 = 0$$

C12 - 7.0 - Exponentials Notes

Bananas have a half life of 4 days.



Definition

Half Life: Time to decay to half of the remaining mass.

Fraction Remaining	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{256}$
Mass (grams)	$100\ g$	$50\ g$	25	12.5	6.25	3.125	1.5625	0.78125	
$Time\ (days)$	0	4	8	12	16	20	24	28	32
# Half Lives	0	1	2	3	4	5	6	7	8

How much after 28 days?

$$F = P(r)^{\frac{t}{T}}$$

$$F = 100\left(\frac{1}{2}\right)^{\frac{28}{4}}$$

$$F = 0.78125\ g$$

How long till 3.125 g?

$$F = P(r)^{\frac{t}{T}}$$

$$3.125 = 100\left(\frac{1}{2}\right)^{\frac{t}{4}}$$

$$0.03125 = 0.5^{\frac{t}{4}}$$

$$t = 20\ d$$

How long till $\frac{1}{256}th$ $of\ original?$

$$F = P(r)^{\frac{t}{T}}$$

$$\frac{1}{256} = 1\left(\frac{1}{2}\right)^{\frac{t}{4}}$$

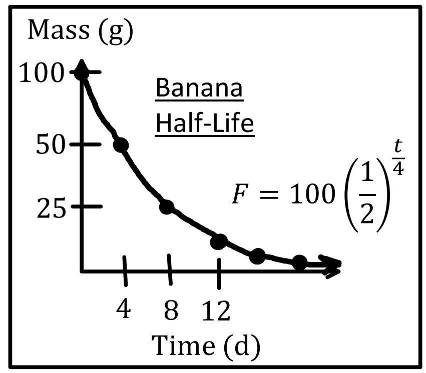
$$2^{-8} = (2^{-1})^{\frac{t}{4}}$$

$$(2^{-8})^4 = \left((2^{-1})^{\frac{t}{4}}\right)^4$$

$$2^{-32} = 2^{-t}$$

$$t = 32\ d$$

t	g
0	100
4	50
8	25
12	12.5
16	6.25
20	3.125
24	1.5625
28	0.78125
32	0.390625



C12 - 7.0 - Exponentials Notes

If you deposit $1000 in the bank at 8% interest how much interest will you have after 5 years?

$$F = P(1 \pm r)^t$$
$$F = 1000(1 + 0.08)^5$$
$$F = \$1469.32$$
$$I = F - P$$
$$I = 1469.32 - 1000 = \$469.32$$

t	*F*
0	*1000*
1	*1080*
2	*1166.40*
3	*1259.71*
4	*1360.49*
5	*1469.32*

1000
Enter
×
1.08
Enter
Enter

Find the rate to triple your money in 10 years.

$$F = P(1 + r)^t$$
$$3 = 1(1 + r)^{10}$$
$$(3)^{\frac{1}{10}} = \left((1 + r)^{10}\right)^{\frac{1}{10}}$$
$$1.116 = 1 + r$$
$$r = 0.1116$$
$$r = 11.6\%$$

If you deposit $5000 in the bank at 8% interest, compounded quarterly, how much will you have after 6 years?

$$F = P\left(1 \pm \frac{r}{n}\right)^{tn}$$
$$F = 5000\left(1 + \frac{0.08}{4}\right)^{6\times 4}$$
$$F = \$8042.19$$

Find the present value of deposit worth $2000 in the bank at 10% interest how much will you have after 4 years?

$$F = P(1 \pm r)^t$$
$$2000 = P(1 + 0.1)^4$$
$$2000 = P(1.4641)$$
$$P = \frac{2000}{1.1641}$$
$$P = \$1366.03$$

Find the rate of a $1000 deposit worth $1100 after 2 years.

$$F = P(1 \pm r)^t$$
$$1100 = 1000(1 + r)^2$$
$$\frac{1100}{1000} = (1 + r)^2$$
$$1.1 = (1 + r)^2$$
$$(1.1)^{\frac{1}{2}} = \left((1 + r)^2\right)^{\frac{1}{2}}$$
$$1.0488 = 1 + r$$
$$r = 0.0488$$
$$r = 4.9\%$$

How long to quadruple your money at 8%

$$F = P(1 \pm r)^t$$
$$400 = 100(1 + 0.08)^t$$
$$\frac{400}{100} = 1.08^t$$
$$4 = 1.08^t$$
$$t = 18.01\ yrs$$

OR "logs"

$y_1 = y_2$

If you deposit $100 in the bank, how long will it take to grow to $6400 if it doubles each year?

$$F = P(r)^{\frac{t}{T}}$$
$$6400 = 100(2)^{\frac{t}{1}}$$
$$\frac{6400}{100} = 2^t$$
$$64 = 2^t$$
$$2^6 = 2^t$$
$$t = 6s$$

If a population starts at 1000 and triples every 4 hours, how large will the population grow in 25 hours?

$$F = P(r)^{\frac{t}{T}}$$
$$F = 1000(3)^{\frac{25}{4}}$$
$$F = 959417\ pop$$

Light diminishes by 10% every 5 meters. Find the depth of 1% light.

$$F = P(1 \pm r)^{\frac{t}{T}}$$
$$1 = 100(1 - 0.1)^{\frac{d}{5}}$$
$$0.01 = 0.9^{\frac{d}{5}}$$
$$0.01 = 0.9^{\frac{d}{5}}$$
$$d = 218.5\ m$$

If the population starts at 300 and grows continuously at a rate of 0.06, how large will it grow after 20 days?

$$F = Pe^{kt}$$
$$F = 300e^{0.06\times 20}$$
$$F = 996.03\ pop$$

How many times as intense is an earthquake of 6.0 than 3.0?

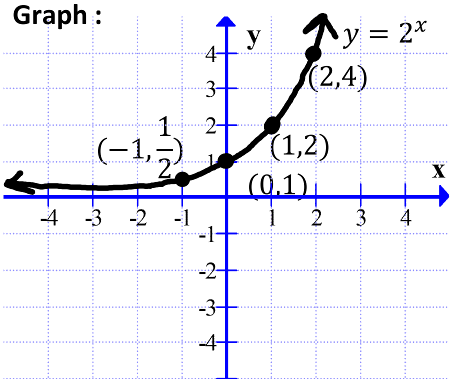
$$I = 10^{b-s}$$
$$I = 10^{6-3}$$
$$I = 10^3$$
$$I = 1000\ times$$

An earth quake in California of Richter 8.5 Magnitude was 100 times as strong as an earth quake in Vancouver of what Richter Magnitude.

$$I = 10^{b-s}$$
$$100 = 10^{8.5-s}$$
$$10^2 = 10^{8.5-s}$$
$$2 = 8.5 - s$$
$$s = 6.5\ R$$

C12 - 7.0 - Exponentials Notes

Graph :



x	y
-1	$\frac{1}{2}$
0	1
1	2
2	4

$D : x \in \mathrm{I}$

$R : y \geq 0$

$t^* \geq 0$

End Behavior

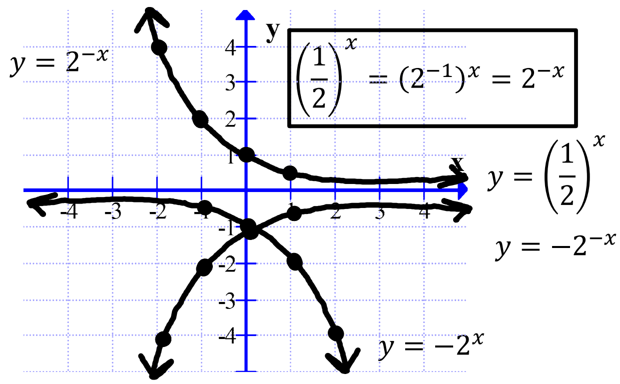
$x \to +\infty \quad x \to -\infty$

$y \to +\infty \quad y \to 0$

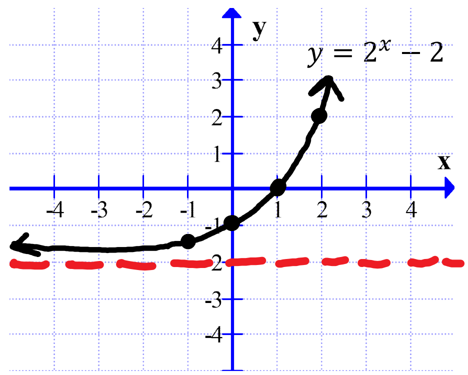
$HA: y = 0$

HR/VR :

$\left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x}$



$y = a(C)^{b(x-h)} + k$

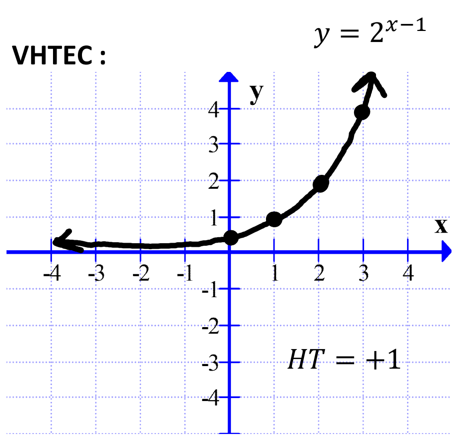


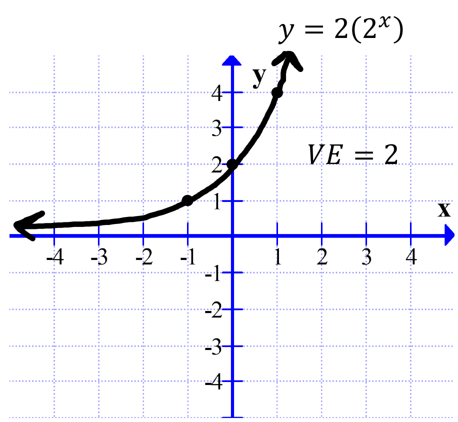
$VT = -2$

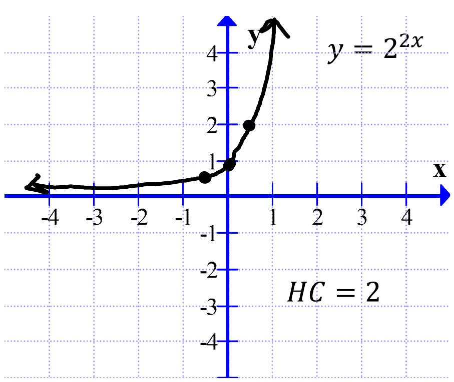
$HA : y = -2$

$R : y > -2$

VHTEC :







C12 - 7.0 - Geometrics Notes

$t_1 =$ 1st term (aka: "a or u_1")
$r =$ common ratio
$t_n =$ term n, every term
$n =$ Term #, or # of terms

Write the first terms 5 of the sequence

$t_1 = 2, r = 3$

Geometric: r must always be the same

$\times 3$

2 , ___ , ___ , ___ , ___

t_1 t_2 t_3 t_4 t_5

2,6,18,54,162

$t_2 = 9, t_5 = 243$

$5 - 2 = 3$

$9r^3 = 243$
$r^3 = 27$
$\sqrt{r^3} = \sqrt{27}$
$r = 3$

3, 9, 27, 81, 243

$t_1 = 2, t_5 = 162$

$5 - 1 = 4$

$2r^4 = 162$
$r^4 = 81$
$r = \pm 3$

2, 6, 18, 54, 162

2, −6, 18, −54, 162

$t_2 = 4, t_4 = 16$

$4 - 2 = 2$

$\div r$ $\times r$ $\times r$ $\times r$

___ , 4 , ___ , 16 , ___

t_1 t_2 t_3 t_4 t_5

$4r^2 = 16$
$r^2 = 4$
$\sqrt{r^2} = \sqrt{4}$
$r = \pm 2$

$t_n = ar^{n-1}$
$4 = ar^{2-1}$
$4 = ar^1$
$a = \frac{4}{r}$

$t_n = ar^{n-1}$
$16 = ar^{4-1}$
$16 = ar^3$
$16 = \left(\frac{4}{r}\right) r^3$
$16 = 4r^2$
…

$\div +2$ $\times +2$ $\times +2$ $\times +2$

2 , 4 , 8 , 16 , 32

$\div -2$ $\times -2$ $\times -2$ $\times -2$

−2 , 4 , − 8 , 16 , − 32

2,4,8,16,32

−2,4,−8,16,−32

$\underline{x+2}$ $\underline{2x+1}$ $\underline{4x-3}$

$r = \frac{2x+1}{x+2}$ $r = \frac{4x-3}{2x+1}$

$\frac{2x+1}{x+2} = \frac{4x-3}{2x+1}$
…
$4x^2 + 4x + 1 = 4x^2 + 5x - 6$
…
$x = 7$

9,15,25 Check

$r = \frac{5}{3}$ $r = \frac{5}{3}$

$\underline{x}$ $\underline{x+5}$ $\underline{x+9}$

$r = \frac{x+5}{x}$ $r = \frac{x+9}{x+5}$

$\frac{x+5}{x} = \frac{x+9}{x+5}$
…
$x^2 + 10x + 25 = x^2 + 9x$
$x = -25$

−25, −20, −16

$r = \frac{-20}{-25} = 0.8$ $r = \frac{-16}{-20} = 0.8$

C12 - 7.0 - Geometrics Notes

$\_\_\_\_, \_\_\_\_, \_\_\_\_ \cdots \_\_\_\_ \cdots \_\_\_\_$

3,6,12 ...

$r = ?$ $\quad t_n = ?$ $\quad t_5 = ?$ $\quad t_n = 768, n = ?$

$\times 2 \quad \times 2$

$\underset{t_1}{3}, \underset{t_2}{6}, \underset{t_3}{12}, \underset{t_4}{?} \cdots \underset{t_4}{48} \cdots\cdots \underset{t_n}{768}$

$n = 1 \quad n = 2 \quad n = 3 \quad n = 4 \quad n = 5 \quad n = ? = 9$

$t_1 = 3$ $\quad r = \frac{6}{3}$, $r = 2$ $\quad r = \frac{12}{6}$, $r = 2$

Ratio : $r = \frac{t_n}{t_{n-1}}$
Find "r" twice

Find the General term $t_n = ?$

$t_n = t_1 r^{n-1}$
$t_n = 3(2)^{n-1}$

$t_n = t_1 r^{n-1}$
General term formula

The number 768 is what term? $t_n = 768, n = ?$

$t_n = 3(2)^{n-1}$
$768 = 3(2)^{n-1}$ *Divide both sides by 3*
$256 = 2^{n-1}$ *Change of base:* $256 = 2^8$
$2^8 = 2^{n-1}$ *Same Base, exponents are equal*
$8 = n - 1$
$n = 9$ *Check*: 3,6,12,24,48,96,192,384,768

What is the fifth term t_5? $t_5 = ?, n = 5$.

$t_n = 3(2)^{n-1}$
$t_5 = 3(2)^{n-1}$
$t_5 = 3(2)^{5-1}$
$t_5 = 3(2)^4$
$t_5 = 48$

Or, Start from beginning
$t_n = t_1 r^{n-1}$
$t_5 = 3(2)^{5-1}$
$t_5 = 48$

Check : 3,6,12,24,48 ✓

Remember: You could have also multiplied by the common ratio repeatedly

Given :
$S_8 = 765$
$t_1 = 3$
$r = 2$
find n.

$s_n = \frac{t_1(1 - r^n)}{1 - r}$
$765 = \frac{3(1 - 2^n)}{1 - 2}$
$-1 \times 765 = \frac{3(1 - 2^n)}{-1} \times -1$
$\frac{-765}{3} = \frac{3(1 - 2^n)}{3}$
$-255 = 1 - 2^n$
$2^n = 1 + 255$
$2^n = 256$
$2^n = 2^8$
$n = 8$

What is the sum of the first eight terms s_8? $s_8 = ?, n = 8$.

$s_n = \frac{t_1(1 - r^n)}{1 - r}$
$s_8 = \frac{3(1 - 2^8)}{1 - 2}$
$s_8 = 765$ *Check* : $3 + 6 + 12 + 24 + 48 + 96 + 192 + 384 = 765$

$s_n = \frac{t_1(1 - r^n)}{1 - r}$ Sum of "n" terms formula (if number of terms is known)

$s_n = \frac{t_1 - rt_n}{1 - r}$
$s_8 = \frac{3 - 2(t_8)}{1 - 2}$
$s_8 = \frac{3 - 2(384)}{1 - 2}$ ←
$s_8 = 756$

OR

$t_n = 3(2)^{n-1}$
$t_8 = 3(2)^{8-1}$
$t_8 = 3(2)^7$
$t_8 = 3(128)$
$t_8 = 384$

$s_n = \frac{t_1 - rt_n}{1 - r}$ Sum of "n" terms formula (if last term t_n is known)

What is the sum of an infinite number of terms? $r = 2$

$\therefore$ Divergent
$r > 1, \therefore$ *no sum* ✓

Check : $3 + 6 + 12 + 24 + 48 + 96 + 192 + 384 + 768 + 1536 \ldots = \infty$

What is the sum of the infinite sequence?

8,4,2 ...

$s_\infty = ?$

$t_1 = 8$ $\quad r = \frac{4}{8}$, $r = \frac{1}{2}$ $\quad r = \frac{2}{4}$, $r = \frac{1}{2}$

$-1 < r < 1$
$-1 < \frac{1}{2} < 1$
$\therefore$ Convergent, has sum

$s_\infty = \frac{t_1}{1 - r}$
$s_\infty = \frac{8}{1 - \frac{1}{2}}$
$s_\infty = \frac{8}{\frac{1}{2}}$
$s_\infty = \frac{8}{1} \times \frac{2}{1}$
$s_\infty = 16$

Check : $8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 15.9375 \approx 16$ ✓

$s_\infty = 8, r = \frac{1}{2}, t_1 = ?$

$s_\infty = \frac{t_1}{1 - r}$
$8 = \frac{t_1}{1 - \frac{1}{2}}$
$8 = \frac{t_1}{\frac{1}{2}}$ Flip & Multiply
$8 = t_1 \times \frac{2}{1}$
$\frac{8}{2} = \frac{2t_1}{2}$
$t_1 = 4$

$4 + 2 + 1 + \frac{1}{2} + \cdots \approx 8$

C12 - 7.0 - Geometrics Notes

_____ , _____ , _____ ... _____ ... _____

Find the sum of the terms

Steps
Put in k = bottom number the equation
Put in $k+1$ (bottom # plus 1)
Repeat until k = top number

Geometric

$$\sum_{k=2}^{6} 8\left(\frac{1}{2}\right)^{k-1} = ?$$

Always r

Not Always t1

k=2	k=3	k=4	k=5	k=6
4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

$3\left(\frac{1}{2}\right)^{k-1} =$ $\quad 8\left(\frac{1}{2}\right)^{2-1} = 4$

$3\left(\frac{1}{2}\right)^{k-1} =$ $\quad 8\left(\frac{1}{2}\right)^{3-1} = 2$

...

$3\left(\frac{1}{2}\right)^{k-1} =$ $\quad 8\left(\frac{1}{2}\right)^{6-1} = \frac{1}{4}$

$4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} = 7.75$

$n = 6 - 2 + 1$

$n = 5$

$$s_n = \frac{t_1(1 - r^n)}{1 - r}$$

$$s_n = \frac{t_1(1 - r^n)}{1 - r}$$

$$s_5 = \frac{4\left(1 - \left(\frac{1}{2}\right)^5\right)}{1 - \left(\frac{1}{2}\right)}$$

$$s_5 = 7.75$$

Infinite Geometric

$$\sum_{k=2}^{\infty} 3(2)^{k-1} = ?$$

4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, ...

$r = \frac{2}{4}$

$r = \frac{1}{2}$

$r = \frac{1}{2}$

$-1 < r < 1$

$-1 < \frac{1}{2} < 1$

$\therefore$ Convergent, has sum

$$s_\infty = \frac{t_1}{1 - r}$$

$$s_\infty = \frac{4}{1 - \left(\frac{1}{2}\right)}$$

$$s_\infty = \frac{4}{\frac{1}{2}}$$

$$s_\infty = 4 \times \frac{2}{1}$$

$$s_\infty = 8$$

$t_1 + t_2 = 48$

$a + ar = 48$

$a(1 + r) = 48$

$a = \frac{48}{1 + r}$

$t_3 + t_4 = 432$

$ar^2 + ar^3 = 432$

$\frac{48r^2}{1 + r} + \frac{48r^3}{1 + r} = 432$

$48r^2 + 48r^3 = 432(1 + r)$

$48r^2 + 48r^3 = 432 + 432r$

$\frac{48r^3}{48} + \frac{48r^2}{48} - \frac{432r}{48} - \frac{432}{48} = 0$

$r^3 + r^2 - 9r - 9 = 0$

...

$r = \pm 3$

$ar^2 + ar^3 = 432$

$r^2(a + ar) = 432$

$r^2(48) = 432$

$r^2 = 9$

$r = \pm 3$

$a = \frac{48}{1 + 3}$

$a = 12$

12,36,108,324

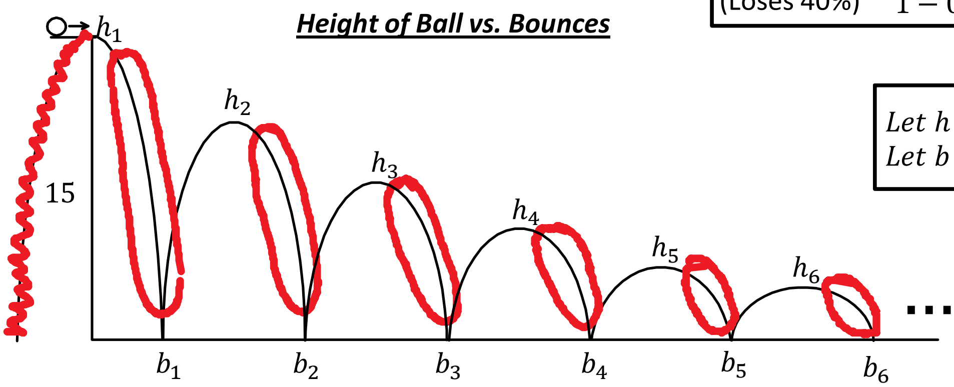
$12 + 36 = 48$ $\qquad 108 + 324 = 432$

C11 - 7.0 - Bouncing Ball Notes (up 60%)

A ball rolls off a building 15 m tall. After each bounce, it rises to 60% of the previous height.

(Loses 40%) $1 - 0.4 = 0.6$

Height of Ball vs. Bounces



Let h = Fall height
Let b = Bounce Number

$r = 60\%$
$r = 0.6$

$\times 0.6 \quad \times 0.6 \quad \times 0.6 \quad \times 0.6 \quad \times 0.6$

$\underline{15}_{t_1}, \underline{9}_{t_2}, \underline{5.4}_{t_3}, \underline{3.24}_{t_4}, \underline{1.94}_{t_5}, \underline{1.17}_{t_6}$

Terms are Fall Heights!

$r = \frac{9}{15}$ $\quad r = .6$

$r = \frac{5.4}{9}$ $\quad r = .6$

How high does the ball bounce after the 1st, 2nd bounce?

Height After 1st Bounce

$15 \times 0.6 = 9\,m$

Height After 2nd Bounce

$9 \times 0.6 = 5.4\,m$

$1 \rightarrow 2!$
$2 \rightarrow 3!$

After $1st = t_2$
After $2nd = t_3$

How high does the ball bounce after the nth bounce? (Find the general formula)

$t_n = t_1 r^{n-1}$

$t_n = t_1 r^{n-1}$

$t_n = 15(0.6)^{n-1}$

How high does the ball bounce after the 4th bounce. t_5

$t_n = t_1(r)^{n-1}$
$t_5 = 15(0.6)^{5-1}$
$t_5 = 15(0.6)^4$
$t_5 = 1.94\,m$

$4 \rightarrow 5!$

After $4th$ bounce $= t_5$

How high does the ball bounce after the 10th bounce. t_{11}

$t_n = t_1 r^{n-1}$
$t_{11} = 15(0.6)^{11-1}$
$t_{11} = 15(0.6)^{10}$
$t_{11} = 0.09\,m$

$10 \rightarrow 11!$

After $10th$ bounce $= t_{11}$

What is the total vertical distance the ball has travelled when it hits the ground for the 5th bounce? $s_5 = ?$

$$s_n = \frac{t_1(1 - r^n)}{1 - r}$$

$$s_5 = \frac{15(1 - (.6)^5)}{1 - .6}$$

$$s_5 = \frac{15(0.87)}{.4}$$

$$s_5 = 34.6\,m$$

$34.6 \times 2 - 15 = 54.2\,m$

Count it

15
$+9 \times 2$
$+5.4 \times 2$
$+3.24 \times 2$
$+1.94 \times 2$
54.2

If it bounces forever, what is the total vertical distance travelled? $s_\infty = ?$

$$s_\infty = \frac{t_1}{1 - r}$$

$$h_\infty = \frac{h_1}{1 - r}$$

$$h_\infty = \frac{15}{1 - 0.6}$$

$$h_\infty = \frac{15}{0.4}$$

$$h_\infty = 37.5\,m$$

$r = 0.6 \qquad r < 1$

$37.5 \times 2 - 15 = 60m$

Double it to account for rise heights and subtract the initial height (double counted)